TRANSVERSE-ULTRASONIC-ATTENUATION DETERMINATION
OF THE ENERGY GAP BETWEEN THE NORMAL AND
SUPERCONDUCTING STATES OF ZINC

by

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CHAPTER I

INTRODUCTION

The phenomena of superconductivity was discovered by H. Kamerlingh Onnes¹ between 1911 and 1913. He observed that the electrical resistance of mercury, lead, and tin sharply decreased to essentially zero at definite cryogenic temperatures which were characteristic of each material. This temperature came to be called the critical or transition temperature (T_c). Onnes recognized that the samples had passed into a new state below T_c which he named the "superconductive state." It is now called the superconducting state as opposed to the normal state which occurs for all temperatures above T_c . He later discovered that the superconducting state can be destroyed by an electrical current density which exceeds a certain value, called the critical current density, or by the application of a magnetic field which exceeds a certain value, called the critical magnetic field. Both the critical current density and the critical magnetic field are characteristic of the superconducting material. In addition, they are both functions of temperature below the critical temperature, that is, they increase with decreasing temperature.

Today there are numerous materials including both elemental metals and alloys which have been found to be

superconductors.² However, the better conductors at room temperature (Cu, Ag, and Au) have not been found to exhibit superconducting properties. The highest critical temperature found for an element is 9.3 K for niobium. Other elemental metal superconductors range in critical temperature from less than 1 K up to the 9.3 K for niobium. Certain alloys and metallic compounds have been produced which have critical temperatures in the 18-25 K range. Also, compounds have been produced with large values for the critical current density and the critical magnetic field. One of these is a lead-molybdenum-sulfur compound (PbMo_5S_6) which was found to have a critical magnetic field of about 500,000 gauss.³

Meissner and Ochsenfeld⁴ discovered in 1933 that superconductors also possess the property of perfect diamagnetism. This means that a superconductor will not allow magnetic flux to exist within its interior. As a consequence of this, all electrical currents flowing within it are confined to the surface. This property came to be known as the Meissner effect. However, the phenomenological theory proposed by F. and H. London⁵ in 1935 predicted that the magnetic flux decreased exponentially with penetration depth from the surface into the superconductor. It decays to $(1/e)$ th of its surface value in a characteristic length called the London penetration depth (λ_L). This

length decreases with decreasing temperature and reaches a limiting value of approximately 10^{-6} cm at a temperature of 0 K. This theory was first experimentally verified by Shoenberg in 1940.⁶

Although there were various phenomenological theories⁷ to explain the macroscopic properties of superconductors, it was not until after 1950 that a successful microscopic theory began to emerge. J. Bardeen⁸ stated in 1950 that superconductivity could depend on an interaction between the free electrons and the crystal lattice vibrations (phonons). At the same time H. Fröhlich⁹ showed by theoretical calculations that this type of interaction could lead to superconductivity at a temperature of absolute zero. Serin, Reynolds, and Nebsitt¹⁰ published in 1950 a summary of their experimental measurements on various isotopic mixtures of mercury. They found that the critical temperature (T_c) appeared to vary indirectly as the square root of the isotopic mass. This variation came to be known as the isotope effect and it showed experimentally that the electron-phonon interaction was fundamental to superconductivity.

B. Goodman¹¹ suggested in 1952, from his work on measuring the thermal conductivity of superconducting tin, that the superconducting electronic specific heat at low temperatures might decrease exponentially with temperature. This form of variation indicated a gap in allowed energies be-

tween the electrons in the normal and the superconducting states. He predicted the magnitude of this gap to be on the order of $k_B T_C$ per electron, where k_B is Boltzmann's constant. This was confirmed by specific heat measurements on vanadium¹² in 1956. Glover and Tinkham¹³ published in 1957 the first direct evidence of the energy gap from their measurements of infrared and millimeter microwave radiation absorption in thin film superconductors. They found a gap in the electronic excitation spectrum of the superconductors of approximately $3 k_B T_C$. Shortly thereafter in 1957, the presently accepted microscopic theory of superconductivity was published by J. Bardeen, L. Cooper, and J. Schrieffer¹⁴ (hereafter referred to as BCS).

I.1 BCS Theory

The BCS theory was derived for an isotropic, weak-coupling ($T_C \ll$ Debye temperature) ideal superconductor. However, the concepts behind the theory may be applied to all superconductors. The mathematical details can be found in many places.^{14,15,16,17} Therefore, only the concepts and the results will be discussed.

The BCS theory considers an electron-virtual phonon interaction which can lead to an attractive force between electrons bringing about a reduction in the total energy of the electron system in the metal. The electrons in a

metal are fermions obeying Fermi-Dirac statistics. The magnitude of the energy reduction in going from the normal to the superconducting state is on the order of $k_B T_C \approx 10^{-4}$ eV per electron. This implies that electrons within about $k_B T_C$ energy above and below the Fermi surface are involved in the interaction. If two of these fermions have equal and opposite linear momenta and opposite spins, then they can interact via a virtual phonon from thermal lattice vibrations and form a pair (Cooper pair¹⁸) with a total linear momentum of zero. The interaction that creates the Cooper pair is attractive which causes a lowering of the total energy of the two fermions. Cooper pairs essentially behave as bosons. That is, Cooper pairs are all in the same quantum state and have the same energy. This is basic to superconductivity.

If the total interaction is to be attractive, there must be a strong interaction between the electrons and phonons. The better conductors at room temperatures have weak electron-phonon interactions and have not been found to become superconductors; whereas, some poor conductors do become superconductors.

The energy gap (superconducting energy gap) can be defined as the minimum energy per electron required to completely disassociate a Cooper pair into two fermions. This energy gap is a function of temperature below the critical

temperature and is denoted by $\Delta(T)$. It varies from zero at $T = T_c$ to a maximum value at $T = 0$ K. The BCS theory predicts a value for the average (many Cooper pairs are considered) energy gap at $T = 0$ K given by

$$\overline{\Delta(0)} = 1.76k_B T_c \quad (1)$$

The average energy gap of an anisotropic superconductor at $T = 0$ K may be a function of crystallographic direction and may depart from the BCS value. This can be expressed by

$$\overline{\Delta(\vec{q}, 0)} = Ak_B T_c \quad (2)$$

where A is a constant to be experimentally determined for each crystallographic direction and $\vec{q}$ ($2\pi/\lambda$) is the phonon wave vector propagating parallel to that crystallographic direction.

Multiple average energy gaps are possible. They can be due to anisotropy of the Fermi surface or to multiple energy surfaces.

I.2 Transverse Ultrasonic Attenuation in Normal Metals

Ultrasonic waves propagating through a normal metal can lose energy (be attenuated) by interactions with both the crystal lattice vibrations and the free electrons (phonon-phonon and electron-phonon interactions). A. Pippard¹⁹

published a theory in 1955 accounting for the electronic attenuation of transverse ultrasonic waves in normal metals. He considered the metal to be isotropic and to conform to the free electron model with a well defined relaxation time (τ). Pippard then showed that the impurity limited (dominant scattering is due to impurities) electronic attenuation (α_N) of the transverse ultrasonic wave is given by

$$\alpha_N = \frac{Nm_e}{\rho_0 v'_S \tau} \left(\frac{1-g}{g} \right) \quad (3)$$

where

$$g = \frac{3}{2(q\ell_e)^2} \left\{ \frac{(q\ell_e)^2 + 1}{q\ell_e} \tan^{-1}(q\ell_e) - 1 \right\}. \quad (4)$$

In the above N is the number of electrons per unit volume, m_e is the mass of a free electron, ρ_0 is the density of the metal, v'_S is the transverse wave velocity, q is the wave number of the ultrasonic wave ($2\pi/\lambda$), and ℓ_e is the electron mean free path.

Three regions must be considered for a study of transverse ultrasonic attenuation in normal metals. They are

$$\alpha_N = \frac{Nm_e v_F^2 \tau}{5\rho_0 v'_S} \omega^2 \left\{ 1 - \frac{8}{35}(q\ell_e)^2 + \dots \right\} \quad q\ell_e \ll 1 \quad (5)$$

$$\alpha_N \approx \frac{4Nm_e v_F}{3\pi\rho_0 v_s'^2} \omega \quad q\ell_e \gg 1 \text{ \& } \nu < 10^9 \text{ Hz} \quad (6)$$

$$\alpha_N \approx \frac{Nm_e}{\rho_0 v_s' \tau} \quad q\ell_e \gg 1 \text{ \& } \nu > 10^9 \text{ Hz} \quad (7)$$

where v_F is the Fermi velocity and ω is the angular velocity ($2\pi\nu$) of the wave. These equations can be obtained from Eq. (3) with Eq.(4). If $q\ell_e \approx 1$, Eq.(3) with Eq.(4) must be used. In the present work the frequency (ν) will be less than 10^9 Hz. Therefore, for $q\ell_e \ll 1$ the normal state electronic attenuation should be dependent upon the value of $q\ell_e$ and be proportional to the square of the frequency. For $q\ell_e \gg 1$ the normal state electronic attenuation should be proportional to the first power of the frequency and should be independent of $q\ell_e$. Since this theory was developed for an isotropic metal these relations may not hold for a real, anisotropic normal metal. This was discussed by Morse²⁰ in 1959 and again by Pippard²¹ in 1960.

It should be stated that the normal state electronic attenuation is a constant value below the critical temperature for weak-coupling, impurity limited superconducting metals.²² Zinc, cadmium, and molybdenum can belong to this category.

I.3 Transverse Ultrasonic Attenuation in Superconductors

The mechanism of ultrasonic attenuation in a metal crystal is assumed to be the same in both the normal and superconducting states. If the frequency of the wave is less than 10^9 Hz, the energy associated with each phonon is less than 10^{-6} eV. The minimum energy required to break up a Cooper pair into two fermions is $2\Delta \approx 3k_B T_C \approx 10^{-4}$ eV. Therefore, these ultrasonic wave phonons will not interact with the Cooper pairs. However, 2Δ is temperature dependent and approaches 10^{-6} eV within about one millikelvin of the critical temperature. As the temperature is lowered below the critical temperature, more of the electrons become Cooper pairs and consequently cannot interact with the ultrasonic phonons. This means that there will be fewer unpaired electrons to interact with and the attenuation will decrease (less energy will be removed from the ultrasonic wave). The attenuation will depend only on the probability distribution function for the normal electrons. BCS¹⁴ used this distribution function to calculate the ratio of superconducting to normal state electronic attenuation at any temperature for longitudinal ultrasonic waves in the region $q\ell_e > 1$. It is given by

$$\frac{\alpha_S(T)}{\alpha_N(T)} = 2f(\Delta(T)) = \frac{2}{e^{\Delta(T)/k_B T} + 1} \quad (8)$$

where $f(\Delta(T))$ is the Fermi function. Tsuneto²³ showed in 1960 that this result held for all values of $q\ell_e$ for impurity limited attenuation.

In 1964 Claiborne and Morse²⁴ observed a somewhat different dependence for transverse ultrasonic attenuation in aluminum. A rapid fall attenuation region was found very close to the critical temperature followed by a residual attenuation region which conformed to the BCS expression. This rapid fall region was observed to occur over a temperature range of several millikelvin near the critical temperature. This behavior has been reported by other investigators^{25,26,27} and is believed to be associated with the Meissner effect.

I.4 Zinc

Zinc is an elemental superconductor with a critical temperature of about 0.85 K and a critical magnetic field at absolute zero of about 53 gauss. It was discovered to be a superconductor by W. Keesom²⁸ in 1933 at the Kamerlingh Onnes Laboratory at Leiden.

Measurements of thermal conductivity,²⁹ electronic specific heat,^{30,31} microwave absorption,³² and depression of the critical temperature by nonmagnetic impurities³³ have suggested a large anisotropy of the average energy gap. Previous longitudinal^{27,34,35,36} and transverse²⁷ ultrasonic

attenuation measurements of the average energy gap have also indicated some anisotropy. The most comprehensive longitudinal ultrasonic study to date indicates a large anisotropy consistent with the non-ultrasonic measurements and the possibility of two average energy gaps in certain crystallographic directions.³⁶ Furthermore, the value of the average energy gap in some cases was found to be dependent on the magnitude of $q\ell_e$.³⁶

An earlier experiment³⁷ with transverse ultrasonic waves propagating parallel to the (0001) crystallographic direction has indicated the presence of some amplitude dependence. Almond, Birks, Lea, and Dobbs³⁸ have stated that if the propagation and polarization directions both lie in the basal plane of zinc, then there will be no amplitude dependence.

This investigation is mainly concerned with the continued study of the pure zinc single crystal of the previous longitudinal ultrasonic attenuation work.³⁶ It was undertaken to determine the anisotropic average energy gap(s) at absolute zero by a study of transverse ultrasonic attenuation. These values can then be used along with the previous work³⁶ to gain a better understanding of superconductivity in this zinc crystal.

Experimental measurements have been conducted to de-

termine the attenuation of transverse ultrasonic waves with the wave vector $\vec{q}$ and the particle motion polarization vector $\vec{s}$ simultaneously parallel to the following crystallographic directions;

$\vec{q}$	$\vec{s}$
(0001)	(10 $\bar{1}$ 0)
(0001)	(11 $\bar{2}$ 0)
(10 $\bar{1}$ 0)	(11 $\bar{2}$ 0)
(11 $\bar{2}$ 0)	(0001)

The λ_e values previously measured are used in this study.³⁶ The data were obtained with a continuously measuring ultrasonic system and a conventional He³ refrigeration-cryostat to achieve the necessary stable temperatures below 1 K.

CHAPTER II

EXPERIMENTAL

The experimental equipment described in this chapter was used to measure the temperature dependence of transverse ultrasonic attenuation in the normal and superconducting states of a zinc sample. This equipment consists of an ultrasonic system which continuously measures small transverse attenuation changes in solids at low temperatures. The stable low temperatures (below 1 K) were produced by a conventional He^3 refrigerator-cryostat system. A carbon thermometry system was used to continuously measure these very low temperatures. This chapter will describe the zinc sample, the pulse-echo method of measuring ultrasonic attenuation, the He^3 refrigerator-cryostat and associated vacuum equipment, and the thermometry system. Related topics to be discussed are the transducers, their bonding to the sample, and the elimination of the earth's magnetic field from the sample.

II.1 Sample

As was stated earlier, this investigation is concerned with the continued study of the pure (99.9999%), single crystal zinc sample used in a previous study in this laboratory.³⁶ The aspects of this previous study necessary to this investigation are restated here. The structure of

crystalline zinc is hexagonal close-packed (hcp)³⁹ as indicated in Figure 1. Three independent, crystallographic directions were chosen for study ((0001) or c-axis, $(10\bar{1}0)$, and $(11\bar{2}0)$). Since these three directions are mutually orthogonal the sample could be prepared in the form of a rectangular parallelepiped. The physical dimensions of the sample are given in centimeters at room temperature as follows:

$$(0001) - 0.4198$$

$$(10\bar{1}0) - 0.5786$$

$$(11\bar{2}0) - 0.7870$$

The average mean-free-path (λ_e) at impurity limited temperatures was measured by the previous work³⁶ to be approximately 0.1 cm for the c-axis, 0.006 cm for the $(10\bar{1}0)$ axis, and 0.005 cm for the $(11\bar{2}0)$ axis.

During the course of the investigation amplitude dependence was observed for sound propagation parallel to the c-axis. Therefore, the sample was etched to clean its surface and then annealed in order to remove some of the dislocations which produced the amplitude dependence. The etching solution was 4%, by volume, of concentrated nitric acid in methanol. The sample was annealed under vacuum in a Thermolyne electric furnace type 10500 for 24 hours at 340°C (the melting point of zinc is approximately 420°C).

Figure 1

Hexagonal Close-Packed Structure

The new physical dimensions of the sample in centimeters at room temperature are as follows:

(0001) - 0.3894

(10 $\bar{1}$ 0) - 0.5472

(11 $\bar{2}$ 0) - 0.7544

II.2 Measurement of Attenuation by the Pulse-Echo Method

The method used to obtain the transverse ultrasonic attenuation in the zinc sample was the standard single transducer pulse-echo technique.⁴⁰ This method consists first of bonding a transverse piezoelectric transducer (quartz in this investigation) onto a flat sample surface. Then a brief pulse (~ 3 μ sec) of radio frequency (R.F.) electrical energy is generated and impressed across the faces of the transducer. If this frequency is the fundamental or an odd harmonic of the transducer, then the electrical pulse will be converted into an ultrasonic wave which propagates into and through the sample. The wave is reflected from the opposite, parallel sample face and returns to the transducer. The transducer converts a small part of the wave to an electrical signal and reflects the rest back into the sample. These electrical signals are amplified, processed, and displayed on the cathode ray tube of an oscilloscope as an initial pulse with an echo train following it. This process is repeated until the energy of the

original ultrasonic wave is attenuated to essentially zero. The cathode ray tube will display the original power pulse and a train of echo pulses that are attenuated approximately exponentially in amplitude. Each pulse will be separated by the time required for the ultrasonic wave to travel through the sample and return to the transducer. It will be shown that the attenuation change of the ultrasonic wave as a function of temperature can be determined by measuring the change in amplitude of any one pulse.

Consider a plane ultrasonic wave that is being attenuated as it travels through a homogeneous sample. The attenuation factor (α) is given by:⁴⁰

$$\alpha = \frac{1}{(x_2 - x_1)} 20 \text{Log}_{10} \left\{ \frac{u(x_1)}{u(x_2)} \right\} \quad (9)$$

where α is measured in decibels (dB) per unit length, u is the amplitude of the wave as a function of distance (x), and x_1 and x_2 are two points along the sound path with $x_2 > x_1$. The voltage (V) on the transducer is proportional to the amplitude of the plane wave at the transducer. Therefore, the attenuation can be expressed by:

$$\alpha = \frac{20}{2(n-m)L} \text{Log}_{10} \left\{ \frac{V_m}{V_n} \right\} \quad (10)$$

where the (m)th echo at the transducer producing a voltage

(V_m) corresponds to x_1 , a later (n)th echo producing a voltage (V_n) corresponds to x_2 , and L is the sample length. The functional dependence of α on temperature is as follows.

Let V_0 represent the initial voltage pulse. Then

$$\alpha(T) = \frac{20}{2nL} \text{Log}_{10} \left\{ \frac{V_0(T)}{V_n(T)} \right\} \quad (11)$$

However, Eq.(11) contains non-electronic background attenuation that is assumed to be temperature independent. This can be expressed by:

$$\alpha(T) = \frac{20}{2nL} \text{Log}_{10} \left\{ \frac{V_0(T)}{V_n(T)} \right\} - \alpha_B \quad (12)$$

where α_B represents all of the non-electronic attenuation. $\alpha(T)$ now represents only the attenuation due to the electrons interacting with the ultrasound.

In the experiment, $V_0(T)$ is taken as an arbitrary reference voltage (V_0) during any one temperature cycling. The superconducting attenuation and the normal attenuation, defined as $\alpha_s(T)$ and $\alpha_N(T)$ respectively, can now be written as follows:

$$\alpha_s(T) = \frac{20}{2nL} \text{Log}_{10} \left\{ \frac{V_0}{V_{ns}(T)} \right\} - \alpha_B \quad (13)$$

and

$$\alpha_N(T) = \frac{20}{2nL} \log_{10} \left\{ \frac{V_0}{V_{nN}(T)} \right\} - \alpha_B \quad (14)$$

$V_{ns}(T)$ and $V_{nN}(T)$ represent the voltages of the (n)th echo at a given temperature in the superconducting and normal states, respectively.

BCS assumes that at a temperature of zero Kelvin there are no unpaired electrons left to attenuate the ultrasound. Therefore, at zero Kelvin

$$\alpha_S(0) = 0 \quad (15)$$

or

$$\alpha_B = \frac{20}{2nL} \log_{10} \left\{ \frac{V_0}{V_{ns}(0)} \right\} \quad (16)$$

Substitution of Eq.(16) into Eqs.(13) and (14) and the determination of the ratio of $\alpha_S(T)$ to $\alpha_N(T)$ yields:

$$\frac{\alpha_S(T)}{\alpha_N(T)} = \frac{\log_{10} \left\{ \frac{V_{ns}(0)}{V_{ns}(T)} \right\}}{\log_{10} \left\{ \frac{V_{ns}(0)}{V_{nN}(T)} \right\}} \quad (17)$$

As was stated earlier, the normal state attenuation of zinc in the impurity limited region is independent of temperature. Therefore, Eq.(17) can be rewritten as:

$$\frac{\alpha_s(T)}{\alpha_N(T)} = \frac{\text{Log}_{10} \left\{ \frac{V_{ns}(0)}{V_{ns}(T)} \right\}}{\text{Log}_{10} \left\{ \frac{V_{ns}(0)}{V_{nN}(T_c)} \right\}} \quad (18)$$

Eq.(18) gives the ratio of superconducting to normal state attenuation in terms of the experimental quantities to be measured. It should be compared to the BCS expression given by Eq.(8).

Also, the limiting or total electronic attenuation (α_e) can be obtained from Eq.(14). It is given by:

$$\alpha_e = \alpha_N(T) = \frac{20}{2nL} \text{Log}_{10} \left\{ \frac{V_{ns}(0)}{V_{nN}(T_c)} \right\} \quad (19)$$

II.3 Electronic Equipment

A detailed discussion of this continuously measuring ultrasonic system may be found elsewhere,^{41,36,42,43} so only a brief description will be given here. A block diagram of the complete ultrasonic system is shown in Figure 2. This system makes use of phase sensitive detection to increase the signal-to-noise ratio in the amplification of the echoes. This is necessary for the measurement of small changes in attenuation.

Referring to Figure 2 the first Hewlett-Packard model 214A pulse generator was operated on internal trigger at a

Figure 2

Block Diagram of Electronic Equipment

frequency of 800 Hz. It produced a trigger signal which was sent to the Hewlett-Packard model 175A oscilloscope and a +4 volt pulse of 3-5 μ sec duration which was sent to the binary frequency divider and the AND gate. The four-stage binary frequency divider produced a 5 volt, 50 Hz reference signal from the 800 Hz input signal. This signal was simultaneously sent to the AND gate and to the PAR model 121 lock-in amplifier as the reference frequency. The AND gate produced a -5 volt signal output of 800 Hz gated at 50 Hz. This signal was used to trigger the second Hewlett-Packard model 214A pulse generator. This generator produced either a positive or a negative pulse at less than 4 volts. These pulses, varying at 800 Hz and gated at 50 Hz, were used to externally trigger the Arenberg model PG-650C pulsed oscillator which produced R.F. pulses with a width of 1-3 μ sec and a peak to peak voltage (PPV) of several volts into 93 ohms. The frequency of these pulses was tunable over the required range (5-200 MHz) by using the appropriate coil in the oscillator circuit. The various frequencies below 100 MHz were measured on a Hewlett-Packard model 5245L electronic counter.

For low frequency R.F. pulses in the range of 5 to 65 MHz, the power pulse and its echo train were first sent to a tunable Arenberg model PA-620SN pre-amplifier. The signal then went to an Arenberg model WA-600-D wide band amplifier.

For high frequency pulses in the range of 65 to 200 MHz, the signal was sent to an Arenberg model VR-720 VHF receiver. After video amplification in either of these two systems, the signal was displayed on the cathode ray tube of a Hewlett-Packard model 175A oscilloscope which was equipped with a Hewlett-Packard model 1782A display scanner plug-in. The maximum video signal strength of any echo to be sampled was kept at less than 10 volts to preserve the linearity of either of the two amplifiers.

The display scanner is used to sample the amplitude of a point on the cathode ray tube trace. This point is set to coincide with the peak of one certain echo. Then the output of this display scanner is a 50 Hz wave proportional in amplitude to the voltage of the echo peak being sampled. This signal is then sent to the lock-in amplifier where it is compared with the 50 Hz reference signal. The amplifier then produces a dc output signal which is still proportional to the voltage of the sampled echo peak. This output is displayed on the vertical axis of the model 2X-2A Moseley X-Y chart recorder.

The display scanner was calibrated at 200 millivolts/cm. The lock-in amplifier unit contains a calibrated suppression voltage ranging from 0 to ± 100 volts. This allows large voltage signals with small voltage changes to be measured. The time constant was normally set at 0.3 seconds

and the sensitivity was run at either 10, 20, or 50 millivolts, depending on how much amplification was needed. The recorder was set at 1 volt/inch.

The binary frequency divider and the AND gate will be discussed in more detail. A circuit diagram with input and outputs is shown in Figure 3. The 800 Hz input is dropped to about +2.5 volts by the zener diode. The binary frequency divider divides the frequency by 16 (from 800 Hz to 50 Hz). A 5 volt, 50 Hz signal is obtained at the reference output. The 2N4120 transistor is the basis of the AND gate. It produces the 800 Hz output gated at 50 Hz as shown in Figure 3.

II.4 He³ Refrigerator-Cryostat System

A conventional He³ refrigerator-cryostat system was used to maintain stable temperatures below the temperature of pumped liquid He⁴ (about 1.4 K). The design of this system is discussed elsewhere.⁴⁴ However, the cryostat was modified to incorporate an R.F. transmission line for the generation of the ultrasonic pulses. A description of this can also be found elsewhere.^{36,42,43} Therefore, only a brief description will be given here.

A diagram of the vacuum system is shown in Figure 4. It consists of a high vacuum system to insulate the He³ can, a large (105 cubic feet per minute) vacuum pump to lower

Figure 3

AND Gate and Binary Frequency Divider

Figure 4

Vacuum Systems

the temperature of the liquid He^4 , and the isolated He^3 vacuum system. The He^3 pump cut-off valve, coarse valve, and micrometer needle valve were always closed to prevent the loss of the He^3 gas unless the He^3 cryostat was ready for operation. The He^3 gas consisted of approximately four to five liters and was maintained at a pressure of less than 1 atmosphere as measured by the Ashcroft vacuum gauge. The bellows was installed to prevent the vibrational energy of the running Welch 5.6 CFM Duo-Seal model 1402 vacuum pump from heating the cryostat.

The He^3 refrigerator-cryostat is shown in Figure 5. The process of preparing the system to be put into operation will now be discussed. The crystal sample was mounted as shown in Figure 5 with the high-vacuum can removed. The quality of the echoes were examined by submerging the lower part of the cryostat in liquid nitrogen. After allowing the sample to reach room temperature, the high-vacuum can was sealed to the lid by using Wood's metal solder. The complete system was then connected as shown in Figure 4. The connecting valve and high-vacuum isolation valve were both opened and the He^4 exchange gas valve was closed. The Welch Duo-Seal rotary forepump was then turned on. When the pressure reached about 1.00 Torr as measured on the Wallace and Tiernan (W & T) model FA-160 absolute pressure gauge, the liquid nitrogen cold trap was filled. When the

Figure 5

 He^3 Cryostat

pressure reached 10-25 microns as measured by the Hasting model GV-38 thermocouple vacuum gauge, the connecting valve was closed. This isolated the He^3 system from the high vacuum system. After the connecting valve was closed, the Distillation Products 6.0 inch diameter oil diffusion pump was turned on. The pressure of the high vacuum system was monitored by a Consolidated Vacuum Corporation (CVC) type GPH-100A cold cathode ionization vacuum gauge. The pressure would drop to 10^{-5} Torr in approximately three days. During this time the inner dewar was pumped out and back-filled with He^4 gas. Also, the inner dewar jacket was pumped down to a pressure of about 5 microns and then sealed.

When the pressure in the high vacuum system reached about 10^{-5} Torr, the outer dewar was filled with liquid nitrogen. This lowered the temperature of the sample to about 78 K in 24 hours. Cryopumping dropped the high vacuum system pressure to about 6×10^{-6} Torr. Liquid He^4 at 4.2 K could then be transferred into the inner dewar. After the inner dewar was filled, the vent valve was closed simultaneously with the opening of the He^4 pump cut-off valve. The Stokes 105 cubic feet per minute vacuum pump then lowered the pressure above the liquid He^4 to less than 5.0 Torr (1.4 K). The high-vacuum isolation valve was then closed after looking for superfluid He^4 leaks. This allowed the high vacuum can to cryopump and lower the pressure still

further (to approximately 10^{-9} - 10^{-10} Torr).

The Hoke coarse valve and the Whitey type 316 micrometer needle valve were slowly opened to allow the He^3 gas to flow into the He^3 line and can. The He^3 gas cooled and then condensed into liquid He^3 in the He^3 can. This required about an hour and a half. At this time the sealed Welch He^3 vacuum pump was turned on. The He^3 pump cut-off valve was slowly opened. This lowered the vapor pressure above the liquid He^3 , and thus the temperature of the He^3 can and the sample were lowered. The pumping rate could then be stabilized at a temperature by controlling the amount of feedback of He^3 gas through the coarse and micrometer needle valves. The lowest temperature was obtained by closing both of these valves. The temperature range of superconducting zinc could be controlled by the micrometer needle valve with the coarse valve closed. A small motor and power supply was used to turn this valve in some of the experiments. This removed the need to turn the valve by hand, resulting in more consistent changes.

After finishing an experiment, the liquid He^3 was boiled off by the heater on the He^3 can and pumped back into its container. The He^4 pump cut-off valve was closed and the inner dewar was back-filled with He^4 gas. It was then left vented to the atmosphere. Also, the high-vacuum isolation valve was re-opened.

II.5 Thermometry System

The primary temperature reference in this investigation was the He^3 vapor pressure as measured by the W & T absolute pressure gauge. This gauge measured the absolute pressure in the range of 20.00 Torr to 0.10 Torr with an absolute accuracy of 0.33% of full scale anywhere within this range. This amounts to an error of approximately 70 microns at any pressure. Therefore, this gauge was used only to measure pressures greater than 1.00 Torr. The temperature of the He^3 liquid and can could then be determined by finding the temperatures corresponding to a measured He^3 vapor pressure in the T_{62} He^3 Thermometry Tables prepared by Sherman, Sydoriak, and Roberts.⁴⁵

Two carbon resistance thermometers (CRT) were employed as secondary thermometers. One thermometer, labeled as CRT A, was a one-half watt Allen-Bradley carbon resistor with a room temperature resistance of 2.7Ω . The other thermometer, labeled CRT B, was also a one-half watt Allen-Bradley carbon resistor, but its room temperature resistance was 5.1Ω . The resistance of these thermometers varied considerably with temperature at less than 1 K. The resistance was determined by the four-terminal resistance measurement technique as shown in Figure 2. A constant current source supplied either a 1 μamp or a 10 μamp current to either a 1 K Ω or a 10 K Ω precision 0.05% standard resistor (SR) in series with

the CRT. The voltage drop across the SR could be continuously monitored by a Hewlett-Packard model 3440A digital voltmeter.

It was determined that if the He^3 vapor pressure was in the range of 1.00 to 20.00 Torr, the CRT A and CRT B would be in thermal equilibrium with the He^3 can. Therefore, the resistance of CRT A and CRT B could be directly calibrated to the temperature as expressed by the W & T gauge. This was done by connecting another Hewlett-Packard digital voltmeter across the CRT. The data were taken at 10 μamp until self-heating occurred, and then at 1 μamp . However, this temperature range was only from 1.3 K to 0.65 K. In order to extend this calibration into the range from 0.65 K to 0.38 K, the original calibration data were fitted to the Clement-Quinnell⁴⁶ equation:

$$\text{Log } R + K/\text{Log } R = B(1/T) + A \quad (20)$$

In the above R is the resistance of the CRT, T is the absolute temperature corresponding to that resistance, and A, B, and K are experimental constants to be determined.

A computer program for the IBM VS/370 computer was used to determine the values of A, B, and K to best fit the experimental data to Eq.(20). The least squares fit was used. When these constants were found, a complete table of resistance versus temperature values could be printed out

using Eq.(20).

It was assumed that the thermometers remained in thermal equilibrium with the sample at all temperatures. The voltage output of a CRT was amplified by a Leeds and Northrup model 9835-B dc microvolt amplifier and used to drive the horizontal axis of the X-Y chart recorder. The current used in these measurements was 1.000 μ amps. The CRT B was used in the region of the critical temperature (1.2 K to about 0.6 K). Then the CRT A was used for the lower temperatures. The impedance as seen by the microvolt amplifier was kept less than 10 K Ω .

II.6 Transducers and Transducer Bonding

The piezoelectric quartz transducers used in this investigation were obtained from Valpey Crystal Corporation. These transducers were cut in such a manner that they would vibrate in the transverse mode, "Y-cut." They had highly parallel faces and were cut as thin cylinders with a diameter of about 3/16 inch. A slot was cut on the edge which indicated the direction of the transverse wave polarization. The thickness corresponded to a quarter wavelength of a 10 MHz sound wave. They could also be vibrated at odd harmonics of this fundamental. Both faces were coated with a thin layer of gold over a thin layer of chrome.

The transducers were bonded to the sample face with a

thin layer of Dow Corning DC-11 stopcock silicon grease. This bonding material becomes rigid at about 200 K. It formed a good bond that rarely broke due to the thermal contractions caused by cooling to low temperatures.

II.7 Elimination of the Earth's Magnetic Field

It was desired to eliminate the effects of the earth's magnetic field on the sample.²⁴ This was done to prevent the earth's magnetic field from affecting the temperature dependence of the attenuation in the rapid fall region. Current-carrying coils were constructed to produce horizontal and vertical magnetic fields. The vertical field was produced by Helmholtz coils that were slipped over the outer dewar. The horizontal field was produced by a larger pair of coils mounted at right angles to the north-south magnetic field direction. Both sets of coils were located so that the sample was at the center of each. Also, the current directions were set so that the magnetic fields produced would oppose the components of the earth's magnetic field. Each coil consisted of 20 turns of #14 copper enamel wire on a neoprene base. The average diameter of the Helmholtz coils was 9 1/4 inches, and the average separation was 4 5/8 inches. The average diameter of the larger coils was 14 inches, and the average separation was 9 1/2 inches.

A constant current source with two variable outputs was used as a power supply for these coils. Refer to Figure 6 for a circuit diagram. The current in each set of coils could be separately set at any value between 0.0 and 1.0 amps. The 5 K Ω pot in each output was used to set this current. The 741C operational amplifier was the basis for either of the two outputs. Each output contains an ammeter to measure the current flowing in each set of coils. It was determined that if the current in the coils producing the vertical magnetic field was 0.18 amps and the current in the coils producing the horizontal magnetic field was 0.30 amps, then the total magnetic field on the sample would be less than 0.05 gauss. This measurement was made using the flat, directional probe of the RFL Universal Gaussmeter model 3265.

CHAPTER III

RESULTS AND INTERPRETATIONS

The He^3 refrigerator-cryostat was used to cycle the temperature of the sample in the approximate temperature range of 1.00 K to 0.38 K. Plots of the voltage of an echo peak versus CRT resistance (temperature) were recorded on the Moseley X-Y recorder during this cycling. Recordings were taken for both decreasing temperature and for increasing temperature. A complete temperature cycling required from twenty to thirty minutes. This slow cycling rate was utilized so that the sample would remain in thermal equilibrium with the CRTs. To obtain data for the normal state at these very low temperatures it is essential to suppress the superconducting properties with a magnetic field. This was accomplished by the application of a 50 gauss magnetic field produced by the superconducting solenoid surrounding the sample as shown in Figure 5. Each plot was usually taken in three segments with a fixed voltage suppression on the PAR lock-in amplifier for each segment. These segments have been combined in Figure 7 to show the experimental data that were obtained. The voltages expressed in Eq.(18) and in Eq.(19) are also shown in Figure 7. The experimental data can be used in conjunction with Eq.(8) to determine $\overline{\Delta(\vec{q},0)}$

Figure 7
Experimental Data

and with Eq.(19) to determine the electronic attenuation (α_e).

The dashed curve in Figure 7 represents another temperature dependent attenuation effect differing from the electron-phonon interaction. This effect depends on the amplitude of the ultrasonic wave and is known as amplitude dependence. It varies non-linearly with temperature and affects the experimental data more at lower temperatures. Amplitude dependence is thought to be caused by an interaction between the ultrasonic wave and the dislocations in the crystal sample. It tends to increase the superconducting attenuation and thus, appears in the data as an opposite effect to the electron-phonon interaction. The values of the amplitude of the ultrasonic wave were kept as low as possible in order to minimize this effect.

III.1 Analysis of Data

Eq.(8) can be rewritten in a more convenient form as follows:

$$\text{Log}_e \left\{ 2 \frac{\alpha_N(t)}{\alpha_S(t)} - 1 \right\} = \frac{\overline{\Delta(\vec{q}, 0)}}{k_B T_C} \frac{G(t)}{t} \quad (21)$$

where $G(t) = \overline{\Delta(t)}/\overline{\Delta(0)}$ and $t = T/T_C$, the reduced temperature. Here the function $G(t)$ represents the temperature dependence of the average energy gap. Mühlischlegel⁴⁷ has

tabulated values for the function $G(t)$ for an ideal superconductor utilizing the BCS theory. However, it has been found to be more convenient to use a power series expansion reported by Clem⁴⁸ which is given by:

$$G(t) = 1.7367(1-t)^{1/2}\{1-0.4095(1-t)-0.0626(1-t)^2\} \quad (22)$$

This expansion will reproduce Mühlischlegel's values of $G(t)$ with an error of less than 0.1% for $t > 0.40$. For the experimental data considered here, the minimum value of t was about 0.44.

A plot of $\text{Log}_e\{2 \alpha_N(t)/\alpha_S(t) - 1\}$ versus $G(t)/t$ should yield a straight line with a slope of $\overline{\Delta(\vec{q},0)}$ which is expressed in units of $k_B T_C$. Also, the straight line should pass through the origin, that is the point (0,0). If multiple average energy gaps do exist, they should appear as different slopes in this plot. Therefore, the slopes of these plots should yield the value(s) of A in Eq.(2). It is possible that the BCS theory may not give a unique functional dependence of the ratio of $\alpha_S(t)/\alpha_N(t)$. Refer to Appendix A.

In order to analyze the data the rapid fall (if present) and the residual attenuation regions must be separated. This can be done by defining a $V'_{ns}(T_C)$ as shown in Figure 7. The value of $V'_{ns}(T_C)$ can be determined by actual inspection of the curve or by using a computer program. The latter

method was employed here. Once this voltage is determined, Eq.(18) will become

$$\frac{\alpha_s(T)}{\alpha_N(T)} = \frac{\alpha_s(t)}{\alpha_N(t)} = \frac{\text{Log}_{10} \left\{ \frac{V_{ns}(0)}{V_{ns}(T)} \right\}}{\text{Log}_{10} \left\{ \frac{V_{ns}(0)}{V'_{ns}(T_c)} \right\}} \quad (23)$$

The electronic attenuation of the residual region (α'_e) can be determined from Eq.(19) as:

$$\alpha'_e = \frac{20}{2nL} \text{Log}_{10} \left\{ \frac{V_{ns}(0)}{V'_{ns}(T_c)} \right\} \quad (24)$$

The ratio $\alpha_s(t)/\alpha_N(t)$ and α'_e cannot be determined unless $V_{ns}(0)$ and $V'_{ns}(T_c)$ are known for each experimental curve. Therefore, a program for the IBM VS/370 computer was used to analyze the data. Refer to Appendix B. The value of $V_{ns}(0)$ was varied to minimize the standard deviation of a least squares fit with $V'_{ns}(T_c) = V_{nN}(T_c)$. This would yield a value for $\overline{\Delta(\vec{q}, 0)}$ and a large, positive value for the $\text{Log}_e \{ 2 \alpha_N(t)/\alpha_s(t) - 1 \}$ intercept. The value of $V'_{ns}(T_c)$ was increased, and a new $V_{ns}(0)$ was determined. This process was repeated until values for $V_{ns}(0)$ and $V'_{ns}(T_c)$ were found to give a $\text{Log}_e \{ 2 \alpha_N(t)/\alpha_s(t) - 1 \}$ intercept of approximately zero and a minimum standard deviation for the least squares fit to Eq.(21). The slope or $\overline{\Delta(\vec{q}, 0)}$

could then be found. If more than one slope is found to occur, the $V_{ns}(0)$ is determined from the slope occurring at the lower reduced temperatures. This slope then corresponds to $\overline{\Delta(\vec{q},0)}$. The slope at the higher reduced temperatures, called $\overline{\Delta^*(\vec{q},0)}$, is then calculated from the same $V_{ns}(0)$.

III.2 Results of Analysis

The critical (or transition) temperature of the zinc sample was found to be $0.840 \text{ K} \pm 0.005 \text{ K}$. This value was independent of either the crystallographic direction or the frequency of the ultrasonic wave.

The effects of the earth's magnetic field on the sample were eliminated for most of the attenuation data. However, this data showed essentially no difference in the value of the critical temperature or in the shape of the rapid fall attenuation region than that taken without eliminating the earth's magnetic field. This was believed to be due to the sensitivity of the CRT near the critical temperature (approximately 1 millikelvin).

Results for $\vec{q}|| (0001)$ and $\vec{s}|| (10\bar{1}0)$

Plots of $\text{Log}_e(2\alpha_N/\alpha_S - 1)$ versus $G(t)/t$ at frequencies of 9.1, 30.8, and 50.1 MHz for $\vec{q}|| (0001)$ and $\vec{s}|| (10\bar{1}0)$ are shown in Figure 8. These plots are representative of the data taken at each of the indicated frequencies. The actual data used to make these plots is presented in Appendix

Figure 8

Plot of $\text{Log}_e(2\alpha_N/\alpha_S - 1)$ Versus $G(t)/t$ for
 $\vec{q} || (0001)$ and $\vec{s} || (10\bar{1}0)$

C, along with representative data from the other crystallographic directions and polarizations.

The values of $\overline{\Delta(\vec{q},0)}$, α_e , and α'_e are tabulated in Table 1 for each of the frequencies under the heading $\vec{q}||$ (0001) and $\vec{s}||(10\bar{1}0)$. It was observed that the plots could be fit with one slope over the entire temperature range for each frequency. The stated error shown with each value of $\overline{\Delta(\vec{q},0)}$ has been estimated from the scatter between data sets. The error associated with α_e and α'_e has been estimated in the same manner to be about ± 1.00 dB/cm.

Results for $\vec{q}||$ (0001) and $\vec{s}||(10\bar{1}0)$ Annealed

Representative plots of $\text{Log}_e(2\alpha_N/\alpha_s - 1)$ versus $G(t)/t$ at frequencies of 10.1, 30.7, and 48.5 MHz for $\vec{q}||$ (0001) and $\vec{s}||(10\bar{1}0)$ with the annealed zinc sample are shown in Figure 9. Plots of α_e and α'_e versus frequency are shown in Figure 10. The values of $\overline{\Delta(\vec{q},0)}$, α_e , and α'_e are tabulated for each frequency in Table 1 under the heading of $\vec{q}||$ (0001) and $\vec{s}||(10\bar{1}0)$ Annealed.

It was found that the plots indicated one value of $\overline{\Delta(\vec{q},0)}$ for each frequency. The total electronic attenuation (α_e) was found to vary linearly with frequency. The value of $(\alpha_e - \alpha'_e)$ was found to increase with frequency. It was observed that the process of removing some dislocations from the sample (annealing) lowered the average energy gap by ap-

Table 1

Electronic Attenuation and Average Energy Gaps

Freq (MHz)	α_e (dB/cm)	α'_e (dB/cm)	$\overline{\Delta^*(\vec{q},0)} (k_B T_c)$	$\overline{\Delta(\vec{q},0)} (k_B T_c)$
$\vec{q} (0001)$ and $\vec{s} (10\bar{1}0)$				
9.1	2.9	2.3	---	1.71 ± 0.20
30.8	5.6	3.7	---	1.62 ± 0.20
50.1	6.7	4.2	---	1.69 ± 0.20
$\vec{q} (0001)$ and $\vec{s} (10\bar{1}0)$ Annealed				
10.1	2.4	1.8	---	1.43 ± 0.10
30.7	4.6	3.3	---	1.56 ± 0.10
48.5	7.5	6.1	---	1.44 ± 0.10
$\vec{q} (0001)$ and $\vec{s} (11\bar{2}0)$ Annealed				
10.8	3.0	2.5	---	1.49 ± 0.20
29.8	6.7	5.2	---	1.47 ± 0.20
48.0	8.6	6.2	---	1.48 ± 0.20
$\vec{q} (10\bar{1}0)$ and $\vec{s} (11\bar{2}0)$				
10.7	1.5	1.3	1.65 ± 0.10	1.41 ± 0.20
30.6	4.5	3.6	---	1.53 ± 0.20
50.0	5.6	4.3	---	1.63 ± 0.10
$\vec{q} (11\bar{2}0)$ and $\vec{s} (0001)$				
10.7	2.0	1.3	2.10 ± 0.30	1.48 ± 0.20
30.0	2.3	2.3	---	1.53 ± 0.20
49.5	2.5	2.3	---	1.52 ± 0.30
70.0	2.4	1.3	---	1.61 ± 0.30
89.2	4.3	3.5	2.27 ± 0.40	1.79 ± 0.30

Figure 9

Plot of $\text{Log}_e(2\alpha_N/\alpha_s - 1)$ Versus $G(t)/t$ for
 $\vec{q} || (0001)$ and $\vec{s} || (10\bar{1}0)$ Annealed

Figure 10
 Plot of Electronic Attenuation Versus Frequency for
 $\vec{q} || (0001)$ and $\vec{s} || (10\bar{1}0)$ Annealed

proximately $0.20 k_B T_c$ for each frequency.

Results for $\vec{q}|| (0001)$ and $\vec{s}|| (11\bar{2}0)$ Annealed

Representative plots of $\text{Log}_e(2\alpha_N/\alpha_S - 1)$ versus $G(t)/t$ at frequencies of 10.8, 29.8, and 48.0 MHz for $\vec{q}|| (0001)$ and $\vec{s}|| (11\bar{2}0)$ with the annealed zinc sample are shown in Figure 11. Plots of α_e and α'_e versus frequency are shown in Figure 12. The values of $\overline{\Delta(\vec{q},0)}$, α_e , and α'_e are tabulated for each frequency in Table 1 under the heading of $\vec{q}|| (0001)$ and $\vec{s}|| (11\bar{2}0)$ Annealed.

Only one value of $\overline{\Delta(\vec{q},0)}$ was indicated for each frequency. α_e can be seen to vary approximately linearly with frequency. The value of $(\alpha_e - \alpha'_e)$ was found to increase with frequency.

Results for $\vec{q}|| (10\bar{1}0)$ and $\vec{s}|| (11\bar{2}0)$

Representative plots of $\text{Log}_e(2\alpha_N/\alpha_S - 1)$ versus $G(t)/t$ at frequencies of 10.7, 30.6, and 50.0 MHz for $\vec{q}|| (10\bar{1}0)$ and $\vec{s}|| (11\bar{2}0)$ are shown in Figure 13. Plots of α_e and α'_e versus frequency are shown in Figure 14. The values of $\overline{\Delta^*(\vec{q},0)}$, $\overline{\Delta(\vec{q},0)}$, α_e , and α'_e are tabulated for each frequency in Table 1 under the heading of $\vec{q}|| (10\bar{1}0)$ and $\vec{s}|| (11\bar{2}0)$.

It was found that the data for 10.7 MHz indicated two different slopes. The value $\overline{\Delta^*(\vec{q},0)}$ was found to occur in the higher temperature range for $0.98 > t > 0.80$. A smaller slope was found for $t < 0.80$. $\overline{\Delta^*(\vec{q},0)}$ tends to the value of

Figure 11

Plot of $\text{Log}_e(2\alpha_N/\alpha_s - 1)$ Versus $G(t)/t$ for
 $\vec{q} || (0001)$ and $\vec{s} || (11\bar{2}0)$ Annealed

Figure 12

Plot of Electronic Attenuation Versus Frequency for $\vec{q} || (0001)$ and $\vec{s} || (11\bar{2}0)$ Annealed

Figure 13

Plot of $\text{Log}_e(2\alpha_N/\alpha_s - 1)$ Versus $G(t)/t$ for
 $\vec{q} || (10\bar{1}0)$ and $\vec{s} || (11\bar{2}0)$

Figure 14

Plot of Electronic Attenuation Versus Frequency for $\vec{q} || (10\bar{1}0)$ and $\vec{s} || (11\bar{2}0)$

$\overline{\Delta(\vec{q},0)}$ as the frequency is increased above 10.7 MHz. The slopes at 30.6 and 50.0 MHz are each represented by only one value of $\overline{\Delta(\vec{q},0)}$. The value of $\overline{\Delta(\vec{q},0)}$ increases with frequency. For this direction and polarization the value of ql_e is on the order of unity. The plot of α_e versus frequency shows that, within the indicated error, α_e varies with the square of the frequency. The value of $(\alpha_e - \alpha'_e)$ was found to increase with frequency.

Results for $\vec{q} || (11\bar{2}0)$ and $\vec{s} || (0001)$

Representative plots of $\text{Log}_e(2\alpha_N/\alpha_s - 1)$ versus $G(t)/t$ at frequencies of 10.7, 30.0, and 49.5 MHz are shown in Figure 15 for $\vec{q} || (11\bar{2}0)$ and $\vec{s} || (0001)$. Figure 16 shows the representative plots at frequencies of 70.0 and 89.2 MHz. Plots of α_e and α'_e versus frequency are shown in Figure 17. A plot of $\overline{\Delta(\vec{q},0)}$ versus frequency for this case is shown in Figure 18. The values of $\overline{\Delta^*(\vec{q},0)}$, $\overline{\Delta(\vec{q},0)}$, α_e , and α'_e are tabulated for each frequency in Table 1 under the heading of $\vec{q} || (11\bar{2}0)$ and $\vec{s} || (0001)$.

The best fit for the plots at 10.7 and 89.2 MHz required two average gaps. $\overline{\Delta^*(\vec{q},0)}$ was found to occur at higher reduced temperatures in the range of $0.98 > t > 0.88$ for both frequencies. The lower value $\overline{\Delta(\vec{q},0)}$ was found to occur for $t < 0.88$. The plots at other frequencies were found to require one value of $\overline{\Delta(\vec{q},0)}$ over the entire tem-

Figure 15

Plot of $\text{Log}_e(2\alpha_N/\alpha_S - 1)$ Versus $G(t)/t$ for
 $\vec{q} || (11\bar{2}0)$ and $\vec{s} || (0001)$

Figure 16

Plot of $\text{Log}_e(2\alpha_N/\alpha_s - 1)$ Versus $G(t)/t$ for
 $\vec{q} || (11\bar{2}0)$ and $\vec{s} || (0001)$

Figure 17

Plot of Electronic Attenuation Versus Frequency for

$\vec{q} || (11\bar{2}0)$ and $\vec{s} || (0001)$

Figure 18
 Plot of $\Delta(\vec{q}, 0)$ Versus Frequency for $\vec{q} || (11\bar{2}0)$ and $\vec{s} || (0001)$

perature range. The values of $\overline{\Delta(\vec{q}, 0)}$ were found to increase with frequency.

Figure 17 shows a complicated dependence of α_e on frequency. Within the indicated error, both α_e and $\overline{\Delta(\vec{q}, 0)}$ increase with frequency. However, the frequency dependence is unlike that expressed by Eq.(4) for $q\ell_e \approx 1$. Based on the previous longitudinal data for $\vec{q} || (11\bar{2}0)$,³⁶ a "hump" is indicated. More importantly, the value of $(\alpha_e - \alpha'_e)$ does not increase with frequency. Also, the data taken at 30.0 MHz does not show any rapid fall region of attenuation.

Results for $\vec{q} || (10\bar{1}0)$ and $\vec{s} || (0001)$

One attempt was made to obtain experimental data for $\vec{q} || (10\bar{1}0)$ and $\vec{s} || (0001)$. However, no data was obtained due to bond deterioration between the transducer and the sample. Some information for this direction and polarization can be found elsewhere.²⁷

Results for $\vec{q} || (11\bar{2}0)$ and $\vec{s} || (10\bar{1}0)$

Some attenuation data were obtained for $\vec{q} || (11\bar{2}0)$ and $\vec{s} || (10\bar{1}0)$ at a frequency of about 10 MHz. The electronic attenuation was found to be consistent with that expressed elsewhere.³⁸ However, no average energy gaps could be determined.

III.3 Interpretation of Results

Electronic thermal conductivity measurements²⁹ have

indicated a maximum average energy gap of about $1.75k_B T_C$ and a minimum of about $1.2k_B T_C$ in superconducting zinc. Microwave absorption measurements³² have shown that an average energy gap of about $1.50k_B T_C$ is associated with most of the Fermi surface of zinc. However, a small part can be associated with an average energy gap of about $2.0k_B T_C$. Other non-ultrasonic measurements^{30,31,33} have indicated an anisotropy consistent with the above values.

Several longitudinal attenuation measurements of the average energy gap of superconducting zinc have been made. Lea and Dobbs³⁴ have found values of 1.71, 1.90, and 1.82 in units of $k_B T_C$ for $\vec{q} || (0001)$, $\vec{q} || (10\bar{1}0)$, and $\vec{q} || (11\bar{2}0)$, respectively. Bohm and Horwitz³⁵ have found values of $1.70 k_B T_C$ and $1.90k_B T_C$ for $\vec{q} || (10\bar{1}0)$ and $\vec{q} || (11\bar{2}0)$, respectively. However, it should be noted that both of the above measurements were made using adiabatic demagnetization refrigerators. Work done by Goncz and Neighbors³⁶ indicates an average energy gap of about $1.60k_B T_C$ for $\vec{q} || (0001)$. Cleave-
lin and Marshall³⁶ (refer to Table 2) have previously measured the limiting average energy gaps of this zinc sample to be 1.50, 1.41, and 1.90 in units of $k_B T_C$ for $\vec{q} || (0001)$, $\vec{q} || (10\bar{1}0)$, and $\vec{q} || (11\bar{2}0)$, respectively. Lou and Bömmel²⁷ have obtained an average energy gap of approximately $1.50 k_B T_C$ for either longitudinal waves with $\vec{q} || (0001)$ and $\vec{q} || (10\bar{1}0)$ or for transverse waves with $\vec{q} || (10\bar{1}0)$ and $\vec{s} || (0001)$.

Table 2

Cleavelin and Marshall (Reference 36)

Electronic Attenuation and Effective Energy Gaps

Average $q\ell_e$	Freq(MHz)	α_e (dB/cm)	$2\Delta^*(\vec{q},0)(k_B T_C)$	$2\Delta(\vec{q},0)(k_B T_C)$
$\vec{q} (0001)$				
180	9.0	4.5	---	3.00 ± 0.10
650	32.0	19.8	---	3.00 ± 0.10
1010	50.3	33.0	---	3.02 ± 0.10
$\vec{q} (10\bar{1}0)$				
0.39	9.0	0.3	$*3.48 \pm 0.20$	2.74 ± 0.10
1.15	31.5	1.8	---	2.78 ± 0.10
1.92	51.1	3.9	---	2.94 ± 0.20
2.69	70.6	6.4	---	2.82 ± 0.10
$\vec{q} (11\bar{2}0)$				
0.13	9.4	0.6	3.58 ± 0.20	2.96 ± 0.10
0.38	31.5	0.2	---	3.20 ± 0.10
0.64	53.0	4.4	4.66 ± 0.20	3.52 ± 0.10
0.90	70.6	6.5	4.80 ± 0.20	3.98 ± 0.10
1.15	90.5	7.3	5.00 ± 0.20	4.02 ± 0.20
1.41	110	6.9	4.42 ± 0.20	3.96 ± 0.20
1.67	130	6.2	---	3.76 ± 0.10
1.92	150	7.5	---	3.74 ± 0.10
2.18	170	9.2	---	3.80 ± 0.10

* Error - should be 2.48 ± 0.20

These measurements were made on their zinc sample with the smallest concentration of impurities.

The work by Cleavelin and Marshall³⁶ should be consulted for a comparison of their work with the other results given above. The interpretation of the results for the average energy gaps of this zinc sample as measured by transverse ultrasonic attenuation consists of comparing them with the previous work on the sample by Cleavelin and Marshall³⁶ utilizing longitudinal waves.

The limiting average energy gaps for this work can be determined from Table 1. A value of $(1.48 \pm 0.20)k_B T_C$ was found for $\vec{q} || (0001)$ with $\vec{s} || (10\bar{1}0)$ or with $\vec{s} || (11\bar{2}0)$; $(1.63 \pm 0.10)k_B T_C$ was found for $\vec{q} || (10\bar{1}0)$ and $\vec{s} || (11\bar{2}0)$; $(1.61 \pm 0.30)k_B T_C$ was found for $\vec{q} || (11\bar{2}0)$ and $\vec{s} || (0001)$. The values for $\vec{q} || (0001)$ agree with the previous longitudinal work on this zinc sample.³⁶ However, the values for $\vec{q} || (10\bar{1}0)$ and $\vec{q} || (11\bar{2}0)$ appear to be in disagreement.

J. Leibowitz⁴⁹ has discussed which electrons on the Fermi surface can enter into an interaction with either a longitudinal or a transverse ultrasonic wave. It should be noted that the Fermi velocity is directed normally outward from the Fermi surface. For a longitudinal ultrasonic wave the interaction is weighted toward electrons having components of Fermi velocity perpendicular to the propagation vector ($\vec{q}$). This selectivity increases as the value of $q\ell_e$

becomes much larger than unity. This suggests that the average energy gap and the electronic attenuation of an anisotropic superconductor, such as zinc, could depend on the value of $q\ell_e$. Leibowitz states that the above holds equally well for transverse ultrasonic waves. However, the transverse interaction is more "selective" because some of the electrons can make an additional contribution. In order to do this they must have a component of Fermi velocity parallel to the polarization vector ($\vec{s}$). A comparison can now be made between the longitudinal and transverse measurements of the average energy gaps of the zinc sample. The value for $\vec{q}|| (0001)$ should be the same for either longitudinal or transverse ultrasonic measurements. The transverse value obtained for $\vec{q}|| (10\bar{1}0)$ and $\vec{s}|| (11\bar{2}0)$ should then be higher than the longitudinal value for $\vec{q}|| (10\bar{1}0)$ and lower than the longitudinal value for $\vec{q}|| (11\bar{2}0)$. Also, the transverse value for $\vec{q}|| (11\bar{2}0)$ and $\vec{s}|| (0001)$ should be higher than the longitudinal value for $\vec{q}|| (0001)$ and lower than the longitudinal value for $\vec{q}|| (11\bar{2}0)$. These observations were made by studying the Fermi surface of zinc as presented by Gibbons and Falicov.⁵⁰ The average energy gaps determined in this work do show this effect.

The assignment of multiple average energy gaps for transverse ultrasonic attenuation measurements is very tentative. The values of $\overline{\Delta^*(\vec{q},0)}$ shown in Table 1 should only

be considered as possibilities. If they do indeed exist, they should be attributable to the anisotropy of the Fermi surface of zinc.³⁶

All plots of the total electronic attenuation versus frequency, except $\vec{q}||(\bar{1}1\bar{2}0)$ and $\vec{s}||(\bar{0}001)$, were found to fit the functional form given by Pippard within the indicated error. This plot for $\vec{q}||(\bar{1}1\bar{2}0)$ and $\vec{s}||(\bar{0}001)$ was believed to be the "hump" as observed by Cleavelin and Marshall.³⁶ Claiborne and Morse²⁴ have indicated that $(\alpha_e - \alpha'_e)$ should increase with frequency. This was found to be true for all the data except $\vec{q}||(\bar{1}1\bar{2}0)$ and $\vec{s}||(\bar{0}001)$. No rapid fall regions of attenuation were expected for directions where $q\lambda_e < 1$. However, rapid fall regions of attenuation were observed for $\vec{q}||(\bar{1}1\bar{2}0)$ and $\vec{s}||(\bar{0}001)$ at all frequencies where $q\lambda_e < 1$ except 30.0 MHz. More investigation is indicated to adequately explain this feature of the results.

III.4 Summary

This investigation was undertaken to determine the average superconducting energy gap(s) by transverse ultrasonic attenuation for three propagation directions in a pure, single crystal zinc sample. These values were then to be compared to those measured by longitudinal ultrasonic attenuation for the three propagation directions in the same zinc sample. The two sets of values have been shown to be consistent with each other.

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APPENDIX A

THE t^3 FUNCTIONAL DEPENDENCE

Experimental data obtained in this laboratory from longitudinal ultrasonic attenuation measurements on superconducting lead, zinc, cadmium, and molybdenum have indicated another functional form of the temperature dependence.³⁶ This is given by the following for $t < 0.80$:

$$\frac{\alpha_s}{\alpha_N} = Kt^3 \quad (A-1)$$

where α_s is the superconducting attenuation, α_N is the normal state attenuation at T_c , K is a constant, and t is the reduced temperature ($t = T/T_c$). The value of K appears to be the same for each superconductor. It should be noted that this dependence can also be expressed by $\text{Log}_{10}\{V_{ns}(T)\}$ being proportional to t^3 .

The experimental data obtained from transverse ultrasonic attenuation measurements on superconducting zinc also indicate this dependence. Representative plots of α_s/α_N (as given by Eq.(23)) versus t^3 are shown in Figure 19 for $\vec{q} || (10\bar{1}0)$ and $\vec{S} || (11\bar{2}0)$. The data from other propagation directions and polarizations also showed this t^3 dependence.

Figure 19
 Plot of α_S/α_N Versus t^3 for $\vec{q}||(\overline{1010})$ and $\vec{s}||(\overline{1120})$

APPENDIX B

COMPUTER PROGRAM

The program for the IBM VS/370 computer that was used to analyze the experimental data will be presented here. Statement symbols for the input and output will be defined after the program is given.

```

      DOUBLE PRECISION ANORM(100),DELTA(100),ALPHAN,DBCM(100)
      DOUBLE PRECISION SLOPE3(1000),YNCP3(1000),SIGMA3(1000)
      DOUBLE PRECISION SLOPI3,YNCPF3,SIGMI3,VZCUB,PATLEN,ECONO
      DOUBLE PRECISION VZERO(1000),SIGMA2(1000),SLOPE2(1000),
1YNCP2(1000)
      DOUBLE PRECISION VTCINL,VTCFNL,V(100),G(150),ALPHAS(100),
1ALN(150)
      DOUBLE PRECISION D(100),SLOPE,YNCP,SIGMA,T(100),TIN(100)
      DOUBLE PRECISION SIGMA1,SLOPE1,YNCP1,VZINL,VZFNL,ASQR,
1VTC
      DOUBLE PRECISION A(500),SIGMA0(800),YNCP0(300),SLOPE0
1(100)
      COMMON MAX
      READ 20,NTIMES
20  FORMAT(1I2)
      DO 21 KKN=1,NTIMES
      READ 15
15  FORMAT(80H
1
      )
      READ 1,NPTS,NPT1I,NPT1F,NPT2I,NPT2F,VZINL,VZFNL,VTCINL,
1VTCFNL,ASQR,VZCUB,PATLEN,ECHONO
1  FORMAT(5I2,6F5.2,F8.4,F2.0)
      READ 2,(V(I),I=1,NPTS)
2  FORMAT(13F6.2)
      MAX=NPTS
      T(1)=1.D0
      DO 4 I=1,NPTS
      D(I)=1.7376D0*(1D0-ASQR)*DSQRT(1D0-T(I))*(1D0-(.4095D0-
11.362D0*ASQR)*(1D0-T(I))-(6.26D-2+.318D0*ASQR)*(1D0-T(I))
2**2)
      G(I)=D(I)/T(I)
4  T(I+1)=T(I)-2.D-02
      VTC=VTCINL
      IF(VTCINL-VTCFNL)99,26,99

```

```

99 NN=1
   A(1)=1.D3
   PRINT 40
80 N=2
   SIGMA0(1)=1.D2
   VZERO(N)=VZINL
   ALN(1)=0.0
81 DO 82 II=2,NPTS
   ALPHAN=2.D1*DLOG10(VZERO(N)/VTC)
   ALPHAS(II)=2.D1*DLOG10(VZERO(N)/V(II))
82 ALN(II)=DLOG(2D0*ALPHAN/ALPHAS(II)-1D0)
   CALL FITX(G,ALN,NPT1I,NPT2F,SLOPE,YNCP,SIGMA)
   SLOPE0(N)=SLOPE
   YNCP0(N)=YNCP
   SIGMA0(N)=SIGMA
   PRINT 91,N,SLOPE0(N),YNCP0(N),SIGMA0(N)
91 FORMAT(/1I3,3F10.5)
   IF(SIGMA0(N)-SIGMA0(N-1))83,83,84
83 IF(VZERO(N)-VZFNL)85,86,86
85 VZERO(N+1)=VZERO(N)+5.D-02
   N=N+1
   GO TO 81
84 NN=NN+1
   A(NN)=YNCP0(N-1)
   IF(A(NN))89,86,86
86 IF(VTC-VTCFNL)87,88,88
87 VTC=VTC+1.D-1
   GO TO 80
88 VTC=VTCINL+1.005D-01
   GO TO 89
89 VTC=VTC-1D-1
   GO TO 26
26 M=2
   SIGMA2(1)=1D1
   VZERO(M)=VZINL
   ALN(1)=0.0
9 DO 3 I=2,NPTS
   ALPHAN=2.D1*DLOG10(VZERO(M)/VTC)
   ALPHAS(I)=2.D1*DLOG10(VZERO(M)/V(I))
3 ALN(I)=DLOG(2D0*ALPHAN/ALPHAS(I)-1D0)
   CALL FITX(G,ALN,NPT2I,NPT2F,SLOPE,YNCP,SIGMA)
   SLOPE2(M)=SLOPE
   YNCP2(M)=YNCP
   SIGMA2(M)=SIGMA
   IF(SIGMA2(M)-SIGMA2(M-1))22,22,23
22 IF(VZERO(M)-VZFNL)7,8,8
7 VZERO(M+1)=VZERO(M)+1.D-02
   M=M+1
   GO TO 9

```

```

8  MM=2
   SIGMA3(1)=1.D1
   VZERO(MM)=VZINL
   ALN(1)=0.0
28 DO 29 J=2,NPTS
   ALPHAN=2.D1*DLOG10(VZERO(MM)/VTC)
   ALPHAS(J)=2.D1*DLOG10(VZERO(MM)/V(J))
29 ALN(J)=DLOG(2.D0*ALPHAN/ALPHAS(J)-1.D0)
   CALL FITX(G,ALN,NPT1I,NPT2F,SLOPE,YNCP,SIGMA)
   SLOPE3(MM)=SLOPE
   YNCP3(MM)=YNCP
   SIGMA3(MM)=SIGMA
   IF(SIGMA3(MM)-SIGMA3(MM-1)) 50,52,52
50 IF(VZERO(MM)-VZFNL) 51,57,57
51 VZERO(MM+1)=VZERO(MM)+1.D-02
   MM=MM+1
   GO TO 28
52 PRINT 40
   PRINT 15
   IF(MM.EQ.2) MM=3
   ALPHAN=2.D1*DLOG10(VZERO(MM-1)/VTC)/(PATLEN*ECHONO)
   DO 56 JJ=2,NPTS
   ALPHAS(JJ)=2.D1*DLOG10(VZERO(MM-1)/V(JJ))/(PATLEN*ECHONO)
   DBCM(JJ)=ALPHAN-ALPHAS(JJ)
   ALN(JJ)=DLOG(2.D0*ALPHAN/ALPHAS(JJ)-1.D0)
   ANORM(JJ)=ALPHAS(JJ)/ALPHAN
   TLN(JJ)=T(JJ)*ALN(JJ)
56 DELTA(JJ)=TLN(JJ)/D(JJ)
   PRINT 31,NPT1I,NPT1F,NPT2I,NPT2F,ASQR,VZCUB,PATLEN
   PRINT 54
54 FORMAT(/5X,4H VTC,5X,6H VZERO,4X,7H ALPHAN,4X,7H SLOPE3,
14X,6H YNCP3,4X,7H SIGMA3/)
   PRINT 55,VTC,VZERO(MM-1),ALPHAN,SLOPE3(MM-1),YNCP3(MM-1),
1SIGMA3(MM-1)
55 FORMAT(6F11.4)
   PRINT 13
   PRINT 14,(T(I),DBCM(I),ANORM(I),TLN(I),DELTA(I),G(I),
1ALN(I),I=2,NPTS)
   PRINT 40
   PRINT 15
   PRINT 41
   CALL AGRAPH(G,ALN)
   GO TO 57
57 PRINT 40
   PRINT 15
   PRINT 31,NPT1I,NPT1F,NPT2I,NPT2F,ASQR,VZCUB,PATLEN
   ALPHAN=2.D1*DLOG10(VZCUB/VTC)
   DO 63 K=2,NPTS
   ALPHAS(K)=2.D1*DLOG10(VZCUB/V(K))

```



```

        ANORM(K)=ALPHAS(K)/ALPHAN
63  ALN(K)=DLOG(2.D0/ANORM(K)-1.D0)
    PRINT 13
    PRINT 75,(T(K),G(K),ALN(K),K=2,NPTS)
75  FORMAT(1F10.5,40X,2F10.5)
    PRINT 40
    PRINT 15
    PRINT 41
    CALL AGRAPH(G,ALN)
    GO TO 21
23  IF(M.EQ.2) M=3
    ALPHAN=2.D1*DLOG10(VZERO(M-1)/VTC)/(PATLEN*ECHONO)
    DO 30 I=2,NPTS
    ALPHAS(I)=2.D1*DLOG10(VZERO(M-1)/V(I))/(PATLEN*ECHONO)
    DBCM(I)=ALPHAN-ALPHAS(I)
    ALN(I)=DLOG(2.D0*ALPHAN/ALPHAS(I)-1.D0)
    ANORM(I)=ALPHAS(I)/ALPHAN
    TLN(I)=T(I)*ALN(I)
30  DELTA(I)=TLN(I)/D(I)
    CALL FITX(G,ALN,NPT1I,NPT1F,SLOPE,YNCP,SIGMA)
    SLOPE1=SLOPE
    YNCP1=YNCP
    SIGMA1=SIGMA
    CALL FITX(G,ALN,NPT1I,NPT2F,SLOPE,YNCP,SIGMA)
    SLOPE13=SLOPE
    YNCP13=YNCP
    SIGMA13=SIGMA
    PRINT 40
40  FORMAT(1H1)
    PRINT 15
    PRINT 31,NPT1I,NPT1F,NPT2I,NPT2F,ASOR,VZCUB,PATLEN
31  FORMAT(//8H NPT1I =,1I3,10H    NPT1F =,1I3,10H    NPT2I =,
    11I3,10H    NPT2F =,1I3,10H    ASOR =,1F5.2,10H    VZCUB =,
    21F5.2,11H    PATLEN =,F8.4)
    PRINT 10
10  FORMAT(///120H    VTC    VZERO    ALPHAN    SLOPE1
    1YNCP1    SIGMA1    SLOPE2    YNCP2    SIGMA2    SLOPE3
    2  YNCP3    SIGMA3    )
    PRINT 11,VTC,VZERO(M-1),ALPHAN,SLOPE1,YNCP1,SIGMA1,
    1SLOPE2(M-1),YNCP2(M-1),SIGMA2(M-1),SLOPE13,YNCP13,SIGMA13
11  FORMAT(/12F10.5)
    PRINT 13
13  FORMAT(/4X,2H T,7X,6H DBCM,7X,6H AS/AN,3X,7H TLN( ),4X,
    16H DELTA,2X,7H G(T)/T,5X,6H LN( ))
    PRINT 14,(T(I),DBCM(I),ANORM(I),TLN(I),DELTA(I),G(I),
    1ALN(I),I=2,NPTS)
14  FORMAT(7F10.5)
    PRINT 40
    PRINT 41

```

```

41 FORMAT(60X,33H LN(2ALPHAN/ALPHAS - 1) VS G(T)/T//)
   CALL AGRAPH(G,ALN)
   GO TO 8
21 CONTINUE
   CALL EXIT
   END

SUBROUTINE FITX(GX,ALNX,NPTXI,NPTXF,SLOPEX,YNCPX,SIGMAX)
DOUBLE PRECISION GX(150),ALNX(150),ALNFIT(150)
DOUBLE PRECISION XSUM,YSUM,XXSUM,XYSUM,ANPTS,DEV,SLOPEX,
1YNCPX
DOUBLE PRECISION SIGMAX,DEVSQ,SDEVSQ
XSUM=0.
YSUM=0.
XXSUM=0.
XYSUM=0.
ANPTS=NPTXF-NPTXI+1
DO 5 I=NPTXI,NPTXF
XSUM=XSUM+GX(I)
YSUM=YSUM+ALNX(I)
XXSUM=XXSUM+GX(I)**2
5 XYSUM=XYSUM+GX(I)*ALNX(I)
SLOPEX=(ANPTS*XYSUM-XSUM*YSUM)/(ANPTS*XXSUM-XSUM**2)
YNCPX=(YSUM*XXSUM-XSUM*XYSUM)/(ANPTS*XXSUM-XSUM**2)
DEV=0.
SDEVSQ=0.0
DO 6 I=NPTXI,NPTXF
ALNFIT(I)=SLOPEX*GX(I)+YNCPX
DEV=DEV+DABS(ALNFIT(I)-ALNX(I))
DEVSQ=(DEV**2)
6 SDEVSQ=SDEVSQ+DEVSQ
SIGMAX=DSQRT(SDEVSQ/ANPTS)
RETURN
END

```

AGRAPH(G,ALN) is a plotting subroutine.

Input

NPTS = Number of points

NPT1I = Number of initial point, first interval (high t's)

NPT1F = Number of final point, first interval

NPT2I = Number of initial point, second interval (low t's)

NPT2F = Number of final point, second interval

VZINL = $V_{ns}(0)$ initial

VZFNL = $V_{ns}(0)$ final

VTCINL = $V'_{ns}(T_c)$ initial

VTCFNL = $V'_{ns}(T_c)$ final

ASQR = square of anisotropy factor

VZCUB = $V_{ns}(0)$ as determined by t^3 plot

PATLEN = Sound path length in centimeters

ECHONO = Echo number

V = Values for $V_{ns}(T)$ as t decreases in intervals of 0.02

Output

VTC = Value of $V'_{ns}(T_c)$

VZERO = Value of $V_{ns}(0)$

ALPHAN = Electronic attenuation (α'_e)

SLOPE = Slope

YNCP = Y-intercept

SIGMA = Standard deviation

T = t

DBCM = dB/cm

AS/AN = α_s/α_N

DELTA = Slope

$G(T)/T = G(t)/t$

$LN() = \text{Log}_e(2\alpha_N/\alpha_s - 1)$

APPENDIX C

EXPERIMENTAL DATA

The representative experimental data in this appendix were taken from X-Y recorder plots. It consists of voltage data (V) versus reduced temperature (t) in intervals of 0.02. This data were read into the computer program given in Appendix B. The values obtained from the computer program were presented in Chapter III.

Table C-1

 $\vec{q} || (0001)$ and $\vec{s} || (10\bar{1}0)$ 9.1 MHz Echo No. 3

t	V (volts)	t	V (volts)
1.00	11.30	.72	23.76
.98	16.26	.70	24.12
.96	17.33	.68	24.43
.94	18.21	.66	24.69
.92	18.95	.64	24.95
.90	19.44	.62	25.23
.88	19.95	.60	25.53
.86	20.47	.58	25.85
.84	21.00	.56	26.15
.82	21.54	.54	26.47
.80	22.06	.52	26.72
.78	22.53	.50	---
.76	22.97	.48	---
.74	23.37	.46	---

Table C-2

 $\vec{q} || (0001)$ and $\vec{s} || (10\bar{1}0)$ 30.8 MHz Echo No. 2

t	V (volts)	t	V (volts)
1.00	3.90	.72	12.70
.98	7.00	.70	13.00
.96	8.13	.68	13.28
.94	8.91	.66	13.54
.92	9.32	.64	13.73
.90	9.65	.62	13.99
.88	10.00	.60	14.23
.86	10.32	.58	14.45
.84	10.65	.56	14.73
.82	11.00	.54	14.95
.80	11.34	.52	15.13
.78	11.68	.50	15.27
.76	12.02	.48	15.35
.74	12.36	.46	---

Table C-3

 $\vec{q} || (0001)$ and $\vec{s} || (10\bar{1}0)$ 50.1 MHz Echo No. 1

t	V (volts)	t	V (volts)
1.00	16.90	.72	24.54
.98	19.88	.70	24.77
.96	20.67	.68	24.93
.94	21.20	.66	25.06
.92	21.50	.64	25.22
.90	21.81	.62	25.37
.88	22.17	.60	25.53
.86	22.57	.58	25.71
.84	22.90	.56	25.87
.82	23.23	.54	26.03
.80	23.51	.52	26.18
.78	23.78	.50	26.31
.76	24.05	.48	26.43
.74	24.30	.46	26.52

Table C-4

 $\vec{q} || (0001)$ and $\vec{s} || (10\bar{1}0)$ Annealed 10.1 MHz Echo No. 1

t	V (volts)	t	V (volts)
1.00	26.40	.72	30.70
.98	28.48	.70	30.82
.96	28.88	.68	30.95
.94	29.17	.66	31.07
.92	29.33	.64	31.15
.90	29.50	.62	31.23
.88	29.68	.60	31.31
.86	29.87	.58	31.39
.84	30.00	.56	31.47
.82	30.13	.54	31.55
.80	30.25	.52	31.63
.78	30.36	.50	31.70
.76	30.47	.48	31.72
.74	30.58	.46	---

Table C-5

 $\vec{q} || (0001)$ and $\vec{s} || (10\bar{1}0)$ Annealed 30.7 MHz Echo No. 1

t	V (volts)	t	V (volts)
1.00	15.25	.72	19.88
.98	17.60	.70	20.00
.96	18.08	.68	20.11
.94	18.36	.66	20.20
.92	18.52	.64	20.29
.90	18.67	.62	20.37
.88	18.80	.60	20.43
.86	18.92	.58	20.51
.84	19.03	.56	20.59
.82	19.19	.54	20.66
.80	19.38	.52	20.72
.78	19.53	.50	20.77
.76	19.64	.48	---
.74	19.76	.46	---

Table C-6

 $\vec{q} || (0001)$ and $\vec{s} || (10\bar{1}0)$ Annealed 48.5 MHz Echo No. 1

t	V (volts)	t	V (volts)
1.00	7.00	.72	12.14
.98	9.03	.70	12.33
.96	9.71	.68	12.50
.94	10.01	.66	12.64
.92	10.30	.64	12.76
.90	10.54	.62	12.83
.88	10.78	.60	12.87
.86	10.98	.58	13.05
.84	11.17	.56	13.17
.82	11.34	.54	13.25
.80	11.50	.52	13.38
.78	11.63	.50	13.45
.76	11.78	.48	13.53
.74	11.97	.46	---

Table C-7

 $\vec{q} || (0001)$ and $\vec{s} || (11\bar{2}0)$ Annealed 10.8 MHz Echo No. 2

t	V (volts)	t	V (volts)
1.00	7.30	.72	10.56
.98	8.64	.70	10.67
.96	8.90	.68	10.78
.94	9.09	.66	10.90
.92	9.28	.64	11.00
.90	9.46	.62	11.07
.88	9.64	.60	11.16
.86	9.81	.58	11.25
.84	9.96	.56	11.33
.82	10.08	.54	11.40
.80	10.18	.52	11.45
.78	10.27	.50	11.49
.76	10.35	.48	---
.74	10.46	.46	---

Table C-8

 $\vec{q} || (0001)$ and $\vec{s} || (11\bar{2}0)$ Annealed 29.8 MHz Echo No. 1

t	V (volts)	t	V (volts)
1.00	11.20	.72	17.74
.98	14.00	.70	17.96
.96	14.63	.68	18.18
.94	15.00	.66	18.30
.92	15.35	.64	18.43
.90	15.65	.62	18.57
.88	15.94	.60	18.72
.86	16.23	.58	18.85
.84	16.47	.56	19.01
.82	16.70	.54	19.17
.80	16.89	.52	19.32
.78	17.07	.50	19.45
.76	17.28	.48	19.55
.74	17.50	.46	---

Table C-9

 $\vec{q} || (0001)$ and $\vec{s} || (11\bar{2}0)$ Annealed 48.0 MHz Echo No. 1

t	V (volts)	t	V (volts)
1.00	5.90	.72	10.68
.98	8.06	.70	10.83
.96	8.56	.68	10.99
.94	8.80	.66	11.14
.92	9.00	.64	11.28
.90	9.23	.62	11.38
.88	9.43	.60	11.49
.86	9.63	.58	11.61
.84	9.82	.56	11.72
.82	10.00	.54	11.83
.80	10.15	.52	11.95
.78	10.28	.50	12.04
.76	10.40	.48	12.09
.74	10.53	.46	---

Table C-10

 $\vec{q} || (10\bar{1}0)$ and $\vec{s} || (11\bar{2}0)$ 10.7 MHz Echo No. 8

t	V (volts)	t	V (volts)
1.00	4.40	.72	14.33
.98	6.86	.70	14.75
.96	8.00	.68	15.13
.94	8.87	.66	15.46
.92	9.56	.64	15.74
.90	10.05	.62	16.03
.88	10.58	.60	16.37
.86	11.06	.58	16.80
.84	11.57	.56	17.04
.82	12.05	.54	17.40
.80	12.52	.52	17.75
.78	13.00	.50	18.06
.76	13.45	.48	18.35
.74	13.89	.46	---

Table C-11

 $\vec{q} || (10\bar{1}0)$ and $\vec{s} || (11\bar{2}0)$ 30.6 MHz Echo No. 2

t	V (volts)	t	V (volts)
1.00	4.15	.72	10.19
.98	6.13	.70	10.43
.96	6.75	.68	10.66
.94	7.21	.66	10.87
.92	7.47	.64	11.04
.90	7.72	.62	11.26
.88	8.00	.60	11.46
.86	8.26	.58	11.67
.84	8.57	.56	11.88
.82	8.86	.54	12.07
.80	9.14	.52	12.24
.78	9.44	.50	12.36
.76	9.71	.48	12.42
.74	9.95	.46	---

Table C-12

 $\vec{q} || (10\bar{1}0)$ and $\vec{s} || (11\bar{2}0)$ 50.0 MHz Echo No. 2

t	V (volts)	t	V (volts)
1.00	6.30	.72	20.30
.98	11.10	.70	20.88
.96	12.50	.68	21.48
.94	13.45	.66	21.90
.92	14.10	.64	22.30
.90	15.00	.62	22.78
.88	15.65	.60	23.23
.86	16.23	.58	23.70
.84	16.93	.56	24.16
.82	17.57	.54	24.56
.80	18.17	.52	24.97
.78	18.70	.50	25.34
.76	19.23	.48	25.63
.74	19.76	.46	---

Table C-13

 $\vec{q} || (11\bar{2}0)$ and $\vec{s} || (0001)$ 10.7 MHz Echo No. 1

t	V (volts)	t	V (volts)
1.00	12.50	.72	16.70
.98	14.60	.70	16.80
.96	15.03	.68	16.90
.94	15.38	.66	16.97
.92	15.62	.64	17.03
.90	15.76	.62	17.10
.88	15.90	.60	17.15
.86	16.00	.58	17.20
.84	16.14	.56	17.27
.82	16.23	.54	17.34
.80	16.34	.52	17.41
.78	16.45	.50	17.45
.76	16.56	.48	17.48
.74	16.63	.46	17.55

Table C-14

 $\vec{q} || (11\bar{2}0)$ and $\vec{s} || (0001)$ 30.0 MHz Echo No. 1

t	V (volts)	t	V (volts)
1.00	12.00	.72	16.02
.98	13.10	.70	16.19
.96	13.40	.68	16.35
.94	13.70	.66	16.50
.92	14.00	.64	16.61
.90	14.30	.62	16.78
.88	14.50	.60	16.92
.86	14.75	.58	17.02
.84	14.96	.56	17.12
.82	15.16	.54	17.24
.80	15.37	.52	17.33
.78	15.54	.50	17.43
.76	15.70	.48	17.50
.74	15.88	.46	---

Table C-15

 $\vec{q} || (11\bar{2}0)$ and $\vec{s} || (0001)$ 49.5 MHz Echo No. 1

t	V (volts)	t	V (volts)
1.00	12.90	.72	18.00
.98	14.77	.70	18.18
.96	15.08	.68	18.39
.94	15.37	.66	18.53
.92	15.65	.64	18.65
.90	15.92	.62	18.77
.88	16.22	.60	18.89
.86	16.48	.58	19.03
.84	16.76	.56	19.17
.82	17.00	.54	19.30
.80	17.19	.52	19.40
.78	17.38	.50	19.53
.76	17.60	.48	19.66
.74	17.80	.46	---

Table C-16

 $\vec{q} || (11\bar{2}0)$ and $\vec{s} || (0001)$ 70 MHz Echo No. 1

t	V (volts)	t	V (volts)
1.00	8.20	.72	11.93
.98	10.68	.70	11.99
.96	10.82	.68	12.05
.94	10.96	.66	12.11
.92	11.08	.64	12.15
.90	11.20	.62	12.21
.88	11.29	.60	12.25
.86	11.39	.58	12.30
.84	11.48	.56	12.33
.82	11.57	.54	12.38
.80	11.65	.52	12.40
.78	11.70	.50	12.42
.76	11.79	.48	---
.74	11.88	.46	---

Table C-17

 $\vec{q} || (11\bar{2}0)$ and $\vec{s} || (0001)$ 89.2 MHz Echo No. 1

t	V (volts)	t	V (volts)
1.00	4.90	.72	9.32
.98	6.39	.70	9.44
.96	6.98	.68	9.55
.94	7.42	.66	9.66
.92	7.72	.64	9.75
.90	7.92	.62	9.85
.88	8.13	.60	9.93
.86	8.29	.58	10.02
.84	8.46	.56	10.09
.82	8.60	.54	10.16
.80	8.77	.52	10.24
.78	8.91	.50	10.31
.76	9.05	.48	10.36
.74	9.20	.46	---