A STATISTICAL EVALUATION OF THE CAUSE-EFFECT
RELATIONSHIP BETWEEN MONEY AND INCOME

by

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A DISSERTATION

IN

ECONOMICS

Submitted to the Graduate Faculty
of Texas Tech University in
Partial Fulfillment of
the Requirements for
the Degree of

DOCTOR OF PHILOSOPHY

Approved

Accepted

August, 1974

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1974

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ACKNOWLEDGMENTS

I am deeply indebted to Professor Gene Uselton for his inspiration and guidance during my graduate career and the preparation of this dissertation. Any merit found in this dissertation must be shared with him. The other members of my committee, Professor James Jonish, Professor Robert Rouse, Professor John Wittman and Professor Howard Balsley, freely contributed their time and made many helpful suggestions. Professor Terry Watkins provided many helpful comments on the statistical techniques used in this study.

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CHAPTER I

INTRODUCTION

Statement of the Issue

Economic theoreticians and empirical analysts have long been concerned with the nature of the relationship between the money stock and economic activity or one of its proxies, aggregate income. Four postulates exhaust the possibilities: (1) money and income are not causally related; (2) causality is unidirectional from income to money; (3) causality is unidirectional from money to income; (4) a two-way causal relationship exists between money and income. Few economists claim that money does not matter or that money is purely passive as suggested in the first two postulates. Most economic analyses proceed on the basis of one of the last two. All four possibilities must be considered in an exhaustive study of the causal relationship between money and income.

Importance of the Issue in Monetary Economics

The economic policy significance of the causality issue is implicit in the four postulates. If changes in the money stock had no significant causal effect on income, then monetary policy actions could not help to raise economic welfare (as measured by income levels). Conversely, the monetary authorities could be certain that fluctuations

in income were not responsible for variations in the money stock. On the other hand, if the money stock was solely determined by economic conditions, then the monetary policy makers would be powerless. If, however, changes in the money stock had a significant causal effect on changes in income, then monetary policy actions would be very powerful. If a two-way causal relationship between money and income characterized the economic system, then the monetary policy makers should consider that relationship. A bidirectional causal relationship between money and income would imply neither that monetary policy is powerless nor that it only is effective. Such a relationship would mean that monetary policy would need to be formulated with special consideration of the feedback effects of income on money.

The cause-effect relationship between money and income also has important implications for empirical analysis. Time-series analysis is employed as a means of testing theoretical constructs on which policy recommendations are based and many such analyses make implicit assumptions about cause-effect relationships. The existence and direction of causality are important considerations in regression analysis, a common tool of empirical economic analysis. If money and income had no causal connection, then regressions of income on money or of money on income might identify only spurious relationships that may be unrelated to the

theory being tested. If causality was unidirectional from money to income, then regressing income on money would be a perfectly valid operation for testing the true causal relationship and might elucidate the timing and strength of the connection. Regressions of money on income would be difficult to explain. If causality was unidirectional from income to money, then the reverse would be true--regressions of money on income would be valuable whereas regressions of the reversed relations would be questionable. A two-way relationship between money and income would not negate the analytical value of regressions based on the two variables, but it would create a number of problems which might discredit conclusions based upon such regressions. Specifically if causality does not run one-way from a list of exogenous variables to an endogenous variable, then single-equation estimates of regression coefficients may be seriously biased. The interpretation of such coefficients as being indicative of a causal relationship, such as a temporal distributed-lag pattern, is questionable. Thus; if the assumed causal relationship between money and income is not consistent with the true causal structure, then a correct theory may be rejected on empirical grounds or, failing that, the statistical results may be misleading. Monetary policy actions based on such regressions and lag distributions may be weakened.

Purpose and Scope of Study

The substantive issue of the causal relationship between money and income thus merits an increasing amount of attention by economists. Very little effort has been devoted to this issue in the past, however; the direction of causal influence between money and income has usually been based on untested assumptions. The purpose of this study is to investigate statistically the causal relationship between money and income. Earlier attempts have produced questionable results because of the use of statistical tests that are sensitive to nonnormality. The statistical technique employed in this study obviates this major difficulty.

The basic theoretical framework for the analysis was suggested by C. W. J. Granger (1963, 1969). Causality in the Granger sense is based on the concept of predictability. Within the Granger criterion of causal ordering, two variables are said to be causally related if there is significant correlation between past values of one variable X and that part of another variable Y which cannot be predicted from Y 's own past. The Granger criterion may be used to test the causal relationship between two variables in a distributed-lag model by regressing the endogenous variable on past, current, and future values of the exogenous variable. If causality is unidirectional from the exogenous to the

endogenous variable, then the future coefficients of the exogenous variable should not be significantly different from zero as a group. Relying on the concept that "the future cannot cause the past" (Granger, 1969, p. 428), nonsignificant future coefficients imply an absence of feedback effects from the endogenous to the exogenous variable.

Obviously, the Granger criterion has limitations. First, it is not capable of isolating instantaneous or contemporaneous feedback. If, for example, X_t influences Y_t with a distributed lag but Y_t influences X_t contemporaneously, then this test will not detect the Y to X feedback (Sims, 1972, p. 542). However, most time series are characterized by serially correlated residuals; hence this failing of the test may be unimportant. The Y to X feedback will rarely manifest itself solely contemporaneously. But in the absence of serial correlation in both series, exclusive contemporaneous causality might occur and a single-equation model would misspecify the relationship.

Second, a relationship that is characterized by a simple causal structure (that is, one that lacks instantaneous causality) may appear to be nonsimple because of the length of the data series employed. If the sampling period is so long that the details of causality cannot be separated, then a simple causal structure may appear to be a feedback mechanism (Granger and Hatanaka, 1964, pp. 119-121).

Finally, this procedure is incapable of detecting an omitted variable, another type of specification error. If a third variable influences both of the variables under consideration, then the Granger criterion may yield inconsistent results. The criterion cannot be substituted for sound theory.

Ignoring these philosophical difficulties, the Granger definition of causal ordering is still no more powerful than the statistical technique chosen to evaluate it. The statistical procedure used with the Granger criterion must, therefore, be powerful, discriminating, and robust in view of the possible nonnormality of economic time-series data. A technique which meets these requirements was suggested by Quenouille (1949, 1956) and developed by Tukey (1958) and by Gray and Schucany (1972). The technique, referred to as the "jackknife" because of its wide range of applicability, is a means of reducing the bias in a statistic and for producing approximate confidence intervals on the asymptotically normal "jackknife" statistic.

The jackknifing technique has one major disadvantage: it requires a large number of computations for any given statistical analysis. Therefore, jackknifing is feasible only if a modern high-speed digital computer and an efficient analysis program are available. Since no such computer program was available, it was necessary to develop

one. This jackknifed multiple regression (JLS) program is presented in Appendix A to this study in order to put the jackknifing technique within easy reach of other researchers.

To summarize, the objectives of the present study are:

1. to identify the theoretical and empirical issues involved in the debate of the causal relationship between money and income,
2. to adopt the Granger criterion of causal ordering as a means of evaluating causal relationships,
3. to demonstrate the need for a more discriminating statistical test than the ones which have been used with the Granger criterion and to offer the jackknifing technique as an attractive alternative,
4. to develop a computer program which combines ordinary least-squares regression with the jackknifing procedure,
5. to demonstrate the superiority of the tests associated with the jackknifing procedure over conventional hypotheses tests, and
6. to explore the implications of the results of the empirical tests for economic analysis and policy recommendations.

Organization and Methodology

A study of the relationship between money and income should be organized within the context of sound economic theory. Chapter II deals with some of the theoretical frameworks which underlie the hypothesized causal relationships between money and income. It is argued that conflicting theories of causality often follow from the simplifying assumptions made to complete the system of equations. The causal connection between money and income is discussed with reference to the simple and modern versions of the quantity theory of money and the simple and modern versions of the income-expenditure theory.

Numerous studies have dealt with the relationship between money and income, although few have treated the precise issue of causality. The review in Chapter III of the major empirical studies attempts to clarify the main issues involved and to demonstrate the need for a test for causal influence. These empirical studies are classified in terms of the alternative theoretical frameworks presented in Chapter II.

The nature of the concept of causality is examined in Chapter IV in order to demonstrate the intuitive appeal of the criterion of causal ordering suggested by C. W. J. Granger. Against this background, the Granger criterion is presented and the importance of evaluating the

significance of groups of regression coefficients with reference to the Granger criterion is emphasized. The last two sections of Chapter IV discuss the problems of hypothesis testing in linear regression models which may arise due to the possible nonnormality of economic time-series data. The possible lack of robustness of conventional hypothesis-testing techniques is explored by examining the relevant literature. As an alternative technique, the jackknifing procedure is presented and developed in terms of its use in linear multiple regression. The hypothesis test associated with the jackknife statistic is evaluated vis-à-vis the conventional F and t tests by means of computer simulation experiments.

Chapter V tests the alternative hypotheses suggested by the four theoretical constructs discussed in Chapter II. The Granger criterion is used as a basis for identifying causal influence; the jackknife statistic is employed in the statistical tests. The null hypothesis is that causality is unidirectional from money to income. The alternative hypotheses are (1) causality is unidirectional from income to money, and (2) a two-way causal relationship exists between money and income.

Chapter VI is an evaluation of the results of the empirical tests and of their relationship to previous studies of causality. The chapter concludes the study

with policy recommendations and suggestions for further work in cause-effect identification in macroeconomics.

The jackknife least-squares multiple regression program is presented in Appendix A. Appendix B is an algorithm by which jackknifed regression coefficients may be obtained from ordinary least-squares estimates.

CHAPTER II

A THEORETICAL FRAMEWORK FOR ANALYZING THE CAUSAL RELATIONSHIP BETWEEN MONEY AND INCOME

Empirical studies of the cause-effect relationship between money and income are based on the assumption that a theoretical connection exists between the two variables. Such a mechanism transmits variations in one variable to the other so as to produce either a one-way causal relationship or a mutual interaction of the two variables. However, few of the empirical studies of the causality issue have specified the theoretical model which underlay their research. The lack of theoretical foundations in the causality debate has weakened the impact of the empirical results and has clouded the nature of the issues involved. The need for theoretical models in the debate was recognized by Friedman (1965) when he noted that: ". . . no assortment of facts, however numerous, have any explanatory value unless they can be organized by a theory [p. xxvi]."

The purpose of this chapter is to specify the general framework for what may be a "common model" and to explore the differences between economists which may account for the different causal interpretations of the model.¹ The

¹The idea of developing a model which contains elements common to most economists was suggested by Milton Friedman

basic model dealt with in this chapter is static. Although static models cannot be used to explain the adjustment mechanisms in a complex economy, they do offer insights into the relationships between key variables which are expected to be important in the real world. To date, dynamic models have added little to the modest accomplishments of comparative statistics.

A Simple Structural Model

A simple short-run, income determination, macroeconomic model can be stated as six behavioral relationships and identities:

$$\frac{C}{P} = c\left(\frac{Y}{P}, r\right), \quad (2-1)$$

$$\frac{I}{P} = i(r), \quad (2-2)$$

$$\frac{Y}{P} = \frac{C}{P} + \frac{I}{P}, \quad (2-3)$$

$$\frac{M^D}{P} = m^d\left(\frac{Y}{P}, r\right), \quad (2-4)$$

(1970, 1971). Since Friedman's original work was published, many economists have argued that his model is not "common" at all (for example, see Brunner and Meltzer, 1972). These objections notwithstanding, the general framework suggested by Friedman is useful as a means of classifying the basic differences among economists regarding the causality debate. The model presented in the present chapter is similar to Friedman's "common model," although the analysis of the model is an extension of Friedman's work.

$$M^S = m^S(z, w), \quad (2-5)$$

$$M^D = M^S. \quad (2-6)$$

The first three equations represent the real goods sector; the last three, the money market. Equation (2-1) is the consumption function which states that real consumption is a function of real income and the interest rate. Equation (2-2) is the investment function which states that real investment is a function of the interest rate. The capital stock is not included in this equation since the model is based on the short-run which assumes a fixed capital stock. Equation (2-3) is the income identity defining real income to be equal to the sum of real consumption and real investment. It may be viewed as the equilibrium, market-clearing equation for the real goods sector.

Equation (2-4) is the so-called liquidity preference function stating that the demand for real balances is a function of real income and the interest rate. Equation (2-5) is the supply function of nominal money. It states that the money supply is determined by a set of factors, z , which are directly controlled by the Federal Reserve System and a set of factors, u , which are outside their control. Equation (2-6) is the money market clearing equation which states that the supply of and the demand for money are equal in equilibrium.

A Reduced Form Model

There are, of course, other markets which could be included in the model. In particular, the bond market is not explicit in this system. It has become a generally accepted practice to exclude one market from the system by virtue of Walras' Law (which states that if $n-1$ of n interrelated markets are in equilibrium, then the remaining market must be in equilibrium). However, even such a simplified model is hardly ever used. Instead, the model is reduced to a smaller set of equations by substituting where possible and by making certain assumptions. It has been argued by Friedman that the assumptions made in order to complete the system of equations constitute a fundamental theoretical point of difference between economists (Friedman, 1970, p. 234).

A reduced form model similar to that suggested by Friedman (1970, 1971) may be stated as:

$$y = c(y, r) + i(r), \quad (2-7)$$

$$M = P \cdot m(y, r), \quad (2-8)$$

$$Y = P_y, \quad (2-9)$$

where M is the nominal exogenously determined money stock.²

²The money stock is assumed to be exogenous. This assumption implies that disturbances to the monetary system

Equation (2-7) is the IS curve which is the locus of all combinations of real income and interest rates which are consistent with equilibrium in the real goods and services market. Equation (2-8) is the LM curve which is the locus of all combinations of real income and interest rates which are consistent with equilibrium in the money market. Equation (2-9) is an identity which relates nominal income (Y) to real income (y) and the price level (P). The assumed derivatives are $0 < c_y < 1$, $c_r < 0$, $i_r < 0$, $m_y > 0$, $m_r < 0$. The model contains four endogenous variables-- Y , P , r , and y --and three equations. It is, therefore, indeterminate.

The Simple Quantity Theory Formulation

One way to complete the system of equations is to assume that there exists a vast system of additional but unspecified relationships (a complete Walrasian system of demand and supply equations) which determine real income. This assumption may be viewed as the distinguishing characteristic of the simple quantity theory version of the

are the result of actions by the monetary authorities only. Since the model is a static equilibrium model, it must, by definition be at an initial position of rest. Thus, exogenous shocks to the system must emanate from the monetary authorities. Such an assumption does not preclude the existence of offsetting or bidirectional feedback effects from income to money which may be generated in response to autonomous changes in the nominal money stock. The exogeneity of money assumption is often (mistakenly) construed as an assumption of one-way causality from money to income.

"common" macro model (Friedman, 1970, p. 219). With real income determined outside the system, the model has a solution. Equation (2-7) can be solved for the equilibrium interest rate:

$$y_0 - c(y_0, r) = i(r) = r_0, \quad (2-10)$$

where y_0 is the predetermined level of real income or real output and r_0 is the equilibrium interest rate. Equation (2-8) can then be solved for the equilibrium price level:

$$P = \frac{M}{m(y_0, r_0)}, \quad (2-11)$$

which is the classical quantity equation as seen by dividing the numerator and the denominator of the right-hand side of (2-11) by y_0 and replacing $m(y_0, r_0)/y_0$ by its equivalent, $1/V$ (Friedman, 1970, p. 220). Dropping the subscripts, this manipulation yields the popular expression of the quantity theory of money:

$$M = \frac{Py}{V}. \quad (2-12)$$

By this formulation, there is no causal relationship between money and real income or money and the interest rate since both are determined by "real" variables. This version represents, therefore, the classical dichotomy.

Additional insight into the model can be obtained by differentiating (2-12) with respect to changes in nominal money, M , and expressing the result in terms of elasticities (Teigen, 1972):

$$E(V,M) = E(y,M) + E(P,M) - 1, \quad (2-14)$$

where $E(V,M)$, $E(y,M)$ and $E(P,M)$ represent the elasticity of velocity, the elasticity of real income, and the elasticity of prices, respectively, with respect to changes in the nominal money stock. These total elasticities (denoted by E) take into account the response of the entire structure as distinguished from partial elasticities (denoted by ϵ) which are calculated from one function only. Equation (2-14) is useful in later analysis.

Differentiating equation (2-7) with respect to the nominal money stock yields:

$$\frac{dy}{dM} = \frac{\partial c}{\partial Y} \frac{dy}{dM} + \left(\frac{\partial c}{\partial r} + \frac{\partial i}{\partial r} \right) \frac{dr}{dM}. \quad (2-15)$$

However, if y is a predetermined variable so that $\frac{dy}{dM} = 0$, then it must be true that $\left(\frac{\partial c}{\partial r} + \frac{\partial i}{\partial r} \right) \frac{dr}{dM} = 0$, which implies that $\frac{dr}{dM} = 0$. Differentiating (2-8) with respect to the nominal money stock yields:

$$\frac{1}{P} - \frac{M}{P^2} \frac{dP}{dM} = \frac{\partial m}{\partial y} \frac{dy}{dM} + \frac{\partial m}{\partial r} \frac{dr}{dM}. \quad (2-16)$$

If $\frac{dy}{dM} = \frac{dr}{dM} = 0$, then equation (2-16) reduces to

$$\frac{1}{P} = \frac{M}{P^2} \frac{dP}{dM}$$

or

$$\frac{M}{P} \frac{dP}{dM} = E(P, M) = 1. \quad (2-17)$$

Substituting this result into equation (2-14) yields $E(V, M) = 0$. Thus, under the assumption made to close the system for the simple quantity theory, the velocity of money is not responsive to changes in the nominal money stock. The result of a stable monetary velocity is consistent with statements by Friedman that velocity may be considered to be a stable function of a specified set of variables (Friedman, 1956). Since velocity is the reciprocal of the demand function for real balances, a stable velocity implies a stable demand for money function. Such a demand function creates a very tight unidirectional relationship between money and nominal income, although the possibility of long-run feedback effects of income on money is not excluded. If, however, the additional assumption of an interest inelastic money demand function is made, then one-way causal influence from money to income is guaranteed.³ The interest elasticity of the

³Critics have attributed to the monetarists an assumption of zero interest elasticity of the demand for money function. Friedman has denied that he ever made such an assumption (Friedman, 1966).

demand for money function is, therefore, an important part of the causality issue and accounts for the great deal of attention that has been devoted to empirical investigations of money demand.⁴ Note, however, that the general result of the simple quantity theory formulation does not depend on any assumptions about the interest elasticity of the demand for money. The result requires only that the system be in equilibrium with all markets clearing.

The Simple Income-Expenditure Formulation

If the system of equations is completed by assuming that prices are constant, as in the simple income-expenditure case (Friedman, 1970, p. 220), then a sequential solution is not possible. Solving the partial derivatives with respect to M of equations (2-7), (2-8), and (2-9) simultaneously for

⁴The nature of the demand for money function is also important from the standpoint of policy recommendations. The combination of a zero interest elastic money demand function with an exogenously determined money stock produces the case in which permanent changes in income are not possible unless supported by changes in the nominal money stock. Under such conditions, increases in government expenditures financed by a bond issue simply increase interest rates and reduce private consumption by an amount equal to the increase in government expenditure so that total expenditure remains constant. Such an effect, referred to as the "crowding-out of private expenditure," has long been recognized (for example, see Keynes, 1936, pp. 119-120) and has been relied on to demonstrate the ineffectiveness of fiscal policy (Spencer and Yohe, 1970).

$\frac{dy}{dM}$ (under the assumption that $P = P_0$) and expressing the result in elasticities⁵ yields:

$$\frac{M}{Y} \frac{dy}{dM} = E(y, M) = \frac{1}{V} \frac{1}{\epsilon(l, y) - \left[\frac{(\epsilon(c, y) - 1) \cdot \epsilon(m, r)}{(\epsilon(c, r) + \epsilon(i, r))} \right]}$$

which may be rewritten as:

$$\frac{M}{Y} \frac{dy}{dM} = E(y, M) = \frac{1}{V} \frac{1}{\epsilon(m, y) + \left[\frac{\epsilon(s, y) \cdot \epsilon(m, r)}{\epsilon(c, r) + \epsilon(i, r)} \right]} \quad (2-18)$$

by virtue of the identity $\left(1 - \frac{\partial c}{\partial y} = \frac{\partial s}{\partial y}\right)$, where s stands for saving. Equation (2-18) expresses the elasticity of real income with respect to changes in the nominal money stock. The first term in the denominator is assumed to be positive. The numerator of the term in brackets is negative since it is the product of a positive elasticity, $\epsilon(s, y)$, and a negative elasticity, $\epsilon(m, r)$. Likewise, the denominator of the bracketed term is assumed to be negative since it is the sum of two negative elasticities. Thus, the term in brackets is a positive quantity, making the entire

⁵The concept of partial elasticity is introduced in order to take into consideration that the variables may not be measured in the same units. Each partial derivative is converted to a partial elasticity by applying the following formula to each: $\epsilon(y, x) = \frac{x \partial y}{y \partial x}$.

expression for $E(y,M)$ positive. Substituting this result into equation (2-14) produces:

$$E(V,M) = E(y,M) - 1, \quad E(y,M) > 0, \quad (2-19)$$

since $E(P,M)$ is assumed to be zero. If $E(y,M)$ is greater than or equal to unity, then $E(V,M)$ will be positive⁶ or zero, respectively, implying that causality is unidirectional from money to income. If $E(y,M)$ is less than unity, then $E(V,M)$ will be negative; that is, changes in the money stock will produce opposite changes in velocity and a tendency for offsetting feedback effects to be generated, resulting in a two-way causal relationship between money and income.

The implications of the simple income-expenditure model for the causality issue are mixed and depend, in the final analysis, on the interest elasticity of the demand for money. As long as the demand for money is interest elastic, the possibility of offsetting or feedback effects from income to money exists. The causality issue therefore reduces to an empirical question under the simple income-expenditure theory.

⁶This result is consistent with the work by Baumol (1952). Under Baumol's assumptions, the average transactions velocity of circulation will vary exactly in proportion with the money stock (Baumol, 1952, p. 551).

The Modern Income-Expenditure Formulation

The foregoing simple income-expenditure version of the short-run macroeconomic model has been classified by some economists as a "vulgar" version of post-Keynesian thinking. Many writers in the post-Keynesian tradition reject the idea that the assumption of rigid prices is an integral part of their analytical framework.⁷ The modern Keynesians emphasize the imperfections in the labor market. The usual assumption made in connection with the labor market is that money wages are inflexible downwards. Instead of assuming that prices are fixed as a means of closing the macro-model, modern Keynesians introduce an aggregated labor market into the analysis. This addition may be viewed as the Keynesian equivalent to the Walrasian system of equations (Teigen, 1972, p. 15). The inclusion of the labor market in the macroeconomic model means that real output is an endogenous variable, reducing the system to three unknowns.

A simple aggregated labor market may be represented by three equations (Teigen, 1972):

$$y = e(N), \quad (2-20)$$

$$N = n(w), \quad (2-21)$$

⁷ Examples of modern post-Keynesian static models in which the price level is an endogenous variable may be found in Bailey (1971, Chapter 3), Modigliani (1963), and Smith (1956).

$$w = \frac{\bar{W}}{P}. \quad (2-22)$$

Equation (2-20) represents an aggregate production function which states that real output or income is a function, e , of the level of employment. The assumed derivative is $e_N > 0$ with $e_N' < 0$. Equation (2-21) is a labor market summary equation which states that the supply of labor is infinitely elastic over a wide range of employment at the prevailing money wage rate so that the level of employment is determined by the demand for labor. The assumed derivative is $n_w < 0$. Equation (2-22) is a definition which states that the real wage rate is the ratio of the money wage rate to the price level.

The complete macroeconomic model under the modern post-Keynesian formulation therefore consists of the original IS-LM equations (2-7) and (2-8) plus the labor market equations (2-20), (2-21), and (2-22). The response of this system to changes in the money stock is found by differentiating each equation with respect to changes in the nominal money stock. The differentiation and simultaneous solution of the labor market relations yields:

$$\frac{dy}{dM} = - \frac{\partial e}{\partial N} \frac{\partial n}{\partial w} \left(\frac{\bar{W}}{P^2} \frac{dP}{dM} \right). \quad (2-23)$$

The first term on the right-hand side of equation (2-23) represents the response of real output to a change in

employment, the second term represents the response of employment to a change in the real wage, and the third term may be viewed as the elasticity of prices with respect to the money stock times the constant term ($\bar{W}/PM$). Solving (2-15), (2-16), and (2-23) simultaneously for $\frac{dy}{dM}$, multiplying both sides by $\frac{M}{y}$ in order to express the result as an elasticity, and expressing the partial derivatives as partial elasticities produces:

$$\frac{M}{y} \frac{dy}{dM} = E(y, M) = \frac{1}{\bar{V}} \frac{1}{\left[\epsilon(m, y) - \frac{1}{\epsilon(e, n) \cdot \epsilon(n, w)} \right] + \left[\frac{\epsilon(s, y) \cdot \epsilon(m, r)}{\epsilon(c, r) + \epsilon(i, r)} \right]}. \quad (2-24)$$

The first term in the first set of brackets in the denominator of equation (2-24), $\epsilon(m, y)$, is assumed to be positive. The second term in these brackets is expected to be negative since $\epsilon(e, n)$ is assumed to be positive and $\epsilon(n, w)$ is assumed to be negative. The algebraic sum of the two terms yields a positive quantity for the first bracketed term. As discussed above, the second bracketed term is also assumed to be positive. Thus, the entire expression for the elasticity of real income with respect to the nominal money stock is positive--the same result as found with the simple income-expenditure theory.

Solving equations (2-15), (2-16), and (2-23) simultaneously for $\frac{dP}{dM}$ and expressing the result in terms of elasticities yields:

$$\frac{M}{P} \frac{dP}{dM} = E(P, M) = - \frac{1}{\frac{P\bar{W}}{M} [\epsilon(e, n) \cdot \epsilon(n, w)] \left[\frac{\epsilon(s, y) \cdot \epsilon(m, r)}{\epsilon(c, r) + \epsilon(i, r)} + \epsilon(m, y) \right] - 1}. \quad (2-25)$$

The first term in brackets in the denominator of equation (2-25) is expected to be negative since $\epsilon(e, n)$ is positive and $\epsilon(n, w)$ is negative. As in equation (2-24), the second term in brackets is expected to be positive, although this expression now contains an additional positive term, $\epsilon(m, y)$. The product of the two terms in brackets is therefore negative. The algebraic sum of the resulting negative product and the negative unity term will yield a negative denominator. When multiplied by the leading negative sign, the result is positive. Thus, the elasticity of prices with respect to the nominal money stock will be positive unless either $\epsilon(e, n)$ or $\epsilon(n, w)$ is zero, in which case $E(P, M)$ will be unity.

The combination of equations (2-24) and (2-25) permits the evaluation of the elasticity of velocity with respect to the nominal money stock, $E(V, M)$. Substituting these two equations into the expression for $E(V, M)$, equation

(2-14), reveals that $E(V,M)$ now depends on all the partial elasticities in the system. If either $\varepsilon(e,n)$ or $\varepsilon(n,w)$ is zero, then real output will not be responsive to changes in the real wage brought about by changes in the nominal money stock; hence, $E(y,M) = 0$ and $E(P,M) = 1$, resulting in $E(V,M) = 0$, a stable velocity situation. If either $\varepsilon(e,n)$ or $\varepsilon(n,w)$ is extremely large, then $E(P,M)$ approaches zero. (This would be the expected result if the system was assumed to be initially at a situation of high unemployment.) The response of $E(V,M)$ in this case now depends on the behavior of $E(y,M)$. An examination of equation (2-24) reveals that the larger the absolute value of $[\varepsilon(s,y) \cdot \varepsilon(m,r)]$ relative to the absolute value of $[\varepsilon(c,r) + \varepsilon(i,r)]$, the more likely is $E(y,M)$ to approach zero and $E(V,M)$ to be negative. The same is true for large values of $\varepsilon(m,y)$. That is to say, the larger the elasticity of saving or money demand with respect to real income or the larger the elasticity of the demand for money with respect to interest rates, the more likely it is that offsetting feedback effects will result from changes in the money stock. In general, whether $E(V,M)$ is positive or negative depends on whether $E(y,M)$ is greater or smaller than unity, which, in turn depends on the relative magnitudes of the partial elasticities which compose it.

As with the simple quantity theory and the simple income-expenditure formulations, the degree to which money can be expected to exert a significant unidirectional causal effect on income depends, to a large extent, on the empirical behavior of the demand for money function. Contrasted with the previous two versions of the model, the modern income-expenditure formulation is quite complex. Whether or not unidirectional causal influence is from money to income depends on the relative magnitudes and signs of a great many partial elasticities. On the basis of the theoretical framework employed by modern post-Keynesians, it is no wonder that they view the issue of the causal influence of money on income as one wrought with a great many theoretical and empirical problems. Their general conclusion that causality is probably not unidirectional from money to income is understandable, for there are a great many places for the relationship between money on income to falter.

The Modern Quantity Theory Formulation

Each of the approaches discussed above is based on at least one restrictive assumption and is therefore subject to question. In an attempt to circumvent the problems associated with these versions of the "common model," Friedman has offered still another way of completing the systems of equations. He referred to the approach as a

theory of nominal income (Friedman, 1971, pp. 34-40).

Friedman views his approach as a theory which embodies the key ideas from the two main schools of thought in economics and, as such, constitutes an intermediate theory vis-à-vis the others. The idea borrowed from Fisher is that the market rate of interest is composed of the real rate of interest plus an adjustment for the rate of change in prices. The Keynesian contribution is largely determined by the rate that is expected to prevail over a longer period. The combination of these two ideas produces the result that the difference between the anticipated real rate of interest and the anticipated real rate of growth is determined outside the system. Friedman assumed that the current market rate of interest and the expected market rate are kept equal by the actions of asset holders, that is, their difference is constant. By assuming that the expected real rate of interest, the expected rate of growth of real output, and the expected rate of growth of nominal income are all determined outside the system, the market rate of interest is also determined outside the system. Thus, the modern quantity theory version assumes that the exogenous variable is the market rate of interest, although the assumption is not explicit.

In order to investigate the implications of this assumption on the reduced-form model, the three equations

(2-7), (2-8), and (2-9) are once again differentiated with respect to the nominal money stock. The result is equation (2-15), equation (2-16), and equation (2-26):

$$\frac{dY}{dM} = P \frac{dy}{dM} + Y \frac{dP}{dM}. \quad (2-26)$$

Solving the three equations simultaneously for $\frac{dy}{dM}$ and expressing the result as an elasticity produces the result $E(y, M) = 0$. This result is consistent with Friedman's assumption that the real rate of growth is determined by real factors--factors that are outside the system of equations. Hence, one result of assuming that the rate of interest is an exogenous variable is to produce a version of the classical dichotomy.

Solving the three equations simultaneously for the change in nominal income with respect to the nominal money stock yields:

$$\frac{dY}{dM} = V, \quad (2-27)$$

the same result found by differentiating the quantity theory expression, $Y = MV$, with respect to M . The result implies that velocity does not respond to changes in money so that there are no feedback effects from income to money. Equation (2-27) demonstrates that the result of the assumptions made in the modern quantity theory version is the

establishment of a direct link between nominal money and nominal income.

Solving the three equations for the change in prices with respect to the nominal money stock and expressing the result as an elasticity yields:

$$E(P,M) = \frac{M}{P} \frac{1}{y} = \frac{1}{V}. \quad (2-28)$$

Equation (2-28) states the relationship which determines the elasticity of prices with respect to nominal money. It indicates that both the current level of real income and the real value of the nominal money stock are important determinants of $E(P,M)$. If asset holders are assumed to be rational and attempt to maintain a fairly constant real value of cash balances, then $E(P,M)$ can be expected to be a fairly constant positive quantity.

In terms of the equation for the elasticity of velocity with respect to nominal money,

$$E(V,M) = E(y,M) + E(P,M) - 1,$$

the modern quantity theory produces the result:

$$E(V,M) = 0 + E(P,M) - 1, \quad 0 < E(P,M) < 1. \quad (2-29)$$

Since $0 < E(P,M) < 1$, $E(V,M)$ will be a small negative quantity between zero and minus one and approaching zero as velocity decreases. Thus, equation (2-29) indicates

that changes in velocity induced by autonomous changes in nominal money may be expected to exert small offsetting effects on the changes in M . However, the link between money and income will still be direct since the value of $E(V,M)$ is approaching zero. The strongest unidirectional effects occur when $E(V,M) = 0$, a limiting case; $E(V,M)$ can never be positive. The result of this analysis can be regarded as the theoretical basis on which Friedman stated ". . . the effect on [velocity] is empirically not to absorb the change in M , as the Keynesian analysis implies, but often to reinforce it . . . [Friedman, 1970, p. 217]," although the last part of the statement is not justified by his theoretical framework.

A Comparison of the Four Models

The four methods to make the "common model" determinate form the theoretical basis for either unidirectional or bi-directional causality between money and income. The variations of the basic model clearly indicate that the division of the two main schools of thought on the causality issue is correctly classified as either Keynesian or monetarist. The implications of causal influence are inherent in their respective theoretical frameworks. It is, of course, an oversimplification to assert that the method by which the two schools of thought complete a set of equations is the

only difference between them. There are many other theoretical and philosophical differences which are important for some issues but relatively unimportant for the causality issue. It is important to note that the foregoing theoretical models were based on the assumption of an exogenously determined money stock. Thus, all the results of the models must be interpreted very carefully. The two quantity theory versions stated that an autonomous change in the nominal money stock can be expected to exert a one-way causal influence on income in the short-run. The absence of offsetting feedback effects was found to be due to a zero money elastic (or nearly so) velocity function which produced a stable demand for money function. As long as the demand for money remains stable, feedback effects are not immediately generated in response to monetary stimuli.

The mechanism which could cause a feedback effect under the income-expenditure versions was a velocity function that was elastic with respect to the nominal money stock. Under the two income-expenditure formulations, a stable velocity and demand for money function could exist, but the necessary conditions were numerous and, therefore, not expected to prevail in the real world.

Thus, all four models are dependent on the nature of the demand for money function, an empirical issue. The

importance of money demand in this context justified the vast amount of empirical work that has been carried out in the area. Unfortunately, the results of these studies are largely inconclusive, although they have established that the case for a zero interest elastic money demand function is not corroborated by the data. But since the quantity theory versions do not rely on an interest inelastic demand function anyway, these empirical results have not permitted any of the theoretical models to be rejected or corroborated.

There are certain factors not embodied in the various reduced-form models that are also important in the causality issue. Another important determinant of the extent to which monetary impulses may or may not be empirically independent of economic activity is the nature of the process by which the nominal money stock is determined. In all of the models, it was assumed that the monetary authorities could exert absolute control over the nominal money stock. A theoretical framework which explicitly considers the determinants of the money stock is discussed in the following section.

A Simple Model of the Money Supply Process

A significant contribution to the money supply literature was by Brunner and Meltzer (1964; 1968). The derivation of a theoretical framework of the money supply

process similar to theirs depends only on a few basic definitions. First, the money stock, $M1$, is defined to be equal to the product of the money supply multiplier, m , and the monetary base (an asset supplied by the Federal Reserve and the Treasury to the economic units that make up the economy). Thus, the multiplier may be stated as:

$$m = \frac{M1}{B}. \quad (2-30)$$

Second, the money stock is defined as cash in hands of the public, C^P , plus demand deposits, D^P , and the monetary base is defined as total reserves, R , plus C^P . Thus:

$$m = \frac{C^P + D^P}{R + C^P}. \quad (2-31)$$

Third, total reserves is defined to be the actual fraction of reserves R times the sum of D^P plus time deposit liabilities, T . Equation (2-31) may, therefore, be rewritten as:

$$m = \frac{C^P + D^P}{R(D^P + T) + C^P}. \quad (2-32)$$

Dividing equation (2-32) by D^P yields:

$$m = \frac{\frac{C^P}{D^P} + 1}{R \left[1 + \frac{T}{D^P} \right] + \frac{C^P}{D^P}}. \quad (2-33)$$

Fourth, the actual fraction of reserves is defined to be equal to the sum of total required reserves and total excess reserves expressed as ratios to the sum of the public's deposit claims ($D^P + T$), denoted as r and e , respectively. The ratio of currency in hands of the public to D^P is denoted by k . Thus:

$$m = \frac{1 + k}{(r + e)(1 + t) + k}. \quad (2-34)$$

Finally, if the deposits of the Treasury and bank borrowing are included (both expressed as ratios to private demand deposits, denoted by d and b respectively), the expression for the money multiplier becomes:

$$m = \frac{1 + k}{(r + e - b)(1 + t + d) + k}. \quad (2-35)$$

The expression for $M1$ is therefore:

$$M1 = \frac{1 + k}{(r + e - b)(1 + t + d) + k} \cdot B. \quad (2-36)$$

The importance of the expression for the money multiplier lies in the insight it offers into the nature of the

money supply process. Equation (2-36) is a specific form of the functional relationship given in equation (2-5). The monetary base is the specific counterpart of the functional z in (2-5) and the expression for the money multiplier is the specific counterpart for the functional u , the factors which are outside the control of the monetary authorities. Partial differentiation of (2-35) with respect to each of its parameters gives the direction of influence each is expected to exert on the money multiplier. Letting m_j denote the partial derivative of the money multiplier with respect to the j^{th} component, the results are as follows:

$$m_k < 0, \frac{8}{7} m_r < 0, m_e < 0, m_b > 0, m_t < 0, m_d < 0.$$

Thus, a change in the monetary base will tend to generate offsetting changes in the money multiplier and therefore in the money stock as the public adjusts its currency ratio and its time deposit ratio and as the banking public adjusts its excess reserves and its borrowing position. The extent to which these adjustments actually offset the actions of the monetary authorities is an empirical question. The magnitude of the behavioral reactions of economic units to a

⁸This result does not follow directly from the differentiation which is indeterminate. Empirical data are required to corroborate the result (Burger, 1971, p. 35).

monetary disturbance (such as an injection of base money) determines whether or not monetary actions have a unidirectional causal effect on economic activity. This theory clearly indicates that some feedback effects can be expected to exist.

The theory of the money supply process emphasizes that there are determinants of the money stock which are outside the control of the monetary authorities. Empirically, however, the variables which comprise the money multiplier are not of a sufficient magnitude to render the monetary authorities powerless; the money stock need not necessarily be completely passive because of the public's attempts to keep their portfolio in balance. Empirical studies have demonstrated that changes in the monetary base dominate changes in the money stock and that the money multiplier behaves in a predictable manner (for examples, see Cagan, 1965, Chapter 2; Meltzer, 1969; Kalish, 1970). Since the monetary base does dominate changes in the stock and since approximately three-fourths of the monetary base is composed of Federal Reserve owned government securities, the monetary authorities do have the ability to control the nominal money stock. The ability of the authorities to control the money stock is not, however, the essence of the causality issue. Rather, the issue is whether or not significant feedback effects are generated in the economy

in response to changes in the money stock. The four theoretical models were found to be inconclusive on the issue since the validity of each depends on the empirical relevance of a simplifying assumption. Empirical studies which relate to the causality issue are reviewed in Chapter III.

CHAPTER III

EMPIRICAL STUDIES OF THE CAUSAL RELATIONSHIP

BETWEEN MONEY AND INCOME: A SURVEY

A strong impetus for much of the recent interest in the issue of the causal relationship between money and income was the publication of a comment by John Culbertson on the work of Milton Friedman and Friedman's response (Culbertson, 1960; Friedman, 1961). In his attempt to sort out the issues implied by the lag in effect of monetary policy as suggested by Friedman (Friedman, 1959, p. 87), Culbertson isolated the issue of causality as a keystone in the debate. He noted: "The Friedman analysis implies that monetary change has been an exogenous variable and that causation runs only from monetary change to economic development [Culbertson, 1960, p. 620]." In disagreeing with the unidirectional causality hypothesis which he attributed to Friedman, Culbertson (1960) noted that:

. . . the interpretation of past data is complicated by the fact that causation also has run in the other direction. Not only has money growth caused business expansion, but business expansion has caused money growth. This means that a simple reading of lag on the assumption of one-directional causality will not do [p. 621].

Friedman's response in 1961 was an attempt to clarify the issues and to offer additional empirical evidence in support of his position. Friedman emphasized that he had

not asserted one-way causality from money to income to be the only possible interpretation of the historical record. He attempted to clarify his position as recognizing that ". . . changes in the money stock are a consequence as well as an independent cause of changes in income and prices," but also interpreting the historical record as providing a great deal of evidence that ". . . even during business cycles the money stock plays a largely independent role [Friedman, 1958, p. 249]." It was, in fact, the two-way relationship between monetary change and economic activity that Friedman offered as one explanation of the long and variable lag in the effect of monetary policy (Friedman, 1961, p. 449). Additional evidence for the position taken by Friedman followed the short interchange in the form of a comprehensive historical study under the auspices of the National Bureau of Economic Research.

The Studies of the National Bureau
of Economic Research

The largest mass of evidence relating to the causal relation between money and economic activity is the detailed study by Friedman and Schwartz (1963a). This study was not designed to be a specific investigation of the causality question, but rather a detailed historical narrative of the role played by money in the United States. The authors were able, nevertheless, to draw some quite specific inferences about the causality question.

In broad summary form, they found that during the period of study (1867-1960):

- (1) Changes in the behavior of the money stock have been closely associated with changes in economic activity, money income, and prices.
- (2) The interrelation between monetary and economic change has been highly stable.
- (3) Monetary changes have often had an independent origin; they have not been simply a reaction of changes in economic activity [p. 676].

All three points are, of course, related to the issue of causality between money and income, although point number three is the most powerful statement.

More specifically, Friedman and Schwartz found that real income tends to vary over the business cycle in the same direction as money, while the secular movements in the money stock and in real income are much less close. They rationalized their finding in the following way.

Apparently, the forces determining the long-run rate of growth of real income are largely independent of the long-run rate of growth of money, so long as both proceed fairly smoothly. But marked instability of money is accompanied by instability of economic growth [p. 678].

Friedman and Schwartz recognized that the close relationship between changes in the stock of money and changes in aggregate income was not evidence about the origin of either or about the direction of causal influence between the two variables. The authors took great care in their explanation, however, to elucidate the nature of the causal relationship. In general, they concluded that

monetary changes have often been independent of changes in economic activity. In support of this conclusion, they cited many specific historical examples. Of special significance, they cited the dates of the three most restrictive policy actions since the establishment of the Federal Reserve System--January-June 1920, October 1931, and July 1936-January 1937. All three occasions were followed by severe contractions in economic activity. These instances were referred to by the authors as "quasi-controlled experiments" (p. 689) and therefore of special significance to students of the causality issue.

Friedman and Schwartz also supported their contention of causal influence by analyzing the alternative hypothesis: What if causal influences had been from income to money? They contended that if reverse causation had been present, then the general relationship between money and economic activity would have changed when our monetary institutions changed in 1914. They found little difference in the relationship between the two classes of variables in the pre-1914 and post-1914 periods.

Friedman and Schwartz did not conclude that "only money matters," a statement often attributed to them. A great deal of the misinterpretation of their work comes from the (apparently) infrequently recognized perspective of their study. The following statements are important in this context.

While the influence running from money to economic activity has been predominant, there have clearly also been influences running the other way, particularly during the shorter-run movements associated with the business cycle. . . . Mutual interaction, but with money rather clearly the senior partner in longer-run movements and in major cyclical movements, and more nearly an equal partner with money income and prices in shorter-run and milder movements--this is the generalization suggested by our evidence [p. 695].

The authors made similar statements in a summary article of their work (Friedman and Schwartz, 1963b). On the basis of the historical evidence and a theoretical sketch of the mechanism transmitting monetary changes to the economy, they noted:

The final picture that might ultimately develop out of this sketch could be of a partly self-generating cyclical mechanism. Disturbances in the rate of change in the money stock set in train a cyclical adjustment mechanism including a feedback in the rate of change in money itself [p. 64].

Thus, it is clear that Friedman and Schwartz did not conclude that money is the only causal influence of economic activity. It is the variable they would isolate as exerting a strong and independent influence on economic activity, especially over a long period and during major cyclical movements. For shorter-run periods and periods of mild cyclical fluctuations, they did not find the data to be inconsistent with a two-way causal relationship between money and economic activity.

The Friedman and Schwartz study was primarily concerned with the importance of the demand for money instead of the

supply of money. A study designed to approach the issue from the supply side was conducted by Phillip Cagan under the auspices of the National Bureau of Economic Research. The original purpose of the study was to "provide raw material for . . . other studies to combine with evidence from the demand side [Friedman, 1965, p. xxvii]." Cagan discovered, however, that approaching the causality issue from the supply side was a very powerful technique, one which provided a great deal of substantive evidence on the relationship between money and economic activity.

Cagan's stated purpose of the study was to investigate the determinants of the money supply and the factors that influence the determinants and account for changes in the money stock. The three major determinants were identified as the monetary base (high-powered money in Cagan's terminology), the banking public's reserve ratio, and the non-banking public's currency ratio. The implicit model for this analysis is appealing since it encompasses the behavior of the monetary authorities, the banking public and nonbanking public.¹

¹The precise theoretical model underlying Cagan's empirical work was not specified. A model which was clearly similar to that implied in Cagan's work was formalized by Brunner and Meltzer (1964, 1968).

In the course of his analysis, Cagan squarely confronted the issue of causality (Cagan, 1965, Chapter 6). Friedman recognized the causal inferences in the study as being the main contribution of Cagan's book: "Probably the most important issue to which [Cagan's] monograph contributes is the long-standing dispute about the causal relation between money and prices [Friedman, 1965, p. xxiv]." Although Cagan concentrated most of his efforts on the causal relationship between money and prices, he was also able to draw inferences about the relationship between money and output. Thus, he succeeded in analyzing, at least indirectly, the substantive issue of the causal relationship between money and nominal income.

Another important contribution of Cagan's study was the recognition of the need for dividing analyses of monetary effects on economic activity into short-term and long-term periods. In examining the historical record, Cagan observed that during short periods of time, there appeared to be a great many cyclical-type factors operating to influence prices and economic activity. Over a longer period, however, these influences tended to cancel each other out since they were cyclical in nature.

In the context of this cyclical-secular framework, Cagan found that there was an extremely high correlation between long-run price movements and changes in the money

stock. His attempt to discover the direction of causation implicit in the correlation involved the analysis of the possible effects of price changes on the long-run movements in the three determinants of the money stock. Since he had demonstrated that changes in the monetary base accounted for nine-tenths of the overall growth in the money stock during the period of study, 1875 to 1955 (Cagan, 1965, Chapter 2), Cagan concentrated his analysis on the relationship between the monetary base and prices. His interpretation of the historical evidence led him to conclude that the two important components of the monetary base, the gold stock and Federal Reserve Credit, were not dominated by secular movements in prices. Thus, he inferred that causality was unidirectional from the money stock to prices over the long period. He noted:

By this interpretation, therefore, the effect of prices on gold flows and production cannot explain the positive correlation [between the gold stock and price movements] [p. 245]. . . .

.
On the whole, price effects on Federal Reserve Credit outstanding seem unable to account for the very high correlation for this period [p. 249].

For the short-run, Cagan found a high covariation between the rate of change in the money stock and cyclical fluctuations in economic activity (Cagan, 1965, Chapter 1). He recognized that this correlation could have resulted from passive movements in the money stock induced by fluctuations in economic activity combined with the simultaneous

influence of money on prices and output. In an attempt to isolate the primary direction of causal influence, Cagan divided his short-run analysis into periods of severe and mild cycles. He found that severe cycles were always accompanied by similar movements in the money stock, with changes in money leading changes in economic activity. For mild cycles, Cagan found clear evidence of the influence of economic activity on the quantity of money. The feedback mechanism primarily responsible for this mutual dependence between money and economic activity was the nonbanking public's currency ratio which accounted for roughly half of the cyclical fluctuations in the quantity of money.

Thus, Cagan found that the causal relationship between money and economic activity depended on the length of the period being analyzed. In a short-run, cyclical analysis, the two series appeared to be bidirectionally related, that is, mutually dependent due to the effects of monetary and nonmonetary variables. Also, in the short run, he noted that changes in the money stock may have tended to induce offsetting variations in the demand for money which impeded the causal effects of money on the economy. But since these reactions were only temporary, they were not important in the long run. The demand for money always returned to its long-run value since it is primarily determined by long-term factors such as permanent

income. Thus, over a long period, the short-run effects tended to cancel each other, revealing a unidirectional causal connection between money, prices, and output.

Although Cagan's work was very obviously in the tradition of the work by Friedman and Schwartz there were a few subtle differences. Most importantly, Cagan seemed to be more willing than Friedman to emphasize the possibility of significant influences of economic activity on money. He did not imply that one-way causality from money to income was the rule but rather recognized the importance of their mutual interaction over the short cycle. It has been suggested that Cagan's findings may be the strongest body of evidence against the Friedman and Schwartz conclusions emphasizing the largely independent role of money (Davis, 1969, p. 120). A careful reading of the two studies, however, reveals that they are in close agreement.

The studies from the National Bureau of Economic Research produced one unequivocal result--they established beyond question the substantial gross association between the money supply and economic activity. The studies did not, however, convince a majority of economists of the importance of money as a prime, causal force affecting economic activity. Additional work in the area was necessary and was not long in coming. Some of the additional work emanated from the St. Louis Federal Reserve Bank.

The Studies of the St. Louis
Federal Reserve Bank

For years, the St. Louis Federal Reserve Bank has been known as a bastion of the strong monetarist position--the predominant causal influence runs from money to economic activity. Perhaps the greatest boon to the causality debate was the publication of "Monetary and Fiscal Actions: A Test of Their Relative Importance in Economic Stabilization" by Anderson and Jordan (1968). As the title indicates, the purpose of this study was to demonstrate which class of policy variables had the most important impact on economic activity. The research produced some very strong statements about the direction of causal influence between economic activity and money.

Anderson and Jordan approached the problem by estimating single-equation regressions of gross national product (GNP) on various proxies for monetary and fiscal actions. The resulting equations were used to test three propositions: (1) fiscal actions exert a larger influence on economic activity than do monetary actions; (2) the response of economic activity to fiscal actions is more predictable than the response to monetary influence; (3) the influence of fiscal actions on economic activity occurs faster than that of monetary actions. On the basis of the relative magnitudes of the linear regression coefficients, they rejected

all three hypotheses. The importance of their article for the causation-reverse-causation argument stems from the criticisms and additonal tests which followed the study.

The main theme of most of the criticisms of the Anderson-Jordan study centered around their use of single-equation regression techniques to estimate the reduced form model which they employed. The nature of the statistical problem was emphasized by De Leeuw and Kalchbrenner (1969). They contended that there are two criteria for choosing exogenous policy variables to use in single-equation estimates such as those used by Anderson and Jordan: (1) the variables must be under the control of the policy makers; and (2) the variables must not be "terribly sensitive to current movements in the endogenous variables [De Leeuw and Kalchbrenner, 1969, pp. 6-7]." They emphasized that "failure to meet this second requirement has been a major criticism of regressions of GNP on the money supply [p. 7]." They were referring to the statistical requirement that the independent variables in a regression be independent of the error terms. If this requirement is not satisfied, it is an indication that the independent variables are not truly exogenous, a type of specification error. Such an error results in the loss of a great many of the desirable properties of least-squares regression coefficients and causes hypothesis tests on these coefficients to be questionable.

Of course, this problem had been previously recognized for single-equation regressions (for example, see Ando and Modigliani, 1965, pp. 711-713).

In order to stay within the spirit of the Anderson-Jordan work, De Leeuw and Kalchbrenner did not attempt to alter the basic single-equation approach but, rather, attempted to develop improved proxies for monetary influences which were relatively free from feedback effects (their procedure was a simplified version of the work by Patric Hendershott, 1968, and utilized the general analytical procedure employed by Cagan, 1965). They concluded that the monetary base could be made truly exogenous by extracting from it borrowed reserves and possibly cash in the hands of the public.

The important contribution of De Leeuw and Kalchbrenner was their recognition that independent variables must be strictly exogenous if regression results are to be treated as indicative of a causal relation. De Leeuw and Kalchbrenner were not successful, however, in developing an acceptable test of exogeneity. Their testing procedure suffered from the same weakness as the procedure employed by Anderson and Jordan--it consisted solely of simple regressions of the dependent variables on the independent variables. Such equations may be influenced by two-way causality between the independent and dependent variables

which weakens the results. De Leeuw and Kalchbrenner emphasized that the issues were whether or not autonomous changes in money exerted unidirectional influences on income and if the proper surrogates for money and economic activity could be discovered.

The Anderson and Jordan study and the De Leeuw and Kalchbrenner comments served to identify the important issues and to lay the groundwork for further research. The causality issue was raised in the St. Louis Review a few months after the comments by De Leeuw and Kalchbrenner. In the August 1969 issue, three articles appeared under the general heading "The Influence of Economic Activity on the Money Stock." The most important article in the series in reference to the causality issue was by Leonall Anderson (1969). Anderson investigated the proposition that causation runs primarily from economic activity to money. He did so by statistically testing three hypotheses: (1) changes in GNP have a greater influence on changes in money than do changes in the monetary base; (2) the effect of a change in GNP on the money multiplier dominates the effect of changes in the monetary base; (3) the monetary base responds in a significant manner to changes in GNP. On the basis of single-equation, distributed-lag regressions, he was able to reject the first two hypotheses. The results of tests on the third hypothesis indicated a contemporaneous

relationship between changes in the monetary base and changes in GNP, but he observed that "the direction of causation is not clear [p. 22]." Additional tests of this proposition utilizing an F test for homogeneity of coefficients over time led Anderson to conclude that the influence of GNP on the monetary base was not significant, that is, that

variations in the strength of this relationship are not accompanied by corresponding changes in the regression coefficients relating changes in GNP to changes in the money stock. . . . As a consequence, money may be treated as an exogenous variable [p. 23].

The aforementioned work at the St. Louis Federal Reserve Bank provided the empirical basis on which the St. Louis Federal Reserve Bank reduced form macroeconomic model was formulated (Anderson and Carlson, 1970). Believing that they had evidence of unidirectional causality from money to income, the model builders estimated single-equation regressions of GNP on money and interpreted the results as causal relationships. The precise theoretical framework underlying the model was, however, not specified. The variables designated as dependent or independent were chosen on the basis of the "authors' own version" of how the economy operates (Anderson and Carlson, 1970, p. 8).

Other Studies

Two other studies directly concerned with causality recently appeared in the literature. These two separate studies are important because of the radically different approaches which were used.

A study by Lombra and Torto (1973) investigated the causality issue by analyzing the actions of the Federal Reserve Board itself to see if the criteria on which they made decisions could be discovered. Their hypothesis was that the actions of the monetary authorities have been primarily defensive in nature, that is, their actions have been an endogenous variable. To test that hypothesis, they regressed the change in the Federal Reserve's holdings of government securities on the sum of the sources of the monetary base, the change in member-bank borrowing, the change in required reserves, and the change in currency. They found that the sum of the components of the monetary base were partially offset by the monetary authorities selling government securities. They interpreted their overall results to be consistent with the hypothesis that central bank action was primarily "defensive" in the 1953-1968 period. Although their statistical estimates revealed both offsetting and accommodating actions on the part of the monetary authorities, they emphasized that "such accommodating behavior in the monetary base implies that the

monetary base is endogenous [p. 51]." Thus, in a somewhat indirect manner, Lombra and Torto were able to conclude that there exists bidirectional causality between money and income. Concerning statistical estimates, they noted that

such a conclusion implies that the practice of using the monetary base as a measure of the importance of money in controlling the economy may result in a serious overestimation of its importance [pp. 53-54].

The implication for financial model builders was clear: the practice of assuming the exogeneity of the monetary base must be abandoned and explicit treatment of the behavior of the Federal Reserve must be included in such models.

The methodology of the study by Lombra and Torto was an important contribution. Perhaps even more important, the study emphasized an apparent misunderstanding among some researchers concerning the exact nature of the causality issue. It appears that Milton Friedman, the National Bureau of Economic Research, the St. Louis Federal Reserve Bank, and other independent researchers have interpreted the issue differently and have been dealing with different aspects of the issue. Friedman and the National Bureau of Economic Research have been concerned with identifying the causal effect on income of independent changes in the money stock. That is, assuming that some monetary change is

independent of economic activity, they were concerned with the extent to which the changes in the money stock cause changes in income and the extent to which offsetting influences are generated in the economy (Friedman, 1961, p. 257). Researchers at the St. Louis Federal Reserve Bank and most others apparently chose to analyze the extent to which all monetary changes can be designated as independent of economic activity.

The empirical issue of the direction of causal influence between money and economic activity is clearly whether or not it is possible statistically to isolate autonomous monetary changes. Friedman noted that isolating the effects of autonomous monetary changes is usually evaded by relying on the averaging out of the effects of other changes (Friedman, 1961, p. 456). That is, the usual procedure is to "take the average relation between the actual changes in money and income as an estimate of the relation to be expected between an autonomous change in money and the resultant change in income [Friedman, 1961, p. 456]." The point at issue is, thus, the so-called identification problem usually discussed in relation to simultaneous equation systems. The issue as it relates to causality is whether or not it is possible to evade the problems associated with isolating the effects of monetary changes through the use of average relations as suggested by Friedman. Thus, the

causality issue can therefore be treated as a statistical question: Are there statistically significant feedback effects in the universe that would invalidate regressions of money and income? In order to answer this question, some criterion for identifying unidirectional causality and an acceptable statistical test must be found.

One attempt to statistically test the causal relationship in question was by C. A. Sims (1972). Sims recognized that simple regressions could not be interpreted as causal relations but that it was possible to combine regression models in such a way as to develop a test for causal influence. Sims did so within the framework of the causality criterion by Granger (1969). The bulk of his work was concerned with causality between nominal GNP and M1 (currency in the hands of the public plus demand deposits).

Sims' research represents an important contribution to the literature but it contains some weaknesses. The most serious defect is the reliance upon conventional F and t statistics for testing the significance of the regression equation and the coefficients, respectively. Sims concluded that causality was unidirectional from money to income, but the validity of this finding depends, first, upon the appropriateness of the Granger criterion as a basis for identifying cause and effect and, second, upon the legitimacy of the test statistics. Chapter IV is a

discussion of the theoretical basis on which cause-effect relationships between the variables of economic models may depend and a discussion of the problems associated with the use of conventional test statistics in models based on economic time-series data. An alternative statistical procedure is also suggested in Chapter IV.

CHAPTER IV

A STATISTICAL METHODOLOGY FOR ISOLATING CAUSAL INFLUENCES

The problem of differentiating between simple associations and causal relationships among observed phenomena has long plagued researchers. One method of identifying causal relationships was given by John Stuart Mill (1843):

Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation [p. 263].

This canon of causation, called the "Method of Concomitant Variation," implies that the correlation of two or more variables is indicative of a causal relationship. It does not, however, specify the means by which "the fact of causation" can be discovered. Two variables A and B may be connected by a unidirectional relationship from A to B or from B to A, a two-way relationship between A and B, or A and B may be different effects of a common cause.

If two variables are characterized by concomitant variation, Mill suggested a method by which the precise fact of causation could be discovered:

Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents [p. 260].

This "Method of Residues" suggests that the removal of all known causal factors from an observed phenomenon permits the identification of previously unknown causal factors.

The general criterion of causality offered by Mill provides a logical framework which permits the analysis of causal relationships. Mill, and other philosophers who developed similar criteria of causation, were only concerned with the philosophical implications of causality and did not, therefore, investigate the procedures by which such criteria could be used in practice.

A Testable Definition of Causality

A causality criterion which may be viewed as an extension of the ideas embodied in Mill's canons to time-series analysis was given by C. W. J. Granger (1963, 1969). Granger's criterion is thus not unique nor is it inconsistent with many other concepts of causality. A similar criterion was suggested by Weiner (1956). However, the Granger criterion is stated in such a way that it can be statistically evaluated. It was adopted to economic time-series analysis by Sims (1972).

Granger's definition of causal ordering is based on the idea that a significant correlation (that is, a concomitant variation) between past values of one variable X and that part of another variable Y which cannot be predicted from Y 's own past (that is, the residue of Y)

implies causal influence from X to Y . More precisely, X_t is said to cause Y_t if Y_t can be better predicted using all available information than if the information apart from X_t had been used. Denoting the optimum, unbiased, least-squares predictor of Y_t using the values X_t by $P_t(Y|X)$, and the predictive error series by $\varepsilon_t(Y|X) = Y_t - P_t(Y|X)$, and the variance of $\varepsilon_t(Y|X)$ by $\sigma^2(Y|X)$, then X_t is said to cause Y_t , denoted by $X_t \Rightarrow Y_t$, if $\sigma^2(Y|U) < \sigma^2(Y|\overline{U-X})$, where U represents all the relevant information in the universe accumulated since $t-1$ (Granger, 1969, p. 428). Thus, the definition of causality is relevant only to stochastic variables and is based entirely on the predictability of some series.

A testable analogue of this definition is given in the following linear model. Let X_t and Y_t be two stationary time series with zero means. Then:

$$X_t = \sum_{j=1}^m a_j X_{t-j} + \sum_{j=1}^m b_j Y_{t-j} + \varepsilon_t,$$

$$Y_t = \sum_{j=1}^m c_j X_{t-j} + \sum_{j=1}^m d_j Y_{t-j} + \eta_t,$$

where ε_t and η_t are taken to be two uncorrelated white-noise series, i.e., $E[\varepsilon_t \varepsilon_s] = 0 = E[\eta_t \eta_s]$, $s \neq t$, and $E[\varepsilon_t \varepsilon_s] = 0$ for all t and s (Granger, 1969, p. 431). Although m can

equal infinity, m will be assumed to be finite and shorter than the given time series due to the finite length of available data series.

The definition of causality given above implies that Y_t is causing X_t provided some b_j is not zero. Similarly, X_t is causing Y_t if some c_j is not zero. If both of these events occur, there is said to be a feedback relationship between X_t and Y_t . For example, in the case of a two-variable model with leading and lagged terms of the "independent" variable included in the model, bidirectional causality would be represented schematically as

$$X_{t-1} \Rightarrow Y_t \Rightarrow X_{t+1},$$

or alternatively as

$$Y_{t-1} \Rightarrow X_t \Rightarrow Y_{t+1},$$

depending upon the viewpoint of the observer. The case of unidirectional causality from X_t to Y_t would be represented as

$$X_{t-1} \Rightarrow Y_t \not\Rightarrow X_{t+1},$$

or alternatively as

$$Y_{t-1} \not\Rightarrow X_t \Rightarrow Y_{t+1}.$$

In the bidirectional case, the two series X and Y are interdependent over time--a feedback effect exists between

the two series. In the unidirectional case, X is independent of Y --no feedback effect exists.

A test for the occurrence of either of the events is to add lagged or leading values of X_t to the right-hand side of the regression equation $Y_t = f(X_t)$. If X_t is truly exogenous, then either the lagged or leading X values should have zero coefficients as a group, that is, they should make no significant contribution toward the predictability of the dependent variable Y_t . Ordinarily, the occurrence of significant groups of lagged values of X would be interpreted as an indication of a distributed-lag relationship. The presence of significant coefficients on future values of X , however, would be an indication that the "independent" variables were not truly exogenous, that is, that the relationship has been misspecified. Causality is either unidirectional in the opposite direction or the cause-effect relationship is bidirectional.

One approach to choosing between the two possibilities would be to reverse the positions of the two variables and to perform another set of regressions with and without future and lagged terms. Thus, one statistical test for unidirectional causality involves regressing Y_t on current, past, and future values of X_t and then computing the significance of the reduction in residual sums of squares (i.e., the improvement in prediction) attributable to the

past values and to the future values of X_t taken separately. If the future values do not significantly improve predictions of Y_t , the conclusion would be that causation runs from X to Y only.

The usual way of testing for the significance of a subset of the vector of regression coefficients is to partition the vector and to compare the resulting sets of mean squared error (MSE) by an F test (MSE in this case is a measure of predictive ability). The empirical validity of the Granger criterion of causality is, therefore, dependent on the power of the statistical test of predictability. Since the statistical application of the Granger causality criterion depends upon the reliability of the hypothesis tests employed, the best possible tests should be chosen. Some of the disadvantages of the standard F and t tests are discussed in the next section of this chapter. The last section in this chapter suggests a nonparametric alternative.

The Problems Introduced into Conventional Hypothesis Tests by Nonnormality

The most commonly used method for estimating regression equations is ordinary least squares (OLS). The least-squares technique has traditionally been justified on the basis of two arguments: (1) that it produces maximum likelihood estimators and, (2) that of the class of all linear unbiased estimators, the least-squares estimators

exhibit minimum variance. In order for the first argument to be universally true, the distribution of the error terms must be normal. The second argument, known as the Gauss-Markov result, is based on the assumption that the expected value of the error terms is zero and that the distribution of the error terms possesses finite second moments. These two conditions must obtain if the least-squares estimates are to possess both desirable characteristics.

Even though all of the desirable properties of the least-squares estimators do not depend on assumptions of normality, tests of hypotheses on these estimators do. Consider the commonly used $\underline{t}$ test. A ratio of two randomly selected independent variates is distributed as a central $\underline{t}$ only if the numerator is normally distributed and the denominator is distributed as a chi-square. Thus, in the case of applying $\underline{t}$ tests to linear regression coefficients, it must be true that the ratio

$$\frac{\hat{\beta} - \beta}{\sigma_{\hat{\beta}}}$$

has a numerator and a denominator which are drawn from a normal and a chi-square distribution respectively. The conditions under which the distribution of $\hat{\beta}$ will be normally distributed are apparent from the general linear hypothesis:

$$y = X\beta + e,$$

where X is fixed, β is fixed, e is a random component, and y is a variable. If e is distributed $N(0, \sigma^2 I)$, then y will be normally distributed since e is the only random component of y . Likewise, the least-squares estimate of β will be normally distributed since $\hat{\beta}$ is linear in y . If e is not normally distributed, then neither y nor $\hat{\beta}$ can be assumed to be normally distributed.

The conditions under which the denominator is distributed as a chi-squared variate also depend on the assumption of the general linear hypothesis that e is distributed $N(0, \sigma^2 I)$. If y (given X) is not normal, then the least-squares estimator ($\hat{\sigma}^2$) of the population variance (σ^2) is no longer the best quadratic unbiased estimator (Theil, 1971, p. 128). It will continue to be unbiased as long as the error terms are uncorrelated, but it will not necessarily be minimum variance. It follows that the variance of each regression coefficient $\left(\hat{\sigma}_{\hat{\beta}_i}^2 = \hat{\sigma}^2 (X'X)^{-1}_{ii} \right)$ may not be distributed as a chi-squared variate. Under conditions of nonnormality, therefore, the ratio of the least-squares regression coefficient to its standard error may not be distributed as a t because of problems in both the numerator and the denominator of the t ratio. The validity of inferences based on such a test under nonnormality is, therefore, questionable.

The same general conclusion is true for $\underline{F}$ tests of the hypothesis that the entire vector of regression coefficients (or a subset of the vector of coefficients) is not significantly different from zero. The hypothesis is usually tested by examining the ratio of the mean square due to regression to the residual mean square. Such a ratio is distributed as $\underline{F}$ only if the numerator and the denominator are distributed as independent chi-squares (Theil, 1971, p. 81). Since nonnormally distributed error terms may result in an estimate of the variance which is not distributed as a chi-square, it follows that the estimates of mean square error (the square of the standard error of the estimate adjusted for degrees of freedom) or the estimate of the mean square due to regression may not be distributed as chi-squares. In such cases, the ratio of the two mean squares may be a noncentral $\underline{F}$ and inferences based on conventional central $\underline{F}$ tables are questionable.

The Nonrobustness of Conventional Test Statistics

The extent to which conditions of nonnormal error terms damage conventional $\underline{t}$ and $\underline{F}$ tests depends on the "robustness" of the tests. A robust statistical procedure is one which shows little sensitivity relative to the assumptions concerning the behavior of the stochastic variables of the model for which the assumptions are imposed (Malinvaud, 1966,

p. 250). Robust procedures are very important in regression analysis since stochastic assumptions are an integral part of confidence interval estimation.

Traditionally, both the t and F tests have been assumed to be somewhat robust in view of nonnormality, although there is a growing body of evidence to the contrary. As early as 1931, it was recognized that the F test is extremely sensitive to nonnormality (Pearson, 1931). Pearson investigated the problems introduced into the F test by deviations from normality as characterized by nonnormal values of the coefficients of skewness and kurtosis. They are defined, respectively, as:

$$\beta_1 = \mu_3^2/\sigma^6; \beta_2 = \mu_4/\sigma^2,$$

where μ_3 and μ_4 are the third and fourth order moments about the mean. Pearson found that the F test has reduced power when β_2 assumes large values.

Gayen (1950) investigated the effects on the F test of various combinations of β_1 and β_2 which differed from the normal case of $\beta_1 = 0$, $\beta_2 = 3$. He found that the significance level of a rejection was sensitive to the values of β_1 and β_2 . Large values of β_1 and β_2 were found to cause theoretical significance levels to increase, resulting in an increased probability of Type I errors

(incorrect rejections). However, for small variations of β_1 and β_2 from nonnormality, Gayen found that the levels of significance were little affected.

The behavior of the $\underline{t}$ test under conditions of nonnormality was investigated by Geary (1947). He found extreme variations of significance levels for one-tailed $\underline{t}$ tests under nonnormal values of β_1 and β_2 . Large values of β_2 resulted in radical increases in Type I errors.

The robustness of $\underline{F}$ tests on regression coefficients was studied by Box and Watson (1962). They demonstrated that robustness depends on the nature of the empirical distribution of the exogenous variables and that the theoretical significance levels of the $\underline{t}$ and $\underline{F}$ tests are valid only when the empirical distribution in question is approximately normal. They also found that the degree to which regression tests are sensitive to nonnormality depends, to a large extent, on the numerical values of the regression variables. The most sensitive tests were found to be those subjected to regressor vectors in which one or two elements were quite different in magnitude from the remainder of the vector. (Such a condition of "outliers" is frequently found in economic time-series data.)

The conditions giving rise to outliers and the effects of outliers on hypothesis tests were investigated by Tukey (1960). He considered the case where the distribution of

errors is a mixture of two normal distributions with the same mean and different variances. A normal distribution with a greater variance was offered as one explanation of outlying observations which preserves the normality assumption. Tukey found that under these conditions the common estimates of the variance were poor estimates of the true dispersion of an error distribution characterized by a large β_2 coefficient. Tukey suggested that other variance estimating procedures might be preferable in such cases; for example, a procedure by which the variance is calculated after some of the outlying observations have been deleted. Such a procedure, he suggested, might also improve the efficiency of the regression coefficients themselves.

Although the studies that have been concerned with the robustness of conventional hypothesis testing procedures are not conclusive, they have produced a few common results. In general, these studies have demonstrated that tests which concern first moments, such as $\underline{t}$ tests, are not as sensitive to departures from normality as are tests which concern second moments, such as $\underline{F}$ tests. Also, in the case of $\underline{F}$ tests, deviations from normality seem to have a relatively small effect on inferences about means but may have serious effects on inferences about variances, especially where the kurtosis of the error distribution differs significantly from three (Sheffé, 1959, pp. 331-345). The possibility that the classical hypothesis testing techniques may break

down under nonnormality requires that alternative statistical procedures be employed when deviations from normality are suspected.

A Nonparametric Alternative: The Jackknife

A statistical technique which is not subject to the problems introduced by nonnormal error terms was suggested by Tukey (1958). The technique, known as the "jackknife," is an extension of an idea by Quenouille (1949, 1956). The jackknife is a statistical tool for reducing the bias in any estimator and for obtaining a measure of the variance of the estimator. An important advantage of the jackknife is that it does not depend on assumptions of normality.

Any statistic can be jackknifed. The procedure does not necessitate multiple observations but is, nevertheless, based on the concept of replication. Since the lack of replication in nonexperimental situations is one of the arguments against the universal use of regression analysis on nonexperimental data, the jackknife may be viewed as one means of strengthening classical multiple regression. The strength of the jackknife arises from the results of the central-limit theorem. This theorem states that the mean of a sampling distribution of means will be normally distributed regardless of the distribution of the original observations. Since the jackknife relies on an average

of many "replicates," the statistic produced by the jackknife procedure is not, therefore, subject to the problems of nonnormality which may be inherent in conventional test statistics.

The jackknife procedure as applied to ordinary least-squares regression (hereafter referred to as jackknifed least squares, JLS) consists of utilizing the random sample $X_1, X_2, \dots, X_n$ in an initial regression to obtain the least-squares estimate of each regression coefficient, $\hat{\beta}$. Then the sample is partitioned into N subsets of size M such that $NM = n$. A separate regression is then performed on each of the subsets producing, in effect, N estimates of β . This "replication" of the experiment is analogous to the replication which characterizes experimental statistics. The jackknife procedure satisfies the conditions necessary to rely on the central-limit theorem by taking a weighted average of the N estimates. If $\hat{\theta}$ is the estimator of θ based on the total sample and $\hat{\theta}_i$ is the estimator of θ based on the subsample which arises by deleting the i^{th} subset, then the jackknifed estimator is defined as

$$J(\hat{\theta}) = \frac{1}{N} \sum_{i=1}^N N\hat{\theta} - (N-1)\hat{\theta}_i = N\hat{\theta} - (N-1)\bar{\hat{\theta}}_i. \quad (4-1)$$

The i^{th} term $[N\hat{\theta} - (N-1)\hat{\theta}_i]$ is referred to as the pseudo-value and is denoted by $J_i(\hat{\theta})$.

Consider the general linear model $y = X\beta + e$. The first step in the jackknifing procedure is to estimate the least-squares values of the coefficients using the entire sample, that is, the entire $n \times k$ matrix of X values. Denote the results of the regression as

$$y_0 = a_0 + a_1X_1 + \dots + a_jX_j, \quad j = 0, 1, \dots, k. \quad (4-2)$$

Then by successively deleting the i^{th} row of X , $i=1, \dots, n$, the i^{th} set of least-squares regression coefficients is obtained.¹ Denote these coefficients as

$$y_i = a_{i0} + a_{i1} + \dots + a_{ij}, \quad i = 1, \dots, N. \quad (4-3)$$

The N coefficients are then combined in the form $[Na_j - (N-1)a_{ij}]$ to produce the pseudo-regression coefficients (b_{ij}). Thus, the j^{th} coefficient in the i^{th} regression is defined as

$$b_{ij} = Na_j - (N-1)a_{ij}. \quad (4-4)$$

¹The n sets of regression coefficients can be obtained by either inverting the $(X'X)$ matrix each time the i^{th} observation is deleted or by adapting an identity given by Bartlett (1951) which requires only one matrix inversion. A multiple regression program which utilizes the former method is given in Appendix A. The identity which permits use of the latter method is given in Appendix B.

The jackknifed regression coefficients (referred to as the jackknife statistics) for the j^{th} variable are therefore

$$J(\hat{\beta}_j) = \frac{1}{N} \sum_{i=1}^N b_{ij}, \quad (4-5)$$

and the jackknifed regression equation is

$$y = J(\hat{\beta}_0) + J(\hat{\beta}_1)X_1 + \dots + J(\hat{\beta}_j)X_j, \quad (4-6)$$

since the jackknife is a linear operator (Gray and Schucany, 1972, p. 130).

By virtue of the central limit theorem, each jackknifed coefficient is normally distributed.² Thus, the quantity

$$\frac{J(\hat{\beta}_j) - \beta [\sqrt{N(N-1)}]}{\sqrt{\sum_{i=1}^N [\hat{\beta}_{ij} - J(\hat{\beta}_j)]^2}} \quad (4-7)$$

will be distributed as a t variate with $N-1$ degrees of freedom (Gray and Schucany, 1972, p. 138). Tests of hypotheses that $J(\hat{\beta}_j) = \beta$ are thus not dependent on assumptions of normality so there is no need to question the robustness of the jackknife statistic.

²The proof that the jackknife statistic is asymptotically normal is given in Gray and Schucany, 1971, pp. 137-186.

The jackknifing procedure is general and can be applied to any estimator. Another straightforward application of the procedure would be to jackknife the $\underline{F}$ test that is usually relied on in regression analysis to test the hypothesis that the entire vector of regression coefficients is zero. The procedure would be to compute the $\underline{F}$ statistic based on the total regression and then to compute the i^{th} $\underline{F}$ statistic which arises when the i^{th} set of observations is deleted. The N $\underline{F}$ statistics would be combined in the form $[N\underline{F} - (N-1)\underline{F}_i]$ to produce the pseudo- $\underline{F}$'s. The arithmetic mean of the pseudo- $\underline{F}$'s would be the jackknifed $\underline{F}$ statistic. To test the hypothesis that $J(\underline{F}) = \theta$, where θ is the expected value of the $\underline{F}$ ratio under the null hypothesis, the ratio given in (4-7) would be employed.

The jackknifed $\underline{F}$ test could also be used to test the hypothesis that some subset of the vector of regression coefficients is zero or the hypothesis that the coefficients remain homogeneous throughout the sample period. In both cases, the desired $\underline{F}$ ratio would be jackknifed in the foregoing manner and the test ratio in (4-7) would be employed to test the hypothesis that the resulting jackknifed statistic is equal to some hypothesized value. Thus, the jackknife procedure is general and can be used in numerous ways in regression analysis.

A Comparison of Classical Hypothesis Tests and the Jackknife

It has been emphasized that the best tests of variances are those based on the F distribution as long as the underlying populations are normal. The studies cited in the second section of this chapter indicated that such tests are, however, extremely sensitive to nonnormality. Since the jackknife has been offered as an alternative test which is not sensitive to nonnormality, it is appropriate to compare the behavior of the F test and the jackknife.

The earliest theoretical and empirical investigation of the jackknife and the F test was by Miller (1968). Miller concentrated on the small sample power functions for tests of the hypothesis

$$H_0: \sigma_X^2 = \sigma_Y^2.$$

The simulation procedure consisted of generating one thousand pairs of samples of twenty-five pseudo-random numbers from uniform, normal, double exponential, skewed double exponential, and sixth power distributions. Based on the outcome of these simulations, Miller concluded:

The F test is extremely non-robust. Its actual significance level under the null hypothesis is much smaller than indicated for short-tailed distributions (uniform), and it gives far too many significant results for long-tailed distributions (double exponential, sixth power) [p. 577].

Concerning the jackknife (with M , the deletion size, equal to one), he noted that it was "the most powerful test in the group [p. 577]."³ Miller also demonstrated that a deletion size of one gives the best results, although the power loss due to larger deletion sizes is quite small.

A simulation study by Arveson and Schmitz (1970) produced similar results. They generated one thousand sets of variance components from the uniform, double exponential, and normal distributions. The results demonstrated that the power distribution of the jackknife agreed with the power distribution of the $\underline{F}$ test only on normal data (Arveson and Schmitz, 1970, p. 681). Under conditions of nonnormality, the $\underline{F}$ test was not robust with respect to theoretical significance levels. The $\underline{F}$ test was found to be extremely nonrobust for the double exponential distribution but robust for the uniform distribution. Even in the latter case, however, the results indicated that the jackknife performed "slightly better" (Arveson and Schmitz, 1970, p. 681).

The aforementioned two articles represent the most significant recent work that has been published concerning

³Miller also found that another alternative to the $\underline{F}$ test, the Box-Anderson test, was equally as powerful as the jackknife. The Box-Anderson test has little empirical appeal, however, due to the vast amount of tedious calculation involved.

the relative empirical performance of the jackknife vis-à-vis alternative techniques. These articles corroborate two theoretical postulates that have appeared in the literature: (1) the $\underline{F}$ test may be extremely nonrobust under certain conditions, and (2) the jackknife is quite robust in the face of the conditions which weaken the $\underline{F}$ test.

These studies concentrated on the use of the $\underline{F}$ statistic to test the equality of two variances. The $\underline{F}$ test employed in regression analysis is, however, concerned with tests of means instead of variances. Although the two tests are analogous, they are theoretically different. In order to test the relative performance of the $\underline{F}$ and $\underline{t}$ tests against that of the jackknife in a regression situation, a simulation experiment was performed by the writer. Various linear models were used in the experiments, but the results were insensitive to alternative specifications. The results of the simplest model are presented below. The model used to generate the data was:

$$y = 100 + 3X + e,$$

where X was a vector of independent, stationary values and e was a vector of independent error terms generated from specified distributions with a known mean and variance. One hundred sets of twenty-four observations of X and y were generated on which ordinary least-squares regression

(OLS) and jackknifed least-squares regression (JLS) were performed. In the JLS procedure, the regression coefficients as well as the $\underline{t}$ tests and $\underline{F}$ tests were jackknifed (with $M = 1$). Many simulations were performed using various distributions. Results which are representative of these simulations are presented below.

The primary purpose of the study was to reveal the empirical distributions of the $\underline{t}$ test, the $\underline{F}$ test (which is merely t^2 in the bivariate case), and the jackknife statistic ($\underline{J}$) which is asymptotically a $\underline{t}$. The evaluation procedure involved dividing the $\underline{t}$ distribution into intervals of .05, counting the number of computed $\underline{t}$ values which fell into each interval, and evaluating the results with a chi-square goodness-of-fit test. The null hypothesis was that the distribution of

$$\frac{\hat{\beta} - \beta}{\sigma_{\beta}}$$

is distributed as a central $\underline{t}$.

In the case of error terms from a double exponential distribution with a mean of zero and a standard deviation of twenty, it was found that for the hypothesis $\beta = 3$, the hypothesis that the $\underline{t}$ tests were distributed as a central $\underline{t}$ was rejected. The chi-square value giving rise to this rejection was $\chi^2_{19} = 38.8$ which is significant at the .05

level. The $\underline{t}$ test of the jackknife statistic which resulted from the jackknifed regression coefficients was distributed as a $\underline{t}$. The chi-square value for the goodness-of-fit test was $\chi^2_{19} = 25.2$ which is not significant at the .01 level. These results clearly indicate that the $\underline{t}$ and $\underline{F}$ tests may break down under conditions of nonnormality but that the $\underline{t}$ statistic of the jackknife statistic is robust. Furthermore, since the double exponential resembles quite closely the normal distribution, these results are especially important. The implication is that deviations from normality may not have to be extremely large in order to weaken the classical $\underline{t}$ and $\underline{F}$ tests. The jackknife, however, performs very well under nonnormality even for small sample sizes.

An important question which has not been addressed in the literature concerns the effects of the jackknifing procedure on the characteristics of the regression coefficients themselves. If it were the case that jackknifing severely damaged the regression coefficients, then the well-behaved distributions of hypothesis tests on those coefficients would be overshadowed. The results of the simulations also provided some interesting insights into this question.

The relative "goodness" of the OLS and JLS estimators was evaluated by examining the bias, variance, mean square

error (MSE), and root mean square error (RMSE) of the two classes of estimators. Mean square error may be defined as expected squared error of the estimator and expressed as:

$$E(\hat{\theta} - \theta)^2 = E(\theta - E\hat{\theta})^2 + (E\hat{\theta} - \theta)^2. \quad (4-8)$$

The right-hand side of (4-8) is simply the expansion of the statement for MSE. The MSE has the desirable characteristic of encompassing both the variance of the estimator $\hat{\theta}$, $E(\hat{\theta} - E\hat{\theta})^2$, and the bias of the estimator $\hat{\theta}$ (specifically, the square of the bias, $(E\hat{\theta} - \theta)^2$).

The results for the double exponential distribution with mean zero and variance 400 are presented in Table 1. The results indicate that the JLS estimators of the regression coefficient, the variance of the regression coefficient and the standard error of the estimate are all slightly inferior on the basis of mean, variance, bias, MSE, and RMSE in this case. This was to be expected since least-squares estimators are known to be the best linear unbiased estimators from the class of all linear estimators. However, the JLS estimates were not significantly different than the OLS estimates based on a Wilcoxon signed-ranks-matched-pairs test. In the case of the estimates of the regression coefficient and the variance of the coefficient, no significant difference was found between the OLS and JLS estimates. The Wilcoxon statistics were -1.355 and -1.021,

TABLE 1*

SIMULATION RESULTS FOR OLS AND JLS REGRESSIONS WITH ERROR TERMS
FROM A DOUBLE EXPONENTIAL DISTRIBUTION WITH
MEAN = 0 AND VARIANCE = 400

Estimated Value	θ		$E\hat{\theta}$		$\sigma_{\hat{\theta}}^2$		Bias of $\hat{\theta}$		MSE of $\hat{\theta}$		RMSE of $\hat{\theta}$	
	OLS	JLS	OLS	JLS	OLS	JLS	OLS	JLS	OLS	JLS	OLS	JLS
β	3.0	3.0	3.035	3.048	1.213	1.225	0.035	0.048	1.214	1.227	1.102	1.107
σ_{β}^2	1.214	1.225	1.757	1.879	1.001	1.635	0.544	0.655	1.298	2.065	1.139	1.437
σ	20.0	20.0	19.307	19.764	22.964	24.076	2.307	2.764	28.287	31.717	5.319	5.631

*The notation in this table is as follows: β is the regression coefficient; σ_{β}^2 is the variance of the estimate; σ is the standard error of the estimate; θ is the true value being estimated; $E\hat{\theta}$ is the arithmetic mean of the estimated values; $\sigma_{\hat{\theta}}^2$ is the sampling variance of the estimator; Bias, MSE, and RMSE of θ are defined in equation (4-8).

respectively, neither of which is significant at the .05 level. There was, however a significant difference between OLS and JLS estimates of the standard error of the estimate. The Wilcoxon statistic was -8.682 which is significant at the .001 level. An examination of Table 1 reveals that this difference is in favor of the JLS estimate: on the average, the JLS estimator of the standard error of the estimate is closer to the true value than the OLS estimate.

Within all classes of estimators the least-squares estimator may not be minimum variance. This possibility was revealed in the case of simulations with error terms from a skewed double exponential distribution with a mean of 18 and a variance of 400. The OLS t test of the hypothesis $\hat{\beta} = 3$ was found to differ from a central t distribution. The chi-square value for the goodness-of-fit test was 28.0, a value which allows rejection of the null hypothesis that the distribution is a central t at the .10 level. The distribution of the jackknifed values was found to be a t . The chi-square value was 16.4. The comparative characteristics of the OLS and JLS estimators are given in Table 2.

The results presented in Table 2 demonstrate that under some conditions the JLS estimate of the regression coefficient may possess a smaller variance than the OLS estimate. The JLS coefficient is not only characterized by a smaller variance than the OLS coefficient, it is also

TABLE 2*

SIMULATION RESULTS FOR OLS AND JLS REGRESSIONS WITH ERROR TERMS
FROM A SKEWED DOUBLE EXPONENTIAL DISTRIBUTION WITH
MEAN = 18 AND VARIANCE = 400

Estimated Value	θ		$E\hat{\theta}$		$\sigma_{\hat{\theta}}^2$		Bias of $\hat{\theta}$		MSE of $\hat{\theta}$		RMSE of $\hat{\theta}$	
	OLS	JLS	OLS	JLS	OLS	JLS	OLS	JLS	OLS	JLS	OLS	JLS
β	3.0	3.0	3.162	3.159	1.599	1.586	0.162	0.159	1.625	1.611	1.275	1.269
σ_{β}^2	1.599	1.585	2.003	2.336	1.294	4.199	0.404	0.749	1.457	4.762	1.207	2.182
σ	20.0	20.0	20.253	20.733	20.739	21.734	0.253	0.733	20.804	22.271	4.561	4.719

*Notation is the same as in Table 1.

closer to the true value of θ on the average, that is, it also has a smaller bias.

Although these simulation results are by no means conclusive evidence of the superiority of the jackknifed least-squares regression estimators vis-à-vis the ordinary least-squares regression estimators, they do indicate that the jackknifed estimators are generally no worse than the OLS estimators in terms of mean squared error. More importantly, the results demonstrate that the tests of hypotheses of the jackknifed coefficients are much more reliable than the usual t and F tests. In economic time-series regression analysis where nonnormality may be the rule, a hypothesis testing procedure such as the jackknife is preferable to the classical tests. The question remaining is whether or not economic time-series data are characterized by nonnormality.

The Possible Nonnormality of Economic Time-Series Data

One of the first to note the apparent nonnormality of price data was Wesley C. Mitchell (1915). He noted that the empirical distributions of price changes were usually too peaked to have been drawn from Gaussian populations. Additional evidence of this possibility was given by Frederick C. Mills (1927).

Most of the recent studies of nonnormality in economic time-series data have been concerned with stock market

prices. Mandelbrot (1963a) studied the variation in certain speculative prices and concluded that they seemed to be characterized by distributions possessing infinite second moments. The evidence supporting the nonnormality of price data is strong enough to merit "a radically new approach to the problem of price variation [p. 395]" in Mandelbrot's opinion. He investigated a few possible alternative approaches in another article (Mandelbrot, 1963b). The nonnormality of stock market price data was also investigated by Fama (1965). His conclusions were much the same as those of Mandelbrot.

A study which dealt with the nonnormality problem in economic time-series data other than price data was a study by Zeckhauser and Thompson (1970). Among other data, they investigated the possible nonnormality of (1) a time-series regression of consumption on the supply of money for the years 1897-1958, and (2) a time-series regression of national income on the money supply for the years 1897-1958. These regressions are similar to those referred to by Friedman (1961).

By using maximum-likelihood estimates of these regressions, they concluded that:

. . . in each case the distribution of the error term that gives the greatest likelihood is one with more density in the tails in comparison to the central hump than is consistent with any normal distribution [p. 283].

Their finding of error term distributions which were relatively flat with long tails is entirely consistent with the findings of Mandelbrot and Fama.

Although the work in the area of nonnormal error terms is by no means complete, it has served to emphasize the need for additional study in this area and for the development of alternative techniques to the classical procedures. The best alternative procedure would be one which performs well when the data are normally distributed and is robust in the face of nonnormality. The computer simulations demonstrated that this is a characteristic of the jackknife. The jackknife procedure was, therefore, used in the present causality analysis of money and income time-series data for the United States. The results are presented in Chapter V.

CHAPTER V

THE STATISTICAL ANALYSIS

In order to test empirically various theoretical propositions concerning the question of causality between money and income, the Granger criterion was adopted and the cause-effect statements evaluated by ordinary least-squares regression combined with the jackknifing technique. The tests were for the null hypothesis that causality is unidirectional from money to income.

The expression of bivariate relationships between selected proxies for the money stock and income provided the basic set of models to which the statistical tests were applied. Reducing the complexities of a market system to a simple bivariate, distributed-lag relationship is, admittedly, a gross oversimplification and a misspecification of the true relationship. However, this research in no way excludes the possibility of other important influences on the level of economic activity. The question to be explored by these models is whether or not money can be held to exert an important, independent, unidirectional causal influence on income, not whether money is the only source of variation in income.

Since the test of causality developed by Sims (1972) and used in this study is based on the predictability of variables as evaluated by tests of significance, it is also

possible to make inferences about the temporal form of the "causality lag" (Granger and Hatanaka, 1964, p. 118) between changes in the money stock and changes in income. For example, if the successive addition of lagged money terms to a distributed-lag regression of aggregate income on money did not improve the predictability of the income series, the inference would be that the nonsignificant lagged terms did not exert a statistically identifiable influence on income. The set of lagged terms which did improve the predictability of income may be indicative of a proximate timing relationship between changes in money and changes in income. Similarly, the point at which future values of money in the regression became significant is an indication of the approximate time at which feedback effects became significant. Thus, the test used in this study permits inferences about both the significance of causal relationships and the approximate temporal form of such relationships.

The Data

The data employed in the causality analysis were the commonly used measures of money and income. Three different series of money supply data were chosen for the analysis: M1, the total of private demand deposits plus currency in hands of the public; M2, the total of M1 plus private time deposits in commercial banks; and MB, the monetary base

which is the basic monetary asset supplied by the Federal Reserve System. The data for M1 and M2 were obtained from the Federal Reserve Bulletin (1970, pp. 896-898). The data for MB were obtained from a series supplied by the Federal Reserve Bank of St. Louis.

The two series chosen to represent income were gross national product (GNP) in current dollars and gross national product in constant (1958) dollars (RGNP). The implicit price deflator (PGNP) used in the construction of the RGNP series was also used as a variable in one class of models. All three GNP series were obtained from The National Income and Product Accounts of the United States, 1929-1965 (1966) and various issues of the Survey of Current Business. All data were seasonally adjusted at the source.

It is well known that economic time-series data contain very strong trend components. The effect of a strong trend is to cause the residuals of the series to be autocorrelated. Autocorrelation weakens linear regression models and must, therefore, be eliminated or, at least, be minimized. The most common procedure is to use first differences of the data (or log first differences) instead of the raw data. This operation is often effective in removing first-order autocorrelation but may leave significant second- and higher-order autocorrelation. Following Sims (1972) in order to remove first- and second-order auto-

correlation and to produce as nearly as possible "white-noise" residuals, the first-differencing process was performed twice, each time with coefficients chosen to minimize the autocorrelation. This procedure may be described as "prefiltering" the data (Malinvaud, 1966, pp. 378 and 415). It produced the prefiltering transformation:

$$\log X_t - 1.5 \log X_{t-1} + .5625 \log X_{t-2}.$$

In order to insure that the trend component was removed and to insure that all seasonal components had been eliminated, a linear time-trend series and three seasonal dummy variables were included in each model, again, the technique employed by Sims (1972). The basic model used in the analysis therefore consisted of the variable designated as endogenous regressed on the current, eight past, and four future values of the variable designated as exogenous, plus three seasonal dummy variables and a time-trend component. Consistent with Sims' study, the various models were fitted to quarterly data for the years 1947I through 1969IV, a total of ninety-two observations. Some of these observations were lost, however, in the preliminary data manipulation and lagging process. The prefiltering process absorbed the first two, the eight lags absorbed the next eight, and the four future values absorbed the last four observations. The actual time period remaining after data

transformations was 1949III through 1968IV, a total of seventy-eight observations.

The Statistical Procedure

The same procedure was used to evaluate causality in each model. The first step was to regress the dependent variable for each model on the current, eight lag, four future, three seasonal, and time-trend values. The second step was to partition the vector of the resulting coefficients in order to examine the contribution of various subsets of the coefficients. The partitioning procedure involved a second regression of the same dependent variable regressed on the subset of the variables which, under the null hypothesis, explains all of the variation accounted for by the total model. The regression sum of squares for the second regression was then subtracted from the regression sum of squares for the first regression producing the regression sum of squares due to the remaining subset of coefficients. The ratio of the result to the residual sum of squares for the total model, both adjusted for degrees of freedom, produced the conventional F test of the hypothesis that the remaining subset of the vector of coefficients is not significantly different from zero. For example, the basic regression model contained 17 independent variables and was estimated on 78 observations, resulting in 17 degrees of freedom for the regression and 60 residual

degrees of freedom. A second regression of the dependent variable on the 4 trend and seasonal variables provided the estimate of the regression sum of squares due to the trend and seasonals. The regression sum of squares due to the remaining 13 variables is, by identity:

$$SSR(13) = SSR(17) - SSR(4),$$

where the numbers in parentheses represent degrees of freedom. The $\underline{F}$ test of the hypothesis that the 13 variables are zero as a group is, therefore:

$$\frac{SSR(13)/13}{SSE(60)/60},$$

where the ratio is distributed as a central $\underline{F}$ under the null hypothesis with 13 and 60 degrees of freedom. A similar procedure was followed for the other partitions. The procedure to this point is consistent with that used by Sims (1972).

The third step was to jackknife each of the two regressions by successively deleting one of the n observations and then combining the resulting $n + 1$ sets of regression sums of squares to produce the jackknifed regression coefficients and the jackknifed equivalent of the $\underline{F}$ tests. Since the jackknifed regression coefficients were not used in the causality test, the jackknifed $\underline{F}$ tests are referred to as the jackknife statistics in the following analysis.

The test of the significance of each vector of coefficients (that is, of each jackknife statistic) was based on the ratio

$$t = \frac{J(\hat{\theta}) - \theta [\sqrt{N(N-1)}]}{\sqrt{\sum_{i=1}^N [\hat{\theta}_i - J(\hat{\theta})]^2}},$$

where θ is zero under the null hypothesis. These t tests provided the basis for evaluating the hypotheses suggested by the Granger criterion.

The Relationship Between Nominal GNP and Money

The Granger-Jackknife testing procedure was first applied to models based on nominal gross national product (hereafter referred to as GNP), M1, M2, and MB. Six separate functional relationships were investigated: (1) $GNP = f(M1)$, (2) $GNP = f(M2)$, (3) $GNP = f(MB)$, (4) $M1 = f(GNP)$, (5) $M2 = f(GNP)$, and (6) $MB = f(GNP)$. The results of the significance tests for each model are presented in Table 3. The first column in Table 3 contains the F tests from the ordinary least-squares regressions (OLS) of the null hypothesis that the independent variables except the trend and seasonals are zero as a group for each model. The second column contains the jackknifed F tests, $\underline{J}$, which resulted from the jackknifed least-squares (JLS) regression procedure.

TABLE 3

SUMMARY OF OLS AND JLS REGRESSIONS BASED ON
NOMINAL GNP, M1, M2, AND MB

Model ^{a/}	Significance of Total Model		Summary Statistics		
	$F^b/$ (13,60)	$J^c/$	$\bar{R}^2$	Durbin- Watson Statistic	Standard Error of Estimate
GNP = f(M1)	1.365	1.168 (1.141)	.778	2.518	.01056
M1 = f(GNP)	4.224**	7.343 (3.914)**	.845	1.739	.00405
GNP = f(M2)	1.416	1.358 (1.378)	.778	2.419	.01052
M2 = f(GNP)	5.755**	10.755 (6.059)**	.951	1.675	.00413
GNP = f(MB)	15.573**	1.939 (2.094)**	.786	2.299	.01038
MB = f(GNP)	5.480**	9.810 (4.673)**	.866	2.627	.00427

^{a/} Each model included the current value, eight past lags and four future values of the exogenous variable plus three seasonal dummy variables and a time-trend variable. An intercept was also estimated. In all cases, the time trend was significant and the seasonals were not.

^{b/} The F tests result from OLS regressions and are for the null hypothesis that all independent variables except trend and seasonals are zero as a group.

^{c/} The J statistics result from JLS regression. The values in parentheses below the J statistics are the t tests (at 77 degrees of freedom) of the null hypothesis stated in ^{b/}.

* Significant at the .10 level.

** Significant at the .05 level.

The values in parentheses below the jackknife statistics are the $\underline{t}$ tests of the null hypothesis that the $\underline{J}$ statistics (and therefore the group of independent variables except the trend and seasonals) are zero. The significance of each test statistic is denoted by asterisks: one asterisk denotes significance at the .10 level and two asterisks denotes significance at the .05 level. The last three columns in Table 3 contain the summary statistics for each model.

A comparison of the conventional $\underline{F}$ and jackknife $\underline{t}$ statistics reveals that the two are in agreement concerning the hypothesis that all independent variables except the trend and seasonals are zero as a group. At the .10 level, neither test rejects the hypothesis for the GNP on M1 or for the GNP on M2 regression. Both the $\underline{t}$ and $\underline{F}$ tests reject the hypothesis if the level of significance is lowered to the .20 level. The fairly large $\bar{R}^2$ statistics for the two regressions corroborates the alternative hypothesis that most of the explanatory power of the two models is due to the trend and seasonal components. The standard error of the estimate for these two models is large relative to the other models, indicating that they are characterized by relatively large variances. For the GNP on MB regression, both the $\underline{F}$ and $\underline{t}$ tests reject the hypothesis at the .05 level. Of the three regressions of GNP on money, the most

significant relationship is, therefore, between GNP and the monetary base.

The reverse regressions of money on GNP all identify significant relationships. The F tests and the jackknife t tests are in agreement for the three reversed regressions. In each case the hypothesis that only the trend and seasonals are significant is rejected at the .05 level by both tests. The $\bar{R}^2$ statistics for these three equations are much higher than those for the GNP on money equations and the standard errors of the estimates are smaller, implying that the reversed regressions are more significant relationships and are characterized by relatively smaller variances.

The direction of causal influence in the models is evaluated by examining Table 4 which contains the results of partitioning the models into their four future, eight lag, four lag, and two lag groups of coefficients. Under the null hypothesis, each GNP on money relationship is characterized by unidirectional causality from the independent to the dependent variable. The Granger criterion suggests that if the future coefficients in the model are not significantly different from zero and the future coefficients in the reversed regression are, then the hypothesis of unidirectional causality from the independent to the dependent variable cannot be rejected. If, on the other hand, the significance of the future coefficients differs from this pattern, then the hypothesis must be rejected.

TABLE 4
SIGNIFICANCE TESTS OF THE FUTURE AND PAST LAG PARTITIONS FOR
REGRESSIONS BASED ON NOMINAL GNP, M1, M2, AND MB

Model	Four Future Coefficients		Current and Eight Lag Coefficients		Current and Four Lag Coefficients		Current and Two Lag Coefficients	
	$F_a/$ (4,60)	$J_b/$	F (9,60)	J	F (5,60)	J	F (3,60)	J
GNP = f(M1)	0.369	-0.992 (-1.252)	1.702	1.608 (-1.078)	2.563**	3.596 (1.639)	2.904**	4.319 (1.392)
M1 = f(GNP)	4.806**	9.080 (3.002)**	4.294**	7.854 (3.782)**	6.480**	13.042 (3.617)**	8.299**	17.245 (3.684)**
GNP = f(M2)	0.539	-0.701 (-0.589)	1.734*	1.982 (1.562)	2.582**	4.036 (1.938)*	2.412**	3.417 (1.186)
M2 = f(GNP)	10.350**	22.324 (4.729)**	4.302**	8.066 (4.851)**	6.938**	14.310 (4.837)**	7.587**	15.920 (4.132)**
GNP = f(MB)	0.192	-0.974 (-1.804)*	1.538	1.808 (1.619)	1.535	1.752 (1.284)	1.881*	2.573 (1.370)
MB = f(GNP)	8.212**	16.532 (3.614)**	5.586**	10.622 (4.356)**	9.005**	18.611 (4.291)**	4.733**	9.183 (2.126)**

a/ Values in parentheses below the F headings indicate the degrees of freedom for a particular F test. Each F test is for the null hypothesis that all coefficients in a particular lead or lag partition are not significantly different from zero.

b/ The J statistics are the jackknifed F tests of the null hypothesis in a/. The values in parentheses below each J statistic are t tests of the null hypothesis that each J statistic is not significantly different from zero at 77 degrees of freedom.

*Significant at the .10 level.

**Significant at the .05 level.

Although the GNP on M1 regression was dominated by the time trend and seasonal components, the tests of significance of the leading and lagged coefficients are interesting. At the .10 level, the jackknife $\underline{t}$ does not reject the hypothesis of zero coefficients for any of the partitions. The $\underline{F}$ test rejects the hypothesis for both the current and two and the current and four lag partitions. (By the evidence presented in Chapter IV, these rejections are an indication of the nonnormality of the data.) If, however, the significance level of the model, .20, is adopted to evaluate the partitions of the model, the jackknife $\underline{t}$ rejects the hypothesis of zero coefficients for the current and four lag coefficients and the four future coefficients and the $\underline{F}$ test rejects for the current and eight lag coefficients. Since the four future coefficients in the GNP on M1 regression are significantly different from zero by the jackknife $\underline{t}$ test, the more reliable of the two, and since the four future coefficients in the M1 on GNP regression are also significant, the hypothesis that causality is unidirectional from M1 to GNP is rejected at a low level of significance. The alternative hypothesis that causality is bidirectional between M1 and GNP would be consistent with this result.¹

¹The rejection of the hypothesis of unidirectional causality from M1 to GNP conflicts with Sims' conclusions

The causality-lag distributions implied by these regressions are also interesting. In the GNP on M1 regression, the jackknife t test indicates that the four past values of M1 are the most significant lagged values. Thus, it appears that changes in M1 may exert a significant causal influence on GNP within four quarters and then decline in importance thereafter. The feedback effects from GNP to M1 appear to occur almost immediately (within two quarters based on the regression of M1 on GNP) and remain important for at least six additional quarters.

Examining the significance of the coefficients within the GNP on M2 regression is not useful since the regression was not a significant relationship at any reasonable level. Considering that the M2 series is simply M1 plus time deposits in commercial banks, this result is surprising in light of the results of the GNP on M1 regression. Such a result might occur if nominal GNP showed no consistent pattern of influence by the components of M2 during the sample period. Similarly, the effects of M2 on GNP could have been swamped by extraneous influences (Sims, 1972, p. 543). The former postulate can be tested

which were based on conventional F tests (Sims, 1972). Sims' results were quite similar to the F tests presented in Tables 3 and 4. As discussed in Chapter IV, the jackknife tests are expected to be more reliable than conventional tests, a result which casts doubt on the validity of Sims' conclusions that causality is unidirectional from money to GNP.

by examining the division of the influence of GNP on M2 according to the real and price components of GNP. This is done in the next two sections of the present chapter for all three money proxies. The latter postulate would need to be tested within the framework of a large simultaneous model and is, therefore, outside the scope of this study.

The most unambiguous findings of the tests of causality result from using the monetary base as the proxy for money since both the GNP on MB and the MB on GNP regressions are significant at the .05 level (Table 3). For the GNP on MB partitions, the $\underline{F}$ and $\underline{t}$ tests produce conflicting results. The $\underline{F}$ test rejects the hypothesis of zero coefficients for the current and two lag partition at the .10 level, whereas the $\underline{t}$ does not reject for any of the lag partitions at the .10 level. At a slightly lower level of significance (approximately the .15 level), the $\underline{t}$ and the $\underline{F}$ tests reject the hypothesis that the current and eight lag coefficients are zero as a group but only the $\underline{F}$ test rejects for the current and four lag partition. The hypothesis that the four future values of MB are not significantly different from zero is rejected at the .10 level by the jackknife $\underline{t}$ but not by the $\underline{F}$ test. Since the jackknife $\underline{t}$ is expected to yield the more reliable result, it suggests that the hypothesis of unidirectional causality from MB to GNP must be rejected.

Such an inference is corroborated by the reversed regression of MB on GNP. Table 4 reveals that both the $\underline{F}$ and the jackknife $\underline{t}$ tests reject the hypothesis of zero coefficients at the .05 level for the future and all past lag partitions. The alternative hypothesis that there is a two-way causal relationship between nominal GNP and MB is consistent with these results. This alternative hypothesis is important since the monetary base is quite often considered by many to be an exogenous money series.

The causality-lag distributions implied by the tests suggest that changes in the monetary base influence nominal GNP over a period of at least two years. The feedback effects of income on the monetary base appear to become significant within one year.

The existence of a bidirectional causal relation between money and income suggests that the relationship between the two variables may be unstable over time. In order to test for a changing structure, the sample was split in half and a test for the homogeneity of the coefficients was performed. The test consisted of comparing the sum of the regression sum of squares for the two subperiods to the regression sum of squares for the total sample period by means of an $\underline{F}$ test and its jackknifed

equivalent.² The results presented in Table 5 indicate that the hypothesis of homogeneous coefficients for the total period 1947I through 1969IV cannot be rejected for all models by the F test. There are, however, two points of disagreement between two tests. The F test rejects the hypothesis of homogeneous coefficients for the GNP on the M1 model at the .05 level, whereas the jackknife t does not reject at any reasonable level. Since the F test tends to reject too often under conditions of nonnormality, the jackknife t is expected to be the more reliable result. The implication of these tests is that the two-way relationship between money and GNP was stable during the period of study.

The results of the causality test for models based on money and nominal GNP indicate that the choice of the proxy for money is important. The most significant relationship is between GNP and MB followed by GNP and M1. Adopting the monetary base as the best proxy of the three to evaluate the relationship between money and nominal GNP, the hypothesis that causality is unidirectional from money to income is rejected. The alternative hypothesis that causality is bidirectional is accepted.

²The derivation of the original F test for the homogeneity of coefficients was by Chow (1960). The jackknifed version of Chow's test is an extension of that test.

TABLE 5

TESTS OF THE HOMOGENEITY OF COEFFICIENTS FOR
REGRESSIONS BASED ON NOMINAL
GNP AND M1, M2, AND MB

Model ^{a/}	$F_{\underline{b/}}$ (18,42)	$J_{\underline{c/}}$
GNP = F(M1)	2.147**	-1.060 (-0.473)
M1 = f(GNP)	1.082	-0.230 (-0.200)
GNP = f(M2)	1.325	-0.382 (-0.326)
M2 = f(GNP)	1.345	-0.770 (-0.762)
GNP = f(MB)	0.967	-0.761 (-0.798)
MB = f(GNP)	1.001	-1.167 (-0.423)

^{a/} Each model included the current term of the exogenous variable as well as eight lagged values and four future values of the exogenous variable, three seasonal dummy variables and a linear time-trend term.

^{b/} The F tests are for the null hypothesis that all independent variables including trend and seasonals remained the same over the entire sampling period.

^{c/} The J statistics are the jackknifed F tests explained in ^{b/}. The values below the J statistics are t tests of the null hypothesis that the J statistics are not significantly different from zero, a null hypothesis which is equivalent to that given in ^{b/}.

*Significant at the .10 level.

**Significant at the .05 level.

The Relationship Between Real GNP and Money

It was suggested earlier that one possible way to gain insight into the nature of the two-way relationship between money and income would be to examine how the relationship is divided between the real and price effects. Both the simple and modern versions of the quantity theory suggest that money exerts an important influence on nominal income only, real income being determined by other factors. Alternatively, the Keynesian and post-Keynesian income-expenditure theories suggest that money could influence both real and nominal income.

These two alternative theories were tested within the framework of the Granger-Jackknife causality criterion by substituting real GNP (RGNP) for nominal GNP in one set of tests and then by substituting the GNP price deflator (PGNP) in another set of tests. The results of the former group of tests is presented in Table 6.

Table 6 indicates that the $\underline{F}$ tests and the jackknife $\underline{t}$ tests are in agreement regarding rejection of the hypothesis that all independent variables except the trend and seasonals are zero as a group. Both the $\underline{t}$ and $\underline{F}$ tests indicate that none of the three RGNP on money regressions are significant at the .10 level when the seasonal and trend components are removed. The nonsignificance of the

TABLE 6^{a/}

SUMMARY OF OLS AND JLS REGRESSIONS BASED
ON REAL GNP, M1, M2, AND MB

Model	Significance of Total Model		Summary Statistics		
	F (13,60)	J	$\bar{R}^2$	Durbin- Watson Statistic	Standard Error of Estimate
RGNP = f(M1)	1.137	0.830 (1.050)	.546	2.611	.01121
M1 = f(RGNP)	2.489**	4.682 (3.673)**	.818	1.517	.00441
RGNP = f(M2)	1.049	0.661 (0.716)	.539	2.557	.01129
M2 = f(RGNP)	4.250**	7.618 (5.229)**	.982	1.438	.00447
RGNP = f(MB)	1.049	0.661 (0.716)	.539	2.557	.01129
MB = f(RGNP)	3.093**	5.047 (3.986)**	.825	2.498	.00488

^{a/} Notation is the same as that used in Table 3.

RGNP on money models is also indicated by the summary statistics in the last three columns of Table 6. The $\bar{R}^2$ statistics are low for the RGNP on money models (.54 on the average) and the standard errors of the estimates are high relative to the reversed regressions. The regressions of money on RGNP are significant. When the trend and seasonal components of the models are removed, both the $\bar{F}$

tests and the jackknife $\underline{t}$ tests reject at the .05 level the hypothesis that the explanatory power of each model is due to the trend and seasonal variables.

The significance of the various future and past lag partitions of each model are presented in Table 7. Since none of the RGNP on money regressions are significant relationships, the significance of the partitions of these models would be difficult to interpret. For the regression of money on RGNP, Table 7 indicates that the hypothesis of zero future and past lag coefficients is rejected at the .05 level by both tests. The only exception is the failure of the jackknife $\underline{t}$ test to reject the hypothesis for the current and four lag partition.

The lag distribution of the effect of RGNP on money implied by the significance of the jackknife $\underline{t}$ statistics is the same for all three money on RGNP regressions. Real GNP appears to influence the three measures of money almost immediately (within two quarters), decline in importance over the next two quarters, and then increase in importance over the next four quarters (or longer). The feedback effect from RGNP to money becomes significant within four quarters.

Taken together, the regressions of RGNP on money and the reversed regressions of money on RGNP imply that a two-way causal relationship between real GNP and money may

TABLE 7^a/₁

SIGNIFICANCE TESTS OF THE FUTURE AND PAST LAG PARTITIONS FOR REGRESSIONS
BASED ON REAL GNP, M1, M2, AND MB

Model	Four Future Coefficients		Current and Eight Lag Coefficients		Current and Four Lag Coefficients		Current and Two Lag Coefficients	
	F (4,60)	J	F (9,60)	J	F (5,60)	J	F (3,60)	J
RGNP = f (M1)	0.623	-0.382 (-0.444)	1.230	0.801 (0.689)	1.402	1.250 (0.801)	1.874	2.089 (0.874)
M1 = f (RGNP)	2.566**	4.464 (2.431)**	3.274**	5.767 (3.631)**	3.224**	5.935 (2.805)**	5.054**	10.028 (3.089)**
RGNP = f (M2)	0.785	-0.117 (-0.956)	1.191	0.907 (0.782)	1.917**	2.594 (1.445)	2.170*	2.784 (1.001)
M2 = f (RGNP)	6.751**	14.272 (4.436)**	3.783**	6.827 (4.285)**	2.837**	4.744 (2.019)**	5.733**	11.352 (3.237)**
RGNP = f (MB)	0.189	-1.320 (-3.830)**	1.134	0.953 (0.947)	1.557	1.427 (0.634)	1.748	1.940 (0.813)
MB = F (RGNP)	4.325**	8.159 (2.977)**	3.364**	6.081 (4.159)**	2.378**	3.320 (1.178)	4.473**	8.914 (2.520)**

^a/ Notation is the same as that used in Table 4.

exist, but that it is quite weak. The relationship is apparently dominated by a unidirectional causal relationship running from real GNP to money. This result may account for the low level of significance which characterized the relationships between nominal GNP and money.

The stability of the relationship between real GNP and money was examined by splitting the sample in half and testing the hypothesis that the coefficients are homogeneous over time. The results of the conventional $\underline{F}$ test and the jackknifed version of that test are presented in Table 8. Both the $\underline{F}$ and $\underline{t}$ tests of the hypothesis reveal similar results except in the case of the RGNP on M1 regression. The $\underline{F}$ test rejects at the .05 level the hypothesis of homogeneous coefficients for this regression while the jackknife $\underline{t}$ test fails to reject the hypothesis. For all relationships, the hypothesis of homogeneous relationships over the entire sample period cannot be rejected on the basis of the jackknife $\underline{t}$ test.

The Relationship Between the GNP Price Deflator and Money

Table 9 indicates that the significance levels differ for the $\underline{F}$ and $\underline{t}$ tests of the hypothesis that only the trend and seasonals are significant. The only models which are significant at the .05 level by the jackknife $\underline{t}$ test are the PGNP on M1 regression and the reversed regression of

TABLE 8^{a/}

TESTS OF THE HOMOGENEITY OF COEFFICIENTS FOR
REGRESSIONS BASED ON REAL
GNP AND M1, M2, AND MB

Model	F (18, 42)	J
RGNP = f(M1)	2.071**	-0.368 (-0.211)
M1 = f(RGNP)	1.618	-1.029 (-0.552)
RGNP = f(M2)	1.522	-0.431 (-0.446)
M2 = f(RGNP)	1.716*	-1.588 (-0.762)
RGNP = f(MB)	0.929	-0.367 (-0.399)
MB = f(RGNP)	1.405	-1.064 (-0.439)

^{a/} All notation is the same as that used in Table 4 except RGNP which denotes real gross national product.

M1 on PGNP. The F test rejects the hypothesis at the .10 level for the M1 on PGNP regression only.

The partitions of the models are presented in Table 10. Within the PGNP on M1 regression, the future and all past lag partitions are significantly different from zero at the .05 level on the basis of the jackknife t. By the Granger criterion, this result implies a two-way causal relationship between money and prices.

TABLE 9^{a/}

SUMMARY OF OLS AND JLS REGRESSIONS BASED ON
THE GNP PRICE DEFLATOR, M1, M2, AND MB

Model	Significance of Total Model		Summary Statistics		
	F (13,60)	J	$\bar{R}^2$	Durbin- Watson Statistic	Standard Error of Estimate
PGNP = f(M1)	1.485	2.006 (2.715)**	.604	2.634	.00589
M1 = f(PGNP)	1.690*	2.020 (2.123)**	.783	1.069	.00480
PGNP = f(M2)	0.739	0.429 (0.986)	.549	2.678	.00629
M2 = f(PGNP)	1.321	1.283 (1.347)	.914	0.824	.00546
PGNP = f(MB)	0.509	-0.081 (-0.215)	.528	2.731	.00643
MB = f(PGNP)	0.100	0.484 (0.471)	.759	1.724	.00572

^{a/} Notation is the same as that used in Table 3.

The lag distribution implied by the significance of the various groups of coefficients is also important. The most significant lag partitions in the PGNP on M1 relation are the current and two past lags. The effect of money on prices declines in the following two quarters and increases slightly in the next four quarters. The most significant effect of prices on M1 occurs within the current and four

TABLE 10^a/SIGNIFICANCE TESTS OF THE FUTURE AND PAST LAG PARTITIONS FOR REGRESSIONS
BASED ON THE GNP PRICE DEFLATOR, M1, M2, AND MB

Model	Four Future Coefficients		Current and Eight Lag Coefficients		Current and Four Lag Coefficients		Current and Two Lag Coefficients	
	F (4,60)	J	F (9,60)	J	F (5,60)	J	F (3,60)	J
PGNP = f(M1)	1.459	2.440 (2.679)**	1.706	2.384 (2.366)**	2.310	3.595 (2.094)**	3.771**	7.412 (2.588)**
M1 = f(PGNP)	3.339**	5.010 (1.900)*	1.082	1.162 (1.710)*	1.412	2.069 (1.853)*	1.287	1.844 (1.382)
PGNP = f(M2)	1.078	1.650 (1.955)*	0.934	0.711 (1.089)	1.667	2.092 (1.599)	2.336*	3.867 (1.836)*
M2 = f(PGNP)	2.967**	4.255 (1.584)	0.607	0.178 (0.284)	0.184	-0.576 (-1.315)	0.205	-0.679 (-1.006)
PGNP = f(MB)	0.369	-0.372 (-0.523)	0.521	0.001 (0.027)	0.419	-0.129 (-0.236)	1.733	-0.852 (-1.689)*
MB = f(PGNP)	1.126	0.691 (0.437)	0.957	0.453 (0.390)	0.355	-0.454 (-0.848)	0.216	-1.121 (-1.608)

^a/ Notation is the same as that used in Table 4.

past lag groups of coefficients. This timing relationship agrees with the relationship between nominal GNP and M1, implying that the price effect is dominant.

The relationships between prices and money appear to have been quite stable over the sampling period. The results of the test for the homogeneity of the coefficients are presented in Table 11. Table 11 indicates two points of disagreement between the $\underline{F}$ and the $\underline{t}$ tests. The $\underline{F}$ tests

TABLE 11^{a/}

TESTS OF THE HOMOGENEITY OF COEFFICIENTS FOR
REGRESSIONS BASED ON THE GNP PRICE
DEFLATOR AND M1, M2, AND MB

Model	F (18,42)	J
PGNP = f (M1)	0.797	-1.454 (-0.726)
M1 = f (PGNP)	1.856*	-0.241 (-0.122)
PGNP = f (M2)	0.973	-1.694 (-0.784)
M2 = f (PGNP)	1.325	-0.382 (-0.326)
PGNP = f (MB)	0.651	-2.370 (-0.924)
MB = f (PGNP)	2.126**	-0.388 (-0.221)

^{a/} All notation is the same as that in Table 4.

reject the hypothesis of homogeneous coefficients for the M1 on PGNP and for the MB on PGNP regressions; the jackknife $\underline{t}$ statistics are not significant at any reasonable level. In no case is the hypothesis of homogeneous coefficients rejected by the jackknife $\underline{t}$ tests.

A Synthesis of the Empirical Results

In general, the regressions employed in the present analysis indicate that a two-way causal relationship between money and nominal GNP prevailed during the period of study. The regressions of money on the real and price components of GNP revealed that practically all of the causal influences of money on nominal GNP were through the price component; real GNP was relatively unaffected by money. There were some slight differences, however, depending upon the proxy for money used in the causality test.

The distributed-lag relationship between M1 and nominal GNP was not highly significant; however, the two-way relationship between the two variables was readily apparent. The causal influence of M1 on nominal GNP appeared to operate almost entirely through the price component of GNP while the feedback effects of nominal GNP on M1 operated through both the real and price components of GNP.

The relationship between M2 and nominal GNP was not readily apparent. Most of the evidence indicated unidirectional causality from M2 to nominal GNP although there

were signs of a weak feedback effect from GNP to M2.

Neither the price component nor the real component of GNP appeared to have been significantly related to M2, although the relationship between the price component and M2 was the more significant of the two. Either the causal relationship between M2 and the real and price components of GNP was not consistent during the period of study or it was swamped by extraneous influences. The M2 series appeared to have been influenced quite differently by the real and price components of GNP, with neither effect dominating for the entire period of study.

The most significant relationship between money and nominal GNP was produced when the monetary base series was used as the proxy for money. The monetary base is often assumed to be the more exogenous series of the three commonly used proxies for money. However, the results of the present study clearly indicate that the monetary base is not statistically exogenous. The causal effects of changes in the monetary base on nominal GNP and the feedback effects from GNP to the monetary base appear almost immediately. The monetary base could not, however, be demonstrated to be significantly related to either the price components or the real components of GNP. Such a result implies that either the relationship between the monetary base and the real and price components of GNP was swamped by extraneous influences

or that neither the real nor the price component dominated the relationship during the sample period.

The reversed regressions of the monetary base on the real and price components of GNP were unambiguous. The only significant feedback effects of GNP on the monetary base were through the real component of GNP; the price effects were not significant. Such a result corroborates the conclusion that the monetary base cannot be properly considered to have been an exogenous variable during the period of study.

CHAPTER VI
SUMMARY, ANALYSIS, AND IMPLICATIONS
OF THE RESULTS

Summary

Previous empirical studies of the relationship between money and income have produced conflicting results for one or both of two reasons: (1) precise, testable definitions of causality have not been used; (2) conventional hypothesis-testing procedures have been used to evaluate empirical data which may not meet the assumptions of such tests.

The purpose of this study was to test the nature of the causal relationship between money and income by using a procedure designed to deal implicitly with both problems. Causality was evaluated within a criterion offered by C. W. J. Granger and the hypotheses tested by combining the jackknifing procedure with ordinary least-squares regression. The Granger criterion was demonstrated to be consistent with other concepts of causality but superior in its empirical applicability. The hypothesis tests associated with the jackknife statistic were demonstrated by computer simulation methods to be highly robust when the data are nonnormal and therefore an attractive alternative to the conventional F and t tests of hypotheses relating to regressions and regression coefficients.

The causality analysis revealed that in distributed-lag regressions of nominal GNP on various proxies for money, the hypothesis of unidirectional causality from money to income was rejected. The alternative hypothesis that causality is bidirectional between money and income was found to be consistent with the results. Similar results were found for models of the relationship between nominal GNP and M1, M2, and MB. The model based on GNP and the monetary base was the most significant relationship and appeared to be characterized by the strongest two-way causal relationship. The weakest overall relationship was between GNP and M1; the weakest two-way causal relationship was between GNP and M2.

In an attempt to discover the division of the two-way relationship of money and income between real and price effects, the three money variables were used in models which replaced nominal GNP with real GNP and then in models based on the GNP price deflator. The causality criterion revealed that real GNP was not significantly influenced by any of the measures of money, whereas real GNP exerted a strong causal influence on money. The regressions of RGNP on money and the reversed regressions of money on RGNP allowed rejection of the hypothesis that causality is unidirectional from money to real GNP. The alternative hypothesis that there is a (weak) bidirectional causal relationship between money and real GNP was accepted.

The regressions based upon money and the GNP price deflator produced mixed results. The only significant pair of relationships was between the price deflator and M1. The significant two-way relationship between M1 and the GNP price deflator implied that prices were the primary means by which M1 and nominal GNP influence each other. The nonsignificant relationship between M2 and the price deflator and MB and the price deflator combined with the discovery of nonsignificant relationships between real GNP and M2 and MB implied that the relationships between these variables was divided between real and price effects but that neither effect dominated.

Analysis of Results

The discovery of a two-way causal relationship between money and income supports previous studies with similar findings, although it does conflict with some studies of the causality question. As an example of the latter, Sims (1972) reported empirical results which were consistent with hypothesis of unidirectional causality from money to income. Since the parametric methodology and data used in this study are comparable to those of Sims, the results of the present study cast doubt on Sims' conclusions. Based on the theory and computer simulations comparing the jackknife method to conventional $\underline{F}$ and $\underline{t}$ tests, it appears that Sims' conclusions

were the result of statistical tests which reject too often under conditions of nonnormality. The more robust hypothesis tests associated with the jackknife statistic combined with the Granger criterion do not support the hypothesis of unidirectional causality from money to income.

The two-way causal relationship discovered in the present study is consistent with many other studies of the causality question. For example, a study by Cagan (1965) found that the short-run relationship between money and economic activity was bidirectional during periods of mild cyclical fluctuations. This has been characteristic of the post-World War II period in the United States.

Embodied in the mild cyclical explanation of the bidirectional causal relationship between money and income is the idea that such a relationship could also have been due to the fact that the monetary authorities have not consistently engaged in strong countercyclical activities in the post-war period (Uselton, 1973, p. 25). The lack of strong action by the monetary authorities has produced the result that "much of the apparent cyclic behavior of the money stock has been unrelated to open market purchases and sales of government securities [Uselton, 1973, p. 23]." This proposition implies that the autonomous changes in the money stock which would be necessary to isolate statistically the precise relationship between autonomous changes

in money and income were not present during the sample period, thus strengthening the discovery of a bidirectional relationship between money and income.

In relation to studies of the lag in the effect of monetary policy, the bidirectional findings of this study are not inconsistent with a lag of either a short or a long duration. Granger and Hatanaka (1963) demonstrated that within the context of the Granger causality criterion, a truly unidirectional causal structure could appear to be bidirectional because of the length of the sampling period employed in the analysis. If, for example, money affected income within a few days or weeks of a change in the money stock, then a truly unidirectional relationship could show up as bidirectional if, say, quarterly data were used in the analysis. However, until monthly or weekly GNP data becomes available, the hypothesis that causality is actually unidirectional and that the causality lag is very short cannot be empirically evaluated.

If the results of this study are indicative of a very short causality lag, then the discovery of a nonsignificant relationship between nominal GNP and lagged money is important. This finding supports other studies with similar results. Laffer and Ranson (1971), for example, noted the absence of a significant relationship between lagged growth rates of the money supply and the growth rate of nominal GNP.

A study by Tanner (1969) also failed to find evidence of a lag relationship between money and income. A short causality lag is not necessarily evidence that money does not exert any causal influence on GNP. Such a finding may be interpreted as evidence that changes in the money supply have an immediate and powerful causal influence on nominal GNP.

The possibility that a unidirectional causal structure between money and income might exist over a very long (secular) period was suggested by Cagan (1965). Cagan found that over a very long period, the cyclical or feedback forces tended to cancel each other out, revealing a unidirectional causal structure running from money to prices and nominal income. Cagan's proposition could be tested within the present framework if the leads and lags of the exogenous variables were stated in terms of much longer periods--say years instead of quarters. Such tests were outside the scope of the present analysis but are suggested for future studies.

Two other explanations of the bidirectional relationship between money and income are also important. The proposition that the bidirectional relationship between money and income was due to the seasonal covariation of money and income is unlikely since great care was taken to insure that all seasonal components were removed from the data. Even if some seasonal components were remaining in

the data, there is evidence that the seasonal variations of GNP and money are distinctly different. Laffer and Ranson, for example, found that GNP and money move in the opposite direction two out of four quarters and that both are characterized by distinctly different magnitudes of movements when they move in the same direction (Laffer and Ranson, 1971, p. 260).

Another frequently advanced explanation for a two-way relationship between money and income may be more important. The possibility that feedback effects operate through the money multiplier was discussed previously (Chapter II). The empirical results of the present study suggest that the determinants of the money supply may indeed be very important factors in a complete analysis of the relationship between money and income. A two-way causal relationship between money and income emphasizes the inherent problems associated with assumptions that the common empirical measures of the money stock can be taken to be statistically exogenous. Thus, it becomes necessary to specify the transmission mechanism which carries the feedback effects from income to money. The theoretical money supply process framework developed by Brunner and Meltzer (1964, 1968) and elaborated by Burger (1971) appears to be a viable means of identifying the nature of the feedback effects. The extent to which the empirical proxies for

the variables in that money supply process framework are exogenous or endogenous could be easily analyzed with the technique used in this study. Subjecting that theory to such an evaluation is suggested for future study.

The division of the relationship between money and income according to real and price effects is also an important finding of the present study. The result that prices are significantly related to the money stock but that real GNP is not is consistent with the quantity theory of money which implies that real variables are unaffected by monetary variables. Such a result suggests that monetary policy may not be useful as an instrument to encourage increases in real output and income; monetary policy can encourage increases in nominal income but increases in prices are likely to follow.

Implications of Results and Suggestions for Additional Study

The results of this study suggest a number of implications for empirical economic analysis and for the use of monetary policy. The result that there is a two-way relationship between money and income implies that single-equation estimates based on money and income cannot be validly interpreted as causal relations. The bidirectional relationship implies that the single-equation assumption that the error terms are independent of the

"independent" variables invalid for regression of GNP on money. Thus, attempts to measure the lag in the effect of monetary policy should not proceed in terms of standard single-equation parametric regressions, such as those used by William Gibson (1970). Either a system of simultaneous equations must be employed or distribution-free procedures must be relied on. However, until detailed theoretical constructs of the feedback mechanisms between income and money are specified, the simultaneous equation approach will not be a viable alternative. Distribution-free techniques such as the jackknife technique used in the present study or the type of procedure employed by Uselton (1973) may be the most promising methods for analyzing the relationship between money and income.

In connection with studies of lags, the discovery of bidirectional causality between money and income also suggests that the common methods of estimating rational lag distributions of the effect of money on income or the reverse relation may be producing erroneous results. The most common method of estimating lag distributions is referred to as the Almon lag technique (from the original article by Shirley Almon, 1965). In the original application of the Almon lag technique, Shirley Almon estimated the distribution of the lag between capital appropriations and capital expenditures--clearly a situation of

unidirectional causal influence. Economists have since come to use the Almon lag technique with the same unquestioning reverence as the assumption of normality-- both quite dangerous practices. Indeed, some popular empirical models are deeply rooted in the Almon lag technique and would possibly prove valueless in its absence. Well-known examples are the St. Louis Federal Reserve Bank study of the relative effectiveness of monetary and fiscal policy (Anderson and Jordan, 1968) and the St. Louis Federal Reserve Bank reduced-form macroeconomic model (Anderson and Carlson, 1970). The findings of the present study pose serious questions about the results of these studies. The importance of the exogeneity of the independent variables when using the Almon lag procedure was recently recognized (Schmidt and Waud, 1973). The need for a test of exogeneity prior to employing the Almon lag technique has not, however, been emphasized in the literature. The Granger-Jackknife causality test proposed in the present study may be useful as a preliminary method of determining whether or not the Almon lag technique can be validly employed in a certain model.

Another commonly used statistical technique which could be invalid unless unidirectional causality from the independent to the dependent variables is present is the Durbin method of correcting for serial correlation (Durbin,

1960). The Durbin method consists of correcting the observed data by deriving correction factors through an application of linear multiple regression. If unidirectional causality from the independent to the dependent variables is not present, the corrected results may be meaningless. Hence, use of the Durbin method of correcting for serial correlation should be preceded by a test for the direction of causality such as the Granger-Jackknife proposed in the present study.

The findings of this study do not necessarily imply that the Federal Reserve authorities have no control over the money stock. The results are direct evidence that during the period of study feedback effects from income to money were significant, but it does not follow that the monetary authorities could not have exercised control if they had so desired. The endogeneity of money can be interpreted to mean either that the monetary authorities did not choose to offset the feedback effects but rather adopted a policy of accommodation or that stabilization policy was a function of GNP and other variables during the period of study. The two-way relationship between money and income does not necessarily imply that "money does not matter." The results do imply that it is incorrect to attribute all fluctuations in the money stock and in economic activity to active monetary policy.

The discovery of a two-way causal relationship between money and income may not be inconsistent with the concept of exogeneity of money employed by some economists (Fand, 1970, pp. 12-16). In monetary theory, some writers interpret the concept of exogeneity to be satisfied by a variable which can be directly controlled by the monetary authorities. However, such a definition of exogeneity is not necessarily consistent with the statistical definition of exogeneity which focuses on the absence of correlation between the exogeneous variables and the error terms of a regression equation. Indeed, the differences of interpretation of exogeneity may have clouded the issue of causality between money and income and unnecessarily lengthened the debate of the issue. On the basis of the results of this study, it must be recognized that even though the monetary authorities may have the potential power to control the money stock, they did not do so during the post-war period--at least not consistently and exclusively for economic stabilization purposes.

The results of this study also have important implications for the use of monetary policy. If the monetary authorities are charged with the responsibility of achieving certain goals through their policy actions, then they must rely on some indicator of their policy actions. Ideally, the chosen indicator of monetary policy should be an

exogenous variable if it is to reflect accurately only the actions of the monetary authorities. The discovery of a two-way causal relationship between GNP and M1, M2, and MB implies that none of these measures of money are strictly exogenous and therefore would make poor indicators of monetary policy actions. The partial endogeneity of M1 and M2 has been recognized by many researchers, while the partial endogeneity of MB is not universally recognized. For example, most of the work from the St. Louis Federal Reserve Bank has been heavily based on the assumption that MB is strictly exogenous. This proposition was attacked by De Leeuw and Kalchenbrenner (1969) but the researchers at the St. Louis bank were not convinced (Anderson, 1969). The evidence presented in the present study supports the contentions of De Leeuw and Kalchenbrenner.

A series that may be truly exogenous, "reserves available to support private demand deposits" (RPD), is being compiled by the Board of Governors of the Federal Reserve System. As soon as a long enough series of RPD is available, the testing procedure employed in the present study could be applied to that series to test its exogeneity. If the RPD series is found to be statistically exogenous, then distributed lag regressions of RPD on income could be validly interpreted as causal relations and the series could be validly used to measure the lag in the effect of

monetary policy. Testing for the exogeneity of RPD within the context of the Granger-Jackknife method is suggested for future study.

Another approach to the construction of a truly exogenous series of monetary action has been suggested by Hendershott (1968). Hendershott's approach to the problem of deriving a "neutralized" money stock by removing the feedback effects from the common measures of money seems to be an attractive approach in light of the present results. Hendershott relied on the predictability of a series as a criterion of causality but failed to extend the criterion to test for unidirectional causality. Thus, many of his parameter estimations may be questionable. A combination of the Granger-Jackknife causality test and the method of Hendershott is suggested for future study.

In general, the discovery of a two-way relationship between money and income implies that the common practice of reducing the complexities of a dynamic monetary economy to a single equation may simply be an incorrect specification. The discovery of a two-way relationship between money and income in the light of the popular single-equation approach to the estimation of their relationship recalls the pre-Marshallian debate concerning whether demand or supply was the most important determinant of price (Marshall, Ninth (Variorum) Edition, 1961, p. 348).

Students of the money-income causality debate can be divided into two main groups: those who hold that the primary influence is the "pull" of income on money, operating through the demand for money function and those who emphasize the importance of the "push" of money on income through the supply of money function. The results of the present study indicate Marshall's astute statement some fifty-four years ago that "[we] might as reasonably dispute whether it is the upper or the under blade of a pair of scissors that cuts a piece of paper [p. 348]" may apply in the case of the issue of the causal relationship between money and income. Such a view would integrate the demand and supply forces, a view that has been more or less overlooked heretofore. One exception is a study by Ronald Teigen (1964). The implication of the simultaneous equation approach for model building would be that neither money nor economic activity should be considered to be an exogenous variable. Advances in this area will necessitate more specific formulations of the transmission mechanisms between money and economic activity and between economic activity and money and improvements in tests for the direction of causal influences.

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APPENDIX

- A. Computer Program JACKKNIFE
- B. An Algorithm to Obtain Jackknifed Regression Coefficients from Standard Least-Squares Linear Regression Programs

APPENDIX A: COMPUTER PROGRAM JACKKNIFE

In an attempt to help make the jackknifing regression procedure readily available to anyone who might wish to employ it, the jackknifed least-squares linear regression program written for the present study is presented below. It requires only three control cards and is quite easy to use. It is written in FORTRAN IV in order to make it easy to follow.

The regression procedure is carried out entirely with matrix manipulation in order to speed-up execution time and to minimize the loss of accuracy which may characterize programs based on the standard algebraic regression procedure. The required matrix inversions are accomplished in SUBROUTINE MATINV which is based on a procedure known as the Doolittle Method (for an explanation of this algorithm, see Graybill, 1961, pp. 149-152). This procedure is extremely accurate (it has performed well with matrices possessing determinants as small as $.1 \times 10^{-30}$) and very efficient. Typical execution time for a model with four independent variables and 25 observations and deleting one observation at a time in the jackknifing process (for a total of 26 regressions) is 10 seconds.

The main weakness of the program in its present form is the inflexibility of input data and the lack of transformations for input data. The data must be punched with

the dependent variable either in the first or the last data field. The position of this variable is indicated on control card number 2. Transformations were not included because of the necessary increased core which they would require. The program now compiles in about 130K. If transformations are required, there are two courses available. A simple program may be written to read the input data, transform it in the desired manner, and punch it onto cards. Alternatively, it is quite easy to write a program to transform the data and to store it on a temporary storage device for later execution by JACKKNIFE.

The options which are available to the user of JACKKNIFE are (1) the choice of suppressing the constant term, and (2) the choice of utilizing the jackknife option--the program may also be used as a standard multiple regression package--and (3) the choice of any part of or all of the output. The coding required for these options appears on control card number 2. The control card set-up and the source listing follow.

```

C      PROGRAM JACKKNIFE      WRITTEN BY M. EDWIN IRELAND      JKN 001
C
C
C      CONTROL CARDS ARE AS FOLLOWS      JKN 002
C
C      CARD NUMBER 1      JKN 003
C      COLUMN      INFORMATION TRANSFERRED      JKN 004
C      1-3      PUNCH 1-999, RIGHT JUSTIFIED, TO INDICATE THE NUMBER OF SETS      JKN 005
C      DATA TO BE PROCESSED ON ONE JOB RUN. IF LEFT BLANK, ONLY      JKN 006
C      ONE DATA SET IS ASSUMED.      JKN 007
C
C      CARD NUMBER 2      JKN 008
C      COLUMN      INFORMATION TRANSFERRED      JKN 009
C      1-3      PUNCH NUMBER OF OBSERVATIONS 1-110, RIGHT JUSTIFIED      JKN 010
C      4-5      PUNCH NUMBER OF VARIABLES 1-24, RIGHT JUSTIFIED      JKN 011
C      6-7      PUNCH NUMBER OF CARDS FOR COMPLETE FORMAT 1-2, RIGHT JUSTIFIED      JKN 012
C      8      PUNCH 1 IF Y VALUES ARE PUNCHED IN THE FIRST DATA FIELD.      JKN 013
C      LEAVE BLANK IF THEY ARE PUNCHED IN THE LAST DATA CARD FIELD.      JKN 014
C      10-20      PUNCH 1 IN EACH COLUMN WHICH INDICATES THE OUTPUT DESIRED *      JKN 015
C      23-24      PUNCH THE NUMBER OF VARIABLE IDENTIFICATION CARDS TO BE      JKN 016
C      USED IF ANY. THIS NUMBER WILL INCLUDE A TITLE CARD ALSO      JKN 017
C      25-26      PUNCH 88 TO INDICATE THAT THE CONSTANT IS TO BE CONSTRAINED      JKN 018
C      TO ZERO.      JKN 019
C      27-29      PUNCH THE NUMBER OF SUBSETS TO BE DELETED IN COMPUTING THE      JKN 020
C      JACKKNIFE ESTIMATORS.      JKN 021
C      30-32      PUNCH THE NUMBER OF OBSERVATIONS TO BE DELETED ON EACH PASS      JKN 022
C      IN THE JACKKNIFE PROCEDURE. IMPORTANT--THE NUMBER OF      JKN 023
C      SUBSETS INDICATED IN COLUMNS 27-29 TIMES THE SIZE OF EACH      JKN 024
C      SUBSET MUST BE EQUAL TO THE TOTAL NUMBER OF OBSERVATIONS.      JKN 025
C
C      CARD NUMBER 3 . . . THE IDENTIFICATION SET, IF USED      JKN 026
C      PUNCH ANY REMARKS OR IDENTIFICATION IN COLUMNS 1-80 ON ANY      JKN 027
C      NUMBER OF CARDS, THE TOTAL NUMBER OF WHICH IS PUNCHED IN      JKN 028
C      COLUMNS 23-24 OF CARD NUMBER 2      JKN 029
C
C      CARD NUMBER 4      JKN 030
C      COLUMN      INFORMATION TRANSFERRED      JKN 031
C      1-72      PUNCH F-TYPE FORMAT SPECIFICATIONS WHICH CORRESPOND TO THE      JKN 032
C      WAY THE DATA IS PUNCHED ON THE CARDS      JKN 033
C
C      CARD NUMBER 5 TO END OF DATA SET - PUNCH AND ENTER DATA HERE      JKN 034
C
C      COLUMN CODES TO SELECT PRINTED OUTPUT DESIRED      JKN 035
C      COLUMN      OUTPUT TO BE PRINTED      JKN 036
C      10      INPUT X VALUES AND INPUT Y VALUES      JKN 037
C      11      FORMAT FOR INPUT X AND Y VALUES      JKN 038
C      12      XIX - X TRANSPOSED TIMES X      JKN 039
C      13      XIX-1 - INVERTED XIX MATRIX      JKN 040
C      14      XIY - TRANSPOSED X TIMES Y      JKN 041
C      15      YIY - Y TIMES TRANSPOSED Y      JKN 042
C      16      ESTIMATED Y, ERROR, PERCENTAGE ERROR, DURBIN-WATSON      JKN 043
C      17      COEFFICIENTS + F-TESTS FOR THE I TH REGRESSION      JKN 044
C      18      PSEUDO REGRESSION COEFFICIENTS + F-TESTS      JKN 045
C      19      COEFFICIENT, VARIANCE, T-TEST, AND F-TESTS      JKN 046
C      20      ESTIMATED Y AND RESIDUALS RESULTING FROM JACKKNIFED COEFFICIENTS      JKN 047
C
C      IMPLICIT REAL*8(A-H,O-Y)      JKN 048
C      REAL *4XX,YY      JKN 049
C      INTEGER Z      JKN 050
C      COMMON INTXV(25)      JKN 051
C      INTEGER*2 IDEL1(110), IDEL2(110)      JKN 052
C      DIMENSION XX(111,25), YY(110), BJ(110,25), X(111,25), Y      JKN 053
C      1(110), IOUT(11), FORM(36), BETA(25), XIX1(25,25), XIX(25,25), XIY(      JKN 054
C      225), UBE(25), BJ8(110,25), DIFF30(25), PFTST(110), CFTST(110), BET      JKN 055
C      38(25)      JKN 056
C      EQUIVALENCE (XIX1(1,1),XIX(1,1))      JKN 057
C      READ (5,1040) IPROBS      JKN 058

```

DO 140 KPROBL=1,IPROBS	JKN 070
IDEM=25	JKN 071
MXCBS=110	JKN 072
MXOBPI=MXCBS+1	JKN 073
FJTST=0.0000	JKN 074
DO 10 J=1,IDEM	JKN 075
DIFFRU(J)=0.0000	JKN 076
UBE(J)=0.0000	JKN 077
BETA(J)=0.0000	JKN 078
DO 10 I=1,MXCBS	JKN 079
BJ3(I,J)=0.0000	JKN 080
BJ(I,J)=0.0000	JKN 081
10 CONTINUE	JKN 082
DO 20 J=1,MXCBS	JKN 083
YY(J)=0.0000	JKN 084
IDEL1(J)=0	JKN 085
20 IDEL2(J)=0	JKN 086
DO 30 I=1,MXOBPI	JKN 087
DO 30 J=1,IDEM	JKN 088
XX(I,J)=0.0000	JKN 089
30 CONTINUE	JKN 090
DO 40 I=1,IDEM	JKN 091
DO 40 J=1,IDEM	JKN 092
40 X1X1(I,J)=0.0000	JKN 093
READ (5,1140) NOBS,NVAR,IFOPM,LOCATY,(ICUT(J),J=1,11),ITITL,IZC,	JKN 094
1 NDEL,MSIZE	JKN 095
IF (NDEL.GT.0) NDEL=NDEL+1	JKN 096
IF (NDEL.EQ.0) MSIZE=0	JKN 097
I1=1	JKN 098
I2=MSIZE	JKN 099
ND=0	JKN 100
DO 50 I=2,NDEL	JKN 101
IDEL1(I)=I1	JKN 102
IDEL2(I)=I2	JKN 103
ND=ND+1	JKN 104
I1=I1+MSIZE	JKN 105
I2=I2+MSIZE	JKN 106
IF (I1.GT.NOBS.OR.I2.GT.NOBS) GO TO 51	JKN 107
50 CONTINUE	JKN 108
51 NDEL=ND+1	JKN 109
KDEL=NDEL-1	JKN 110
NCBST=NOBS	JKN 111
NVART=NVAR	JKN 112
IF (ITITL) 80,80,60	JKN 113
60 WRITE (6,1200)	JKN 114
DO 70 I=1,ITITL	JKN 115
READ (5,1190) (FORM(J),J=1,20)	JKN 116
70 WRITE (6,1210) (FCPM(J),J=1,20)	JKN 117
80 WRITE (6,1150)	JKN 118
IF (IFCRM) 90,90,100	JKN 119
90 IFORM=1	JKN 120
100 IFCFM=IFORM*18	JKN 121
READ (5,1160) (FORM(J),J=1,IFORM)	JKN 122
Z=2	JKN 123
IF (IZC.GT.0) Z=1	JKN 124
IF (IZC.EQ.0) NVART=NVART+1	JKN 125
DO 130 I=1,NOBST	JKN 126
IF (LOCATY-1) 120,110,120	JKN 127
110 READ (5,FORM) YY(I),(XX(I,J),J=Z,NVART)	JKN 128
GO TO 130	JKN 129
120 READ (5,FORM) (XX(I,J),J=Z,NVART),YY(I)	JKN 130
130 CONTINUE	JKN 131
IF (NCBST.GT.NVART) GO TO 140	JKN 132
WRITE (6,1030) KPROBL	JKN 133
GO TO 840	JKN 134
140 DO 150 IJKNF=1,NDEL	JKN 135
Y1Y=0.0000	JKN 136
JDEL=IDEL1(IJKNF)	JKN 137
NOBSTC=0	JKN 138

DO 160 I=1,NOBST	JKN 139
IF (I.GE.IDEL1(IJKNF).AND.I.LE.IDEL2(IJKNF)) GO TO 160	JKN 140
NOBSTC=NOBSTC+1	JKN 141
Y(NOBSTC)=YY(I)	JKN 142
Y1Y=Y1Y+(Y(NOBSTC)*Y(NOBSTC))	JKN 143
DO 150 J=Z,NVART	JKN 144
X(NOBSTC,J)=XX(I,J)	JKN 145
150 CONTINUE	JKN 146
160 CCNTINUE	JKN 147
NOBS=NOBSTC	JKN 148
NVAR=NVART	JKN 149
NVAFM1=NVAR-1	JKN 150
ANDBS=NOBS	JKN 151
VM1=NVAR-1	JKN 152
OMV=NOBS-NVAR	JKN 153
IF (IJKNF.NE.2) GO TO 170	JKN 154
IF (NOBS.GE.(NVAR+2)) GO TO 170	JKN 155
IOBREQ=MSIZE+1	JKN 156
WRITE (6,1020) IOBREQ,KPROBL	JKN 157
GO TO 840	JKN 158
170 JKNF=IJKNF-1	JKN 159
IF (IZC.GT.0) GO TO 190	JKN 160
DO 180 I=1,NOBS	JKN 161
180 X(I,1)=1.0000	JKN 162
190 CCNTINUE	JKN 163
INTXV(1)=0	JKN 164
L=1	JKN 165
IF (IZC.GT.0) L=0	JKN 166
DO 200 I=Z,NVAR	JKN 167
200 INTXV(I)=I-L	JKN 168
IF (IOUT(1)) 210,220,210	JKN 169
210 WRITE (6,860)	JKN 170
WRITE (6,850)	JKN 171
MATRIX=1	JKN 172
DO 215 J=1,NOBS	JKN 173
215 X(J,NVAR+1)=Y(J)	JKN 174
CALL OUTPUT (MXOBP1, IDEM, NOBS, NVAR, X, MATRIX)	JKN 175
220 CCNTINUE	JKN 176
IF (IOUT(2)) 270,280,270	JKN 177
270 WRITE (6,1170) (FORM(J),J=1,IFORM)	JKN 178
280 CONTINUE	JKN 179
DO 290 L=1,NVAR	JKN 180
DO 290 I=1,NVAR	JKN 181
X1X(L,I)=0.0	JKN 182
DO 290 K=1,NOBS	JKN 183
290 X1X(L,I)=X1X(L,I)+(X(K,L)*X(K,I))	JKN 184
IF (IOUT(3)) 300,310,300	JKN 185
300 WRITE (6,860)	JKN 186
WRITE (6,890)	JKN 187
MATRIX=2	JKN 188
CALL OUTPUT (IDEM, IDEM, NVAR, NVAR, X1X, MATRIX)	JKN 189
310 CONTINUE	JKN 190
JCALL=1	JKN 191
CALL MATINV (X1X, NVAR, DETER, IDEM)	JKN 192
IF (JDEL.GT.0) GO TO 320	JKN 193
IF (DETER.LE.1.0D-30) WRITE (6,1010) IJKNF, JCALL, DETER	JKN 194
320 IF (IOUT(4)) 330,340,330	JKN 195
330 WRITE (6,860)	JKN 196
WRITE (6,910)	JKN 197
MATRIX=2	JKN 198
CALL OUTPUT (IDEM, IDEM, NVAR, NVAR, X1X1, MATRIX)	JKN 199
340 CCNTINUE	JKN 200
IF (IOUT(5)) 350,360,350	JKN 201
350 WRITE (6,900)	JKN 202
360 DO 370 I=1,NVAR	JKN 203
X1Y(I)=0.0	JKN 204
DO 370 J=1,NOBS	JKN 205
X1Y(I)=X1Y(I)+(X(J,I)*Y(J))	JKN 206
370 CONTINUE	JKN 207

	IF (IOUT(5)) 380,400,380	JKN 208
380	DO 390 I=1,NVAR	JKN 209
	WRITE (6,880) X1Y(I)	JKN 210
390	CONTINUE	JKN 211
400	IF (IOUT(6)) 410,420,410	JKN 212
410	WRITE (6,1130) Y1Y	JKN 213
420	DO 450 I=1,NVAR	JKN 214
	DO 450 J=1,NVAR	JKN 215
	IF (JDEL) 430,430,440	JKN 216
430	BETA(I)=BETA(I)+(X1X1(I,J)*X1Y(J))	JKN 217
	GO TO 450	JKN 218
440	BJ(JKNF,I)=BJ(JKNF,I)+(X1X1(I,J)*X1Y(J))	JKN 219
450	CONTINUE	JKN 220
	DO 480 I=1,NVAR	JKN 221
	IF (JDEL) 460,460,470	JKN 222
460	BETA(I)=BETA(I)	JKN 223
	GO TO 480	JKN 224
470	BETA(I)=BJ(JKNF,I)	JKN 225
480	CONTINUE	JKN 226
	SSB=0.0	JKN 227
	DO 490 I=1,NVAR	JKN 228
490	SSB=SSB+(BETA(I)*X1Y(I))	JKN 229
	SSRES=Y1Y-SSB	JKN 230
	RESMS=SSRES/OMV	JKN 231
	IF (JDEL.GT.0) GO TO 510	JKN 232
	WRITE (6,990)	JKN 233
	WRITE (6,860)	JKN 234
	WRITE (6,920)	JKN 235
	WRITE (6,860)	JKN 236
	DO 500 I=1,NVAR	JKN 237
	S2BI=RESMS*X1X1(I,I)	JKN 238
	SBI=DSQRT(DABS(S2BI))	JKN 239
	SSBI=(DABS(BETA(I)))/SBI	JKN 240
500	WRITE (6,990) INTXV(I),BETA(I),S2BI,SBI,SSBI	JKN 241
	WRITE (6,860)	JKN 242
510	CONTINUE	JKN 243
	NVAFM1=NVAR-1	JKN 244
	DO 520 I=1,NVAFM1	JKN 245
	II=I+1	JKN 246
	DO 520 J=1,NVAFM1	JKN 247
	JJ=J+1	JKN 248
520	X1X1(I,J)=X1X1(II,JJ)	JKN 249
	JCALL=2	JKN 250
	CALL MATINV (X1X1,NVAFM1,DETER,IDEM)	JKN 251
	IF (JDEL.GT.0) GO TO 530	JKN 252
	IF (IDETER.LE.1.0D-30) WRITE (6,1010) IJKNF,JCALL,DETER	JKN 253
530	SSREG=0.0000	JKN 254
	DO 550 I=1,NVAFM1	JKN 255
	SREG=0.0000	JKN 256
	DO 540 J=1,NVAFM1	JKN 257
	JJ=J+1	JKN 258
540	SREG=SREG+(BETA(JJ)*X1X1(I,J))	JKN 259
	II=I+1	JKN 260
550	SSREG=SSREG+(SREG*BETA(II))	JKN 261
	SSM=SSB-SSREG	JKN 262
	RSQ=SSREG/(SSREG+SSRES)	JKN 263
	RBARSQ=1.0-(((1-RSQ)*(NOBS-1))/OMV)	JKN 264
	BMS=SSREG/VM1	JKN 265
	FTST=BMS/RESMS	JKN 266
	IF (JDEL) 570,570,560	JKN 267
560	CF1ST(JKNF)=FTST	JKN 268
	PFTST(JKNF)=(DFLOAT(KDEL)*FFTST)-((DFLOAT(KDEL-1))*FTST)	JKN 269
	FJTST=FJTST+PFTST(JKNF)	JKN 270
570	NCMAV=NCBS-NVAR	JKN 271
	NVM1=NVAR-1	JKN 272
	IF (JDEL.EQ.0) FFTST=FTST	JKN 273
	IF (JDEL.GT.0) GO TO 580	JKN 274
	WRITE (6,930) NOBS,Y1Y,NVAR,SSB,SSM	JKN 275
	WRITE (6,940) NVM1,SSREG,BMS,FTST	JKN 276

	WRITE (6,950) NCMNV,SSRES,RESMS,RSQ	JKN 277
	WRITE (6,1000) RBAKSQ	JKN 278
	IF (KDEL.GT.0) GO TO 580	JKN 279
580	IF (ICUT(7)) 590,630,590	JKN 280
590	WRITE (6,860)	JKN 281
	WRITE (6,970)	JKN 282
	WRITE (6,860)	JKN 283
	DWDEN=0.0000000	JKN 284
	DWNLN=C.0000000	JKN 285
	SUMERT=0.0000000	JKN 286
	SMERSQ=0.0000000	JKN 287
	DO 620 I=1,NBRS	JKN 288
	YEST=0.0000000	JKN 289
	DO 600 J=1,NVAR	JKN 290
600	YEST=YEST+(BETB(J)*X(I,J))	JKN 291
	ERT=Y(I)-YEST	JKN 292
	SMERSQ=SMERSQ+(ERT*ERT)	JKN 293
	SUMERT=SUMERT+ERT	JKN 294
	PCTERT=(ERT/Y(I))*100	JKN 295
	IF (IZC.GT.0) GO TO 620	JKN 296
	IF (I.EQ.1) GO TO 610	JKN 297
	DWNUM=DWNUM+((ERT-ERTM1)**2)	JKN 298
610	ERTM1=ERT	JKN 299
	DWDEN=DWDEN+(ERT**2)	JKN 300
620	WRITE (6,980) I,Y(I),YEST,ERT,PCTERT	JKN 301
	WRITE(6,1220) SUMERT	JKN 302
	SQERS=DSQRT(SMERSQ/OMV)	JKN 303
	WRITE(6,1230) SQERS	JKN 304
	IF (IZC.GT.0) GO TO 630	JKN 305
	DW=DWNUM/DWDEN	JKN 306
	WRITE (6,1180) DW,NVM1,NBRS	JKN 307
630	IF (IJKNF.GT.1) GO TO 650	JKN 308
	DO 640 I=1,7	JKN 309
640	ICUT(I)=0	JKN 310
650	CONTINUE	JKN 311
	IF (NDEL.EQ.0) GO TO 840	JKN 312
	ANDEL=KDEL	JKN 313
	QUOTNT=1.0000/ANDEL	JKN 314
	FJTST=(QUOTNT*FJTST)	JKN 315
	FJTD=C.0000	JKN 316
	DO 670 I=1,KDEL	JKN 317
	DO 660 J=1,NVAR	JKN 318
660	BJB(I,J)=BJB(I,J)+((ANDEL*BETA(J))-((ANDEL-1.0000)*BJ(I,J)))	JKN 319
670	CONTINUE	JKN 320
	DO 690 J=1,NVAR	JKN 321
	DO 680 I=1,KDEL	JKN 322
680	UBE(J)=UBE(J)+BJB(I,J)	JKN 323
690	CONTINUE	JKN 324
	DO 700 J=1,NVAR	JKN 325
700	UBE(J)=UBE(J)*QUOTNT	JKN 326
	SQRTND=DSQRT(ANDEL*(ANDEL-1.0000))	JKN 327
	DO 720 I=1,KDEL	JKN 328
	DO 710 J=1,NVAR	JKN 329
710	DIFFBU(J)=DIFFBU(J)+((BJB(I,J)-UBE(J))*2)	JKN 330
720	CONTINUE	JKN 331
	MATRIX=3	JKN 332
	IF (ICUT(8)) 750,750,730	JKN 333
730	WRITE (6,1070) KDEL	JKN 334
	DO 740 J=1,KDEL	JKN 335
740	BJ(J,NVAR+1)=CFTST(J)	JKN 336
	CALL OUTPUT (MXORS,IDEM,KDEL,NVAR,BJ,MATRIX)	JKN 337
	MATRIX=3	JKN 338
750	IF (ICUT(9)) 780,780,760	JKN 339
760	WRITE (6,1080)	JKN 340
	DO 765 J=1,KDEL	JKN 341
765	BJB(J,NVAR+1)=PFTST(J)	JKN 342
	CALL OUTPUT (MXORS,IDEM,KDEL,NVAR,BJB,MATRIX)	JKN 343
	DO 770 J=1,KDEL	JKN 344
770	FJTD=FJTD+((PFTST(J)-FJTST)**2)	JKN 345


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      TTSTF=(FJTST*SQRIND)/DSQRT(FJTD)
      WRITE (6,860)
      WRITE (6,860)
780    IF (IOUT(10)) 840,840,790
790    L=1
      IF (IZC.GT.0) L=0
      WRITE (6,1050)
      WRITE (6,860)
      WRITE (6,920)
      WRITE (6,860)
      DO 800 J=1,NVAR
      N=J-L
      TTEST=(SQRIND*DABS(UBE(J)))/(DSQRT(DIFFBU(J)))
      DIFFBU(J)=DIFFBU(J)/(ANDEL*(ANDEL-1.0000))
      SERJAK=DSQRT(DIFFBU(J))
800    WRITE (6,960) N,UBE(J),DIFFBU(J),SERJAK,TTEST
      WRITE (6,1060) FJTST
      WRITE (6,1090) TTSTF
      WRITE (6,860)
      IF (IOUT(11)) 810,840,810
810    WRITE (6,970)
      WRITE (6,860)
      SUMERJ=0.000000
      SMERJS=0.000000
      NCBS=NCBS+MSIZE
      DO 830 I=1,NOBS
      YESTJ=0.000000
      DO 820 J=1,NVAR
820    YESTJ=YESTJ+(UBE(J)*X(I,J))
      ERJ=0.000000
      ERJ=Y(I)-YESTJ
      SMERJS=SMERJS+(ERJ*ERJ)
      SUMERJ=SUMERJ+ERJ
      PCTERJ=0.000000
      PCTERJ=(ERJ/Y(I))*100
830    WRITE (6,980) I,Y(I),YESTJ,ERJ,PCTERJ
      WRITE (6,1220) SUMERJ
      WRITE (6,1240) SMERJS
      SQRJS=DSQRT(SMERJS/OMV)
      WRITE(6,1230) SQRJS
840    CONTINUE
      CALL EXIT
C
850    FORMAT (1H ,42X,22HMATRIX OF INPUT VALUES)
860    FCRMAT (1H0)
880    FCRMAT (1H ,48X,E15.8)
890    FORMAT (1H ,25X,56HTHE MATRIX PRODUCT OF X TRANSPOSED TIMES INPUT
1X... (X'X))
900    FORMAT(1HC,29X,'THE MATRIX PRODUCT OF X TRANSPOSED TIMES INPUT Y..
Z.',5H(X'Y),/)
910    FORMAT(1H ,75X,2H-1,/46X,30HX TRANSPOSED X INVERSE...(X'X))
920    FORMAT(1H0,10X,'COEFFICIENT',8X,'REGRESSION COEF.',7X,'VARIANCE',
28X,'STANDARD ERROR',8X,'T - STATISTIC')
930    FORMAT(1HC,33X,'ANALYSIS OF VARIANCE',//
1//10X,
70HSOURCE OF DEGREES OF SUM OF SQUJKN
2ARES MEAN SQUARE ,23X,8HF - TEST ,/10X,JKN
39HVARIAION ,4X,7HFREEDOM,/10X,5HTOTAL,I13,11X,E15.8,/10X,5HMCDELJKN
4,I13,11X,E15.8,/10X,4HMEAN,13X,1H1,11X,E15.8)
940    FORMAT(1H ,9X,'REGRESSION ',I4,11X,E15.8,14X,E15.8,18X,E15.8)
950    FORMAT(1H ,9X,
11HRESIDUALJKN
2 ,I7,11X,E15.8,14X,E15.8,/,11X,1H2,/,10X,4HR = ,E15.8)
960    FORMAT (1H ,13X,1HB,I3,12X,E15.8,4X,E15.8,4X,E15.8,6X,E15.8)
970    FORMAT(1H0,9X,'OBSERVATION',7X,'OBSERVED Y',16X,'CALCULATED Y',18XJKN
1,'ERROR',17X,'PERCENTAGE ERROR')
980    FCRMAT (1H ,11X,I3,10X,D15.8,3(12X,D15.8))
990    FORMAT (1H1,50X,'SUMMARY TABLE')
1000    FCRMAT(1H0,10X,'2',/,10X,'% ADJUSTED FOR DEGREES OF FREEDOM =',
1E15.8,/)
1010    FCRMAT(1H0,'**** THE MATRIX APPEARS TO BE SINGULAR FOR JACKKNIFE

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1STEP =',I4,' FOR CALL =',I2,' DETERMINANT = ',D15.8,/) JKN 415
1020 FORMAT(6(/),1X,5(1H$),' THE NUMBER OF OBSERVATIONS IS < THAN OP = JKN 416
1 TO THE NUMBER OF VARIABLES PLUS ',I3,' , THEREFORE JACKKNIFING IS JKN 417
2NOT POSSIBLE . . .',//6X,' PROBLEM NUMBER ',I3,' HAS BEEN TERMINA JKN 418
3TED',/) JKN 419
1030 FORMAT(1H0,6(/),1X,5(1H*),' THE NUMBER OF OBSERVATIONS IS < CR = JKN 420
1TO THE NUMBER OF VARIABLES, THEREFORE REGRESSION IS NOT POSSIBLE . JKN 421
2 . .',//6X,' PROBLEM NUMBER ',I3,' HAS BEEN TERMINATED',/) JKN 422
1040 FORMAT (26I3) JKN 423
1050 FORMAT (1H1,46X,' THE JACKKNIFED VALUES') JKN 424
1060 FORMAT(1H0,10X,' F - TEST =',E15.8) JKN 425
1070 FORMAT(1H0,43X,' VALUES FOR THE I TH REGRESSION, I = 1,',I3) JKN 426
1080 FORMAT(1H0,56X,' THE PSEUDO VALUES') JKN 427
1090 FORMAT (1H0,10X,' T - TEST OF REGRESSION = ',E15.8) JKN 428
1130 FORMAT (1H0,///45X,3H(Y'Y) = ,E15.8) JKN 429
1140 FORMAT (I3,2I2,1I,1X,11I1,2X,2I2,2I3) JKN 430
1150 FORMAT (1H1) JKN 431
1160 FORMAT (18A4) JKN 432
1170 FORMAT (1H0,23H THE VARIABLE FORMAT IS ,3X,18A4,/(1H ,25X,18A4)) JKN 433
1180 FORMAT (1H0,10X,' DURBIN-WATSON =',F8.5,2X,5H(K' =,I3,' , N = ', JKN 434
1I3,')') JKN 435
1190 FORMAT (20A4) JKN 436
1200 FORMAT (1H1,25X,31H IDENTIFICATION OF THE VARIABLES,/) JKN 437
1210 FORMAT (1H0,20A4) JKN 438
1220 FORMAT (1H ,//,11X,' SUM OF RESIDUALS = ',E15.8) JKN 439
1230 FORMAT (1H0,10X,' STANDARD ERROR OF ESTIMATE =',E15.8) JKN 440
1240 FORMAT (1H0,10X,' SUM OF SQUARED RESIDUALS = ',E15.8) JKN 441
END JKN 442
SUBROUTINE OUTPUT (II,JJ,NOBS,NVAR,X1X,MATRIX) OUT 001
REAL *8X1X OUT 002
REAL LEAD1(4), LEAD2(4), LEAD3(25), LEAD4(4) OUT 003
Z , LEAD5(4) OUT 004
COMMON INTXV(25) OUT 005
DIMENSION X1X(II,JJ), CHAR(4) OUT 006
DATA IBLNK / 1H /, CHAR / 1HX, 1H8,1HY, 1HF / OUT 007
DATA LEAD1 / 4H 08, 4HSERV, 4HATIO, 3HN / OUT 008
DATA LEAD2 / 4H D, 4HELET, 4HION ,3H / OUT 009
DATA LEAD5 / 4H ,4H R,4HOW ,3H / OUT 010
IF (MATRIX.NE.2) NVAR=NVAR+1 OUT 011
IF (MATRIX.NE.2) INTXV(NVAR)=1 OUT 012
GO TO (10,10,60) ,MATRIX OUT 013
10 DO 20 I=1,NVAR OUT 014
20 LEAD3(I)=CHAR(1) OUT 015
IF (MATRIX.EQ.1) LEAD3(NVAR)=CHAR(3) OUT 016
IF (MATRIX.EQ.2) GO TO 40 OUT 017
DO 30 I=1,4 OUT 018
30 LEAD4(I)=LEAD1(I) OUT 019
GO TO 90 OUT 020
40 DO 50 I=1,4 OUT 021
50 LEAD4(I)=LEAD5(I) OUT 022
GO TO 90 OUT 023
60 DO 70 I=1,NVAR OUT 024
70 LEAD3(I)=CHAR(2) OUT 025
LEAD3(NVAR)=CHAR(4) OUT 026
DO 80 I=1,4 OUT 027
80 LEAD4(I)=LEAD2(I) OUT 028
90 IM=1 OUT 029
IN=NVAR OUT 030
IF (NVAR-7) 110,110,100 OUT 031
100 IN=7 OUT 032
110 CONTINUE OUT 033
WRITE (6,190) OUT 034
IMI=IM OUT 035
INI=IN OUT 036
120 WRITE(6,180) LEAD4,(IBLNK,LEAD3(KJK),INTXV(KJK),KJK=IMI,INI) OUT 037
WRITE (6,190) OUT 038
DO 130 I=1,NOBS OUT 039
130 WRITE(6,170) I, (X1X(I,J),J=IM,IN) OUT 040
IM=IM+7 OUT 041

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	IN=IN+7	OUT 042
	IF (NVAR-IN) 140,150,150	OUT 043
140	IN=NVAR	OUT 044
150	IF (NVAR-IM) 160,110,110	OUT 045
160	IF (MATRIX.NE.2) NVAR=NVAR-1	OUT 046
	RETURN	OUT 047
C		OUT 048
170	FORMAT (1H ,I9,6X,7(1X,E15.8))	OUT 049
180	FORMAT (1H0,3A4,A3,7(5X,2A1,I2,7X))	OUT 050
190	FORMAT (1H0)	OUT 051
	END	OUT 052
	SUBROUTINE MATINV (ARRAY,N,D,NDEM)	MAT 001
	IMPLICIT REAL*8(A-H,O-Z)	MAT 002
	DIMENSION ARRAY(NDEM,NDEM)	MAT 003
	D=1.0	MAT 004
	DO 100 L=1,N	MAT 005
	RECIP=1./ARRAY(L,L)	MAT 006
	D=D*ARRAY(L,L)	MAT 007
	ARRAY(L,L)=-RECIP	MAT 008
	DO 100 I=1,N	MAT 009
	IF (I-L) 10,100,20	MAT 010
10	R=ARRAY(I,L)*RECIP	MAT 011
	GO TO 30	MAT 012
20	R=ARRAY(L,I)*RECIP	MAT 013
30	CONTINUE	MAT 014
	DO 70 J=I,N	MAT 015
	IF (J-L) 40,70,50	MAT 016
40	ARRAY(I,J)=ARRAY(I,J)-R*ARRAY(J,L)	MAT 017
	GO TO 60	MAT 018
50	ARRAY(I,J)=ARRAY(I,J)-R*ARRAY(L,J)	MAT 019
60	CONTINUE	MAT 020
70	CONTINUE	MAT 021
	IF (I-L) 80,100,90	MAT 022
80	ARRAY(I,L)=R	MAT 023
	GO TO 100	MAT 024
90	ARRAY(L,I)=R	MAT 025
100	CONTINUE	MAT 026
	DO 110 I=1,N	MAT 027
	DO 110 J=I,N	MAT 028
	ARRAY(I,J)=-ARRAY(I,J)	MAT 029
	ARRAY(J,I)=ARRAY(I,J)	MAT 030
110	CONTINUE	MAT 031
	RETURN	MAT 032
	END	MAT 033

APPENDIX B: AN ALGORITHM TO OBTAIN JACKKNIFED REGRESSION
COEFFICIENTS FROM STANDARD LEAST-SQUARES
LINEAR REGRESSION PROGRAMS

If the computer program presented in Appendix A is not used to produce jackknifed regression coefficients, another alternative is available. The output from a standard least-squares linear regression program can be transformed into jackknifed coefficients with little additional computation. If the number of observations and variables is not too large, these calculations may be done by hand. The required matrix manipulations are, however, easily programed on a digital computer. The data required for this transformation method are (1) the inverted matrix of the original data times its transpose, $(X'X)^{-1}$, and (2) the vector of least-squares regression coefficients, $\hat{\beta}$. The method is essentially a means of adjusting the original $(X'X)^{-1}$ matrix when one observation is deleted (this method is limited to a deletion size of one) based on the extension of an identity by Bartlett (1951). The algorithm follows.

Given the general linear model

$$y = X\beta + e, \quad (B-1)$$

the least-squares solution is

$$\hat{\beta} = (X'X)^{-1} X'y. \quad (B-2)$$

Consider the case of partitioning the original X and y matrices as follows:

$$X = \begin{bmatrix} X'_1 \\ X_2 \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \quad (B-3)$$

where X'_1 is a row vector of the deleted observation and y_1 is a scalar of the deleted observation. The matrix X' can now be represented as

$$X' = [X_1 | X'_2], \quad (B-4)$$

and $X'y$ becomes

$$X'y = [X_1 | X'_2] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = X_1 y_1 + X'_2 y_2. \quad (B-5)$$

Manipulation of (B-5) reveals that

$$X'_2 y_2 = X'y - X_1 y_1. \quad (B-6)$$

A similar manipulation of the $X'X$ matrix produces

$$X'_2 X_2 = X'X - X_1 X'_1. \quad (B-7)$$

The vector of regression coefficients which results from the exclusion of the X'_1 row vector from the original X matrix may be written as

$$\hat{\beta}_1 = (X_2' X_2)^{-1} X_2' y_2 \quad (B-8)$$

By virtue of equations (B-5) and (B-7), (B-8) may be written as

$$\hat{\beta}_1 = (X'X - X_1 X_1')^{-1} (X'y - X_1 y_1). \quad (B-9)$$

Substituting this result into the identity by Bartlett (1951) yields:

$$(X'X - X_1 X_1')^{-1} = (X'X)^{-1} + \frac{(X'X)^{-1} X_1 X_1' (X'X)^{-1}}{1 - X_1' (X'X)^{-1} X_1}, \quad (B-10)$$

which implies that

$$\begin{aligned} \hat{\beta}_i = (X'X)^{-1} X'y - (X'X)^{-1} X_i y_i + \frac{(X'X)^{-1} X_i X_i' (X'X)^{-1} X'y}{1 - X_i' (X'X)^{-1} X_i} \\ - \frac{(X'X)^{-1} X_i X_i' (X'X)^{-1} X_i y_i}{1 - X_i' (X'X)^{-1} X_i}, \end{aligned} \quad (B-11)$$

where $\hat{\beta}_i$ represents the vector of regression coefficients which results from deleting the i^{th} observation, X_i denotes the i^{th} deleted row, and y_i denotes the i^{th} deleted dependent variable. The expression (B-11) may be simplified and rewritten as:

$$\hat{\beta}_i =$$

$$\hat{\beta} + \frac{S^{-1} X_i X_i' \hat{\beta}}{1 - X_i' S^{-1} X_i} - S^{-1} \left[I + \frac{X_i X_i' S^{-1}}{1 - X_i' S^{-1} X_i} \right] X_i y_i, \quad (B-12)$$

where S^{-1} denotes the original $(X'X)^{-1}$ matrix and $\hat{\beta}$ denotes the vector of coefficients derived from the original S^{-1} matrix.

The expression (B-12) is extremely useful since it contains only one inverse matrix--the original $(X'X)^{-1}$. Thus, the computation of the jackknifed regression coefficients which result from deleting a row of the original observations necessitates only one inverse matrix for all deletions and a few matrix multiplications and additions in order to adjust that matrix for each deletion. The results of equation (B-12) are combined in the manner described in Chapter IV to produce the pseudo-regression coefficients:

$$J_i(\hat{\beta}) = N\hat{\beta} - (N-1)\hat{\beta}_i, \quad (B-13)$$

the arithmetic mean of which produces the jackknifed regression coefficients:

$$J(\hat{\beta}) = \frac{1}{N} \sum_{i=1}^N \left[N\hat{\beta} - (N-1)\hat{\beta}_i \right]. \quad (B-14)$$

To test the hypothesis $J(\hat{\beta}) = \beta$, the statistic

$$\frac{J(\hat{\beta}) - \beta [\sqrt{N(N-1)}]}{\sqrt{\sum_{i=1}^N [\hat{\beta} - J(\hat{\beta})]^2}} \quad (\text{B-15})$$

is distributed approximately as a Student t with $N-1$ degrees of freedom.