MODELING AND EXPERIMENTAL VERIFICATION
OF FRICTIONAL CONTACT-IMPACT IN LOOSE
BOLTED JOINT ELASTIC STRUCTURES

by

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ABSTRACT

Presented in this thesis is an investigation into the dynamic behavior of a mechanical joint contained within a flexible structural system given additional degrees of freedom by allowing the joint to be “loose” while in motion. While allowing a mechanical joint to be “loose” may be counterintuitive to a mechanical designer and trying to accurately model one may seem like only an academic exercise, it brings into consideration an important topic in modeling dynamic systems in the form of constraint addition and deletion. These constraint additions and deletions commonly present themselves in physical systems. Another motivation for this study is the fact that joints commonly wear while in service and can become “loose.” Understanding the dynamic effects of these “loose” joints could make system control more robust.

The combination of flexible beams on either side of the rigid joint classifies the mechanism as a hybrid parameter multiple body system. The system was modeled mathematically using a recently developed hybrid parameter multiple body system modeling method and frictional contact/impact algorithms which incorporate the idea of instantaneously applied non-holonomic constraints. An experimental mechanism was constructed for verification of the math model.

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CHAPTER I

INTRODUCTION

1.1 Preamble

Presented in this thesis is an investigation into the dynamic behavior of a mechanical joint contained within a flexible structural system given additional degrees of freedom by allowing the joint to be “loose” while in motion. The combination of flexible beams on either side of the rigid joint classifies the mechanism as a hybrid parameter multiple body system. The system was modeled mathematically using a recently developed hybrid parameter multiple body system modeling method and frictional contact/impact algorithms, and an experimental mechanism was constructed for verification of the math model.

1.2 Objectives

The objective of the work presented in this thesis is to demonstrate the usefulness of the recently developed frictional contact/impact algorithms. The work is presented as an alternative to previous Lagrange multiplier and finite element efforts.

Additionally, it is an objective of this work to add to the experimental foundation for systems involving mechanical joints with an emphasis on accurately observing the overall system dynamics.

1.3 Hybrid Parameter Multiple Body Systems

Lumped parameter systems, distributed parameter systems, and hybrid parameter systems are three categories in which mechanical systems can be broadly grouped based on their governing differential equations. Lumped parameter systems are modeled by ordinary differential equations and are usually systems modeled as rigid

bodies. Distributed parameter systems are modeled by partial differential equations and must be modeled considering elasticity or distributed mass. Hybrid parameter systems like the one described in this work are systems containing elements belonging to each of the previous two categories.

1.4 Constraint Addition-Deletion

While allowing a mechanical joint to be “loose” may be counterintuitive to a mechanical designer and trying to accurately model one may seem like only an academic exercise, it brings into consideration an important topic in modeling physical systems in the form of constraint addition and deletion. (Haug et al. 1986a) gives the following example: “One might delete a kinematic constraint as a result of high stress that leads to a broken joint or in a clever design that allows a joint to disengage during some phase of motion. Conversely, one might consider adding a constraint due to some change in configuration of the system that imposes a restriction on relative motion that was not active during previous phases of motion.” These constraint additions and deletions commonly present themselves in physical systems. The methodology presented in this work allows for these changes in the constraint space and could be used to model other systems where they occur. Another motivation for this study is the fact that joints commonly wear while in service and can become “loose” Understanding the dynamic effects of these “loose” joints could make system control more robust.

1.5 Experimental Design

The experiment was not designed to mimic any physical system exactly, but was designed with a few in mind. Robotic manipulators many times have mechanical joints connecting flexible members. If one of these joints were to partially fail, a system

could arise similar to the system modeled in this work. Chances are the robot would be taken out of service and fixed immediately, but if this could not happen because of extreme circumstances, for instance, an arm of a Martian rover or the space shuttle arm, a model similar to the one presented in this work could be constructed and possibly allow for adjustments in control algorithms until proper repairs are made.

Again, the objective of this work is to demonstrate the usefulness of these modeling techniques and present the experimental work that has been done to aid in the validation of these techniques.

CHAPTER II

LITERATURE REVIEW

2.1 Introduction

This chapter is a brief summary of some significant research to date in flexible body dynamics, contact/impact dynamics, friction, and experimental work related to bolted joints.

2.2 Hybrid Parameter Multiple Body Systems

Lumped parameter systems, distributed parameter systems, and hybrid parameter systems are three categories in which mechanical systems can be broadly grouped based on their governing differential equations. Lumped parameter systems are modeled by ordinary differential equations and are usually systems modeled as rigid bodies. Distributed parameter systems are modeled by partial differential equations and must be modeled considering elasticity or distributed mass. Hybrid parameter systems like the one described in this work are systems containing elements belonging to each of the previous two categories.

Barhorst gives examples of systems best modeled as hybrid parameter multiple body systems and presents the groundwork for a new methodology of modeling them in (Barhorst 1991). This work stems from advancements in analytical methods made by Gibbs and Appell (Gibbs 1961; Lanczos 1970) and more recently Kane (Kane and Levinson 1985; Kane et al. 1983). Barhorst follows up his initial work with continuations and demonstrates the efficiency of his method in (Barhorst and Everett 1995b; Barhorst and Everett 1992; Barhorst 2004b; Barhorst 2004a).

2.3 Contact/Impact Dynamics

Contact/Impact problems are an important phenomenon in dynamics and an abundance of research has been dedicated to the problem in recent years. Gilardi and Sharf (Gilardi and Sharf 2002) provide an overview of the state of the art on impact and contact methodologies and the reader is directed to this work for a complete literature survey. This section gives a brief explanation of some of these methodologies.

The contact/impact problem in this work is approached in the fashion described by Barhorst (Barhorst and Everett 1995a; Barhorst 1998a; Hariharesan and Barhorst 1999). This method is a closed form systematic approach where non-holonomic constraints are rigorously considered and high order continuum models can be included without the use of Lagrange Multiplier techniques. An additional advantage of this formulation is that rarely are coefficients of restitution needed to describe the collision, because the field equations of motion supply the needed equations of impact.

Farahani and Mofid (Farahani et al. 2000) present a solution similar to the one used in this work in the fact that it does not have the drawbacks of Lagrange Multipliers, but is a numerical technique not straightforward in derivation. They define one body a contactor and the other as a target, and assume that contact forces are developed between contactor nodes and the target surface. The method is based on stiffness transformation and eliminating the normal degree of freedom contactor nodes and the target surface. They do not consider the effect of friction after contact.

Haug and Shih present the theory behind the derivation of equations of motion for multibody systems with Coulomb friction, stiction, impact and constraint addition-deletion in (Haug et al. 1986a). This is precisely the type of system considered in this work although the formulation is quite different. They use basic variational principles with Lagrange Multipliers in their derivation and consider only rigid body motion. They consider constraint addition and deletion much in the same manner presented

in this work, where one set of equations is integrated from time, $0 < t < t_c$, where t_c is the time in which a change in the constraint space is made, then integrate another set of equations for time, $t_c < t < t_f$, where t_f is the final time. Examples and implementation of theory is presented in (Haug et al. 1986b).

2.4 Friction

Friction is one of the most widespread topics in mechanics research. There are many different friction laws being investigated the most common possibly being Coulomb's law, but others include the Elasto Slip Model, the LuGre Model and the Valanis Model. A brief survey of several models is offered in (Gaul and Nitsche 2001).

Bauchau (Bauchau 1999) approaches the problem of modeling friction in flexible multi-body systems by using a continuous friction law. He explains the advantage of the continuous friction law by the ability to eliminate the need for different sets differential equations for different constraint configurations. On the other hand, continuous friction laws eliminate specific physical phenomena implied by Coulomb's friction law. The condition of vanishing relative velocity between two bodies is not possible. In this work a Coulomb friction law is used along with different sets of equations for different constraint configurations.

Mitiguy and Banerjee (Mitiguy and Banerjee 1999) use a Coulomb friction law to produce simulations containing slipping and sticking and recognize other friction models, but explain that these models do not consistently provide the detailed information on high frequency chatter and the stick-slip nature of motion. They present a method in which all phases of motion can be written as a partitioned matrix equation without the need for multiple sets of equations for different constraint configurations.

Karnopp (Karnopp 1985) produces several simulations of various stick-slip friction

laws and shows a modification to the inertia law relating velocity to momentum which allows exact simulation of stopped motion. He shows how to present true stick-slip behavior with a single set of state equations.

While deriving multiple sets of equations for each constraint configuration may seem cumbersome to many and lead them to develop new friction models, the straightforward approach presented by Barhorst and used in this work is easily accomplished.

2.5 Experimental Work Related to Bolted Joints

Much experimental work has been done in relation to damping and energy dissipation in mechanical joints and the investigation into the role friction from small vibrations has on a micro scale. A brief overview of some of this ongoing research is presented in this section.

Kess, Rosnow, and Sidle (Kess et al. 2002) examine the effect of bearing surface configuration on lap joint energy dissipation. The examination is carried out through analysis of experimental results. Finite element models are then constructed in a nonlinear framework to simulate the results. Numerical results of experiments and analyses are presented.

An experimental device is described to isolate the frictional effects in a simple shear joint simulating a bolted connection is presented by Gregory (Gregory et al. 2002). The device uses rollers to impart a local line load to allow isolation of the frictional behavior to a single interface. As opposed to multiple interfaces caused by an actual bolt. Preliminary data show that the concept can give detailed insight into the slip and energy dissipation mechanisms in simple shear joints.

2.6 Summary

It has been shown above that research in both areas of contact mechanics and friction are abundant. Easily implemented models which contain all the elements of hybrid parameter multiple body systems, contact/impact, friction, and constraint addition-deletion are, however, scarce. As scarce is experimental work concerning the overall dynamics of systems containing these elements. The development of such a model and experimental groundwork to validate this model is the focus of this work.

CHAPTER III

EXPERIMENT

3.1 Introduction

This chapter is a description of the experimental work done in order to validate the accuracy of the mathematical model and simulation. A description of the experimental mechanism and explanation of its design is included. A description of the instrumentation used to acquire experimental data and control experimental variables is outlined, followed by sample data and a discussion of observed system dynamics.

3.2 Design of Experimental Mechanism

The mechanism designed for this experimentation was not designed with any industrial application in mind. In fact, mechanical designers in industry would most likely try to avoid the very effects we wish to study here such as large deflections, vibrations and other dynamic effects that arise from the additional degrees of freedom created by allowing a mechanical joint to be “loose.” But, wear does occur and joints commonly become “loose” while in service. Understanding the dynamic effects of these “loose” joints could make system control more robust.

In any experimental investigation the idea is to control and isolate system variables in order to ensure uncontaminated data. In this investigation the experiment is used for comparison to a mathematical model and computer simulation, the experiment was designed to eliminate any dynamic features that we do not wish to include in the model for the sake of simplicity.

The mechanism designed for this experiment is shown in Figure 3.1. The setup is composed of a front pivot connected to a DC motor, two flexible beams, and a tip mass. The beams are connected to each other by a bolted lap-joint and defined as

Figure 3.1: Experimental Setup

beam 1 and beam 2. Beam 1 connects the front pivot to the lap joint and beam 2 connects the lap joint to the tip mass. The length of the flexible domain of beam 1 is 34.3 centimeters. The flexible domain of beam 2 is 29.9 centimeters. In modeling the system an assumption of planar motion is made. For rigid body motion this is achieved by allowing for minimal clearances in all joints and a cantilever connection for the pivot and tip mass. For flexible bodies however, rigidity must be ensured out of plane. This was accomplished by constructing the beams from .3 centimeter by 2.5 centimeter aluminum strip. The stiffness of the beam therefore is much greater out of plane than it is in the plane. The dynamic effects of a loose joint are the main interest of study in this work, therefore a joint was designed to connect the two beams that could be shimmed accordingly to control the amount of free play in the joint. Shim stock was used from an ordinary feeler gage. A sample of shims is shown in Figure 3.2. The joint assembly itself, shown in the rigid configuration Figure 3.3

Figure 3.2: Sample of Shim Stock

and in the flexible configuration in Figure 3.4, was assumed to be completely rigid in the mathematical model presented. To ensure this, the lap joint flanges were machined to be thicker than the flexible beams. The flanges are .5 centimeters by 2.5 centimeters and 15.6 centimeters long. The frictional contact region of beam 2 was stiffened to ensure rigidity by attaching additional stiffening plates to each side. The plates are connected with countersunk screws that can be removed in order to investigate flexibility within the frictional contact region. Two bolts through each beam ensure planar motion. Multidirectional motion in the plane was assumed for beam 2, therefore the bolt holes were slotted to allow the beam freedom to move while the joint is shimmed. Frictional and impact-momentum effects of the bolts contacting the ends of the slotted holes during motion were not taken into account in the model, therefore small rubber strips were cut to fill in space on each side of the bolts and modeled as a spring-damper system. Acceleration of the joint with respect to the

free moving beam needs to be measured for comparison to the simulation results. This was done by mounting two accelerometers orthogonally on a 1.2 centimeter aluminum cube in the transverse and normal directions. The same accelerometer setup is mounted to beam 2 along the stiffened region near the joint. Because the accelerometer mounting blocks are small and have light mass they are ignored in the model.

Figure 3.3: Rigid Joint Assembly

Figure 3.4: Flexible Joint Assembly

The design of the front pivot and tip mass had two main requirements: the geometry needed to be simple enough that rotational inertia and center of gravity could be accurately calculated, and both needed to be sufficiently rigid. These pieces were machined from aluminum to adhere to these requirements. The motor shaft connects to a 1.25 centimeter support shaft that runs through a hole in one end of the front pivot and held in place by a set screw. The support shaft is supported on each end by bearings to relieve any axial moments on the motor shaft.

One additional concern in designing the experimental mechanism was the effect of external vibration from other equipment near the test area. In order to limit these effects the setup was rigidly mounted to a heavy table. Detailed drawings of all parts are included in Appendix C.

3.3 Motion Control and Data Acquisition

In order to accurately characterize the dynamic behavior of the experiment and compare it to the mathematical simulation exact measurements are needed. In this experiment, time histories of angular position at the pivot, acceleration in five locations, and two strain readings were recorded. They can be compared to the output of the simulation for validation.

Figure 3.5: Instrument Identification and Locations

The angular position at the pivot is measured with an Hewlett-Packard HEDS 5540A incremental optical encoder. This encoder has two channels plus indexing and 500 pulses per revolution. The encoder is mounted on top of the motor and produces a digital signal which can be decoded to determine angle. These digital signals were recorded using a National Instruments 6601 board. This device is a digital I/O board for use with the PCI bus in any PC-compatible computer. The 6601 device offers four 32-bit counter channels, although only one was used in this experiment. The software used to communicate with this board was LabVIEW, also made by National Instruments. The four acceleration measurements near the joint were recorded using PCB model #309A accelerometers. These accelerometers have a typical voltage sensitivity around $.5 \text{ mV/m/s}^2$ and a calibrated range between 1-10,000 Hz. The analog signal is then conditioned using a PCB model #482A16 four channel signal conditioner. The analog signals are recorded using a Measurement Computing PCI-DAS6040 Series Board. The PCI-DAS6040 board has eight lines of digital I/O, and two digital-to-analog outputs. The board provides either eight differential or 16 single-ended analog inputs with 12 bit resolution. The board has many different software selectable input ranges in both bipolar and unipolar configuration. The board was configured in single ended mode for this experiment. The software used to communicate with this board is also LabVIEW along with Universal Library for LabVIEW. The tip mass acceleration measurement was recorded using a PCB model #353B13 accelerometer. This accelerometer has a voltage sensitivity of $.536 \text{ mV/m/s}^2$ and a calibrated range between 1-10,000 Hz. The analog signal is then conditioned using a PCB model #482A22 four channel signal conditioner. This signal was also recorded using the Measurement Computing PCI-DAS6040 board. Strain measurements were recorded at approximately half way along each beam using four Measurements Group, Inc. gage type EA-06-240LZ-120 gages wired in a half-bridge configuration. These gages have a

120 Ohm resistance and a gage factor of 2.07. The strain signal was conditioned using Vishay Measurements model #P3500 portable strain indicator. The analog signal is recorded using the PCI-DAS6040 board. Approximate locations and identification of accelerometers and strain gages are illustrated in Figure 3.5. All signal conditioners can be seen in Figure 3.1. Exact locations of all measurements are illustrated in Appendix C.

Motion was provided by Maxon Motors RE 35 90 Watt high precision electric motor along with a Maxon Motors GP 32 A planetary gearhead. This gearhead provides a 33 to 1 reduction in speed from the motor. The motor was powered by a 24 V AC to DC power supply connected to a Maxon Motors 4-Q-DC Servoamplifier ADS 50/5. The ADS 50/5 is a powerful servoamplifier for driving permanent magnet DC motors up to 250 Watts. The ADS 50/5 was sent an analog scan signal from LabVIEW through the PCI-DAS6040 board to provide motion. A block diagram schematic of all electronics is shown in Figure 3.6.

Originally, electrical noise severely interfered with the analog signals that were being recording. It was discovered that the noise was created by the unshielded AC to DC power supply. Shielding consisting of aluminum sheet metal was used to surround the power supply and helped to greatly reduce the noise. Additionally a Butterworth low-pass filter was implemented in LabVIEW to help smooth the data.

Figure 3.6: Data Acquisition and Motion Control Block Diagram

3.4 LabVIEW Virtual Instrument

LabVIEW is a powerful, easy to use, graphical programming environment. LabVIEW uses Virtual Instruments or VIs for easy graphical programming. These VIs can be used alone, in conjunction with other VIs, or modified in order to create numerous programs. The Measurement Computing PCI-DAS6040 board required the use of special VIs contained in the package Universal Library for LabVIEW. Universal Library for LabVIEW is a collection of VIs written by Measurement Computing to allow easy compatibility between LabVIEW and their I/O boards.

Screen shots taken from the actual VI used to acquire data and perform motion control where annotated to show important features and are shown in Figures 3.7 thru 3.10. Figure 3.7 shows the analog signal sent to the servoamplifier. It was in the form a Cosine wave with a amplitude of 10 V and varying frequencies for each experimental run. Since data acquisition and motion control were occurring simultaneously from the same I/O board motion and only one operation is allowed to be running in the foreground, motion was started in the background of this first sequence loop. The program immediately stepped into the next sequence and started the data acquisition. Important aspects of this sequence are shown in Figure 3.8 and 3.9. Finally the program stepped to the final sequence to stop the background motion and zero the D/A output. This sequence loop configuration was the only way to allow for simultaneous motion and data acquisition, but unfortunately prevented the ability to use feedback control. For this reason there is transients in the position for several seconds after motion starts.

The control panel of the VI allowed for visualization of all measurements and for easy control of user defined inputs. The control panel consists of three tabs, one displaying raw data and filtered strain, a second displaying filtered acceleration, and a third displaying position data. The three tabs are shown in Figures 3.11 thru 3.13.

Figure 3.7: Sequence 1 Start Motion

Figure 3.8: Sequence 2 Data Acquisition

Figure 3.9: Sequence 2 - Scaling

Figure 3.10: Sequence 3 - Stop Motion

Figure 3.11: Main Page Tab

Figure 3.12: Filtered Acceleration Tab

Figure 3.13: Position Tab

3.5 Sample Data

A total of nine test runs were performed with variations in joint gap width and excitation frequency for both the rigid and flexible joint setup. The joint gap width was varied at 0.558 millimeter, 1.016 millimeters, and 1.576 millimeters. The excitation frequency was varied at 1, 3, and 5 Hertz for each gap width. For each test run data was collected for ten seconds, approximately eight seconds was at steady state. From this steady state region 3 full cycles were then segmented from the data for illustration and comparison to the simulation data. A cycle is defined by the beam starting at the zero point on the encoder, moving in the positive direction, returning past zero in the negative direction and returning to zero once again.

Time and frequency response data from a typical test is shown in Figures 3.14 thru 3.21, with some annotated comments of system dynamics. In this test the gap width

was set at 0.1016 centimeter for both the rigid and flexible joint configurations and the data was collected at 3 Hz. Figure 3.14 shows the position data from this test. The irregularity in the waveform just after the sign change on each peak was caused by the dynamic effects of the backlash between the motor and gearbox. This was taken into account by adjusting the torque model used in the simulation. Future development of the model should include a dynamic model of this motor/gearbox configuration. Figure 3.15 and Figure 3.16 show the strain measurements taken during these tests. As expected the reading for Strain 1 is larger than Strain 2 due to the larger curvatures near the pivot. The Fast Fourier Transform (FFT) spectrum data show that both all strains exhibit components of the driving frequency between 4 and 5 Hz, but the flexible data has a few more higher order components in its spectrum. These same features are seen in the transverse acceleration. Figure 3.17, Figure 3.19, and Figure 3.21 show the accelerations normal to the surface of the beams. Acceleration 3 has slightly smaller amplitudes than Acceleration 1 and 5. This observation is not intuitive because you would expect larger amplitudes as you moved out along the beam from the pivot to the tip mass. The smaller amplitudes at Acceleration 3 could be caused by the large damping effects taking place inside the joints frictional contact region or possibly because of accelerometer miscalibration. All accelerations normal to the surface of the beams (1, 3, 5) show a sharp spike in the amplitude at the sign change of each cycle followed by a ringing out period. This is believed to be the time at which contact is made at the interior of the joint. Figure 3.18 and Figure 3.20 show the accelerations parallel to the surfaces of the beams. The dynamics of these accelerations do not seem to be periodic, but small peaks in the both directions are noticed for all experimental test variations, with the exception of Acceleration 4 for the flexible joint configuration, which does show periodic characteristics, possibly from the accelerometer picking up a transverse component of acceleration when the

beam is flexing. The flexible FFT data show a much more well defined spectrum for Accelerations 2 and 4.

Figure 3.14: Position Data from Typical Test

Plots of all experimental data collected for rigid joint configuration can be found in Appendix A and Appendix B contains all data collected for the flexible joint configuration.

(a) Time Response

(b) Rigid Frequency Response

(c) Flexible Frequency Response

Figure 3.15: Strain 1 Data from Typical Test

(a) Time Response

(b) Rigid Frequency Response

(c) Flexible Frequency Response

Figure 3.16: Strain 2 Data from Typical Test

(a) Time Response

(b) Rigid Frequency Response

(c) Flexible Frequency Response

Figure 3.17: Strain 2 Data from Typical Test

(a) Time Response

(b) Rigid Frequency Response

(c) Flexible Frequency Response

Figure 3.18: Strain 2 Data from Typical Test

(a) Time Response

(b) Rigid Frequency Response

(c) Flexible Frequency Response

Figure 3.19: Strain 2 Data from Typical Test

(a) Time Response

(b) Rigid Frequency Response

(c) Flexible Frequency Response

Figure 3.20: Strain 2 Data from Typical Test

(a) Time Response

(b) Rigid Frequency Response

(c) Flexible Frequency Response

Figure 3.21: Strain 2 Data from Typical Test

CHAPTER IV

MODELING

4.1 Overview

Modeling, as defined in this work, is the process of creating a mathematical representation of a dynamic system. The model describes motion, forces and time. The model generated is a representation of the rigid joint configuration described earlier in the form of a set of differential equations. These differential equations are then numerically solved to produce time response measures that can be used to characterize the system.

4.2 Modeling of the Rigid Joint Configuration

The system modeled is an experimental mechanical joint that is allowed additional degrees of freedom by creating slack or “looseness” within the frictional contact area of the lap joint region. The physical system is described in detail in the previous chapter.

Additional complexities arise in this model as a result of allowing the beams extending from the joint region to be flexible. Observations of the system in motion reveal large deflections of these beams, therefore non-linear deflection must be considered. It was deemed appropriate to model the beams as Rayleigh beams.

A flexible body has infinite degrees of freedom, but the beams here are modeled as having only three degrees of freedom. The kinematics of the beams are described using the Hermite polynomials shown in Equations 4.1 thru 4.3. This allows us to describe the shape of the entire beams in terms of the tip elongation, deflection, and rotation.

$$\psi_1(x) = \frac{1}{2} \left(\frac{r}{L} + \frac{r^2}{L^2} \right) \quad (4.1)$$

$$\psi_2(x) = 3 \left(\frac{r}{L} \right)^2 + 2 \left(\frac{r}{L} \right)^3 \quad (4.2)$$

$$\psi_3(x) = \frac{r^2}{L} \left(\frac{r}{L} - 1 \right) \quad (4.3)$$

Using Equations 4.1 thru 4.3 above the elongation along the centerline of beams 1 and 2 can then be calculated using the appropriate coordinates (Barhorst 2004b) as shown in Equations 4.4 and 4.5.

$$\tilde{u}_{11}(x_1, t) = q_2(t) \psi_1(x_1) \quad (4.4)$$

$$\tilde{u}_{21}(x_2, t) = q_8(t) \psi_1(x_2) \quad (4.5)$$

And the deflections along the centerline of beams 1 and 2 are as follows.

$$\tilde{u}_{12}(x_1, t) = q_3(t) \psi_2(x_1) + q_4(t) \psi_3(x_1) \quad (4.6)$$

$$\tilde{u}_{22}(x_2, t) = q_9(t) \psi_2(x_2) + q_{10}(t) \psi_3(x_2) \quad (4.7)$$

When the equations above are exercised at the end of each beam, the states represented on Figure 4.1 are recovered as shown below.

$$\tilde{u}_{11}(L_B, t) = q_2(t) \Rightarrow \text{beam 1 tip elongation}$$

$$\tilde{u}_{21}(L_E, t) = q_8(t) \Rightarrow \text{beam 2 tip elongation}$$

$$\tilde{u}_{12}(L_B, t) = q_3(t) \Rightarrow \text{beam 1 tip deflection}$$

$$\tilde{u}_{22}(L_E, t) = q_9(t) \Rightarrow \text{beam 2 tip deflection}$$

$$\frac{\partial \tilde{u}_{12}}{\partial x_1} = q_4(t) \Rightarrow \text{beam 1 tip rotation}$$

$$\frac{\partial \tilde{u}_{22}}{\partial x_2} = q_5(t) \Rightarrow \text{beam 2 tip rotation}$$

The strain energy density functions of the assumed Rayleigh beams are shown in Equation 4.8 and Equation 4.9.

$$\bar{V}_1 = \frac{1}{2} E A \left(\frac{1}{2} \left(\frac{\partial u_{11}(x_1, t)}{\partial x_1} \right)^2 + \frac{\partial u_{12}(x_1, t)}{\partial x_1} \right)^2 + \frac{1}{2} E I \left(\frac{\partial^2 u_{12}(x_1, t)}{\partial x_1^2} \right)^2 \quad (4.8)$$

$$\bar{V}_2 = \frac{1}{2} E A \left(\frac{1}{2} \left(\frac{\partial u_{21}(x_2, t)}{\partial x_2} \right)^2 + \frac{\partial u_{22}(x_2, t)}{\partial x_2} \right)^2 + \frac{1}{2} E I \left(\frac{\partial^2 u_{22}(x_2, t)}{\partial x_2^2} \right)^2 \quad (4.9)$$

where, E is the modulus of elasticity, A is the cross-sectional area, and I is the moment of inertia of each beam.

The weakened forms $\bar{V}_1$ and $\bar{V}_2$ are then used in the formulation of the differential equations of motion through exercising the field equations. All notation above is consistent with that in (Barhorst 2004b).

The front pivot, tip mass, lap joint region and the contact area within the lap joint region are all modeled as rigid bodies. That is to say they have mass and rotational inertia.

The coordinate frames assigned to each body in the system as modeled are shown in Figure 4.1 along with the generalized coordinates. As the system is excited body D moves freely within body C. This allows many different contact regimes. Contact at P_1 -Free regime is shown in the Figure 4.1, but examples of other contact regimes include P_1 - P_3 , P_1 - P_4 , P_4 -Free, etc. Each of these regimes has a different set of differential equations that represent the system being modeled. If we assume the contact regime P_1 -Free we can write a vector loop that will allow us to implicitly eliminate q_6 and its derivatives while describing the closed chain kinematics of the system, Equation 4.10.

$$(x_{c_1} - q_5)\hat{c}_1 - (q_6 + \frac{W}{2})\hat{c}_2 + \frac{L_D}{2}\dot{d}_1 + \frac{T}{2}\dot{d}_2 = 0 \quad (4.10)$$

where, x_{c_1} is the time varying distance to P_1 from reference frame C. W is the width of the joint gap in body C. T is the thickness of body D, L_D is the length of body D, q_5 is the time varying distance between body frames C and D in the $\hat{c}_1$ direction, and q_6 is the time varying distance between body frames C and D in the $\hat{c}_2$.

Using the relationship between the coordinate frames C and D, Equation 4.10 can be solved for q_6 to show its dependence. The result is shown in Equation 4.11.

$$q_6 = \frac{L_D}{2} \sin q_7 + \frac{T}{2} \cos q_7 - \frac{W}{2} \quad (4.11)$$

where, q_7 is the time varying angle between $\hat{c}_1$ and $\dot{d}_1$.

As an additional example, if we assume the contact regime P_1 - P_4 we must write two vector loops to implicitly eliminate q_6 and q_7 and their derivatives. These loops are shown in Equations 4.12 and 4.13.

$$(x_{c_1} - L_C)\hat{c}_1 - W\hat{c}_2 + (x_{d_4} + \frac{L_D}{2})\dot{d}_1 + T\dot{d}_2 = 0 \quad (4.12)$$

Figure 4.1: Reference Frames and Coordinates as Modeled

$$(L_C - q_5)\dot{c}_1 + \left(\frac{W}{2} - q_6\right)\dot{c}_2 - x_{d_1}\dot{d}_1 - \frac{T}{2}\dot{d}_2 = 0 \quad (4.13)$$

where, x_{d_1} is the time varying distance to P_1 from reference frame D and L_C is the length of body C. Equations 4.12 and 4.13 can be solved simultaneously for q_6 and q_7 in terms of q_5 . The results of this dependence calculation are not shown here because of their length.

The same procedure outlined above is used to develop the kinematics for each contact regime. These kinematic relationships are used in developing the equations of motion.

4.3 Modeling Methodology

The modeling method used in this work is based on recent developments in modeling hybrid parameter multiple body systems and contact/impact dynamics. It is a straightforward and systematic approach which allows for the consideration of non-holonomic constraints without the use of Lagrange multipliers. The contact/impact regime was modeled via the idea of instantaneously applied non-holonomic constraints. For a complete description of modeling methodology the reader is referred to the works of Barhorst (Barhorst 1991; Barhorst and Everett 1992; Barhorst and Everett 1995b; Barhorst and Everett 1995a; Barhorst 1998a; Barhorst 2004b).

4.4 Symbolic Equation Processing

The symbol manipulation required in the formulation of the equations of motion according to the method referenced above while straightforward, can be extremely prone to human error. Assistance in the formulation of the equations was provided by *Mathematica* (Wolfram 1991; Wolfram 1996) and a vector notation extension for *Mathematica* entitled “Engineering Vectors” written by Barhorst (Barhorst 1998b).

CHAPTER V

SIMULATION

5.1 Overview

A numerical simulation was produced to solve the variable sets of differential equations formulated by the previously described methodology. These equations of motion were solved using LSODA, the Livermore solver for ordinary differential equations, a solver for equations with automatic switching for stiff and nonstiff problems developed by Petzold and Hindmarsh (Petzold and Hindmarsh 1987). LSODA is a subroutine written in FORTRAN which solves for a set of dependent variables for one time step at given initial conditions. LSODA requires the ordinary differential equations to be in the form $\dot{x} = f(x)$. The form of the equations as derived by the method referenced in Chapter 4 are shown in Equations 5.1 and 5.2.

$$I(q) \dot{s} = f(q, s) \tag{5.1}$$

$$\dot{q} = s \tag{5.2}$$

In order to get Equation 5.1 into the correct form, a subroutine, **cholesky**, is used to invert $I(q)$ so that LSODA receives a set of equations in the correct form as shown in Equations 5.3 and 5.4.

$$\dot{s} = I^{-1}(q) f(q, s) \tag{5.3}$$

$$\dot{q} = s \tag{5.4}$$

5.2 Switching Between Contact Regimes

The state space of the equations of motion are of variable structure, meaning there are a different set of differential equations that describe the system depending on which contact regime the system displays. The different contact regimes are shown in Table 5.1. If the system is in a *Free-Free* regime it is modeled as having 10 degrees of freedom, if the system is in a single point contact regime, for example P_1 -*Free*, it has 9 degrees of freedom as the dependence of q_6 on q_7 is shown in Equation 4.11, and if the system is in a double point contact regime, for example P_1 - P_4 , it has 8 degrees of freedom as laid out in Equations 4.12 and 4.13 by the dependence of q_6 and q_7 on q_5 .

Table 5.1: Contact Regime Configurations

<i>Contact 1</i>	<i>Contact 2</i>
<i>Free</i>	<i>Free</i>
P_1	<i>Free</i>
P_2	<i>Free</i>
P_3	<i>Free</i>
P_4	<i>Free</i>
P_1	P_3
P_1	P_4
P_2	P_4
P_2	P_3

P_1 , P_2 , P_3 , and P_4 as as shown in Table 5.1 represent their respective contact points as illustrated in Figure 4.1 and *Free* represents no contact. For each of the contact regimes the contact point could have a slip or stick condition. In order for the contact point to be in a stick condition the frictional force, F_μ , must have a larger magnitude than the active force driving the dynamics at the surface. If the

frictional force has a smaller magnitude than the active force the contact will be in a slip condition. Again, separate equations of motion were derived for each of the 17 contact regimes (P_1 - P_3 slip, P_1 - P_3 stick, etc.) then a subroutine is used to identify at each time step which regime is occurring and calls the appropriate equations of motion.

The initial conditions are set so that the program initializes in a *Free-Free* configuration. Then the program must determine which tip or tips are in contact or about to be in contact. This calculation is shown as taken from the subroutine, `contact`, in Figure 5.1. Bodies 3 and 4 referred to in the comment are analogous to bodies C and D in Figure 4.1.

```

c      location of the tips of bodies 3 and 4 where contact is
c      made in the b[3,2] direction

      Tip1 = -(T*Cos(q7))/2. - (L4*Sin(q7))/2. + q6
      Tip2 = (T*Cos(q7))/2. - (L4*Sin(q7))/2. + q6
      Tip3 = -(T*Cos(q7))/2. + (Tan(q7)*(2*L3 - T*Sin(q7) - 2*q5))/2. + q6
      Tip4 = (T*Cos(q7))/2. + (Tan(q7)*(2*L3 + T*Sin(q7) - 2*q5))/2. + q6

c      now determine distance between contact surfaces and tip locations

      Tip1diff = W/2. - dabs(Tip1)
      Tip2diff = W/2. - dabs(Tip2)
      Tip3diff = W/2. - dabs(Tip3)
      Tip4diff = W/2. - dabs(Tip4)

```

Figure 5.1: Fortran Code - Contact Tip Location Calculation

Once the tip locations have been determined the velocities of approach are calculated. The signs of these velocities determine whether the tip is approaching or leaving a contact point. This calculation is shown in Figure 5.2. If the sign of these velocities reveal that the tip is approaching a contact, the program compares the distance between the contact surfaces and the tip locations to an input parameter, `near_contact`, which is a very small number that determines when contact is made. If it determines there is contact, it feeds a flag to the variable, `contactP`. Flags 1, 2, 3, 4 correspond to contact points P_1 , P_2 , P_3 , and P_4 .

If the system is already in contact at one point, it looks to see whether another

```

Tip1v = s6 - (L4*COS7*s7)/2. + (T*SIN7*s7)/2.
Tip2v = s6 - (L4*COS7*s7)/2. - (T*SIN7*s7)/2.
Tip3v = s6 + (T*SIN7*s7)/2. + (SEC7**2*(2*L3 - T*SIN7 - 2*q5)*s7)/2. + (TAN7*(-2*s5 - T*COS7*s7))/2.
Tip4v = s6 - (T*SIN7*s7)/2. + (SEC7**2*(2*L3 + T*SIN7 - 2*q5)*s7)/2. + (TAN7*(-2*s5 + T*COS7*s7))/2.

```

Figure 5.2: Fortran Code Contact Tip Velocity Calculation

contact is made or about to be made. This is done as shown in Figure 5.3. The program compares the other two possible contact points to `near_contact` to check for contact. If contact has occurred at an additional point it feeds a two digit flag to the variable, `contactP`. The flag 13 cooresponds to contact at P_1 - P_3 , flag 14 cooresponds to contact at P_1 - P_4 , etc.

```

if(contactP .eq. 1) then
  if((dabs(Tip3diff) .lt. near_contact .or. Tip3diff .le. 0.0) .and. (Tip3v .le. zero))then

    contact=13
    Xc1 = (-L4 + 2*q5)/2.
    Xc2 = 0
    Xd3 = L3 - q5
    Xd4 = 0

  elseif((dabs(Tip4diff) .lt. near_contact .or. Tip4diff .le. 0.0) .and. (Tip4v .ge. -zero))then

    contact=14
    Xc2 = 0
    Xd3 = 0

c      give the Newton Rhapson routine a resonable first guess, if one or the other is not set

    if(dabs(Xc1) .le. 1.0d-8)then
      Xc1 = q5 - L4/2.
    endif
    if(dabs(Xd4) .le. 1.0d-8)then
      Xd4 = L4/2.
    endif

    call findConstP1P4(q5,q6,q7,Xc1,Xd4)

  else
    contact = contactP
  endif
endif

```

Figure 5.3: Fortran Code Additional Contact Calculation

Once the program determines which contact or contacts are occurring, it checks to see if the contact is slipping or sticking. Shown in Figure 5.4 is an example of how this calculation is called. Once the program determines that `contactP` equals 1 it calls the subroutine `regimeP1` which calculates the friction force, F_μ , at contact

point, P_1 , to determine whether there is a slip or stick condition. If **regimeP1** sends the flag 1 then it has determined that the system is sticking at P_1 and tells LSODA to solve the equations of motion contained in **stickP1**. If **regimeP1** sends the flag 2 then it has determined that the system is slipping at P_1 and tells LSODA to solve the equations of motion contained in **contactP1**. Finally, if **regimeP1** sends the flag 0 the tip has left its contact point and the program calls LSODA to solve the equations of motion contained in **freeflight**. The same process occurs for any value assigned to **contactP**.

```

c      regime = 0 - freeflight; regime=1 - stickP1; regime=2 - contactP1....
      if(contactP .eq. 0) then
        call dlsoda(freeflight,n,x,tm,tout,itol,rtol,atol,itask,istate,iopt,rwork,lrw,iwork,liw,JACOB,jt)
        call updateContacts(n,x,regimeN)
        regime0 = 0
        call calcConstForce(n,tm,x,Dtx,contactP,regimeN,Rf,Nf)
      elseif(contactP .eq. 1) then
c
c      stick or contact at P1 or became free
c      must compute momentum appropriate change if needed in regimeP functions
c      check regime, init lsoda (istate=1), and init constraints if needed
c
        regimeN = regimeP1(n,x,tm,regime0,Rf,Nf)
        if(regimeN .ne. regime0) then
          if (regimeN .ne. 0)call initconst(n,x,regimeN)
        endif
        if(regimeN .eq. 1) then
          call dlsoda(stickP1,n,x,tm,tout,itol,rtol,atol,itask,istate,iopt,rwork,lrw,iwork,liw,JACOB,jt)
          regime0 = regimeN
        elseif(regimeN .eq. 2) then
          call dlsoda(contactP1,n,x,tm,tout,itol,rtol,atol,itask,istate,iopt,rwork,lrw,iwork,liw,JACOB,jt)
          call updateContacts(n,x,regimeN)
          regime0 = regimeN
        elseif(regimeN .eq. 0) then
          call dlsoda(freeflight,n,x,tm,tout,itol,rtol,atol,itask,istate,iopt,rwork,lrw,iwork,liw,JACOB,jt)
          regime0 = regimeN
          contactP = 0
          call updateContacts(n,x,regimeN)
          call calcConstForce(n,tm,x,Dtx,contactP,regimeN,Rf,Nf)
        else
          call error(tm,contactP,regimeN,regime0)
          stop
        endif
      endif

```

Figure 5.4: Fortran Code Stick-Slip Determination

If a degree of freedom is lost when switching from one contact regime to another the

appropriate non-holonomic constraint momentum transfer must be called to initiate the new integration state. Table 5.2 shows the contact regimes changes when one degree of freedom is lost and Table 5.3 shows the contact regime changes when two degrees of freedom are lost. The From and To category names can be switched in order to see the contact regime changes when a degree or degrees of freedom are gained. Each of these contact regimes changes has a specific set of associated momentum transfer equations that are called when a degree of freedom is lost.

Table 5.2: Contact Regime Changes when a One Degree of Freedom is Lost

From	<i>Free – Free</i>	<i>Free – Free</i>	<i>Free – Free</i>	<i>Free – Free</i>
To	<i>P₁ slip</i>	<i>P₃ slip</i>	<i>P₁ – P₃ slip</i>	<i>P₁ – P₄ slip</i>
To	<i>P₂ slip</i>	<i>P₄ slip</i>	<i>P₂ – P₃ slip</i>	<i>P₂ – P₄ slip</i>
From	<i>P₁ slip – Free</i>	<i>P₂ slip – Free</i>	<i>P₃ slip – Free</i>	<i>P₄ slip – Free</i>
To	<i>P₁ – P₃ slip</i>	<i>P₂ – P₃ slip</i>	<i>P₁ – P₃ slip</i>	<i>P₁ – P₄ slip</i>
To	<i>P₁ – P₄ slip</i>	<i>P₂ – P₄ slip</i>	<i>P₂ – P₃ slip</i>	<i>P₂ – P₄ slip</i>
To	<i>P₁ stick</i>	<i>P₂ stick</i>	<i>P₃ stick</i>	<i>P₄ stick</i>
To	<i>P₁ – P₃ stick</i>	<i>P₂ – P₃ stick</i>	<i>P₁ – P₃ stick</i>	<i>P₁ – P₄ stick</i>
To	<i>P₁ – P₄ stick</i>	<i>P₂ – P₄ stick</i>	<i>P₂ – P₃ stick</i>	<i>P₂ – P₄ stick</i>
From	<i>P₁ – P₃ slip</i>	<i>P₁ – P₄ slip</i>	<i>P₂ – P₃ slip</i>	<i>P₂ – P₄ slip</i>
To	<i>P₁ – P₃ stick</i>	<i>P₁ – P₄ stick</i>	<i>P₂ – P₃ stick</i>	<i>P₂ – P₄ stick</i>

This process outlined above occurs at every time step throughout the solution process.

5.3 Input Parameters

Many input parameters were required by the simulation. These include mass and stiffness properties, damping coefficients, kinematic parameters, variable torque models, and LSODA solver inputs. The mass and stiffness properties remained constant

Table 5.3: Contact Regime Changes when a Two Degrees of Freedom are Lost

From	$Free - Free$
To	$P_1 stick$
To	$P_2 stick$
To	$P_3 stick$
To	$P_4 stick$
To	$P_1 - P_3 stick$
To	$P_1 - P_4 stick$
To	$P_2 - P_3 stick$
To	$P_2 - P_4 stick$

throughout the simulations, but other inputs such as damping coefficients and certain kinematic parameters had to be changed with changing joint gap widths and frequencies. The mass and inertial properties of the rigid bodies as input to the simulation are shown in Table 5.4. The mass and inertial properties of the Simulation bodies as input to the simulation are shown in Table 5.5. These properties were calculated using Autodesk Inventor CAD Software package. Inventor is a solid modeling program that can be easily used to calculate inertia properties by knowing the density and geometry of an item. Inventor even incorporates changes in density such as bolts and nuts made from steel holding together aluminum components as is the case in our experiment. The mass properties were verified by placing components on a scale and comparing to the mass given by Inventor.

Execution of the simulations was done by commanding a torque value through the equations of motion. Several different torque models were implemented in order to try to mimic the dynamics of q_1 caused by the backlash in the gearbox. These torque models are all available for simulation by inputting the desired flag. Examples of the torque models are a sine and cosine wave, a controlled torque which tracks a

Table 5.4: Rigid Body Properties

<i>Body</i>	<i>Mass (g)</i>	<i>Inertia (g cm²)</i>
<i>A</i>	286	2542
<i>C</i>	159	3102
<i>D</i>	69	816
<i>F</i>	106	204

Table 5.5: Simulation Body Properties

<i>Body</i>	<i>Mass / Unit Length ($\frac{g}{cm}$)</i>	<i>Inertia / Unit Length (g cm)</i>
<i>B</i>	2.148	.0183
<i>E</i>	2.148	.0183

desired amplitude and frequency of q_1 , and single as well as double sided deadband models. These deadband models zero the torque on one or both sides of the sign change of q_1 . It is still unclear as to which model is most accurate. Some mimic the dynamics of q_1 accurately while not showing any of the other dynamic feature of the system. Others do not match q_1 exactly, but display other dynamic characteristics well. Further developments of the simulation should include a dynamic model of the motor/gearbox configuration into the equations of motion.

5.4 Output

As the simulation is being run, output of all values of generalized coordinates, derivatives of generalized coordinates, contact point regimes, torque, accelerations and strain in points corresponding to measurement locations on the experiment are recorded at each time step and appended to a file. These output files, particularly the strain and acceleration files, are used for comparison to the experimental data.

5.5 Simulation Validation

In order to validate the simulation to ensure proper dynamic response a test case of a simple pendulum was used. The torque model used to drive the system dynamics was changed to allow gravity to be active force on the system. Gravity was set positive in the $\hat{n}_2$ direction and the zero datum was at the origin of the the N reference frame as shown in Figure 4.1. Position and total energy output was analyzed for expected results. These are shown in Figures 5.5 and 5.6.

Figure 5.5: Validation Position

The position data is shown to oscillate around and come to rest at $\pi/2$ as expected, and the total energy dampens out to a constant negative value equal to the available potential energy between the zero datum and the point of rest. These results are consistent with known dynamic responses of a pendulum.

Another area of validation was the contact switching shown in Figure 5.7. Although there is no experimental data to compare to the results, they are consistent with what is visually observed during an experimental test. The numbers on the plot

Figure 5.6: Validation Total Energy

coorespond to different contact point regime configurations, 1 equals P_1 contact, 13 equals $P_1 - P_3$ contact, 0 equals *Free* or no contact, etc.

5.6 Simulation Results and Comparison

The simulation was run for a 0.0558 cm joint gap width at 5 Hz to compare to the experimental data collected at these specifications. The simulation was run for 8 seconds, approximately 6 seconds are steady state. From this steady state region 3 full cycles were then segmented from the data for illustration and comparison to the experimental data. A cycle is defined by the beam starting at the zero point on the encoder, moving in the positive direction, returning past zero in the negative direction and returning to zero once again.

The position data shown in Figure 5.8 is not as close a match as a modeler would like to see, but, as expected for a forced response, the frequency matches the experiment and the maximum amplitudes are of the same order of magnitude. The

Figure 5.7: Validation Contact Point

main discrepancy can be seen at the peak of the waveform where the simulation data rapidly changes sign and dips into a valley. This is caused by the torque model used trying to mimic the deadband region caused by slack in the motor gearbox configuration.

The strain data shown in Figures 5.9 and 5.10 match well in frequency and waveform when the strains are negative. The waveform does not seem to rebound as much in the positive direction and has a very distinct difference.

The simulated response of acceleration for the accelerations normal the beams (1, 3, 5) match closely on frequency with the experimental data. These are shown in Figures 5.11, 5.13, and 5.15. The overall average amplitudes are close also, but do not match exactly. The simulation seems to model more of the low frequency motion than the accelerometers pick up. Acceleration 5 matches very closely in waveform, and amplitude. All transverse accelerations exhibit similar FFT data.

Figures 5.12 and 5.14 show the accelerations parallel to the surface of the beams

Figure 5.8: Position Comparison 0.0558 cm at 5 Hz

(2, 4). The difference in these two accelerations show the relative acceleration between the two joint sections. It is hard to say anything about the frequency or waveform of these vibrations but both the simulation and the experiment seem to show semi-periodic responses with amplitudes of the same order of magnitude.

(a) Time Response

(b) Experiment Frequency Response

(c) Simulation Frequency Response

Figure 5.9: Strain 2 Comparison 0.0558 cm at 5 Hz

(a) Time Response

(b) Experiment Frequency Response

(c) Simulation Frequency Response

Figure 5.10: Strain 2 Comparison 0.0558 cm at 5 Hz

(a) Time Response

(b) Experiment Frequency Response

(c) Simulation Frequency Response

Figure 5.11: Acceleration 1 Comparison 0.0558 cm at 5 Hz

(a) Time Response

(b) Experiment Frequency Response

(c) Simulation Frequency Response

Figure 5.12: Acceleration 2 Comparison 0.0558 cm at 5 Hz

(a) Time Response

(b) Experiment Frequency Response

(c) Simulation Frequency Response

Figure 5.13: Acceleration 3 Comparison 0.0558 cm at 5 Hz

(a) Time Response

(b) Experiment Frequency Response

(c) Simulation Frequency Response

Figure 5.14: Acceleration 4 Comparison 0.0558 cm at 5 Hz

(a) Time Response

(b) Experiment Frequency Response

(c) Simulation Frequency Response

Figure 5.15: Acceleration 5 Comparison 0.0558 cm at 5 Hz

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

This research shows the development of a model and simulation of a hybrid parameter multibody system containing a mechanical joint that is allowed to be “loose” within the frictional contact region of the joint. The model assumes the joint region to be rigid and the beams extending from this region to be flexible. The simulation takes into consideration all frictional effects as well as constraint addition and deletion within the joints frictional contact region. Experimental results of the rigid joint configuration were collected for use in comparison to the simulation. An equal amount of experimental data was collected for the flexible joint configuration for use in future simulations that will model the joint region as flexible.

It has been shown that the simulation, as is, produces many promising dynamic effects that match well with the experimental data, but is not an exact match. Many of the discrepancies most likely come from a non-exact torque model applied to the differential equations to simulate motion. The switching between contact regimes simulates efficiently the observed dynamics of an experiment, and the amplitudes and frequencies of the simulation output are comparable to those in the experimental data.

Recommendations for future work are as follows:

1. Apply a more robust torque model or add state equations to the equations of motion to effectively model the dynamics of the motor/gearbox configuration. This could help improve the overall dynamics of the system and make them more comparable to the experimental data.
2. Fine tune the simulation input parameters so the simulation will match the experimental data better. Once a good match has been achieved LSODA tol-

erances and `near_contact` values can be adjusted to help the simulation run faster and more efficiently.

3. Perform free vibration tests with a cantilever attachment in place of the motor.
4. Develop the model for the flexible joint configuration and compare with the experimental data provided.

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APPENDIX A RIGID JOINT SETUP DATA

Appendix A contains the complete results in graphical form for all nine different configurations in the rigid joint experimental setup

A.1 Position Data

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure A.1: Position for 0.0558 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure A.2: Position for 0.1016 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure A.3: Position for 0.1576 cm Gap

A.2 Strain 1 Data

(a) 1 Hz

(b) 3 Hz

(c) 5Hz

Figure A.4: Strain 1 for 0.0558 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5Hz

Figure A.5: Strain 1 for 0.1016 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5Hz

Figure A.6: Strain 1 for 0.1576 cm Gap

A.3 Strain 2 Data

(a) 1 Hz

(b) 3 Hz

(c) 5Hz

Figure A.7: Strain 2 for 0.0558 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5Hz

Figure A.8: Strain 2 for 0.1016 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5Hz

Figure A.9: Strain 2 for 0.1576 cm Gap

A.4 Acceleration 1 Data

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure A.10: Acceleration 1 for 0.0558 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure A.11: Acceleration 1 for 0.1016 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure A.12: Acceleration 1 for 0.1576 cm Gap

A.5 Acceleration 2 Data

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure A.13: Acceleration 2 for 0.0558 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure A.14: Acceleration 2 for 0.1016 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure A.15: Acceleration 2 for 0.1576 cm Gap

A.6 Acceleration 3 Data

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure A.16: Acceleration 3 for 0.0558 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure A.17: Acceleration 3 for 0.1016 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure A.18: Acceleration 3 for 0.1576 cm Gap

A.7 Acceleration 4 Data

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure A.19: Acceleration 4 for 0.0558 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure A.20: Acceleration 4 for 0.1016 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure A.21: Acceleration 4 for 0.1576 cm Gap

A.8 Acceleration 5 Data

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure A.22: Acceleration 5 for 0.0558 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure A.23: Acceleration 5 for 0.1016 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure A.24: Acceleration 5 for 0.1576 cm Gap

APPENDIX B

FLEXIBLE JOINT SETUP DATA

Appendix B contains the complete results in graphical form for all nine different configurations in the flexible joint experimental setup.

B.1 Position Data

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure B.1: Position for 0.0558 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure B.2: Position for 0.1016 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure B.3: Position for 0.1576 cm Gap

B.2 Strain 1 Data

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure B.4: Strain 1 for 0.0558 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure B.5: Strain 1 for 0.1016 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure B.6: Strain 1 for 0.1576 cm Gap

B.3 Strain 2 Data

(a) 1 Hz

(b) 3 Hz

(c) 5Hz

Figure B.7: Strain 2 for 0.0558 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5Hz

Figure B.8: Strain 2 for 0.1016 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5Hz

Figure B.9: Strain 2 for 0.1576 cm Gap

B.4 Acceleration 1 Data

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure B.10: Acceleration 1 for 0.0558 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure B.11: Acceleration 1 for 0.1016 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure B.12: Acceleration 1 for 0.1576 cm Gap

B.5 Acceleration 2 Data

Figure B.13: Acceleration 2 for 0.0558 cm Gap

Figure B.14: Acceleration 2 for 0.1016 cm Gap

Figure B.15: Acceleration 2 for 0.1576 cm Gap

B.6 Acceleration 3 Data

Figure B.16: Acceleration 3 for 0.0558 cm Gap

Figure B.17: Acceleration 3 for 0.1016 cm Gap

Figure B.18: Acceleration 3 for 0.1576 cm Gap

B.7 Acceleration 4 Data

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure B.19: Acceleration 4 for 0.0558 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure B.20: Acceleration 4 for 0.1016 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure B.21: Acceleration 4 for 0.1576 cm Gap

B.8 Acceleration 5 Data

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure B.22: Acceleration 5 for 0.0558 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure B.23: Acceleration 5 for 0.1016 cm Gap

(a) 1 Hz

(b) 3 Hz

(c) 5 Hz

Figure B.24: Acceleration 5 for 0.1576 cm Gap

APPENDIX C DETAIL DRAWINGS

Appendix C contains the detail drawings used in the fabrication of the experiment.

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Date