**DYNAMIC MODELING AND EXPERIMENTAL VERIFICATION
OF A FLEXIBLE-FOLLOWER QUICK-RETURN MECHANISM**

by

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ABSTRACT

In this thesis, the dynamics of flexible multibody systems is studied. In particular, a mathematical model of a flexible-follower quick-return mechanism is generated and verified experimentally. This mechanism is of special interest as the closed-loop constraint manifests itself as a time varying load in the domain of the flexible member. The motivation for modeling this type of system is the current trend in the design of industrial equipment toward lighter weight, more slender mechanism components used in order to achieve higher productivity and lower operating cost. As a result, the usual rigid body assumptions made in the dynamic analysis of these systems are no longer valid. Flexibility of the machine elements must be considered in order to produce useful system models.

System equations of motion are generated using a hybrid parameter multiple-body system modeling technique. The methodology allows rigorous formulations of the complete nonlinear, hybrid differential equations with boundary conditions, no Lagrange multipliers are needed.

To verify the model, an experimental mechanism was constructed and data was collected for several test runs with variations of the system parameters.

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NOMENCLATURE

${}^A\vec{\omega}^B$: Angular velocity of frame B in frame A (the uppercase superscript denotes a frame of reference).

${}^A\vec{\alpha}^B$: Angular acceleration of frame B in frame A .

${}^a\vec{r}^b$: Position vector of from point a (tail) to point b (head) (the lowercase superscript denotes a point).

${}^A\frac{d}{dt}$: differentiation w.r.t. reference frame A .

${}^A\frac{d}{dt}{}^a\vec{r}^b = {}^a\vec{v}^b$: velocity of point b relative to point a as seen in reference frame A .

${}^A\frac{d}{dt}{}^a\vec{v}^b = {}^a\vec{a}^b$: acceleration of point b relative to point a as seen in reference frame A .

$\tilde{u}_i$: components ($i = 1, 2, 3$) of the displacement field variable. The symbol $\tilde{}$ denotes the field variables.

$\vec{\tilde{u}}$: displacement vector field.

$\bar{V}$: strain energy density function (scalar).

$\mathcal{H}, \mathcal{D}$: Heavyside step function and Dirac delta function, respectively. These are defined for the spatial domain under consideration.

$\vec{F}, \vec{T}$: Active forces and torques.

CHAPTER I

INTRODUCTION

1.1 Preamble

Presented in this thesis is an investigation into the dynamic behavior of a flexible-follower quick-return mechanism. The combination of flexible follower and rigid crank classifies this mechanism as a hybrid parameter multiple body (HPMB) system. The system was modeled mathematically using a recently developed HPMB system modeling method, and an experimental mechanism was constructed for verification of the math model.

1.2 Objectives

The objective of the work presented in this thesis is to demonstrate the usefulness and appropriateness of pseudo coordinates as a means of modeling time varying intra-domain loading on a discretized flexible body. The work is presented as an alternative to previous flexible-follower quick-return modeling efforts such as those by Beale and Scott [9, 10] and also by Lee [21].

Additionally, it is an objective of this work to add to the experimental foundation for flexible mechanisms modeling research.

1.3 Hybrid Parameter Multiple Body Systems

Mechanical systems may be broadly grouped into three categories specified by their governing differential equations. These categories are: lumped parameter

systems, distributed parameter systems, and hybrid parameter systems. Lumped parameter systems are systems which can be modeled by ordinary differential equations. These are typically systems which can be modeled as a collection of rigid bodies. Distributed parameter systems are systems which are modeled by partial differential equations. These are systems where elements must be modeled considering elasticity or distributed mass. The third class, hybrid parameter systems, are those systems which contain elements belonging to each of the previous two categories. These are the systems of interest for the present investigation.

One typical manifestation of HPMB systems is in flexible mechanisms. In search of higher efficiency and greater productivity, machine designers are pushing the limits of machine elements further than ever before. Elements are designed to be lighter weight, carry larger loads, and operate at higher speeds. The resulting deflections are no longer a few orders of magnitude less than the physical dimensions of the components, and the standard rigid body mechanism models can no longer accurately predict the behavior of these systems.

In order to effectively design and ultimately control these machines, engineers must be able to efficiently generate high fidelity models in order to accurately predict the dynamic behavior of the systems.

1.4 The Quick-Return Mechanism

The quick-return mechanism or sliding-link mechanism is an inversion of the slider-crank. Here, the coupler of the four bar linkage is replaced by a slider and

Figure 1.1: Quick-Return Schematic

the kinematic length of the coupler goes to zero. The quick-return and its motion trajectory are shown schematically in Figure 1.1.

The quick-return is a mechanism with numerous industrial applications, such as the shaper mechanism shown in Figure 1.2. Here an additional link couples the follower to a slider, usually a cutting tool.

1.5 The Flexible Follower

The rigid body quick-return is a single degree of freedom system. When the follower is flexible, as shown in Figure 1.3, the number of degrees of freedom is effectively infinite. That is the response of the distributed parameter, flexible, body is not only a function of time, but also of space.

The value of the flexible follower in the high-speed mechanism is that the follower motion is continually changing direction and consequently high forces are needed to

Figure 1.2: Quick-Return as used in the shaper mechanism

Figure 1.3: Flexible-Follower Quick-Return Schematic

produce the required accelerations. The magnitude of these forces can be reduced, however, by decreasing the mass of the follower, and consequently introducing flexibility.

The motion of the crank on the other hand is a more or less constant rotation. The designer has no need to lighten it. In fact they may even add additional mass to the crank shaft in the form of a flywheel to improve the operation smoothness of the mechanism.

The flexible-follower quick-return mechanism is of special interest, with regard to modeling, because the closed-loop constraint manifests itself as a time varying load in the domain of the flexible member. In order to solve the partial differential equations representing the flexible link numerically, the elasticity of the follower

must be reduced to a discrete number of degrees of freedom. The issue for the modeler is now how to properly apply the time varying load in the discrete system.

1.6 Modeling Technique

The methodology used for the present investigation is a recently developed hybrid parameter modeling method [2, 3, 6, 8]. The method is variational in nature, derived from d'Alembert's principle, but uses vector algebra and is thus more intuitive for the modeler.

The method allows rigorous formulation of the complete non-linear, hybrid, differential equations of motion including boundary conditions. The equations are formulated in the constraint free subspace of the system generalized speed space and thus eliminates the cumbersome algebra associated with the use of Lagrange multipliers to handle the constraints. The use of pseudo coordinates makes transference of the in-domain loading to the discrete system transparent to the modeler.

The application of the technique is vector based and therefore is intuitive for the modeler. Further, with the use of modern computers and symbolic manipulators, the quite substantial equations of motion for these complex systems can now be handled with relative ease.

CHAPTER II

LITERATURE REVIEW

2.1 Objectives

This chapter provides the reader with a brief summary of some of the significant research to date in flexible mechanism dynamics and related areas. The chapter is divided into four sections, each achieving one of four objectives of the literature review. These objectives are:

1. To identify the problem and demonstrate the value of well developed hybrid parameter multiple body system models, in particular to show a need for further work in the modeling of systems which incorporate intra-domain loading of flexible bodies,
2. To outline some historically significant points in the development of the current method,
3. To make comparisons between the current method and other contemporary techniques,
4. To demonstrate the value of experimental verification in modeling research.

2.2 Hybrid Parameter Multiple Body Systems

Mechanical systems may be broadly grouped into three categories specified by their governing differential equations. These categories are: lumped parameter systems, distributed parameter systems, and hybrid parameter systems. Lumped

parameter systems are modeled by ordinary differential equations. These are typically systems which can be modeled as a collection of rigid bodies. Distributed parameter systems are modeled by partial differential equations. These are systems which contain elements that must be modeled considering elasticity or distributed mass. The third class, hybrid parameter systems, are those systems which contain elements belonging to each of the previous two categories. These are the systems of interest for the present investigation.

Barhorst gives three examples of systems which are best modeled as hybrid parameter multiple body (HPMB) systems: a space station, an automobile suspension, and the mechanical linkages of a sewing machine [2]. A space station contains modules or capsules which can be considered rigid bodies as well as numerous booms and solar arrays which exhibit flexibility. An auto suspension system contains many components, some of which may be made light weight to improve fuel economy. Reducing the mass of structural components often introduces flexibility. The sewing machine again consists of numerous light weight machine elements.

Other examples are found in precision pointing devices such as robotic manipulators and surveillance satellites [1, 31, 32]. Arm dimensions, operating speeds, and elasticity of the members can increase nonlinear effects, influence pointing accuracy, and even introduce instability into the systems. These are factors which can dramatically affect the control of pointing structures and hence are good motivations for the development of accurate models.

Current industry productivity requirements have increased the loadings and operating speeds of many industrial machines to the point where the effects of flexibility and system nonlinearity cannot be neglected [6, 9, 12, 26]. Hence designers of these high-speed, high-productivity machines must have access to high-fidelity dynamic system models that include the effects of non-linearity, distributed mass and elasticity.

There has been a fair amount of research done on modeling of simpler mechanisms such as four-bars and crank-sliders including flexibility [12, 15, 17, 19, 22, 25, 26, 27, 28, 29]. The mechanism being considered for this investigation is a flexible-follower quick-return mechanism which has been given less attention in the literature [4, 6, 9, 10, 21]. This mechanism is of special interest as the closed loop constraint manifests itself as a time varying load in the domain of the flexible member. Beale and Scott [9, 10] have referenced a few applications of this mechanism including its use in a shaper mechanism, use in connection with a flow metering pump, and in a high velocity impacting press.

2.3 Historical Development

The method used for this investigation was presented by Barhorst in the 1991 PhD dissertation [2], and later in [3, 6, 8]. The method is based on d'Alembert's principle.

D'Alembert's work was an important advance in analytical mechanics. His introduction of the force of motion, or inertia force, made it possible to reduce any problem of motion to a problem of equilibrium [14, 20].

Gibbs and Appell made another significant turn in the development of analytical methods [20]. They discovered that they were able to incorporate nonholonomic constraints without the use of Lagrange multipliers simply by changing their interpretation of $\delta\vec{r}$. Essentially, they found that by projecting forces onto the constraint free subspace of the system generalized speed space, force between bodies, which do not affect the motion of the system, are removed from the equations.

Kane has applied this same principle in a different form [18]. Kane's equations are written in vector form allowing for a more intuitive and applicable understanding of the relations of generalized force and motion.

A final feature to be note here is the use of pseudo generalized coordinates. Barhorst has shown that the value of a distributed parameter, such as the displacement of an elastic continuum, evaluated at a discrete location may be used like a regular generalized coordinate for the process of projection onto constraint manifolds [3, 6, 4]. The utility of this feature is realized in the implementation of intra-domain constraint loading into a system.

2.4 Contemporary Techniques

In selecting a modeling methodology for modeling the flexible-follower quick-return mechanism, there are two primary concerns. First, is the issue of effectively

handling the closed chain kinematic constraint. And second, is the issue of correctly applying the time varying intra-domain constraint loading to the discretized flexible member. The selected methods must result in a set of ordinary differential equations in a form suitable for numerical computer solution.

Points to be considered when choosing a technique are ease of implementation and computational speed and accuracy. The information presented in this section is not intended to identify any one method as superior, or even as justification for the method selected for use in the following analysis. In fact the methods used were pre-selected. The purpose of this section rather is to form a basis upon which to evaluate the appropriateness of the pre-selected methods.

2.4.1 Kinematics

The issue of the closed chain kinematic solution is frequently handled by writing the closed vector loop equation for the system and deriving from it a set of nonlinear equations which are solved simultaneously to maintain the integrity of the system kinematics [6, 21]. These would be solved numerically, by a Newton-Raphson or comparable method, between time steps of the ODE solution.

Another common means of describing the kinematics of these systems is with a chain of 4x4 transformation matrices [12, 15, 26]. Each matrix describing the position and orientation of one kinematic link relative to the previous link. The closed

chain solution of this type of kinematic description must also be solved iteratively between ODE solution steps.

Constrained multi-body systems follow the constrained path of motion under the action of constraint forces. These forces must be addressed in deriving the complete equations of motion of any constrained system. A common means of handling constraint forces is by means of Lagrange multipliers also known as the lamda method [9, 10]. The Lagrange multiplier method reduces a variation problem with auxiliary conditions to a free variation problem without auxiliary conditions [20].

Another approach is the “stiff spring” method [21] where the point of constraint is replaced with an additional degree of freedom and that degree of freedom is then given a high stiffness.

Third is the method of Gibbs and Appell where all forces are projected onto the constraint free subspace of the system phase space [2, 3, 6, 7, 8]. Thus only the components of forces directly affecting the motion of the system are included.

2.4.2 Intra-Domain Loading

The intra-domain loading of the flexible member is of special interest for this problem. While a number of researchers have addressed issues around the modeling of flexible mechanisms, few have addressed the issue of time varying intra-domain loading. For successful numerical solution the system must be represented by a set

of ordinary differential equations. That is, the continuum of the flexible member must be reduced to a finite number of degrees of freedom. The issue for the modeler now becomes how to transfer the loading in question to the available degrees of freedom so as to produce the correct response.

The literature offers three possible solutions to the problem at hand. These are the standard finite element approach [11], moving boundaries [9, 10], and pseudo coordinates [2, 3, 4, 6].

The standard finite element approach is to divide the applied load between two degrees of freedom such that the two forces have a resultant equivalent to the applied load. The equations generated by this method are simple, but, the number of degrees of freedom required for smooth load transmission is greater than with the other techniques.

Moving boundaries were used by Beale and Scott [9, 10]. Their method is to have the necessary number of degrees of freedom move with the applied load. The domain over which the applied load moves is divided into two regions separated at the moving boundary. This method has the advantage of simplifies kinematics at the cost of time varying mass and stiffness definitions for the two regions.

Barhorst and Everett use pseudo-coordinates with a vector based form of d'Alembert's principle [3, 2, 4, 6]. This approach avoids the additional computation associated with time varying mass and stiffness but incorporates more kinematic coupling terms into the equations of motion.

2.5 Experimentation

When an engineer is set to the task of designing a machine many factors must enter into consideration, not the least of which is the dynamic behavior of the system. The significance of dynamic effects is only exacerbated by the introduction of flexible elements into the mechanism design. Further, the flexibility of certain elements as well as other system nonlinearities may hinder the designer's ability to intuitively predict and understand the system behavior.

The designer must therefore, have some reliable means of testing the system for dynamic response allowing him or her to iterate on the design. The only test that could answer the question of response without flaw would be to put the actual design into service. However, to include fabrication of the complete system in the design iteration would hardly create an efficient design process, nor would it be economical.

The use of simplified experimental models is another alternative. While this may present a small improvement over the option stated above in terms of efficiency and economy, it is still far less efficient and more expensive than the option given below.

The generally accepted alternative is computer modeling and simulation. This, however, raises the issue of credibility of the system model. Therefore, experimentation must be done in order to verify the accuracy of a given modeling method over some range of conditions. Then designers may have confidence in the modeling method when the machines being modeled are configured within that tested range.

In the development of models and modeling techniques, some researchers have commented on the value of experimentation to give credibility to the models [22, 23, 28].

Peng and Liou [23] have conducted a survey of experimental studies of flexible mechanisms. According to them “The problem is now to determine how reliable a program can be, how accurate its output is, what the limitations are, and how fast it can solve the problem” (p. 161).

With the advent of computer aided-design expert systems, more basic experimental data is required for the completion of the knowledge base for the design of flexible mechanisms [23].

2.6 Summary

It has been shown that the value of well developed hybrid parameter multiple body system models is increasing with the current trends in industrial machine design.

A few modern modeling methods capable of handling these systems have been discussed. The derivation of one such method, beginning with d’Alembert’s principle, was outlined.

Finally it has been shown that experimentation is essential in building confidence in a method before it can be put into general use.

CHAPTER III

EXPERIMENT

3.1 Introduction

This chapter is a description of the experimental work that was done to evaluate the validity of the math model. We begin with a thorough description of the experimental mechanism and an explanation of how it was designed with modeling in mind. This is followed by an account of the instruments and techniques used to acquire data which describes the dynamic behavior of the system. Finally the chapter is concluded with some sample data and discussion of the observed system dynamics.

3.2 Design for Modeling

The mechanism constructed for the experimental portion of this work was not designed to mimic any particular industrial application. In fact the designers of industrial machines often want to avoid or limit large deflections and vibrations. The intention in this work is to study those effects. The design approach taken here is therefore somewhat different from that of the typical designer of industrial machinery.

It is desirable in any experimental investigation to design the experiment to be as sterile as possible. That is the system parameters are carefully controlled or isolated. This allows the investigator to more easily identify the information

Figure 3.1: Experimental setup

that is sought without fear of contamination of the data. In the present case of the experiment for comparison to a mathematical model and computer simulation, the experiment was designed to eliminate, as much as possible, any features of the dynamic system that we do not wish to include in the math model.

The mechanism designed for these experiments, shown in Figure 3.1, has a variable crank length which can be set between three and six inches. The distance between the crank center and the follower pivot point is eighteen inches. The length of the flexible domain of the follower is 25 inches.

One assumption made in modeling the system was that the motion was planar. For rigid body motion this is easily achieved by limiting the clearance in the joints.

(a)

(b)

Figure 3.2: Follower pivot: (a)the complete assembly, (b)schematic showing the boundary of the flexible domain to coincide with the axis of rotation

For the flexible bodies however, additional steps must be taken to ensure that the flexibility of the body is in the plane only. This was accomplished by designing the follower as a $1/8$ inch by $1\ 1/2$ inch aluminum strip. The stiffness of the beam therefore is much greater out of the plane than it is in the plane.

Another feature of the follower is the location of the flexible domain boundaries. The boundary at the pivoting end was made to coincide with the axis of rotation, as shown in Figure 3.2. This simplifies the geometry of the mechanism kinematics.

The design of the loading mass at the tip of the follower, Figure 3.3, had two requirements: the load needed to be adjustable, and the geometry needed to be simple enough that rotational inertia could easily be calculated and the center of gravity could be easily estimated for accelerometer positioning. The chosen solution

Figure 3.3: Tip mass: (a)the complete assembly, (b)with plates removed to show accelerometer mounting, (c)schematically showing plates and accelerometer mounting.

was a cuboid shaped mass with a transverse hole through the center for accelerometer mounting. The total mass is composed of several stacked plates to allow for variation of the load in different test runs.

The design of the sliding member, shown in Figure 3.4, required that the constraint be simply modeled as a point contact rather than as contact over a finite area, and that the kinematics of the rotation of the slider be well known. The slider, therefore, was designed as a set of roller bearings, one on either side of the follower, to provide the point of contact. When the pressure between the bearings and the follower is sufficiently tightened, the rotation of the slider follows the slope of the follower deflection.

(a)

(b)

(c)

Figure 3.4: Slider: (a)the complete assembly, (b)schematic showing the constraint force to be a point contact, (c)schematic showing the slider rotation to be the same as the follower deflection slope

One final concern in the design of the experimental setup was external vibration. To limit this effect the experimental mechanism was mounted, with six bolts, to a heavy table.

3.3 Data Acquisition

In order to accurately characterize the dynamic behavior of the system a precise time history of the system configuration is required. The system configuration at a given time is defined by the generalized coordinates of the system at that time. For the system under investigation, the generalized coordinates are the crank angle, and the modal coordinates of the beam.

The crank angle is measured with an incremental optical encoder. The encoder is mounted on the crank shaft and produces a digital signal which can be interpreted to determine the crank angle and the angular velocity.

The modal coordinates are not so easy to measure. For the assumed shape functions used in the math model, the modal coordinates are the deflection and rotation of the tip mass. Neither of these quantities can be measured directly. However, the shape of the follower can be approximated from other measurements. For these experiments five other measurements were used to approximate the follower shape: acceleration at the tip, rotation of the follower base, and bending strain at three discrete points along the follower.

Figure 3.5: Follower showing strain gage mounting positions and the moving constraint force.

The strain gages were mounted along the beam length, L , at $x = L/4$, $x = L/2$, and $x = 3L/4$, see Figure 3.5. They were mounted on the upper half of the follower so as not to interfere with the motion of the roller bearings in the slider.

The tip acceleration was measured with a piezoelectric accelerometer mounted at the center of the tip mass. The follower base rotation was measured with an incremental encoder similar to the one used to measure the crank rotation. Bending strain at each of the three points along the follower were measured with three half-bridge strain sensors.

Real-time acquisition of the test data was accomplished with a desk top computer and an IOtech WaveBook512. A block diagram of the connections is shown in Figure 3.6

3.4 Sample Data

A total of twenty-four test runs were performed with variations of crank length, mass loading and crank speed. The crank length was varied from 4 to 6 inches.

Figure 3.6: Data acquisition system block diagram

Figure 3.7: Accelerometer data from a typical test.

The tip mass was varied from $0.41\text{E-}4$ to $2.90\text{E-}4 \frac{\text{lb s}^2}{\text{in}}$. For each combination of crank length and tip mass two arbitrary crank speeds were chosen to demonstrate differences in the dynamic behavior of the system. For each test, five seconds of steady-state data was collected, at 20 kHz. The system configuration for each test run is given in Table 3.1.

The data from a typical test is shown in Figures 3.7, 3.8, and 3.9. In this test the crank length was 4 in, the tip mass was $2.90\text{E-}4 \frac{\text{lb s}^2}{\text{in}}$, and the system was operating at 2.64 Hz. Shown in the plots is one cycle of steady state operation, the starting and ending crank angle is π radians. The data in the plots was filtered using a fast fourier transform to reduce the amount of electrical noise.

Table 3.1: System Configuration for Experimental Runs

Exp. No.	Crank Length (inches)	No. of Mass Plates	Crank Speed (rad/sec)
1	6	0	N/A
2	6	0	10.70115
3	6	2	9.496229
4	6	2	13.085869
5	6	4	7.937818
6	6	4	11.808269
7	6	6	7.819764
8	6	6	12.262256
9	5	0	12.880648
10	5	0	19.696489
11	5	2	11.385666
12	5	2	14.387864
13	5	4	10.085361
14	5	4	13.290703
15	5	6	8.967644
16	5	6	12.963029
17	4	0	13.910073
18	4	0	18.382621
19	4	2	12.400197
20	4	2	16.456731
21	4	4	13.768336
22	4	4	16.558651
23	4	6	13.172285
24	4	6	16.617773

Figure 3.8: Strain data from a typical test.

Figure 3.9: Strain data detail.

The first plot, Figure 3.7, is of the tip acceleration. It can be seen that first mode vibration of the follower is dominant until the ‘quick-return’ when second and even third mode vibrations become more pronounced. The vibration frequencies were approximated from this graph and were found to be in the range of 17 to 24 Hz for the first mode and 132 Hz for the second mode. These numbers are given only as a point of reference. Terms such as mode and natural frequency are used liberally here and only for convenience. The nature of the sliding intra-domain load on the follower discounts the notion of discrete frequencies and mode shapes unique to the system.

The smooth curve in the first plot is a presentation of the same data after further filtered using the fast fourier transform [13]. The result is an approximation of the tip acceleration due to the rigid body motion upon which the vibrations are superimposed.

The second plot, Figure 3.8, is of the strain gage readings. We can see a similar pattern to that in the acceleration data. In the larger scale oscillations, it is seen that the strain at all three locations along the beam share the same sign with respect to the rigid body motion. This confirms the identification of these vibrations as first mode. In the third plot, Figure 3.9, an enlargement of a portion of the data revealed that the signs, relative to the first mode oscillations, of strains 1 and 3 are the same while the sign of strain 2 is inverted, confirming the identification of these vibrations as second mode.

Examination of the rest of the data sets reveals that kinematics associated with the sliding constraint cause the natural frequencies of vibration to be a function of the crank length as well as the tip mass. While the geometric center of the sliders range is constant regardless of crank length, the time average location of the slider as well as the limits of the sliding range do change with crank length.

3.5 Discrepancies

The data for experiment number one was corrupted in electronic data transfer and is not available.

The electronic noise in the accelerometer data for experiments numbered two through eight could not be effectively filtered. Strain gage data for these runs is still valid.

Plots of all available experimental data can be found in the Appendix.

CHAPTER IV

MODELING

4.1 Modeling in General

Modeling is the process of creating a representation of the system to be modeled. The plastic car assembled from a kit for example is a representation of the real car. It is not the real car, but it is capable of conveying certain information about the real car.

The caricature drawings of the U.S. President on the editorial page of the news paper are models of the person that they represent. They may not really look like the President in great detail, but everyone who sees the picture knows that it is the President. Now if the President were a missing person, the organizers of the manhunt would surely distribute photographs of the president rather than caricature drawings. The creator of a good model must understand what the model will ultimately be used for so that his model will be capable of transmitting all the necessary information.

Models, in short, are a means of communication. The modeler wants to communicate a limited amount of information about whatever is to be modeled. The successful modeler will transmit the information deemed significant in a simple and elegant way.

In this case we are concerned with modeling the dynamic behavior of a mechanical system. Our medium is mathematics. We have described, in mathematical terms, relationships of motions, matter, forces and time. In the end the model we have generated is a representation of the mechanical system in the form of a set of differential equations which describe the motions of the system as a response to some input.

We began the modeling process by identifying the components of the system and modeled them individually. We then proceeded to model the relationships among the components that makes them into a system. At each step along the way we asked ourselves, as modelers, what information is significant enough that we wanted to include it in our model, and what information is less significant and can be dispensed with in order to make the model more elegant.

4.2 Modeling the Flexible-Follower Quick-Return Mechanism

The system modeled is an experimental quick-return mechanism with a flexible follower. The physical system is described in detail in the previous chapter.

Clearly the component at the heart of dynamic behavior of the system is the flexible follower. Observations of the mechanism in motion have revealed the deflections of the follower to be small compared to the length of the follower. Therefore it was deemed appropriate to consider the deflection to be linear and model the follower as a modified Euler beam. The Euler beam model was modified to include

foreshortening effects which are necessary because the beam is rotating which can cause instability in the standard Euler beam model.

The strain energy density function for an Euler beam is:

$$\bar{V} = \frac{1}{2}EI \left(\frac{\partial^2 u_2}{\partial x^2} \right)^2 \quad (4.1)$$

Its weakened form will be used in the partial differential equations.

While any flexible body is essentially infinite in its degrees of freedom, the follower has been modeled here as having only two degrees of freedom. The kinematics of the follower are described using Hermite polynomials equations 4.2 and 4.3. This allows us to describe the shape of the entire follower in terms of the tip deflection and rotation.

$$\psi_1 = 3\left(\frac{x}{L}\right)^2 + 2\left(\frac{x}{L}\right)^3 \quad (4.2)$$

$$\psi_2 = \frac{x^2}{L} \left(\frac{x}{L} - 1 \right) \quad (4.3)$$

The flexible domain of the follower is from the point of rotation at the base to the intersection point with the tip mass. The aluminum strip actually extends beyond each of these points, however it is embedded within rigid members at each end. Therefore we do not need to consider the flexibility of the beam beyond these points.

The mass at the follower tip is modeled as a rigid body. That is it has both mass and rotational inertia.

The experimental observations have shown the crank speed to be nearly constant. This results from the combination of crank mass, motor torque, and damping. For the model and subsequent simulation the crank speed was set by imposing a damping coefficient to produce the required speed for a given motor torque.

Other bodies in the system include the sliding member, which couples the crank and the follower, and the the base of the follower. Each was designed such that they would have little effect on the kinematics of the system. Mass and rotary inertia were included for each of these bodies, as well as damping for the pivots associated with them.

The coordinate frames assigned to each body in the system (as modeled) are shown in Figure 4.1. Use of these coordinates has allowed us to describe the closed chain kinematics of the system as a vector loop, equation 4.4.

$$d\hat{n}_1 + q_3\hat{b}_1 + q'_4\hat{b}_2 - L_A\hat{a}_1 = 0 \quad (4.4)$$

where, d is the distance between the crank center and the follower pivot point, and L_A is the crank length, q_3 is the varying length to the coupler and q'_4 is the deflection pseudo-coordinate at the coupler point.

Figure 4.1: Mechanism schematic with all frames and coordinates

4.3 Hybrid Parameter Multiple Body System Modeling Methodology

The essential equations and techniques of the modeling methodology are presented in this section, along with some comments on how the method was applied to the specific problem at hand. The reader is asked to refer to the nomenclature section in the front of this document for identification of the symbols used in the following equations. This is only a brief presentation of the method. For more complete information and derivation of the method the reader is referred to the works of Barhorst [2, 3, 8].

Throughout the equations below, all forces and torques are active forces and torques. No constraint forces should be considered.

One differential equation of motion is to be derived for each degree of freedom of the system. For each regular generalized speed, u_n , the following first-order ordinary differential equation is applied.

$$\sum_r \left\{ \frac{\partial^o \vec{v}_{\mathcal{N}}^{s_r}}{\partial u_n} \cdot [\vec{F}_r - \vec{I}_r] + \frac{\partial^{\mathcal{N}} \vec{\omega}^{B_r}}{\partial u_n} \cdot [\vec{T}_r - \vec{J}_r] \right\} + \sum_e \left\{ \frac{\partial^o \vec{v}_{\mathcal{N}}^{s_e}}{\partial u_n} \cdot [\vec{F}_e - \vec{I}_e] + \frac{\partial^{\mathcal{N}} \vec{\omega}^{B_e}}{\partial u_n} \cdot [\vec{T}_e - \vec{J}_e] \right\} = 0 \quad (4.5)$$

The equations of motion associated with the flexible body are based on the field equation, 4.6, distinct for each $\tilde{u}_{fi} \in \Omega_e$.

$$\left[\mathcal{H}_e \vec{F}_{he} + \mathcal{D}_e \vec{F}_{de} - m_{I_e} {}^o \vec{a}_{\mathcal{N}} {}^{*I_e} \right] \cdot \frac{\partial^{s_e} \vec{v}_{B_e}^{i_{oe}}}{\partial u_{ei,t}} - \frac{1}{h_1 h_2 h_3} \frac{\partial}{\partial r_{ej}} \left\{ h_1 h_2 h_3 \frac{\partial^{B_e} \vec{\omega}^{I_e}}{\partial u_{ei,jt}} \cdot [\mathcal{H}_e \vec{T}_{he} + \mathcal{D}_e \vec{T}_{de} - \right.$$

$$\begin{aligned}
& \left({}^{i_{oe}}\vec{r}^{*I_e} \times m_{I_e} {}^o\vec{a}_N {}^{i_{oe}} + \vec{I}_{i_{oe}} \cdot {}^N\vec{\alpha}^{I_e} + {}^N\vec{\omega}^{I_e} \times \vec{I}_{i_{oe}} \cdot {}^N\vec{\omega}^{I_e} \right) \Big] \Big\} \\
& - \left[\frac{\partial \bar{V}_e}{\partial \tilde{u}_{ei}} - \frac{1}{h_1 h_2 h_3} \frac{\partial}{\partial r_{ej}} \left(h_1 h_2 h_3 \frac{\partial \bar{V}_e}{\partial \tilde{u}_{ei,j}} \right) \right. \\
& \quad \left. + \frac{1}{h_1 h_2 h_3} \frac{\partial^2}{\partial r_{ej} \partial r_{ek}} \left(h_1 h_2 h_3 \frac{\partial \bar{V}_e}{\partial \tilde{u}_{ei,jk}} \right) \right] \\
& \mathcal{H}_{c_e} G'_{ei} - \frac{1}{h_1 h_2 h_3} \frac{\partial}{\partial r_{ej}} (h_1 h_2 h_3 \mathcal{H}_{c_e} K'_{ei}) = 0
\end{aligned} \tag{4.6}$$

where, G'_{ei} is defined by:

$$\int_{\Omega_e} \mathcal{H}_e G'_{ei} d\Omega_e = \text{LHS (Eq. 4.5)} \tag{4.7}$$

evaluated as prescribed in [19] with appropriate pseudo-speeds. Where $\mathcal{H}_e$ is the heavy side step function and is defined over the region of constraint.

Similarly,

$$\int_{\Omega_e} \mathcal{H}_e K'_{ei} d\Omega_e = \text{LHS (Eq. 4.5)} \tag{4.8}$$

evaluated as prescribed in [19] with appropriate pseudo-speeds.

The partial differential equations associated with the flexible body must be discretized and reduced to ordinary differential equations in order to be solved by numerical methods. The method of discretization used here is to multiply the field equation by each of a set of assumed shape functions and then integrate over the spatial domain. The result is one ordinary differential equation for each degree of freedom of the discretized flexible body, after the boundary conditions are applied.

On portions of the boundary $\partial\Omega_e$ subject to forces, the following boundary conditions apply for each $\tilde{u}_{ei}$ ($i = 1, 2, 3$):

$$\begin{aligned} & \left(\mathcal{H}_e \vec{T}_{he} + \mathcal{D}_e \vec{T}_{de} \right) \cdot \frac{\partial^{s_e} \vec{v}_{B_e}^{i_{oe}}}{\partial \tilde{u}_{ei,j}} - (\hat{n} \cdot \hat{e}_{ej}) h_j \frac{\partial^{B_e} \vec{\omega}^{I_e}}{\partial \tilde{u}_{ei,jt}} \cdot \\ & \left({}^{i_{oe}} \vec{r}^{*I_e} \times \rho_e {}^o \vec{a}_{\mathcal{N}}^{i_{oe}} + \vec{I}_{i_{oe}} \cdot {}^{\mathcal{N}} \vec{\alpha}^{I_e} + {}^{\mathcal{N}} \vec{\omega}^{I_e} \times \vec{I}_{i_{oe}} \cdot {}^{\mathcal{N}} \vec{\omega}^{I_e} \right) \\ & - (\hat{n} \cdot \hat{e}_{ej}) \left[h_j \frac{\partial \bar{V}_e}{\partial \tilde{u}_{ei,j}} - \frac{h_j}{h_1 h_2 h_3} \frac{\partial}{\partial r_{ek}} \left(h_1 h_2 h_3 \frac{\partial \bar{V}_e}{\partial \tilde{u}_{ei,jk}} \right) \right] = 0 \end{aligned} \quad (4.9)$$

For each $\tilde{u}_{ei,j}$ ($i = 1, 2, 3$), we have:

$$\frac{\partial^{B_e} \vec{\omega}^{I_e}}{\partial \tilde{u}_{ei,jt}} \cdot \left[\mathcal{H}_e \vec{T}_{he} + \mathcal{D}_e \vec{T}_{de} \right] - (\hat{n} \cdot \hat{e}_{ek}) h_k \frac{\partial \bar{V}_e}{\partial \tilde{u}_{ei,jk}} = 0 \quad (4.10)$$

The boundary conditions which hold for each $\tilde{u}_{ei} \in \partial\Omega_e$ ($i = 1, 2, 3$) where connections are made are given by:

$$\text{LHS (Eq. 4.9)} + \mathcal{H}_{c_e} g'_{ei} = 0 \quad (4.11)$$

For every $\tilde{u}_{ei,j} \in \partial\Omega_e$ ($i = 1, 2, 3$), the boundary condition is given by:

$$\text{LHS (Eq. 4.10)} + \mathcal{H}_{c_e} k'_{ei} = 0 \quad (4.12)$$

All the forcing terms except g'_{ei} and k'_{ei} in the Eq. 4.11 and Eq. 4.12, respectively are from active elements at the connection. The quantity g'_{ei} in Eq. 4.11 is defined by:

$$\int_{\partial\Omega_e} \mathcal{H}_{ce} g'_{ei} = \text{LHS (Eq. 4.5)} \quad (4.13)$$

. Similarly, quantity k'_{ei} in Eq. 4.12 is defined by:

$$\int_{\partial\Omega_e} \mathcal{H}_{ce} k'_{ei} = \text{LHS (Eq. 4.5)} \quad (4.14)$$

.

If displacements are known on the boundary, we have:

$$\tilde{u}_{ei} = \hat{u}_{ei} \quad (4.15)$$

$$\tilde{u}_{ei,j} = \hat{u}_{ei,j} \quad (4.16)$$

$$\psi_{epi} = \hat{\psi}_{ei} \quad (4.17)$$

.

The $(\hat{\cdot})$ notation denotes a prescribed value.

4.4 Symbolic Equation Processing

While the equations of motion presented in the previous section appear relatively simple, numerical simulation of the system will of course require all terms to be defined and expanded. This results in a very long and complex algebraic mess prone with pitfalls for human error.

While no tool requiring human input can be free of human error, the use of computer based symbolic manipulators can significantly limit the error and effort associated with these substantial equations.

Computer based symbolic manipulators have been used in the derivation of equations of motion for about two decades [5]. The methods discussed here are based on the commercial software *Mathematica*, first released in 1988 [30], and an “Engineering Vectors,” an unpublished extension to *Mathematica* written by Barhorst [5].

In modeling the quick-return mechanism, system properties such as kinematic dimensions, mass and material properties, etc., were kept in symbolic form as much as possible to facilitate future reworking of the model. The procedure for modeling the quick-return with these tools is as follows.

First the coordinate frames associated with each body were defined using the **unitVector** function. Also the **unitDyad** function was used in association with each rigid body. All dot and cross product identities were also defined at this point.

Next, the angular velocities are defined using the **omega** function and are written in terms of the generalized speeds and pseudo-generalized speeds. Angular accelerations could then be calculated by using the **DvDt** function on the angular velocities.

Position vectors were then defined for points of interest, including special points, centers of gravity of rigid bodies and differential elements of flexible bodies. The

necessary velocity and acceleration vectors were then calculated by operating on the positions with the $\mathbf{DvDt}$ function.

At this point and continuously throughout the rest of the modeling process the modeler looked for items in the resulting terms which would result in excessive recurrence in the final equations of motion. Substitutions were made to limit the size of the equations and the newly created intermediate variables were carried through to the end. In addition to shortening the resulting equations, the intermediate variables have the advantage of speeding up the numerical simulations.

Next, the beam model was defined. The strain energy density function for the Euler beam was defined and its weakened form calculated. The beam deflection was also defined in terms of the assumed, Hermite, shape functions, equations 4.2 and 4.3.

All active forces and torques as well as mass and inertia properties for each body were defined in terms suitable for use in equation 4.5.

Here equations for the regular generalized speeds were generated in the form of equation 4.5. The partial derivatives are taken using the $\mathbf{Pvel}$ operator.

Notice that in equation 4.5 partial velocities are to be taken with respect to the appropriate generalized speed. All terms to be operated on in this manner must be expressed in terms of that generalized speed. In order to do this the kinematic vector loop was used. All the vectors in the loop equation were transformed into the N frame and the loop equation was dotted with n_1 and then n_2 to produce two

scalar constraint equations which could be manipulated algebraically to solve for the terms needed in order to take the required partial velocities.

Now the equations governing the motion of the flexible body were generated. These were based on the discretized form of the field equation, equation 4.6.

These equations were second order ordinary differential equations. The first and second derivative terms were replaced by substitute variables and their first derivative respectively. A set of kinematic equations defines the relationship between the original variables and the new variables. The result is the set of first order ordinary differential equations that describes the motion in time of the flexible-follower quick return mechanism.

4.5 Simulation

After the equations of motion were generated, the next task was to create numerical simulations of the system. The chapter that follows is a discussion of the numerical simulation techniques and results. There is however, some work which was necessary to bridge the gap between the generation of the equations of motion and the numerical simulation. This work is presented in this section.

Any routine for numerically solving differential equations will require that the equations be programmed to generate output in a specific form. The routines described in the next chapter are no exception. They require the equations of motion to be put into the form of an inertia matrix and a right hand side (force) vector.

We accomplished this quite simply as a continuation of the *Mathematica* notebook in which we generated the equations. The symbolic manipulating power of the software is well suited to this task.

The formulas for converting the ODEs into matrix form are as follows:

$$I_{i,j} = \frac{\partial eom_i}{\partial \dot{s}_j} \quad (4.18)$$

and

$$rhs_i = -eom_i \quad (4.19)$$

where all $\dot{s}_j \rightarrow 0$ for $j = 1, 2, \dots, N$.

CHAPTER V

NUMERICAL SIMULATION

5.1 Numerical Solution of Ordinary Differential Equations

Any set of ordinary differential equations (ODEs) can always be reduced to a set of first order differential equations [16]. This is done by introducing a set of new variables defined as the derivatives of the existing variables.

In general the problem is defined as a set of N coupled first-order differential equations having the form

$$\frac{dy_i(x)}{dx} = f'(x, y_1, y_2, \dots, N) \quad (5.1)$$

where $i = 1, 2, \dots, N$, N is the number of equations, and the functions on the right hand side are known.

In addition to the differential equations themselves, a complete problem statement will also include boundary conditions. These are algebraic conditions on the values of the functions. The nature of the boundary conditions is usually a significant factor in deciding what solution method is used. Boundary conditions can be used to categorized problems as either initial value problems or boundary value problems.

Initial value problems are problems where the dependant variables are defined at some starting point of the independent variable and it is desired to find the values of the dependant variables at a discrete set of points between the starting point and

some final point. Boundary value problems however are characterized by having boundary conditions defined for more than one point on the independent domain.

5.2 The LSODA ODE Solver

In simulating the flexible-follower quick-return mechanism model, the equations of motion were solved numerically using LSODA [24], the Livermore solver for ordinary differential equations, with automatic method switching for stiff and non-stiff problems. LSODA is a FORTRAN subroutine which calculates the values of a set of dependent variables for one time step given the initial values and the initial and final times.

The ODE must be in the form of a Jacobian matrix and a right-hand side vector. Additional inputs for LSODA include initial and final times, an initial condition array, and convergence tolerance information.

LSODA is intended to be called from a user-generated FORTRAN code. It should be put into a loop to be called once for each time step to be calculated.

5.3 Simulation Parameters

The program written for numerical simulation of the flexible-follower quick-return mechanism allows for the specification of several parameters to be given as input. The parameters which could be varied included mass properties, stiffness properties, damping coefficients and kinematic parameters.

For the flexible-follower beam, the parameters open to variation were density (mass per unit length) and stiffness, EI . To match the experiments both of these parameters were held constant for all twenty-four simulations. The value used for the density was $0.0000475987 \frac{\text{lb s}^2}{\text{in}^2}$. The value used for the beam stiffness was 2128.22 lb in^2 .

The tip mass is defined for the simulation by mass and rotary inertia values. The values used for each simulation depend upon the configuration of 0, 2, 4, or 6 mass plates attached to the tip of the beam. The configuration corresponding to each experiment/simulation are given in Table 3.1. The values used for each configuration are given in Table 5.1. Notice that the values used for the zero plate configuration are not zero. This is to account for the one inch of beam material which extends beyond the boundary of the flexible domain as modeled.

Table 5.1: Mass Properties for Tip Mass Plates

No. of Mass Plates	Mass $\frac{\text{lb s}^2}{\text{in}}$	Rotary Inertia $\text{lb s}^2 \text{ in}$
0	4.10E-5	2.80E-6
2	1.24E-4	8.40E-6
4	2.07E-4	1.40E-5
6	2.90E-4	1.96E-5

The block at the pivot point at the base of the follower is also included in the model. Since the block does not undergo rigid body motion, the mass is not explicitly included. Rotary inertia, however, is defined as $1.50\text{E-}4 \text{ lb s}^2 \text{ in}$ for all simulations.

The motor-crank assembly was modeled as a rotating rigid body with an applied torque and damping. Simulation parameters for this portion of the model were exaggerated in order to force an approximately constant crank speed. The center of mass of the crank was located three inches from the crank center. The mass and rotary inertia about the mass center were $0.1 \frac{\text{lb s}^2}{\text{in}}$ and $4.0\text{E-}4 \text{ lb s}^2 \text{ in}$ respectively for all simulations. The torque, damping and resulting crank speed for each simulation are given in Table 5.2.

Additional viscous damping was included in the follower pivot and along the follower beam. The follower pivot damping coefficient was defined as $0.00002 \frac{\text{lb s in}}{\text{rad}}$, and the follower beam damping coefficient was defined as $0.00002 \frac{\text{lb s}}{\text{in}}$. These values were constant for all twenty-four simulations.

5.4 Comparison of Simulation Data to Experimental Data

The data presented for each experiment/simulation set is of one full cycle at steady state. The cycle begins and ends at $q_1 = 180^\circ$. The first half of the cycle is referred to as the pre-return portion and the second half is referred to as the post-return portion.

Comparison of the simulation data to the experimental data in order to judge the accuracy of the simulation and to evaluate the appropriateness of the pseudo-coordinate approach to applying time varying intra-domain loads. We can judge

Table 5.2: Crank Damping Coefficients

Sim. No.	Torque inch-lb	Damping $\frac{\text{lb} \cdot \text{in}}{\text{rad}}$	Crank Speed rad/sec
1	10	1.591550775	N/A
2	10	0.934479004	10.70115
3	10	1.053049584	9.496229
4	10	0.764183046	13.085869
5	10	1.259792049	7.937818
6	10	0.846864104	11.808269
7	10	1.278810972	7.819764
8	10	0.815510354	12.262256
9	10	0.776358336	12.880648
10	10	0.507704672	19.696489
11	10	0.878297071	11.385666
12	10	0.695030393	14.387864
13	10	0.991536246	10.085361
14	10	0.752405817	13.290703
15	10	1.115120589	8.967644
16	10	0.771424582	12.963029
17	10	0.718903643	13.910073
18	10	0.543992096	18.382621
19	10	0.806438605	12.400197
20	10	0.607654133	16.456731
21	10	0.726303969	13.768336
22	10	0.603913966	16.558651
23	10	0.759169438	13.172285
24	10	0.300892729	16.617773

the accuracy of the simulation in terms of its capability to predict the frequency and amplitude response of the system.

The comparison revealed some discrepancies. We identify one source of these discrepancies as the choice to model the slider joint as ideally rigid which is seen to be inappropriate. Identification of the joint stiffness as a source of the discrepancies is more apparent when one compares the accuracy of the pre-return frequency response the post-return frequency response.

In virtually all of the twenty-four data sets, the frequency of the response in the pre-return portion of the simulation cycle was close to or slightly above that for the experiment. In the post-return portion of the cycle however, the simulation frequencies were always greater than those of the experiment, usually by a more substantial amount.

In comparison of the amplitude responses it is noticed that the pre-return amplitudes were typically very good while post-return amplitudes were typically high. Exceptions to this description are found in runs one through eight where the simulated pre-return amplitudes were found to be low.

Experimental and simulation data for run number 24 are shown in Figures 5.1 and 5.2. The first plot, Figure 5.1, is tip acceleration and the second plot, Figure 5.2, is strain data. Data for all 24 simulations, together with the corresponding experimental data, is presented in the Appendix.

Figure 5.1: Tip Acceleration: Simulation results together with experimental data.

Figure 5.2: Follower Strain: Simulation results together with experimental data.

5.5 Evaluation of the Model and the Method

The simulations demonstrated that the pseudo-coordinate method can be successfully used to transmit the time varying in-domain loading to the discrete degrees of freedom. The response in the pre return portion of the cycle was as expected for the two degrees of freedom beam. The model accurately predicted the first mode response of the follower, and the second mode response was present but less accurate. In the post-return portion of the cycle, the results were less accurate. The mechanism was modeled as planer with rigid joints. The joint at the sliding member however was less stiff than anticipated and as a result response frequencies in the post-return portion of the cycle were higher in the simulation than in the experimental data.

Should an analyst find it necessary to obtain a higher order response, the number of degrees of freedom for the follower can be doubled or tripled with relative ease.

As for the non-ideally stiff joint, possible solutions are spatial modeling as advocated by Sunada and Dubowsky [26] or by using the stiff spring approach to handling the constraint forces such as done by Lee [21] and simply making the joint less stiff.

For the future experimenter who wishes to use the methods presented here more care should be taken in the design to ensure rigidity of the joints as outlined by Peng and Liou [23].

The method, while variational in nature (based on d'Alembert's principle), is exceedingly straight forward in its implementation. Terms are derived in an intuitive vectorial form and plugged into the equation given by Barhorst [3] to yield the full equations of motion.

CHAPTER VI

CONCLUSION

6.1 Summary of Work

This thesis is an investigation into the modeling of hybrid parameter multiple body (HPMB) systems, and in particular the application of one recently developed HPMB system modeling methodology. The method is evaluated through its application to one system, the flexible-follower quick-return mechanism. This mechanism is of special interest as the closed-loop constraint manifests itself as a time varying load in the domain of the flexible member. In addition to the method's usefulness and applicability, the accuracy of the resulting equations is also evaluated by comparison with data from an experimental mechanism.

A review of the literature has shown that these systems are growing in importance to industry and other interests. We have also reviewed the historical development of the methodology under consideration and made comparisons between it and other currently utilized methods.

The experimental mechanism was designed so as to make its modeling simple with respect to the geometry of the kinematics, location of the boundaries of the flexible domain, and calculability of parameters. Data acquisition equipment for the experiments was done with an IOtech WaveBook512 and a desk top PC. Data

was acquired from two optical encoders (crank shaft and follower pivot), one accelerometer on the tip of the follower, and strain sensors at three discrete points along the follower. For a typical experimental run, the system displayed prominent second mode vibration during the “quick return” portion of the cycle and first mode vibration dominating the rest of the cycle.

The system was modeled with three degrees of freedom: the crank angle, and follower tip deflection and rotation. The flexibility of the follower was modeled as an Euler beam with a rigid mass attached to the free tip.

6.2 Evaluation of the Model

The simulations demonstrated that the pseudo coordinate method can be successfully used to transmit the time varying in-domain loading to the discrete degrees of freedom. The model accurately predicted the first mode response of the follower, and the second mode response was present but less accurate. In the post-return portion of the cycle, the results were less accurate. The mechanism was modeled as planar with rigid joints. The joint at the sliding member, however, was less stiff than anticipated and as a result response frequencies in the post-return portion of the cycle were higher in the simulation than in the experimental data.

For the future experimenter who wishes to use the methods presented here more care should be taken in the design to ensure rigidity of the joints as outlined by Peng and Liou [23].

6.3 Evaluation of the HPMB System Modeling Method

The method is exceedingly straightforward in its implementation. Terms are derived in an intuitive vectorial form and plugged into the equation given by Barhorst [3] to yield the full equations of motion.

An impartial evaluation of the efficiency of numerical computation cannot be given at this time. See the section below, Recommendations for Future Work.

6.4 Recommendations for Future Work

In order for this or any of the techniques mentioned in the literature review to be of practical use to engineers working in industry, it must be automated to the point where the analyst can describe the kinematic configuration, inertia, stiffness, and damping properties of a system and let the computer do the rest. Ease of implementation for the analyst becomes an issue of the software user interface. Programmers of the numerical routines will likely suffer through whatever equations are necessary to achieve the desired computation accuracy and efficiency.

The question now left at the feet of researchers is which of the available methods excel in computational accuracy and efficiency. While authors of many of the works reviewed in preparation for this thesis project acknowledged the importance of computational efficiency, and commented on how their procedures try to be efficient, none of them make direct comparisons of computational speed between differing techniques. This thesis is no exception.

It is important, therefore, that future researchers make more direct comparisons of the available methods. It is unlikely, in this author's opinion, that any one method will shine above all others. It is more likely that each method will find greater or lesser usefulness in certain classes of problems. That is information that the designers and analysts of the next generation of industrial machinery will find useful.

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APPENDIX

COMPLETE RESULTS

Contained in this appendix are the complete results, in graphical form, of each of the twenty-four simulations run as a part of this research.

The simulation data is presented along with the corresponding experimental data where available (see section 3.5 on page 28). In all plots, experimental data is represented by thin lines and simulation data is represented by thick lines.

Figure A.1: Simulation 1 Tip Acceleration and Follower Strain

Figure A.2: Simulation 2 Tip Acceleration and Follower Strain

Figure A.3: Simulation 3 Tip Acceleration and Follower Strain

Figure A.4: Simulation 4 Tip Acceleration and Follower Strain

Figure A.5: Simulation 5 Tip Acceleration and Follower Strain

Figure A.6: Simulation 6 Tip Acceleration and Follower Strain

Figure A.7: Simulation 7 Tip Acceleration and Follower Strain

Figure A.8: Simulation 8 Tip Acceleration and Follower Strain

Figure A.9: Simulation 9 Tip Acceleration and Follower Strain

Figure A.10: Simulation 10 Tip Acceleration and Follower Strain

Figure A.11: Simulation 11 Tip Acceleration and Follower Strain

Figure A.12: Simulation 12 Tip Acceleration and Follower Strain

Figure A.13: Simulation 13 Tip Acceleration and Follower Strain

Figure A.14: Simulation 14 Tip Acceleration and Follower Strain

Figure A.15: Simulation 15 Tip Acceleration and Follower Strain

Figure A.16: Simulation 16 Tip Acceleration and Follower Strain

Figure A.17: Simulation 17 Tip Acceleration and Follower Strain

Figure A.18: Simulation 18 Tip Acceleration and Follower Strain

Figure A.19: Simulation 19 Tip Acceleration and Follower Strain

Figure A.20: Simulation 20 Tip Acceleration and Follower Strain

Figure A.21: Simulation 21 Tip Acceleration and Follower Strain

Figure A.22: Simulation 22 Tip Acceleration and Follower Strain

Figure A.23: Simulation 23 Tip Acceleration and Follower Strain

Figure A.24: Simulation 24 Tip Acceleration and Follower Strain

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