WIDE BAND MODELING AND PARAMETER IDENTIFICATION
IN MAGNETOSTRICTIVE ACTUATORS

by

DINESH BANDARA EKANAYAKE, B.Sc.

A THESIS

IN

MATHEMATICS

Submitted to the Graduate Faculty
of Texas Tech University in
Partial Fulfillment of
the Requirements for
the Degree of

MASTER OF SCIENCE

Approved

Ram Venkataraman Iyer
Co-Chairperson of the Committee

Wijesuriya Dayawansa
Co-Chairperson of the Committee

Clyde F. Martin

Xiaochang Wang

Accepted

John Borrelli
Dean of the Graduate School

May, 2006

ACKNOWLEDGMENTS

I would like to express my gratitude to my advisors, Professors Iyer and Dayawansa, for providing a great opportunity to transition from electrical engineering experiment-based research to mathematical proof-based research and to learn the value of such research. They have encouraged me to explore new and diverse areas and I am thankful for their guidance and support throughout the duration of this project. I am particularly grateful to Professor Iyer for the extensive knowledge of mathematics, especially control theory, he has shared with me. I also wish to thank Professors Martin and Wang for their willingness to be members of my defense committee.

Additionally, I would also like to thank my beloved, Amy, for her willingness to correct my English and to help me in other various ways.

I would like to acknowledge number of colleagues for offering suggestions, comments and support. My brother, Professor J.B. Ekanayake, has always been a substantial influence on my academic accomplishments, as he gives me inspiration to broaden my interests in many directions. My entire family has always loved and supported me and has served as a constant source of encouragement for which I will always be grateful.

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ABSTRACT

Smart materials, such as magnetostictives, piezoelectrics, and shape memory alloys, all display certain coupling phenomena between applied electromagnetic fields and their mechanical properties. Smart actuators made of these materials can be used to build smart structures with the ability to sense and respond to environmental changes to achieve desired goals. However, the complicated electro-magnetic behaviors of such actuators limit effective use of them. We introduce a model which describes most of the characteristics of magnetostrictive actuators in a large frequency range, from 0-2kHz. Rather than treating each behavior separately, we treat them with arrays of similar structural parameters to minimize difficulties arising from usage of such a model for control purposes.

We develop a low frequency model with DC resistance and two current sinks, which is sufficiently accurate for frequencies less than 500Hz. Even at 500Hz, it is challenging to identify the parameters due to the nonlinearity of the hysteresis losses. To reduced this difficulty we derive a method to fix the hysteresis losses and develop a parameter identification process using sinusoidal input voltage. We then introduce an array of current sinks to improve the model for frequencies up to 800Hz. To accommodate the additional series losses that are significant after 1kHz, we introduce a transfer function which appears before the low frequency model. We experimentally verify that such models are indeed sufficiently accurate.

CHAPTER I

INTRODUCTION

Modelling magneto-motive actuators is a challenge due to the strong coupling between their electric, magnetic and mechanical properties and the hysteresis in the constitutive relation of the magnetic materials. Many approaches have been used for deriving such models, most of which are based on empirical classifications of material properties of the core. Although these approaches, which assume linear Ohmic losses in the winding, are adequate for producing low frequency models such as for 200Hz, they are insufficient for producing models which are accurate over large frequency ranges such as 0-2kHz. A broad-band model should reflect the loss characteristics that are significant over a given frequency range, particularly, the electric and magnetic losses of both the core and the winding. Additionally, a broad-band model may need to incorporate thermal, stray capacitance, and inductance effects, as these phenomena affect the behavior of some actuators.

A significant model was developed by Venkataraman and Krishnaprasad who proposed a ferromagnetic hysteresis model using the Preisach operator which includes classical eddy current losses [7]. This model works well for frequencies less than 200Hz. Later, Iyer and Manservisi introduced a new dimension to the model by suggesting a nonlinear resistance for excessive power loss to more accurately model the behavior at high frequencies [9]. However, due to the complexity, the question of identifying the parameters for a broad-band model has not yet been thoroughly solved, though this step is crucial to the usefulness of the model.

Any attempt to solve for these parameters is further hindered by the need to find the hysteresis loss, which is again nonlinear. In particular, the hysteresis loop is not only a function of peak magnetization, but also of the magnetizing waveform shape. For example, the hysteresis loop under sinusoidal magnetization is different from the loop under square magnetization. Our goal is to present a model derived for an arbitrary input voltage waveform and to develop a parameter-identification

process for a sinusoidal input with a fixed hysteresis loop. We present a modification of the models developed by Venkataraman-Krishnaprasad and Iyer-Manservigi by introducing an array of dependent current sinks for excessive power loss, skin effect and proximity effect at low frequencies. We employ another current sink in place of the resistor, which the above models use to represent eddy current losses. In addition, we identify a transfer function to model the other significant behavior appearing at high frequencies.

The process of modelling can be divided in to three broad categories:

1. the identification of electric and magnetic loss parameters,
2. the identification of the Preisach function of the hysteresis operator, and
3. the identification of magneto-elastic loss parameters.

The second and third categories have been investigated previously by Venkataraman [6], [8] and Della Torre [10]. We primarily address the first category by developing an electro-magnetic loss parameter identification process, independent of the second and third classes. We divide the first class into two smaller classes, namely, identification of core loss parameters and identification of winding loss parameters.

1.1 Core losses

For a broad class of magnetic materials, including ferromagnetic materials, the losses of a magnetized core can be decomposed into the sum of hysteresis (P_{hys}), classical (P_{cl}), and excess (P_{exc}) loss components [11]. In the frequency domain we have the following equation, for sinusoidal voltage inputs:

$$P_{total} = P_{hys} + P_{cl} + P_{exc} = C_0 f + C_1 f^2 + C_2 f^{1.5}. \quad (1.1)$$

Hence, the power loss per cycle is

$$\frac{P_{total}}{f} = C_0 + C_1 f + C_2 \sqrt{f}, \quad (1.2)$$

where the coefficients are functions of the peak magnetization current. Hysteresis power loss per cycle is independent of the frequency while the other two terms depend

on frequency. As we describe in Chapter III, the excessive losses and the eddy current losses can be represented as two current sinks, in an electrical circuit model.

One might expect the losses in the core to depend on many aspects of the magnetization processes. However, most of these aspects turn out to be irrelevant, and only a few dominant features, which are described by Equation (1.1), survive [11]. Also Equation (1.1) applies to a broad class of different magnetic materials with different wave shapes. Bertotti explains the reason for the three terms in the Equation 1.1: they are result of the existence of three scales in the magnetization process [11]. The scale associated with the hysteresis loss is the scale of the Barkhausen effect, where small domain wall segments make localized jumps between local minima of the system's free energy, giving rise to the localized eddy current around the jumping walls. The second contribution is so called classical eddy current loss. The scale associated with this is the scale fixed by the specimen's geometry. The classical loss is the loss calculated by Maxwell's equations for a perfectly homogeneous material with no domain structure. In this case, the boundary conditions of the problem, that is the specimen's geometry, determine the result. Finally, the scale associated with the excess loss is the scale of magnetic domains. The excess loss arises from the eddy currents surrounding the active domain walls in motion under the driving action of the external field.

It is clear that any modelling problem related to magnetic fields needs to address each of the above physical phenomena separately. Using experimental data for soft ferromagnetic materials, Bertotti, Fiorillo and Novikov claimed that the above loss separation is sufficient even for 50 KHz [5], [4]. However, the interaction between time-varying magnetic fields and the related current density in the current-carrying conductors was not taken into the consideration. From their experimental graphs it is noticeable that there is considerable deviation between the theoretical and experimental data for high frequencies. However, with their experiments, they justify that core losses in soft ferromagnetic materials can sufficiently be described by three such physical phenomena.

1.2 Winding losses

It is important to identify the loss separation as two separate problems, one being the core loss separation and other being the winding loss separation. Even though the eddy current losses found in the core is also found in the winding, their resultant losses are different from that of core losses. These phenomena work against the applied current and increases the Ohmic losses. Also effective magnetic field for winding is different from that of the core, leading to difficulties of lumping these losses with core losses.

At high frequencies the flux within the wire changes rapidly. Changing flux induces a voltage drop which develops eddy current within the wire. This eddy current reinforces the main current flow at the surface, but opposes it toward the center of the wire. As a result, the rising frequency causes the current density to increase at the conductor surface and to decrease towards zero at the center. Consequently, wire resistance at high frequencies is many times greater than at low frequencies. This is called skin effect, which is one of the undesirable effect of eddy current in the winding. The proximity effect is similar but it is caused by the changing flux due to the current of nearby conductors. Both the skin effect and the proximity effect cause the current density to be nonuniform in the cross-section of the conductor and cause higher winding loss at higher frequencies.

In 1997, Fiorillo and Bertotti carried out a set of experiments which showed deviation of loss separation above 400 Hz, [14]. They suggest that it is due to the eddy current shielding, which is the skin and proximity effects on the winding, which acts as a shield for the input current.

By suggesting a fundamental model with constant lead resistance, we carried out a set of experiments to observe the effect of winding losses. For low frequencies, we recognize that the only significant winding loss is indeed the DC lead resistance. However, for high Frequencies, the experimental data suggest that the constant resistor is no longer sufficient. We then propose a model with an array of current sinks

and a transfer function to adequately represent the winding losses and any other undesirable behaviors present in series with the input line. Then we demonstrate that such a model can describe the actuator for the frequency band that the actuator can tolerate.

CHAPTER II

FUNDAMENTAL CIRCUIT DIAGRAM

In this chapter we first describe a fundamental circuit diagram for the actuator. Then we discuss the behavior of the average magnetic field, the magnetization, and the induced emf developed across the actuator winding. Furthermore, we extend our studies to a discussion of the functional space for each of the above magnetic parameters corresponding to various input voltages.

2.1 Description of circuit diagram

It is customary to express the winding losses at low frequencies as a series resistor, R . The functional representation of the core losses is a monotone increasing function with respect to induced emf. Using these details, we formulate a fundamental circuit diagram for an actuator, as shown in Figure 2.1.

Figure 2.1: Fundamental circuit diagram of the actuator

Parameters v and i in Figure 2.1 are the input voltage and the current of the actuator winding, respectively. The magnetization current is represented by i_m , and e represents the induced emf due to magnetization. Let function $\varphi(e)$ be the current corresponding to the core losses. Then $e\varphi(e)$ describes the power losses and is a monotone increasing function with respect to $e(t)$. Hence, $\varphi(e)$ is also monotone increasing function of e . Assuming that the winding losses can be described by the DC lead-resistance, R , we can obtain a system of equations to describe the circuit.

To do so, we first apply Kirchoff's voltage and current laws to the circuit in Figure 2.1. We obtain

$$e + iR = v \quad (2.1)$$

$$\varphi(e) + i_m = i. \quad (2.2)$$

The magnetization current, i_m , is proportional to the average axial magnetic field of the coil and is in the form $i_m = cH$, where H is the magnetic field and c is a constant. From Faraday's Law, voltage e can be written as $e = NA \frac{dB}{dt}$, where B is the average magnetic flux density along the axis of the inductor, N is the number of turns in the coil, and A is the cross-sectional area of the rod. The definition of B yields $B = \mu_0(M + H)$, where μ_0 is the permeability of the air and M is the axial magnetization. The classical Preisach operator is employed to represent the rate-independent hysteresis, yielding $M(\cdot) = \Gamma[H(\cdot); \psi_0]$, where $\Gamma[\cdot; \psi_0]$ is the hysteresis operator and ψ_0 is the initial memory curve. Combining Equations (2.1)-(2.2), $B = \mu_0(M + H)$, and the classical Preisach operator, we obtain the following system of equations:

$$e + \alpha\varphi(e) + \beta H = v \quad (2.3)$$

$$\dot{H} + \dot{M} = \gamma e \quad (2.4)$$

$$M(\cdot) = \Gamma[H(\cdot); \psi_{-1}] \quad (2.5)$$

where $\alpha = R$, $\beta = cR$, and $\gamma = \frac{1}{\mu_0 NA}$.

2.2 Existence

We now prove existence and uniqueness of system of equations (2.3)-(2.5) for $v \in L^2(0, T)$. To prove existence we need the following two propositions, the proofs of which are in *Brokate and Sprekels* [1].

Proposition 1. *Let $S(x) = 0$ be a system of nonlinear equations. Let $S : \mathbb{R}^n \rightarrow \mathbb{R}^n$ be continuous and coercive with respect to some $x_0 \in \mathbb{R}^n$. That is,*

$$\lim_{\|x\| \rightarrow \infty} \frac{S(x) \cdot (x - x_0)}{\|x\|} = +\infty \quad (2.6)$$

Then $S(x) = 0$ has a solution in $x \in \mathbb{R}^n$.

Proposition 2. Let $\Omega \cap \mathbb{R}^N$, $N \geq 2$, denote some non-empty, open and bounded set with Lipschitz boundary $\partial\Omega$. Then, if $m \leq N/2$, the imbedding

$$H^m(\Omega) \hookrightarrow C(\bar{\Omega}) \quad (2.7)$$

is continuous and compact, where $H^m(\Omega)$ is the Sobolev space of all functions $u \in L^2(\Omega)$ having generalized partial derivatives $D^\alpha u \in L^p(\Omega)$, for every α satisfying $0 \leq |\alpha| \leq m$

Throughout this section, we use the following general hypotheses:

(H1): $\varphi(e)$ is strictly monotone increasing with respect to e , and $\varphi(0) = 0$.

(H2): The hysteresis operator $\Gamma[\cdot; \psi_{-1}]$ is continuous on $C[0, T]$ and piecewise increasing. Also, $|\Gamma(H; \psi_{-1})(t)| \leq \bar{C} + C_p \|H \cdot 1_{[0,t]}\|_\infty$, where $\bar{C} > 0$ and $C_p > 0$.

We approximate system (2.3)-(2.5) by means of an implicit time discretization scheme as follows. Fix any $K \in \mathbb{N}$, and set $h = T/K$. Consider,

$$\alpha\varphi(e^i) + \beta H^i + e^i = v^i \quad (2.8)$$

$$H^i - H^{i-1} + M^i - M^{i-1} = h\gamma e^i \quad (2.9)$$

$$M^i = \Gamma[H^0, \dots, H^i, \psi_{-1}]. \quad (2.10)$$

where $i = 0, 1 \dots K$.

First we prove the existence and uniqueness of solutions $\{(e^i, H^i, M^i)\}_{i=0}^K$ to system (2.8)-(2.10).

Lemma 1. For a given h and $V = [v^0, \dots, v^K]$, Equations (2.8) - (2.10), with initial conditions H^0 and M^0 , have a unique solution for $\{(e^i, H^i, M^i)\}_{i=0}^K$.

Proof. Let

$$S(e^i, H^i) = \begin{bmatrix} \gamma\alpha h\varphi(e^i) + \gamma\beta h H^i + \gamma h e^i - \gamma h v^i \\ \beta(H^i - H^{i-1}) + \beta(M^i - M^{i-1}) - \gamma\beta h e^i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (2.11)$$

Then we have

$$\begin{aligned}
S(e^i, H^i) \cdot [(e^i - 0) \quad (H^i - H^{i-1})]^T &= \gamma \alpha h \varphi(e^i) e^i + \gamma h (e^i)^2 + \beta (H^i - H^{i-1})^2 \\
&\quad + \beta (M^i - M^{i-1}) (H^i - H^{i-1}) \\
&\quad + \gamma h e^i (\beta H^{i-1} - v^i).
\end{aligned} \tag{2.12}$$

For large (e^i, H^i) , $(e^i)^2$ and $(H^i)^2$ dominates in $S(e^i, H^i) \cdot [(e^i - 0) \quad (H^i - H^{i-1})]^T$. Then for a given pair $(0, H^{i-1})$,

$$\lim_{\|e^i, H^i\| \rightarrow \infty} \frac{S(e^i, H^i) \cdot [(e^i - 0) \quad (H^i - H^{i-1})]^T}{\|e^i, H^i\|} = \infty. \tag{2.13}$$

Hence, $S(e^i, H^i)$ is coercive with respect to the point $(0, H^{i-1})$. Then, by Proposition 1, Equations (2.8) - (2.10) have a solution, (e^i, H^i) . Also Equation (2.10) yields the existence of M^i . Since above argument is true for each i , for given initial conditions V^0 , H^0 , and M^0 , we have solutions for e^i and H^i , $i = 1, \dots, K$. Thus, given v , there exist solutions

$$e = [e^0, e^1, \dots, e^K], \tag{2.14}$$

$$H = [H^0, H^1, \dots, H^K], \tag{2.15}$$

$$M = [M^0, M^1, \dots, M^K]. \tag{2.16}$$

To prove uniqueness, let

$$\begin{bmatrix} e_1 \\ H_1 \\ M_1 \end{bmatrix} = \begin{bmatrix} e^0, e_1^1, \dots, e_1^K \\ H^0, H_1^0, \dots, H_1^K \\ M^0, M_1^0, \dots, M_1^K \end{bmatrix}, \tag{2.17}$$

$$\begin{bmatrix} e_2 \\ H_2 \\ M_2 \end{bmatrix} = \begin{bmatrix} e^0, e_2^1, \dots, e_2^K \\ H^0, H_2^0, \dots, H_2^K \\ M^0, M_2^0, \dots, M_2^K \end{bmatrix} \tag{2.18}$$

be two solutions. Then from Equations (2.8) - (2.10), we get

$$\alpha[\varphi(e_1^i) - \varphi(e_2^i)] + \beta[H_1^i - H_2^i] + [e_1^i - e_2^i] = 0, \quad (2.19)$$

$$[H_1^i - H_2^i] + [M_1^i - M_2^i] - [H_1^{i-1} - H_2^{i-1}] - [M_1^{i-1} - M_2^{i-1}] = h\gamma[e_1^i - e_2^i] \quad (2.20)$$

$$\Gamma[H^0, H_1^1, \dots, H_1^{i-1}, H_1^i, \phi_{-1}] = M_1^i, \quad (2.21)$$

$$\Gamma[H^0, H_2^1, \dots, H_2^{i-1}, H_2^i, \phi_{-1}] = M_2^i. \quad (2.22)$$

Assume $H_1^{i-1} = H_2^{i-1}$ and $M_1^{i-1} = M_2^{i-1}$. Since Γ is monotone increasing, if $H_1^i - H_2^i > 0$, then $M_1^i - M_2^i > 0$, and from Equation (2.20), we have $e_1^i - e_2^i > 0$. From the monotonicity of $\varphi(e)$ we also get $\varphi(e_1^i) - \varphi(e_2^i) > 0$. In the same way, if $H_1^i - H_2^i < 0$, then $e_1^i - e_2^i < 0$ and $\varphi(e_1^i) - \varphi(e_2^i) < 0$. By Equation (2.19), this implies $H_1^i = H_2^i$, $e_1^i = e_2^i$, and $M_1^i = M_2^i$. Since the initial conditions are the same, by induction we have

$$\begin{bmatrix} e_1 \\ H_1 \\ M_1 \end{bmatrix} = \begin{bmatrix} e_2 \\ H_2 \\ M_2 \end{bmatrix}. \quad (2.23)$$

Hence, the solution is unique. $\square$

Next we prove the existence of a solution to system (2.3)-(2.5). We employ Lemma 1 to prove the next theorem.

Theorem 1. *Let hypothesis $\mathcal{H}1$ hold. Suppose $w[\cdot; \psi_{-1}]$ is a family of hysteresis operators satisfying $\mathcal{H}2$. Let $H^0 \in H_0^1(0, T)$ and $v \in L^2(0, T) \cap L^\infty(0, T)$. Then there exists a weak solution (H, M, e) to the system (2.3) - (2.5) satisfying*

$$H \in H^1(0, T) \cap L^\infty(0, T) \quad (2.24)$$

$$M \in H^1(0, T) \cap L^\infty(0, T) \text{ and} \quad (2.25)$$

$$e \in L^2(0, T) \cap L^\infty(0, T) \quad (2.26)$$

Proof. Since $\Gamma[\cdot; \psi_{-1}]$ is piecewise increasing, $H^i - H^{i-1} > 0$ gives $M^i - M^{i-1} \geq 0$. Equation 2.9 then gives $e^i > 0$ and hence $\varphi(e^i) > 0$ by $\mathcal{H}1$. By the same reasoning,

if $H^i - H^{i-1} < 0$, we have $e^i < 0$ and $\varphi(e^i) < 0$. Hence,

$$\varphi(e^i)(H^i - H^{i-1}) \geq 0. \quad (2.27)$$

Multiplying Equation 2.8 by $(H^i - H^{i-1})$, we get

$$\alpha\varphi(e^i)(H^i - H^{i-1}) + e^i(H^i - H^{i-1}) = (v^i - \beta H^i)(H^i - H^{i-1}). \quad (2.28)$$

Equation 2.27 gives

$$e^i(H^i - H^{i-1}) \leq (v^i - \beta H^i)(H^i - H^{i-1}). \quad (2.29)$$

Next multiply Equation 2.9 by $(H^i - H^{i-1})$ to obtain

$$\frac{|H^i - H^{i-1}|^2}{h} + \frac{(M^i - M^{i-1})(H^i - H^{i-1})}{h} = \gamma e^i(H^i - H^{i-1}). \quad (2.30)$$

Since $\Gamma[\cdot; \psi_{-1}]$ is piecewise increasing $\forall t \in [0, T]$, it holds that

$$\frac{(M^i - M^{i-1})(H^i - H^{i-1})}{h} \geq 0 \quad \forall i \quad (2.31)$$

Hence from Equation 2.30 we now have

$$\frac{|H^i - H^{i-1}|^2}{h} \leq \gamma e^i(H^i - H^{i-1}). \quad (2.32)$$

Equation (2.29) gives

$$\frac{|H^i - H^{i-1}|^2}{h} \leq \gamma(v^i - \beta H^i)(H^i - H^{i-1}) \quad (2.33)$$

$$\frac{|H^i - H^{i-1}|^2}{h} + \gamma\beta H^i(H^i - H^{i-1}) \leq \gamma v^i(H^i - H^{i-1}). \quad (2.34)$$

Then, from Young's Inequality,

$$\frac{|H^i - H^{i-1}|^2}{h} + \gamma\beta H^i(H^i - H^{i-1}) \leq \frac{h\gamma^2 v_i^2}{2} + \frac{(H^i - H^{i-1})^2}{2h}. \quad (2.35)$$

Sum both sides of Equation (2.33) from $i = 1$ to $i = k$, where $1 \leq k \leq K$:

$$\sum_{i=1}^k \frac{|H^i - H^{i-1}|^2}{2h} + \sum_{i=1}^k \gamma\beta H^i(H^i - H^{i-1}) \leq \sum_{i=1}^k \frac{h\gamma^2 v_i^2}{2}. \quad (2.36)$$

Note that

$$\sum_{i=1}^k H^i |H^i - H^{i-1}| = \sum_{i=1}^k \frac{|H^i - H^{i-1}|^2}{2} + \frac{|H^k|^2}{2} - \frac{|H^0|^2}{2}, \quad (2.37)$$

and hence,

$$\sum_{i=1}^k \frac{|H^i - H^{i-1}|^2}{2h} + \frac{\gamma\beta}{2} \sum_{i=1}^k \frac{|H^i - H^{i-1}|^2}{2} + \frac{\gamma\beta|H^k|^2}{2} \leq \frac{\gamma^2}{2} \sum_{i=1}^n h v_i^2 + \gamma\beta \frac{|H^0|^2}{2}. \quad (2.38)$$

Taking the maximum over $1 \leq k \leq K$, we get

$$\sum_{i=1}^K \frac{|H^i - H^{i-1}|^2}{2h} + \frac{\gamma\beta}{2} \sum_{i=1}^K \frac{|H^i - H^{i-1}|^2}{2} + \frac{\gamma\beta}{2} \left(\max_{1 \leq k \leq K} |H^k|^2 \right) \leq \frac{\gamma^2}{2} \sum_{i=1}^K h v_i^2 + \gamma\beta \frac{|H^0|^2}{2}. \quad (2.39)$$

Since $v \in L^2[0, T]$, using $v^i = \int_{(i-1)h}^{ih} v(t) dt$

$$\sum_{i=1}^K h v_i^2 \leq \left(\sum_{i=1}^K h \right) \|v\|_{L^2(0, T)}^2 \leq \|v\|_{L^2[0, T]}^2 K h = T \|v\|_{L^2[0, T]}^2 < c_1, \quad (2.40)$$

where c_1 is a constant. Also, $\left| \frac{H^0}{2} \right|^2 + c_1 = c_2$ is constant, so we conclude

$$\frac{h}{2} \sum_{i=1}^K \left| \frac{H^i - H^{i-1}}{h} \right|^2 + \frac{\gamma\beta}{2} \sum_{i=1}^K \frac{|H^i - H^{i-1}|^2}{2} + \frac{\gamma\beta}{2} \max_{1 \leq k \leq K} |H^k|^2 \leq c_2. \quad (2.41)$$

From Lemma 1, for each h there is a unique solution to system (2.8) - (2.10). To indicate dependence on K , we denote the solution of system (2.8) - (2.10) by H_K^i and M_K^i for any $K \in \mathbb{N}$. Then, for a given K ,

$$\frac{h}{2} \sum_{i=1}^K \left| \frac{H_K^i - H_K^{i-1}}{h} \right|^2 + \frac{\gamma\beta}{2} \sum_{i=1}^K \frac{|H_K^i - H_K^{i-1}|^2}{2} + \frac{\gamma\beta}{2} \max_{0 \leq k \leq K} |H_K^k|^2 \leq c_2. \quad (2.42)$$

Define piecewise linear approximations

$$H_K(t) = \tau H_K^{i+1} + (1 - \tau) H_K^i \quad \tau \in [0, 1] \quad (2.43)$$

$$M_K(t) = \tau M_K^{i+1} + (1 - \tau) M_K^i \quad \tau \in [0, 1] \quad (2.44)$$

where $t = (i - 1)h + \tau$ and $i = 1, 2, \dots, K$. Also define the constant piecewise approximations $\tilde{H}_K(t) = H_K^{i+1}(t)$ and $\tilde{e}_K(t) = e_K^{i+1}(t)$ $0 \leq i \leq K - 1$. With this notation,

Equations (2.8) and (2.9) gives

$$\int_0^T \frac{dH_K}{dt} \phi(t) dt + \int_0^T \frac{dM_K}{dt} \phi(t) dt = \gamma \int_0^T \tilde{e}_K \phi(t) dt \quad \forall \phi(t) \in L^2[0, T]. \quad (2.45)$$

Also,

$$\frac{H_K^i - H_K^{i-1}}{h} = \frac{dH_K}{dt} \quad (2.46)$$

and

$$\sum_{i=1}^K h \left| \frac{H_K^i - H_K^{i-1}}{h} \right|^2 = \sum_{i=1}^K h \left| \frac{dH_K}{dt} \right|^2 = \left\| \frac{dH_K}{dt} \right\|_{L^2}^2. \quad (2.47)$$

Furthermore, $\max_K |H_K^k|^2 = \|H_K\|_{L^\infty}^2$. Then, from Equation (2.41), we have

$$\frac{1}{2} \left\| \frac{dH_K}{dt} \right\|_{L^2}^2 + \frac{\gamma\beta}{2} \sum_{i=1}^K \frac{|H^i - H^{i-1}|^2}{2} + \frac{\gamma\beta}{2} \|H_K\|_{L^\infty}^2 \leq c^2. \quad (2.48)$$

Hence, $\|dH_K/dt\|_{L^2}$, $\|H_K\|_{L^\infty(0,T)}^2$, and $\sum_{i=1}^K |H^i - H^{i-1}|^2$ are bounded for all h . In addition, as $K \rightarrow \infty$, we have

$$\|H_K - \tilde{H}_K\|_{L^2} = \frac{T}{3K} \sum_{i=1}^K |H^i - H^{i-1}|^2 \rightarrow 0 \quad (2.49)$$

since $\sum_{i=1}^K |H^i - H^{i-1}|^2$ remains bounded. From Equation (2.8),

$$|e_K^i|^2 \leq |e_K^i + \alpha\varphi(e_K^i)|^2 = |v_K^i - \beta H_K^i|^2. \quad (2.50)$$

Using inequality $|a + b|^2 \leq (a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$ we get

$$|e_K^i|^2 \leq 2 [|v_K^i|^2 + \beta^2 |H_K^i|^2] \quad (2.51)$$

Since

$$v_K^i = \int_{(i-1)h}^{ih} v(t) dt, \quad (2.52)$$

it follows that

$$|v_K^i| \leq \int_{(i-1)h}^{ih} v^2(t) dt = \|v\|_2^2 \quad \text{on } [(i-1)h, ih]. \quad (2.53)$$

Then, on $[(i-1)h, ih]$,

$$\|\tilde{e}_K\|_\infty^2 \leq 2 \left[\|v\|_2^2 + \beta^2 \|\tilde{H}_K\|_\infty^2 \right] \quad (2.54)$$

As $v \in L^2(0, T)$ and $\|\tilde{H}_K\|_\infty \leq C_2$ for all K , we have $\|\tilde{e}_K\|_{L^\infty(0, T)}^2 \leq C$

Furthermore inequalities

$$\|\tilde{e}_K\|_{L^2(0, T)}^2 = \sum_{i=1}^K h |e_K^i|^2 \leq \sum_{i=1}^K 2h |v_K^i|^2 + \sum_{i=1}^K 2\beta^2 h |H_K^i|^2 \quad (2.55)$$

$$\leq \sum_{i=1}^K 2h |v_K^i|^2 + \sum_{i=1}^K 2\beta^2 h \|H_k\|_{L^\infty(0, T)}^2 \quad (2.56)$$

give

$$\|\tilde{e}_K\|_{L^2(0, T)}^2 \leq 2\|v_K\|_{L^2(0, T)}^2 + 2\beta^2 \|H_K\|_{L^\infty(0, T)}^2 \leq C_2. \quad (2.57)$$

From Equation (2.45) and Holder inequality

$$\left| \int_0^T \frac{dM_K}{dt} \phi dt \right| \leq \left| \int_0^T \frac{dH_K}{dt} \phi dt \right| + \left| \int_0^T \tilde{e}_K \phi dt \right| \quad (2.58)$$

$$\leq \left\| \frac{dH_K}{dt} \right\|_{L^2[0, T]} \|\phi\|_{L^2[0, T]} + h\gamma \|\tilde{e}_K\|_{L^2[0, T]} \|\phi\|_{L^2[0, T]} \quad (2.59)$$

$$\leq C_3 \|\phi\|_{L^2[0, T]} \quad \forall \phi \in L^2[0, T]. \quad (2.60)$$

From $\mathcal{H}2$, we also have

$$|M_K| \leq \overline{C} + C_p \|H_K\|_{L^\infty(0, T)} \quad (2.61)$$

Hence, $\|M_K\|_{L^\infty(0, T)} \leq C_4$

In conclusion, there are functions H , $\tilde{H}$, and M such that, possibly after selecting subsequences, as $K \rightarrow \infty$,

$$H_K \rightarrow H \text{ weakly star in } H^1(0, T) \cap L^\infty(0, T) \quad (2.62)$$

$$\tilde{H}_K \rightarrow \tilde{H} \text{ weakly star in } L^\infty(0, T) \quad (2.63)$$

$$M_K \rightarrow M \text{ weakly star in } H^1(0, T) \cap L^\infty(0, T) \quad (2.64)$$

$$\tilde{e}_K \rightarrow e \text{ weakly star in } L^2(0, T) \cap L^\infty(0, T). \quad (2.65)$$

From (2.49), we see that $H = \tilde{H}$. Since $\tilde{v}_K \rightarrow v$ in $L^2[0, T]$, it follows that we may pass the limit $K \rightarrow \infty$ in Equations (2.62) - (2.65) to obtain the desired result,

(2.24)-(2.26) provided that the hysteresis equation (2.5) holds. Define on $[0, T]$ the functions

$$z_K = \Gamma[H_K, \psi_{-1}] \quad (2.66)$$

$$z = \Gamma[H, \psi_{-1}] \quad (2.67)$$

From Proposition 2.2, the compactness of the imbedding of $H^1(0, T)$ in $C(0, T)$ yields that $H_K \rightarrow H$, strongly in $L^2(0, T)$; in particular, $H_K \rightarrow H$ in $C(0, T)$. Thus, using the continuity of Γ assumed in $\mathcal{H}2$, we have $z_K \rightarrow z$ in $C(0, T)$.

Consider another set of functions $\bar{H}_K$ with same standard partition as H_K such that $M_K = [\bar{H}_K; \psi_{-1}]$. Here M_K is piecewise linear and $\bar{H}_K$. In Equation (2.66), H_K is piecewise linear and z_K is not. Now from the continuity of Γ , For a given $\epsilon > 0$ $\exists \delta > 0$ such that

$$\|H_k - \bar{H}_K\|_\infty < \delta \quad \Rightarrow \quad \|M_k - z_K\|_\infty < \epsilon \quad (2.68)$$

which shows that $M_K - z_K \rightarrow 0$ strongly in $L^2(0, T)$. Thus we have shown that $M_K \rightarrow M$ strongly in $L^2(0, T)$ which concludes the proof. $\square$

2.3 Stability and uniqueness

To prove stability and uniqueness of the solution to system of equations (2.3)-(2.5), we need the following proposition, the proof of which is found in *Brokate and Sprekels* [1].

Proposition 3. (*Hilpert's Inequality*) Consider the hysteresis operator Γ given by

$$\Gamma[v; w_{-1}](t) = q(\mathcal{F}_r[v; w_{-1}](t)), \quad 0 \leq t_E, \quad (2.69)$$

with $w_{-1} \in \mathbb{R}$, and where $q \in W_{loc}^{1,\infty}(R)$ is an increasing function. Suppose that $v_1, v_2 \in W^{1,1}(0, t_E)$ and $w_{-1,1}, w_{-1,2} \in \mathbb{R}$ are given, and let $v = v_2 - v_1$, $w = w_2 - w_1$, where $w_i = \Gamma[v_i; w_{-1,i}]$, $i = 1, 2$. Then

$$\frac{d}{dt}w_+(t) \leq w'(t)H(v(t)), \quad a.e. \text{ in } (0, t_E), \quad (2.70)$$

where $w+ := \max\{w, 0\}$, $\mathcal{F}_r$ is the play operator with parameter r , and where H denotes the Heaviside function.

The Preisach operators do not satisfy Hilpert's Inequality since they do not have a scalar memory. We define the Preisach memory, namely operators of the form $\Gamma[H; \psi_{-1}]$, using

$$\Gamma[H; \psi_{-1}](t) = Q(\phi(t)) = \int_0^\infty q(r, \phi(t, \nu)) d\nu(r) + W_{00}, \quad (2.71)$$

where ν is a nonnegative finite measure and

$$q(r, s) = 2 \int_0^s \omega(r, s\sigma) d\sigma \quad (2.72)$$

where $\omega \in L^1_{loc}(\mathbb{R}_+ \times \mathbb{R})$ [1]. Integral (2.71) commutes with the time derivative to which we can apply Hilpert's Inequality. Next, we describe the time evolution of Preisach memory curve. Given any initial memory curve ψ_{-1} , we obtain a finite sequence of functions $\psi_i : \mathbb{R}_+ \rightarrow \mathbb{R}$ through the basic memory update

$$\psi_i(t, r) = \mathcal{F}_r[H(\cdot), \psi_{i-1}(r)] \quad (2.73)$$

where $r \geq 0$, $0 \leq i \leq N$ and $\mathcal{F}_r$ is the play operator with parameter r [1].

First we present the general stability result. In addition to $\mathcal{H}1$ and $\mathcal{H}2$, we need the following hypotheses for the hysteresis operator $\Gamma(\cdot)$:

$\mathcal{H}3$: The operator Γ maps $H^1(0, T)$ into itself and $\exists c_1 > 0$, $c_0 \in L^2(0, T)$ such that

$$|\Gamma[H; \psi^{-1}]'(t)| \leq C_0 + C_1 |\dot{H}(t)| \quad \forall t \in [0, T], \quad H \in H^1(0, T). \quad (2.74)$$

$\mathcal{H}4$: $\Gamma[H; \psi^{-1}]$ is a family of operators that is defined as in Equation (2.71), with q given by (2.72). Both measure ν and density ω are non-negative and it holds that

$$\int_0^\infty \sup_{s \in \mathbb{R}} \omega(r, s) d\nu(r) < \infty. \quad (2.75)$$

In addition, the initial condition ψ_{-1} is measurable and satisfies,

$R_{supp} \circ \psi_{-1} \in L^2(0, T)$. Here, $R_{supp}(\psi) := \sup\{r | r \geq 0, \psi(r) \neq 0\}$.

Now we prove stability of solution.

Theorem 2. (*Stability*) Let $\mathcal{H}1$ - $\mathcal{H}2$ hold. Let $H_1(0), H_2(0) \in \mathbb{R}$ and $v_1, v_2 \in L^2(0, T) \cap L^\infty(0, T)$. Suppose that $\Gamma(H, \psi^{-1})$ satisfies $\mathcal{H}3$ - $\mathcal{H}4$. Then any two sets $(H_1, M_1, e_1), (H_2, M_2, e_2)$ of weak solutions of (2.3)-(2.5) satisfy

$$\begin{aligned} |H_2 - H_1|(t) + |M_2 - M_1|(t) &\leq |H_2(0) - H_1(0)| + \int_0^\infty |w_2(r, 0) - w_1(r, 0)| d\nu(r) \\ &\quad + \int_0^t |v_2 - v_1| dt \end{aligned} \quad (2.76)$$

where $w_i(r, 0) = q(r, \mathcal{F}_r[H_i; \psi_{-1}(r)](0))$, $i = 1, 2$

Proof. From Equation (2.3) and the variation form of Equation(2.4),

$$\alpha[\varphi(e_1) - \varphi(e_2)] + \beta[H_1 - H_2] + [e_1 - e_2] = v_1 - v_2 \quad (2.77)$$

$$\int_0^T \frac{dH_1}{dt} \phi(t) dt + \int_0^T \frac{dM_1}{dt} \phi(t) dt = \gamma \int_0^T e_1 \phi(t) dt \quad (2.78)$$

$$\int_0^T \frac{dH_2}{dt} \phi(t) dt + \int_0^T \frac{dM_2}{dt} \phi(t) dt = \gamma \int_0^T e_2 \phi(t) dt, \quad (2.79)$$

where $\phi \in L^2(0, T)$. Set $\hat{H} = H_1 - H_2$, $\hat{M} = M_1 - M_2$, and $\hat{e} = e_1 - e_2$, where $M_i(t) = \Gamma[H_i(t), \psi_{-1,i}]$, $i = 1, 2$. Let $H_\varepsilon : \mathbb{R} \rightarrow \mathbb{R}$ denote the regularized Heaviside function:

$$H_\varepsilon(x) = \begin{cases} 1 & x \geq \varepsilon \\ x/\varepsilon & 0 \leq x \leq \varepsilon \\ 0 & x \leq 0 \end{cases} \quad (2.80)$$

Taking the difference of the variational Equations (2.78) and (2.79) yields

$$\int_0^t \frac{d\hat{H}}{dt} H_\varepsilon(\hat{H}) dt + \int_0^t \frac{d\hat{M}}{dt} H_\varepsilon(\hat{H}) dt = \gamma \int_0^t \hat{e} H_\varepsilon(\hat{H}) dt, \quad (2.81)$$

where the function $\phi = H_\varepsilon \circ \hat{H}\chi_{(0,t)} = H_\varepsilon(H_1 - H_2)\chi_{(0,t)}$, and where $\chi(0, t)$ is the characteristic function over $[0, t]$. Applying Hilpert's Inequality to the first integrand in (2.81) we find that

$$\int_0^t \frac{d\hat{H}}{dt} H_\varepsilon(\hat{H}) dt \geq \int_0^t \frac{d\hat{H}_+}{dt} dt = \hat{H}_+(t) - \hat{H}_+(0). \quad (2.82)$$

Also from Hilpert's Inequality, we get

$$\begin{aligned} \frac{d\hat{M}}{dt} H_\varepsilon(\hat{H}) dt &= \int_0^\infty \frac{d}{dt} [q(r, \mathcal{F}_r[H_2; \psi_{-1,2}](t)) - q(r, \mathcal{F}_r[H_1; \psi_{-1,1}](t))] H_\varepsilon(\hat{H}) d\nu \\ &\geq \int_0^\infty \frac{d}{dt} [q(r, \mathcal{F}_r[H_2; \psi_{-1,2}](t)) - q(r, \mathcal{F}_r[H_1; \psi_{-1,1}](t))]_+ d\nu. \end{aligned} \quad (2.83)$$

Thus, setting $w_i(r, t) = q(r, \mathcal{F}_r[H_i; \psi_{-1,i(r)}](t))$, $i = 1, 2$, we estimate the second integral in (2.81):

$$\begin{aligned} \int_0^t \frac{d\hat{M}}{dt} H_\varepsilon(\hat{H}) dt &\geq \int_0^\infty (w_1(r, t) - w_2(r, t))_+ d\nu(r) \\ &\quad - \int_0^\infty (w_1(r, 0) - w_2(r, 0))_+ d\nu(r). \end{aligned} \quad (2.84)$$

Since

$$|\hat{M}|(t) \leq \int_0^\infty |w_1(r, t) - w_2(r, t)| d\nu(r), \quad (2.85)$$

we have

$$\begin{aligned} \hat{M}_+ &= \frac{1}{2} [\hat{M} + |\hat{M}|] \\ &\leq \frac{1}{2} \int_0^\infty (w_1 - w_2) + \frac{1}{2} \left| \int_0^\infty (w_1 + w_2) \right| \\ &\leq \frac{1}{2} \int_0^\infty [(w_1 - w_2) + |w_1 - w_2|] \\ &= \frac{1}{2} \int_0^\infty (w_1 - w_2)_+ d\nu(r). \end{aligned} \quad (2.86)$$

we use Equations (2.81)- (2.86) to obtain

$$\begin{aligned} \hat{H}_+(t) - \hat{H}_+(0) + \hat{M}_+(t) \\ - \int_0^\infty (w_1(r, 0) - w_2(r, 0))_+ d\nu(r) &\leq \int_0^t \frac{d\hat{H}}{dt} H_\varepsilon(\hat{H}) dt + \int_0^t \frac{d\hat{M}}{dt} H_\varepsilon(\hat{H}) dt \\ &= \gamma \int_0^t \hat{e} H_\varepsilon(\hat{H}) dt. \end{aligned} \quad (2.87)$$

Since $\hat{H}H_\varepsilon(\hat{H}) \geq 0$ and $\hat{e}$ and $\varphi(e_1) - \varphi(e_2)$ have same signs, Equation (2.77) implies

$$\gamma \int_0^t \hat{e}H_\varepsilon(\hat{H})dt \leq \gamma \int_0^t (v_1 - v_2)H_\varepsilon(\hat{H})dt \quad (2.88)$$

Then from Equation (2.87)

$$\begin{aligned} (H_1 - H_2)_+(t) + (M_1 - M_2)_+(t) &\leq (H_1(0) - H_2(0))_+ \\ &\quad + \int_0^\infty (w_1(r, 0) - w_2(r, 0))_+ d\nu(r) \\ &\quad + \int_0^t \gamma(v_1 - v_2)H_\varepsilon(\hat{H})dt \end{aligned} \quad (2.89)$$

By switching indices 1 and 2 in Equation (2.89) and adding the result to the Equation (2.89) we obtain

$$\begin{aligned} |H_2 - H_1|(t) + |M_2 - M_1|(t) &\leq |H_2(0) - H_1(0)| \\ &\quad + \int_0^\infty |w_2(r, 0) - w_1(r, 0)| d\nu(r) \\ &\quad + \int_0^t |v_2 - v_1| dt \end{aligned} \quad (2.90)$$

□

Now that we have stability, we desire that the solution be unique for a given input voltage. We use the Stability Theorem to prove uniqueness of (e, H, M)

Theorem 3. (*Uniqueness*) *Under the same assumptions as the Stability Theorem, Theorem 2, the weak solution of system of equations (2.3)-(2.5) is unique*

Proof. Since the initial conditions are same, $H_2(0) - H_1(0) = 0$ and $w_2(r, 0) - w_1(r, 0) = 0$. Also the input voltage is the same, thus the Stability Theorem gives,

$$|H_2 - H_1| + |M_2 - M_1| \leq 0. \quad (2.91)$$

Since, both terms on the left hand side of Equation (2.91) are nonnegative, the two terms must be zero. Hence, $H_2 = H_1$, and $M_2 = M_1$. Then from Equation (2.77) $e_1 = e_2$ □

CHAPTER III

EFFECTS OF WINDING LOSSES

In this chapter we first investigate the behavior of the induced emf for sinusoidal input voltages. Then we discuss the accuracy of the fundamental model discussed in Chapter II.

As we will show in Chapter IV, the core losses can be expressed as the sum of exponential functions of induced back emf. Thus the function $\varphi(e)$ satisfactorily models the behavior of the core losses. To this end, a set of experiments has been carried out to observe the effectiveness of such a constant resistance over the frequency range 0 to 1000 Hz.

First we state that, for a given periodic input voltage waveform, the induced emf $e(t)$, the magnetization $M(t)$, the average magnetic field $H(t)$, and the flux density $B(t)$ are all periodic functions. Under hypothesis $\mathcal{H}1$ - $\mathcal{H}4$, both $\varphi(e)$ and $\Gamma[H ; \psi_{-1}]$ are continuous monotone increasing function. Also We have that they are bounded.

Theorem 4. *Let v be a bounded, continuous periodic function on $[0, \infty)$ with period T_p . Assume $\mathcal{H}1$ to $\mathcal{H}4$ Then e , H , and M each asymptotically converge to a continuous periodic function with period T_p .*

Next we prove that for a given e_{rms}/f constant curve, the RMS value of the input current is a monotone increasing function of frequency f . The equation for the winding resistance is

$$v(t) = i(t) R + e(t) \quad \forall t. \quad (3.1)$$

We know that $v(\cdot)$ is a periodic function with period f and, from Theorem 4, $i(\cdot)$ and $e(\cdot)$ are almost periodic functions. So, asymptotically (or for t large enough), we can take inner products on the interval $[0, \frac{2\pi}{f}]$ and get the RMS values of i :

$$i_{rms}^2 = \frac{f}{2\pi} \langle i, i \rangle, \quad (3.2)$$

Using the transform $\tau = f t$, we convert the inner-products to the interval $[0, 2\pi]$ as

$$i_{rms}^2 = \frac{1}{2\pi} \langle i, i \rangle_\tau, \quad (3.3)$$

From Equation (3.1), if we take inner products, we obtain

$$\langle v, v \rangle_\tau = R \langle i, v \rangle_\tau + \langle e, v \rangle_\tau. \quad (3.4)$$

Note that $\langle i, v \rangle_\tau \geq 0$ since it is the power delivered by the power supply. Therefore,

$$\langle v, v \rangle_\tau \geq \langle e, v \rangle_\tau. \quad (3.5)$$

Next, take the inner product of v with e :

$$\langle v, e \rangle_\tau = R \langle i, e \rangle_\tau + \langle e, e \rangle_\tau. \quad (3.6)$$

Now, $\langle i, e \rangle_\tau \geq 0$ since it is the power delivered to the output of the 2-port network (in this case, the magnetostrictive actuator). Hence,

$$\langle v, e \rangle_\tau \geq \langle e, e \rangle_\tau. \quad (3.7)$$

Combining (3.5) and (3.7),

$$\langle v, v \rangle_\tau \geq \langle e, e \rangle_\tau. \quad (3.8)$$

This shows that $v_{rms} \geq e_{rms}$. If $e_{rms} = C f$ for some $C > 0$ then $v_{rms} \geq C f$. Since $V(\cdot)$ is the voltage supply, we can pick its amplitude to change with frequency (otherwise, we cannot stay on the $e_{rms} = C f$ curve). Suppose we choose the amplitude of $v(\cdot)$ so that $v_{rms} = C_1 f$ where $C_1 \geq C$. Using Cauchy-Schwartz Theorem on $\langle v, e \rangle_\tau$, we obtain

$$\langle v, e \rangle_\tau \leq \langle v, v \rangle_\tau^{1/2} \langle e, e \rangle_\tau^{1/2}.$$

Next, take the inner product of Equation (3.1) asymptotically:

$$R^2 \langle i, i \rangle_\tau = \langle v - e, v - e \rangle \quad (3.9)$$

$$= \langle v, v \rangle - 2\langle v, e \rangle + \langle e, e \rangle \quad (3.10)$$

$$\geq \langle v, v \rangle - 2\langle v, v \rangle^{1/2} \langle e, e \rangle^{1/2} + \langle e, e \rangle. \quad (3.11)$$

Hence,

$$R^2 i_{rms}^2 \geq v_{rms}^2 - 2v_{rms}e_{rms} + e_{rms}^2 \quad (3.12)$$

$$= (v_{rms} - e_{rms})^2, \quad (3.13)$$

which implies

$$R i_{rms} \geq v_{rms} - e_{rms}, \quad (3.14)$$

i.e., $i_{rms} \geq \frac{(c_i - c)f}{R}$ for all R . This shows that I_{rms} is a monotone increasing function with respect to f .

3.1 Parameter measurement

To experimentally observe the monotone increasing behavior of i_{rms} for a given e_{rms}/f constant curve, we must measure the resistance R and the e_{rms} . However, $e(t)$ is not directly obtainable. In this section we identify an accurate and feasible method to measure both R and e_{rms} .

3.1.1 Measuring R

For a constant DC current we have $i_m = \text{const.}$ In this case, H remains constant, which implies that $\dot{B} = 0$. Therefore, the induced back emf $e = N A \dot{B} = 0$; thus $\varphi(e) = 0$. The fundamental circuit described in Chapter II reduces to the circuit shown in Figure (3.1).

Figure 3.1: DC model of the fundamental circuit

By applying controlled DC current in the range 0-1A with step size 0.1, we measure the input voltage across the terminal and obtain the graph in Figure (3.2). The approximate gradient of the curve is $R = \frac{\Delta V}{\Delta i} = 4.2\Omega$.

Figure 3.2: DC Load Curve

3.1.2 Measuring RMS values of $e(t)$ and $i(t)$

Consider the circuit in Figure (3.3) with an external circuit connected to the actuator, where R_1 is the potential divider resistance.

Figure 3.3: Circuit for measuring induced emf $e(t)$

For this circuit,

$$v_1 = 2Ri + e \quad (3.15)$$

$$v = Ri + e. \quad (3.16)$$

Therefore, $e = 2v - v_1$. Also, $\tilde{v} = v - \frac{v_1}{2} = \frac{2v - v_1}{2} = \frac{e}{2}$. Hence,

$$|e|_{rms} = \sqrt{\frac{1}{T} \int_0^T e^2 dt} \quad (3.17)$$

$$= \sqrt{\frac{1}{T} \int_0^T 4\tilde{v}^2 dt} \quad (3.18)$$

$$= 2\tilde{v}_{rms} \quad (3.19)$$

Thus we only need to measure the RMS value of $\tilde{v}$, which is directly obtainable. Furthermore, by measuring the RMS voltage across the external resistor R , we can obtain the RMS current.

3.2 Experimental observations

To empirically verify the effectiveness of the fundamental circuit in Figure 2.1, we carried out the following experiments. We applied a sinusoidal voltage to a Terfenol-D actuator, manufactured by Etrema Products, Inc. By varying the applied voltage we varied the magnetic field in the rod and thus changed the electro-magnetic losses in the actuator. We used an AA-050H actuator which has current rating 1.4 A and frequency range 0-3000 Hz. Applying a set of sinusoidal waveforms from 0 to 1000 Hz, experimental RMS values for $\tilde{V}$ and I were obtained. We calculated the induced emf by Equation (3.17), and obtained the surface shown in Figure 3.4.

Figure 3.4: Surface generated by constant e_{rms}/f curves

Figure 3.4 illustrates the relationship between induced emf, applied frequency and input current for constant e_{rms}/f curves from 0.001 to 0.055 V/Hz. We have already shown in this chapter that the input current surface is a monotone increasing function with respect to emf. However, it is apparent in Figure 3.4 that the input current undergoes a significant drop at high frequencies. Figure 3.5 shows three constant e_{rms}/f curves, where the input current drop is large after 800 Hz and where values of e_{rms}/f are large.

Figure 3.5: Frequency vs input current variation for constant e_{rms}/f curves

In Figures 3.4 and 3.5, we notice an unpredicted phenomenon: the input current drops at increased frequencies. This contradicts our proven claim, that i_{rms} is a monotone increasing function of f . Hence, there must be an error in the fundamental circuit diagram. Since our only assumption was that lead resistance is constant, we conclude that, for frequencies greater than 800 Hz, lead resistance must be a function of other parameters. As Ohmic losses of the winding increase with frequency due to the skin and proximity effect, such an explanation is justifiable.

CHAPTER IV

WIDE BAND MODEL

In this chapter we look at the representation of the excessive and eddy current losses in the core and the Ohmic losses in the winding. An array of current sinks with n degrees of freedom is employed to represent core losses. For low frequencies, we model the winding losses with a constant resistance. For high frequencies, we incorporate any changes in winding resistance in a transfer function. Also, we use the Preisach operator to represent the hysteresis, which includes the magnetic hysteresis losses of the core. As we later show, it is sufficient to let two elements represent core losses for low frequency models. Furthermore, for low frequencies, the eddy currents losses in windings can be combined with the eddy current losses in the core. However, as the bandwidth of the model increases, a higher number of current sinks are required. Additionally, losses due to skin effect, proximity effect and stray capacitance effects significantly increase at high frequencies. Although these losses are not just functions of emf $e(t)$, it is sufficient to represent the effect of these in a transfer function and an array of current sinks.

4.1 Model for frequencies less than 500Hz

In the previous chapter, we described a model with a constant series resistor and a parallel current sink which has monotone increasing current with respect to induced emf voltage e . Also we observed some predictable experimental behavior for such a model with frequencies less than 800Hz. Here we demonstrate that such a model is indeed enough to express the behavior of the actuator for low frequencies. Furthermore, we identify the functional representation for current $\varphi(e)$.

We start by finding the eddy current loss of the windings. This loss causes skin and proximity losses. As a result, the effective winding loss increases beyond the DC losses, contrary to the assumption that a constant series resistor represents winding losses. For the purpose of simplifying calculations, we assume that the field remains

constant inside the conductor, which is equivalent to assuming that the skin depth is significantly large compared to the wire diameter. This restriction suffices for frequencies less than 500Hz. Sullivan proves that these losses are approximately proportional to $\dot{B}^2$ [3]. Consider a single loop of the actuator coil. Figure 4.1(a) shows that flux is perpendicular to the loop, allowing us to regard the loop as a straight cylinder with the field parallel to the cross-section. Now consider an eddy current loop of a cylindrical wire distance x away from the symmetric line and with a uniform current density over thickness dx . See Figure 4.1(b).

Figure 4.1: Uniform field across a cylindrical conductor (a) Single loop of actuator coil with perpendicular flux (b) Cross-sectional view of coil

The induced emf due to uniform flux density is given by

$$\frac{d\phi}{dt} = 2xl \frac{dB_i}{dt}, \quad (4.1)$$

where l is the length of the wire. The resistance of the cross-sectional area with width dx shown in Figure 4.1(b) is

$$R = \frac{2l\rho_c}{2\sqrt{\frac{d^2}{4} - x^2}dx}. \quad (4.2)$$

Then total power dissipation over the cross section is given by

$$P(t) = \int_0^{d/2} \left(2xl \frac{dB_i}{dt} \right)^2 \frac{\sqrt{d^2/4 - xx^2}}{l\rho_c} dx \quad (4.3)$$

$$= \frac{\pi l d_c^4}{64\rho_c} \left(\frac{dB_i}{dt} \right)^2 = \bar{k} \left(\frac{dB_i}{dt} \right)^2. \quad (4.4)$$

The total loss due to the eddy current loops in all the conductors of a winding is the spacial average of the above power losses, and is given by

$$P_w(t) = \sum_{i=1}^n \bar{k} \left(\frac{dB_i}{dt} \right)^2 = \bar{k} N \left\langle \left(\frac{dB_i}{dt} \right)^2 \right\rangle. \quad (4.5)$$

Even though flux density in the core is not equal to the spacial average of the flux density, if we assume that they are proportional to each other, we find that $P_w(t)$ is proportional to $\dot{B}^2$. As we mentioned in Chapter II, the induced emf, e , in Figure 2.1 is proportional to $\dot{B}$. Therefore, eddy current losses in the winding can be written $P_w(t) = ke^2$, where k is a constant. Since $P_w(t)$ is monotone increasing with respect to $e(t)$, we can incorporate this loss in the function $\varphi(e)$. The remaining loss in the winding is the DC loss which can be represented by a constant resistor in series with the winding.

4.2 Modelling core losses

In this section we discuss the representation of excessive and eddy current losses in the core. We use the statistical theory of losses developed by Bertotti [5], which has been empirically proven by Fiorillo and Novikov [4]. Classical eddy current loss ($P_c(t)$) and excessive loss ($P_e(t)$) in the time domain can be expressed as

$$P_c(t) = \frac{\sigma k}{12} \left(\frac{dB}{dt} \right)^2 \quad (4.6)$$

and

$$P_e(t) = \sqrt{\sigma G V_0 S} \left| \frac{dB}{dt} \right|^{1.5}, \quad (4.7)$$

with σ being the material conductivity, G a known dimensionless coefficient, V_0 a physical parameter related to the strength and distribution of internal coercive fields,

S the cross-sectional area, and k an empirical parameter which depends on the dimension of the core. Since the induced emf e across the nonlinear inductor is $NA \frac{dB}{dt}$, we find the losses in terms of e as

$$P_c(t) = K_c \frac{e^2}{(NA)^2} \quad (4.8)$$

$$P_e(t) = K_e \frac{e^{1.5}}{(NA)^{1.5}}, \quad (4.9)$$

where $K_c = \sigma k/12$ and $K_e = \sqrt{\sigma G V_0 S}$. We rearrange $P_c(t)$ and $P_e(t)$:

$$P_c(t) = e(k_1 \text{sign}(e)|e|), \quad (4.10)$$

$$P_e(t) = e(k_2 \text{sign}(e)|e|^{0.5}), \quad (4.11)$$

where $k_1 = K_c/(NA)^2$ and $k_2 = K_e/(NA)^{1.5}$. If we define $i_1(e)$ and $i_2(e)$ as

$$i_1(e) = k_1 \text{sign}(e)|e| \quad \text{and} \quad (4.12)$$

$$i_2(e) = k_2 \text{sign}(e)|e|^{0.5}, \quad (4.13)$$

then we can represent $\varphi(e)$ as

$$\varphi(e) = i_1 + i_2 = k_1 \text{sign}(e)|e| + k_2 \text{sign}(e)|e|^{0.5}, \quad (4.14)$$

Therefore, We can decompose the function $\varphi(e)$ into two current sinks opposing $e(t)$: i_1 and i_2 . Hence the model in Figure 4.2 sufficiently describes the low frequency behavior. This model is identical to the fundamental circuit diagram discussed in Chapter II.

Figure 4.2: Model for frequencies <500 Hz

4.3 Improving the representation of $\varphi(e)$

In the previous section we identified a sufficient representation for core losses for frequencies less than 500Hz. We also saw that we can include the eddy current losses of the winding in the function $\varphi(e)$. However, we will see in Chapter V that it is not sufficient to represent $\varphi(e)$ as two current sinks for frequencies in the range 500-1000Hz. Here we improve the functional representation of $\varphi(e)$ by adding a set of functions $\sigma_i(e)$ such that $\sigma_i(0) = 0$:

$$\varphi(e) = i_1 + i_2 + \sum_{j=3}^n \sigma_j(e) \quad (4.15)$$

where i_1 and i_2 are defined in Section 4.1.1. For frequency region 500Hz-1000Hz, we will empirically justify that it is sufficient to express each $\sigma_j(e)$ as an exponential of the back emf $e(t)$. By naming $\sigma_j(e) = k_j i_j$, we get a current sink representation similar to that of eddy and excessive loss representations i_1 and i_2 . Thus, we will identify an array of current sinks with n -degrees of freedom to represent all the losses except the resistance loss and hysteresis loss. Hence, we obtain the circuit shown in Figure 4.3, which models the behavior of the actuator up to 1000Hz.

Figure 4.3: Equivalent circuit for the core and winding losses

Figure 4.3 illustrates the combined equivalent circuit diagram for the winding and core losses for frequencies less than 1000Hz, where i_1 represents the excessive loss and i_2 represents the eddy current losses in the core. The sinking current of the j^{th} element is given by $i_j = k_j \text{sign}(e)|e|^{\alpha_j}$, with constraint $0.5 = \alpha_1 < \alpha_2 < \dots < \alpha_n = 1$. Also, the nonlinear inductor corresponds to hysteresis loss accommodation. For brevity, we refer to the coefficients k_j , $j = 3, \dots, n$ as refinement parameters of $\varphi(e)$.

4.4 Model for high frequencies

We have seen that, after 800Hz, the constant resistor representation fails to model the actuator behavior. At high frequencies, the assumption that skin depth is significantly large compared to the wire dimension is no longer valid. Furthermore, there are several other complicating factors at high frequencies. Skin and proximity effects significantly increase, which cause the winding resistance to increase approximately proportionally to $\sqrt{f}$. Parasitic capacitance of the winding can no longer be neglected since they significantly affect the inductor behavior and are responsible for resonant frequencies. Stray capacitance across the terminals creates undesired effects, as does the behavior of the power supply for large inductive loads at high frequencies. Consequently, it is become tough to identify a model which represents all the behaviors of the actuator at high frequencies. However, if we use appropriate shielding, the current through all the stray capacitors reduces to a value negligible compared to the magnitude of the applied current. Since the proximity and skin effects work against the input current, their representation is a variable resistor in series with the model. Also, the undesirable behavior of the power supply appears before the input current measurement, thus the measured current gives an accurate measurement of the current which reaches the core. Hence, we can describe all the effects as two black boxes which appear before the fundamental circuit that we already developed for low frequencies. (See Figure 4.4).

Figure 4.4: Representation of additional losses

Although the current reaching the fundamental circuit is the measured current, the voltage across the fundamental circuit is not the applied voltage. In Figure 4.4, function $V_e(t)$ represents effective voltage across the low frequency model and V_{in} represents the input voltage. Thus, modelling high frequencies boils down to identifying the transfer function from applied voltage, V_{in} , to effective voltage, V_e . In Chapter II, we proved the existence, uniqueness and stability for the low frequency model, given that $V_e \in L^2(0, T)$. For the existence, uniqueness and stability of the system of equations representing the modified circuit found in Figure 4.4, it remains only to prove that, for a given $V_{in}(t) \in L^2[0, t]$, V_e remains in $L^2[0, t]$. First we introduce parameter $\tilde{R}$ to represent the additional losses describe by the series element in Figure 4.4. We use the assumption that, for a given frequency, the change in $\tilde{R}$ is sufficiently small. The KVL applied to the circuit shown in Figure 4.4 gives

$$V_{in} = i\tilde{R} + V_e, \quad (4.16)$$

and hence

$$\int_0^T V_{in}^2 dt = \int_0^T \tilde{R} V_{in} i dt + \int_0^T V_{in} V_e dt. \quad (4.17)$$

Also,

$$\int_0^T V_{in} V_e dt = \int_0^T \tilde{R} V_e i dt + \int_0^T V_e^2 dt. \quad (4.18)$$

Combining Equations (4.17) and (4.18) we have

$$\int_0^T V_{in}^2 dt = \int_0^T \tilde{R} [V_{in} i + V_e i] dt + \int_0^T V_e^2 dt \quad (4.19)$$

$$\int_0^T V_{in}^2 dt \approx \tilde{R} \int_0^T [V_{in} i + V_e i] dt + \int_0^T V_e^2 dt \quad (4.20)$$

Integral $\int_0^T V_{in} i dt$ is the input power and $\int_0^T V_e i dt$ is the power delivered to the circuit. Thus both quantities are positive. It follows that

$$\int_0^T V_{in}^2 dt \geq \int_0^T V_e^2 dt. \quad (4.21)$$

Hence, $V_e \in L^2(0, T)$, proving existence, uniqueness and stability.

4.5 Block diagram

The discussion thus far can be summarized in the block diagram in Figure 4.5.

Figure 4.5: Block Diagram of the Model

In the figure, V_{in} , v_e , and e stand for the input voltage, the effective voltage seen by the model, and the induced emf of the actuator winding, respectively. The first block represents the transfer function described in the previous section. The next block stands for Ohm's Law, where R is the DC lead resistance and i is the projected input current. Block G_1 corresponds to all the core losses of the winding. A detailed description of block G_1 is found in Figure (4.6).

Figure 4.6: Expanded Block Diagram of the Block G_1

The magnetization current, i_m , is proportional to the average axial magnetic field of the coil, H , and is in the form $H = ci_m$. The block containing C characterizes this relationship. The classical Preisach operator is employed to represent the rate-independent hysteresis and is given by Γ in Figure (4.5). The magnetization of the

actuator, M , is related to the magnetic field, H , by $M(\cdot) = \Gamma[H(\cdot), \psi_0]$, where ψ_0 is the initial memory curve. Induced emf, e , is proportional to the rate of change of the magnetic flux density. Using the equation $B = \mu_0(H + M)$, we can express the rate of change of magnetic flux density as the sum of the rate of change of the magnetic field, $\dot{H}$, and the rate of change of the magnetization, $\dot{M}$. The term bM^2 describes the induced mechanical force from the magnetization, while F_{ext} is the external force. The combined mechanical forces applied on the linear system G_4 give displacement y . Thus G_4 describes the magneto-elastic dynamics of the model.

CHAPTER V

PARAMETER IDENTIFICATION

As we discussed in Chapter III, it is straightforward to identify the DC lead resistance by applying a controlled DC voltage and measuring the input current. The remainder of the parameter identification can be divided into three classes: identification of the fundamental core loss parameters k_1 and k_2 , identification of the refinement parameters k_3 through k_n , and identification of the transfer function discussed in Chapter IV.

5.1 Core-loss parameter identification

We first identify the parameters $\{k_j\}_{j=1}^n$. Parameters k_1 and k_2 are identified by applying sinusoidal input voltages with frequencies less than 500Hz. Then, keeping the frequencies between 500Hz and 1000Hz, we obtain the refinement parameters of $\varphi(e)$. To obtain an appropriate method to estimate the current through the inductor, i_m , we need to prove that e , H , and M are continuous functions.

Lemma 2. *Let $\mathcal{H}2$ - $\mathcal{H}4$ hold. Suppose the model in Figure (4.3) holds. Let $V \in C(0, T) \cap L^2(0, T)$ and $i_j = k_j \text{sign}(e)|e|^{\alpha_j}$, with $0 < \alpha_j \leq 1$.*

Then $(e, H, M) \in C(0, T) \cap L^2(0, T) \times C(0, T) \cap H^1(0, T) \times C(0, T) \cap H^1(0, T)$

Proof. Since $H \in H^1(0, T) \subset W^{1,1}(0, T)$, where $W^{1,1}(0, T)$ is the space of absolute continuous functions, H is continuous. From KVL and KCL,

$$v = e + R \sum_{j=1}^n k_j \text{sign}(e)|e|^{\alpha_j} + cRH. \quad (5.1)$$

Then since $v, H \in C(0, T)$ we have $g(e) = e + R \sum_{j=1}^n k_j \text{sign}(e)|e|^{\alpha_j} \in C(0, T)$. Since $g(e)$ is a strictly monotone increasing continuous function on $(0, T)$, g^{-1} is continuous. Thus, $e \in C(0, T)$. □

As we showed in Chapter III, for frequencies up to 500 Hz, two current sinks and a DC resistor is enough to model the actuator. (Refer to Figure 4.2.) To identify the

unknown parameters in the model, we must obtain the value of the magnetization current, i_m . However, estimating i_m is a challenge due to hysteresis losses. As a result, the balance of the internal energy becomes crucial for our analysis. This gives rise to additional mathematical complications, since hysteresis loss is highly nonlinear. To simplify our analysis we desire to fix the hysteresis loss. The following lemma and theorem suggest an appropriate method for fixing the hysteresis curve.

Lemma 3. *Let $\mathcal{H}2$ - $\mathcal{H}4$ hold. Let $\{V_i(t)\}_{i=1}^n$ be a sequence of bounded continuous periodic input voltages on $[0, \infty)$ with frequencies f_i , $i \in \{1, N\}$ and let $\{e_i(t)\}_{i=1}^n$ be the corresponding induced voltages across the inductor. In the asymptotical sense, if $\frac{|e_i|_{avg}}{f_i} = k$ for some constant k , and for any interval $(a_i, a_i + 1/f_i)$ containing only one zero of $e_i(t)$, where $e_i(a_i) = 0$, then $B_{i(max)} - B_{i(min)} = \hat{k}$ for some constant $\hat{k}$.*

Proof. Equation (2.4) gives

$$\dot{B} = \mu_0(\dot{H} + \dot{M}) = \mu_0\gamma e. \quad (5.2)$$

Therefore, each zero of $e_i(t)$ corresponds to a local extremum of $B_i(t)$. Without loss of generality assume $B_i(a_i) = B_{i(min)}$. From Lemma 2 and Theorem 4, if the input voltage is periodic and continuous, then $B_i(t)$ is periodic and continuous. Hence

$$B_i(a_i + 1/f_i) - B_i(a_i) = \int_{a_i}^{a_i + \frac{1}{f_i}} \mu_0\gamma e_i dt = 0 \quad \forall a \in \mathbb{R}. \quad (5.3)$$

Then

$$\int_a^{a + \frac{1}{f_i}} e_i(t) dt = \int_a^{a + \frac{1}{f_i}} e_i^+ dt - \int_a^{a + \frac{1}{f_i}} e_i^- dt = 0. \quad (5.4)$$

Relation $\frac{|e_i|_{avg}}{f_i} = k$ gives

$$\frac{|e_i|_{avg}}{f_i} = \int_a^{a + \frac{1}{f_i}} |e_i| dt = \int_a^{a + \frac{1}{f_i}} e_i^+ dt + \int_a^{a + \frac{1}{f_i}} e_i^- dt = k. \quad (5.5)$$

From Equations (5.4) and (5.5), we have

$$\int_a^{a + \frac{1}{f_i}} e_i^+ dt = \int_a^{a + \frac{1}{f_i}} e_i^- dt = \frac{k}{2}. \quad (5.6)$$

Let the corresponding zero of $e_i(t)$ in $(a, a + 1/f_i)$ be τ_i such that $B(\tau_i) = B_{i(max)}$. Then

$$B_{i(max)} = \int_a^{\tau_i} \mu_0 \gamma e_i dt + B_{i(min)} \quad (5.7)$$

Since $e_i(t)$ is periodic, $e(a + 1/f_i) = 0$ and, with the conditions in the lemma,

$$\int_a^{\tau_i} e_i dt = \int_a^{a+1/f_i} e_i^+ dt \quad \text{and} \quad (5.8)$$

$$- \int_{\tau_i}^{a+1/f_i} e_i dt = \int_a^{a+1/f_i} e_i^- dt. \quad (5.9)$$

Then from Equations (5.7) and (5.8)

$$B_{i(max)} - B_{i(min)} = \frac{\mu_0 \gamma k}{2} = \hat{k}. \quad (5.10)$$

□

Theorem 5. *Let $\mathcal{H}2$ - $\mathcal{H}4$ hold. Suppose the model in Figure (4.3) holds. Let $\{V_i(t)\}_{i=1}^N$ be sinusoidal input voltages with frequencies f_i , $i \in \{1, \dots, n\}$, and with corresponding induced voltages $\{e_i\}_{i=1}^N$. If initial memory curves ψ_0 are the same and $\frac{|e_i|_{avg}}{f_i} = k$ for all i , then the hysteresis curves are the same.*

Proof. From Theorem 3 we have that e_i , H_i , and M_i are periodic with period $1/f_i$. We will show that for any period $[\tau, \tau + 1/f_i]$ such that $e(\tau) \neq 0$, $e(t)$ has exactly two zeros:

From Equation (5.1) with $e(t) = 0$

$$H = \frac{v}{cR}. \quad (5.11)$$

Also for $e(t) = 0$, Equation (2.4) gives

$$\dot{H} + \dot{M} = 0. \quad (5.12)$$

Since M is monotonically increasing with H , Equation (5.12) implies that $\dot{H} = \dot{M} = 0$. Thus the zeros of $e(t)$ occur at the extrema of $H(t)$. Therefore, all the local maxima and minima lie on the line given by Equation (5.11). From Lemma 2, H is continuous,

and hence each maximum of H must follow a minimum of H . Furthermore, $H \circ v$ is continuous. Since $v(t)$ is periodic, $H(t)$ is also periodic. A sinusoidal waveform can only achieve a given value only twice per cycle. Hence, for $t \in [\tau, \tau + 1/f_i]$, the trace of H in H - V plane is a simple close curve which crosses the line $H = v/cR$ at exactly two points. Thus we have two extrema of $H(t)$ in one cycle. Hence $e(t)$ has exactly two zeros. Now $e(t)$ satisfies the conditions in Lemma 3, yielding $B_{i(max)} - B_{i(min)} = k$ for some constant k . Since the initial memory curves are same we have the same hysteresis loop. $\square$

5.1.1 Identifying k_1 and k_2

Theorem 4 gives us a method to fix the hysteresis losses. Under the condition that hysteresis power-loss per cycle is constant, we can find the average power delivered to the circuit by

$$\int_a^{a+T} v i dt = \int_a^{a+T} i_1 e dt + \int_a^{a+T} i_2 e dt + \int_a^{a+T} i^2 R dt + C_0, \quad (5.13)$$

where T is the period and C_0 is the hysteresis power loss. Then

$$\int_a^{a+T} i(v - iR) dt = k_1 \int_a^{a+T} |e|^2 dt + k_2 \int_a^{a+T} |e|^{1.5} dt + C_0, \quad (5.14)$$

i.e.,

$$\int_a^{a+T} e i dt = k_1 \int_a^{a+T} |e|^2 dt + k_2 \int_a^{a+T} |e|^{1.5} dt + C_0 \quad (5.15)$$

$$= k_1 T e_{rms}^2 + k_2 \int_a^{a+T} |e|^{1.5} dt + C_0. \quad (5.16)$$

As shown in Chapter III, e is directly available for measurements. Since i can also be measured, the number of unknowns in the above equation is three. Thus, we can use three different settings with fixed hysteresis power loss to identify k_1 and k_2 .

5.1.1.1 Measuring absolute average value of emf

Consider, Figure (3.3) where $|e|_{avg} = \int_a^{a+\frac{1}{T}} |e| dt = \frac{1}{T} \int_a^{a+T} |2\tilde{v}| dt = 2|\tilde{v}|_{avg}$. The most popular voltage and current transducers and meters actually measure the average absolute value of the voltage or current and are calibrated for the RMS value

of sine wave inputs. The following analysis shows how to find the calibration factor. Consider

$$(f(x))_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} \sin^2 x dx} = \frac{\sqrt{2}}{2} \quad (5.17)$$

$$|f(x)|_{avg} = \frac{1}{2\pi} \int_0^{2\pi} (\sin x) dx = \frac{2}{\pi}. \quad (5.18)$$

The the calibration factor is $= \frac{(f(x))_{rms}}{|f(x)|_{avg}} = \frac{\frac{\sqrt{2}}{2}}{\frac{2}{\pi}} = \frac{\pi}{2\sqrt{2}}$. Using the reading of a moving coil meter or an OSI model voltage transducer, $|e|_{avg}$ can be found by the following equation:

$$|e|_{avg} = 2 \times (\text{meter reading of } \tilde{v}) \times \frac{2\sqrt{2}}{\pi}. \quad (5.19)$$

5.1.1.2 Procedure and experimental results

We first apply a sinusoidal input voltage with 500Hz frequency and increase the input voltage such that i_{rms} reaches 1A. Then we obtain the $|e|_{avg}/f$ value, the e_{rms} value, and sample data for $e(t)$ and $i(t)$. We simultaneously decrease input voltage $v(t)$ and frequency f to achieve the same $|e|_{avg}/f$ value for different frequencies. Note that it is important to decrease $v(t)$ first and f second so that the largest hysteresis loop we obtain is the loop corresponding to the constant $|e|_{avg}/f$ value. In this manner, we fix the initial memory curve. For the new setting we obtain the e_{rms} value and the sample data of $e(t)$ and $i(t)$. We obtain a third set of sample data likewise. According to Equation (5.16), we need the values of

$$\int_a^{a+T} e i dt \quad \text{and} \quad \int_a^{a+T} |e|^{1.5} dt$$

for a periodic cycle. Since e and i are continuous functions, we can approximate the integral by Riemann sums, which converge with order $\omega(T/N)$ where N is the number of steps and ω is the modulus of continuity. Thus we have

$$\int_0^t e i dt \approx \sum_{k=1}^N h e_k i_k \quad \text{and} \quad \int_0^t |e|^{1.5} dt \approx \sum_{k=1}^N h |e_k|^{1.5}.$$

Since all the function are periodic, any time length t can be expressed as $t = KT + t_0$.

Consider

$$\lim_{t \rightarrow \infty} \frac{T}{t} \int_0^t e idt = \lim_{t \rightarrow \infty} \frac{T}{t} \left[K \int_0^T e idt + \int_0^{t_0} e idt \right] \quad (5.20)$$

$$= \lim_{K \rightarrow \infty} \frac{KT}{KT + t_0} \int_0^T e idt + \lim_{t \rightarrow \infty} \frac{T}{t} \int_0^{t_0} e idt \quad (5.21)$$

$$= \int_0^T e idt. \quad (5.22)$$

Likewise

$$\lim_{t \rightarrow \infty} \frac{T}{t} \int_0^t |e|^{1.5} dt = \int_0^T |e|^{1.5} dt. \quad (5.23)$$

Therefore, we use Reimann sums $\frac{T}{t} \sum_{k=1}^N h e_k i_k$ and $\frac{T}{t} \sum_{k=1}^N h |e_k|^{1.5}$ to calculate the integrals, where t is sufficiently large. Then using Equation (5.16), we obtain three linear equations of k_1 , k_2 and C_0 , which we solve for k_1 and k_2 .

It is important to observe the distribution of k_1 and k_2 for distinct pairs of points. Figure (5.1) shows that all the values lie on a neighborhood with radius 0.002, indicating that the deviations between two values are comparably small.

Figure 5.1: Distribution of k_1 & k_2

We take the sample mean to get values for k_1 and k_2 , which are 0.0076 and 0.0068 respectively.

5.1.2 Refinement parameter identification

To maintain a constant RMS current with increased frequencies, the input voltage must be increased. As a result, e_{rms} also increases. Figure (5.2) shows the frequency verses e_{rms} curve for 1A constant RMS current.

Figure 5.2: Frequency vs e_{rms} graph for $i_{rms} = 1A$

At low frequencies the difference between the actual $\varphi(e)$ and $i_1 + i_2$ is negligible. However, since e_{rms} is increasing with frequency, the difference becomes significant. The graphs in Figure 5.3 give the variation of k_1 and k_2 for different frequencies.

Figure 5.3: Observed deviation of k_1 and k_2 values with frequencies

The Figure 5.3 shows that, as frequency increases, the sample means for k_1 and k_2 also increase, although k_1 and k_2 are suppose to be constants. This is a result of the increased difference between $\varphi(e)$ and $i_1 + i_2$. We must, therefore, refine the method of calculating $\varphi(e)$. We do so by introducing refinement parameters k_3 through k_n with respective current sinks i_3 through i_n . In this case, for simplicity we let $n = 4$ and define $i_3 = k_3 \text{sign}(e)|e|^{2/3}$ and $i_4 = k_4 \text{sign}(e)|e|^{5/6}$ such that the powers of e are equally spaced between $1/2$ and 1 . Then we let $\varphi(e) = i_1 + i_2 + i_3 + i_4$. With the additional current sinks, the power losses per cycle can be written

$$\int_a^{a+T} e i dt = k_1 T e_{rms}^2 + k_2 \int_a^{a+T} |e|^{3/2} dt + k_3 \int_a^{a+T} |e|^{5/3} dt + k_4 \int_a^{a+T} |e|^{11/6} dt + C_0. \quad (5.24)$$

To identify the values k_3 and k_4 , we use the same procedure discussed in Section 5.1. Since we have three unknown parameters, we record three sets of observation data with a fixed hysteresis loop and solve for k_3 and k_4 .

Figure 5.4: Distribution of k_3 & k_4

To ensure more accurate values, we repeat this procedure many times and use the sample means for k_3 and k_4 , which are 0.0011 and 0.0005 respectively.

5.2 Transfer function identification

We must now identify the transfer function for frequencies greater than 1kHz. Since the actuator is designed for frequencies less than 3kHz, we specifically focus on

the range 1kHz-3kHz.

Consider Figure 5.5, where $\tilde{R}$ denotes the additional average series losses and v_e is the effective voltage across the low frequency model. Let $\tilde{v}$ be the voltage across points A and C in the figure. We select potential divider resistors, labelled R_1 , which are significantly larger than the DC resistance R .

Figure 5.5: Circuit for estimating $\tilde{R}$

From KVLs applied to the circuit, we get

$$v_2 = (R + \Delta R + \tilde{R} + R)i + e \quad (5.25)$$

$$v_1 = (\tilde{R} + R)i + e. \quad (5.26)$$

Hence,

$$2v_1 - v_2 = e + (\tilde{R} - \Delta R)i, \quad (5.27)$$

and since $\tilde{v} = v_1 - v_2/2$, we have $2\tilde{v} = e + (\tilde{R} - \Delta R)i$. The average power delivered to the circuit is

$$\begin{aligned} \int_a^{a+1/f} v_1 i dt &= \int_a^{a+1/f} i^2 (R + \tilde{R}) dt + \frac{k_1}{f} e_{rms}^2 \\ &+ k_2 \int_a^{a+1/f} |e|^{3/2} dt + k_3 \int_a^{a+1/f} |e|^{5/3} dt \\ &+ k_4 \int_a^{a+1/f} |e|^{11/6} dt + P_{hys}. \end{aligned} \quad (5.28)$$

From Equations (5.26) and (5.27),

$$v_1 - iR = i\tilde{R} + e = 2v_1 - v_2 + i\Delta R = 2\tilde{v} + i\Delta R. \quad (5.29)$$

Then from Equation (5.28) and (5.29),

$$\int_a^{a+1/f} 2\tilde{v}idt = \int_a^{a+1/f} (\tilde{R} - \Delta R)^2 dt + \sum_{j=1}^4 k_j \int_a^{a+1/f} |2\tilde{v} - (\tilde{R} - \Delta R)i|^{\alpha_j} + P_{hys}, \quad (5.30)$$

where α_j represents the corresponding exponentials of e in Equation (5.28). When $\Delta R \rightarrow \tilde{R}$, Equation (5.30) converges to

$$\int_a^{a+1/f} 2\tilde{v}idt = \sum_{j=1}^4 k_j \int_a^{a+1/f} |2\tilde{v}|^{\alpha_j} + P_{hys}. \quad (5.31)$$

5.3 Procedure and Experimental Results

We use Equation (5.31) to estimate $\tilde{R}$. First we set the frequency of the input voltage to 800Hz and ΔR to 0. Using Equation (5.24) we obtain the corresponding $\frac{|e|_{avg}}{f}$ value and corresponding hysteresis power loss, C_0 . By setting the frequency to any $\bar{f}$, where $\bar{f} > 800Hz$, and by increasing the input voltage, we set the value of $\frac{|2\tilde{v}|_{avg}}{\bar{f}}$ equal to the $\frac{|e|_{avg}}{f}$ value at 800Hz. Since $\tilde{R} > \Delta R = 0$, by Equation (5.27), $|2\tilde{v}|_{avg} > |e|_{avg}$. Therefore, the corresponding value of $\frac{|e|_{avg}}{f}$ at $\bar{f}$ is less than that at 800Hz. From Theorem 5 and Lemma 3, we then have

$$|B_{i(max)} - B_{i(min)}|_{\bar{f}} < |B_{i(max)} - B_{i(min)}|_{800Hz}. \quad (5.32)$$

Hence, the hysteresis loss at $\bar{f}$ is smaller than C_0 , the hysteresis loss at 800Hz. If we estimate the left and right hand sides Equation (5.31) using C_0 , we get

$$\int_a^{a+1/f} 2\tilde{v}idt < \sum_{j=1}^4 k_j \int_a^{a+1/f} |2\tilde{v}|^{\alpha_j} + C_0. \quad (5.33)$$

By slowly increasing ΔR and v_1 in such a way as to keep $\frac{|2\tilde{v}|_{avg}}{f}$ constant, we evaluate

$$\int_a^{a+1/f} 2\tilde{v}idt \quad \text{and} \quad \sum_{j=1}^4 k_j \int_a^{a+1/f} |2\tilde{v}|^{\alpha_j} + C_0. \quad (5.34)$$

When $\Delta R \rightarrow \tilde{R}$, $2\tilde{v} \rightarrow e$. Hence, $P_{hys} \rightarrow C_0$ and the integrals in Equation (5.34) converge to the same value. By obtaining the corresponding value of ΔR , we estimate

the average $\tilde{R}$ for the particular f . From KVL applied to the Figure 5.5

$$v_1 = i\tilde{R} + v_e \quad (5.35)$$

$$v_3 = i(\tilde{R} + \Delta R) + v_e. \quad (5.36)$$

Hence

$$2v_1 - v_3 = i(\tilde{R} - \Delta R) + v_e. \quad (5.37)$$

Let the voltage across A and B be $\hat{v}$. Then

$$2\hat{v} = i(\tilde{R} - \Delta R) + v_e. \quad (5.38)$$

When $\Delta R \rightarrow \tilde{R}$, we have $2\hat{v} = v_e$. By obtaining the peak-to-peak voltages v_1 and $2\hat{v}$, we are able to find the amplitude bode plot.

Figure 5.6 shows the relationship between equivalent Ohmic resistance ($R + \Delta R$) and frequency for 1A peak-to-peak current. Notice that the gradient of the curve is increasing with frequency. This contradicts the classical skin effect formula based on $\sqrt{f}$ whose gradient increases at a rate less than linear. This unpredicted relationship may be the result of ignoring mechanical losses at high frequencies, which are lumped together with resistance in the model.

Figure 5.6: variation of equivalent Ohmic resistance

Figure 5.7 shows the experimental bode plot.

Figure 5.7: Experimental Amplitude Bode plot

Next, assuming that the transfer function is linear, we try to match the experimental bode plot with a bode plot corresponding to a transfer function. The bode plot in Figure 5.8 is for the transfer function $H(s) = \frac{1}{s/9500 + 1}$.

Figure 5.8: Magnitude and Phase plot of $H(s)$

Based on the graphs, we conclude the transfer function for our model is

$$H(s) = \frac{1}{s/9500 + 1}. \quad (5.39)$$

CHAPTER VI

SUMMARY AND CONCLUSION

We have investigated the problem of modelling magnetostrictive actuators for broad-band frequencies.

Most of the previous approaches have been based on empirical classifications of material properties of the core with the assumption that all the series losses can be represented by a constant Ohmic resistance in the winding. By suggesting a fundamental model with constant lead resistance, we recognize that, at low frequencies, the only significant winding loss is indeed the DC lead resistance. However, at high frequencies, we observed that assuming constant resistance is no longer sufficient.

We developed a low frequency model with DC resistance and two current sinks. To identify the parameters, we used the balance of input energy and energy losses. To deal with the non-linearity of the hysteresis loss, we identify a method to fix the hysteresis loss. We observe that the model is adequately accurate for frequencies less than 500Hz.

We then investigated the accuracy of the low frequency model for higher frequencies. We observed a large margin of error, indicating a need to improve the model for high frequencies. To do so, we introduced two more current sink and observed that the model is sufficient for frequencies up to 800Hz.

To accommodate the additional series losses that are significant after 1kHz, we introduced a transfer function which appears before the low frequency model. Using a method of estimation we identify the transfer function.

Due to limited equipment availability, the identification of the transfer function is based on few experimental data. Furthermore, we investigated only the experimental magnitude bode plot. To accurately identify the transfer function, we also need to obtain the experimental phase plot. For any future work, therefore, it is important to gather more data to refine and improve the transfer function.

After we obtain a sufficiently accurate model, future work should be devoted

to understanding the nature of the controller and the behavior of a controller in a network setting.

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