A HELICAL ANTENNA FOR EXCITING AZIMUTHALLY-
ASYMMETRIC ALFVÉN WAVES

by

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ABSTRACT

A new coupler for exciting azimuthally-asymmetric Alfvén waves has been experimentally investigated. The coupler, which consists of four symmetrical helical straps wound around the vacuum vessel, is driven by a four-phase rf source. Waves excited by the coupler vary azimuthally as $e^{jm\theta}$, where m can be chosen to be either +1 or -1. The coupler exhibits a strong propagation directionality and is capable of launching waves whose wavelengths differ considerably from its own. Loading measurements indicate an equivalent coil impedance of 1.2 - 2.0 ohms in vacuum, and 1.2 - 4 ohms with plasma present. This impedance is dependent on the excitation frequency and the sign of m . The $m = -1$ wave was observed to propagate above and below ω_{ci} and was identified as the fast wave. The $m = +1$ wave was observed only below ω_{ci} as the slow wave. No $m = +1$ fast wave or $m = -1$ slow wave was observed.

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LIST OF SYMBOLS

$\hat{a}_r, \hat{a}_\theta, \hat{a}_z$	- unit vectors in cylindrical coordinates
b_r, b_θ, b_z	- time-varying magnetic fields of plasma wave in cylindrical coordinates
B_0	- vector static background (confining) magnetic field
c	- free space velocity of light
C	- circumference of helical antenna (or coupler)
C_λ	- circumference of helical coupler in plasma wavelengths
$C_{1,2}$	- constants
d	- propagation distance across plasma column
E_r, E_θ, E_z	- wave electric field vectors in cylindrical coordinates
E_0	- maximum value of electric field
$E_\perp$	- total transverse electric field
f	- temporal frequency
f_{ci}	- temporal ion cyclotron frequency
I_c	- coupler excitation current
I_p	- plasma preparation current
j	- $\sqrt{-1}$
$J_i(\cdot)$	- Bessel function of order i
k	- scalar wave number
$\vec{k}$	- vector wave number in direction of wave propagation
k_c	- coupler wave number
K	- plasma dielectric constant (Appendix B only)

$\vec{K}$	- vector surface current density
$K_{\max}$	- maximum value of surface current density
$\ell_{1,2}; L_{1,2}$	- functions of magnetoplasma parameters and boundary conditions
m	- azimuthal mode number
m_e	- electron rest mass
m_i	- ion rest mass
n_c	- critical electron density
n_e	- electron density
N_e	- total number of electrons per unit area
p_0	- neutral filling pressure
p_1, p_2	- transverse wave numbers
R_c	- radius of helical coupler
r_c	- coil module resistance
r_r	- water resistor resistance
V_A	- Alfvén velocity
α	- attenuation coefficient
β	- phase coefficient of propagating wave
γ	- complex propagation constant = $\alpha + j\beta$
ϵ_0	- permittivity of free space
θ	- azimuthal angle in cylindrical coordinates
λ	- general wavelength
λ_c	- coupler wavelength
λ_p	- plasma wave wavelength
μ_0	- permeability of free space

π	- 3.14159...
ϕ	- general phase angle
ψ	- $\tan^{-1}$ (pitch angle of helical antenna)
ω	- radian frequency
ω_{ce}	- radian electron cyclotron frequency
ω_{ci}	- radian ion cyclotron frequency
ω_{pe}	- radian electron plasma frequency
Ω	- ω/ω_{ci}

CHAPTER I

INTRODUCTION

The objective of this investigation was to determine experimentally the properties of a new helical antenna designed to launch azimuthally-asymmetric Alfvén waves in a cylindrical deuterium plasma column. This was done by the following measurements: antenna impedance with and without plasma present, excited wave amplitude in the plasma, axial wavelength of the wave in plasma as a function of excitation frequency and plasma parameters, and azimuthal variation of the excited waves. These measurements are in addition to those needed to determine plasma properties: neutral filling pressure, static magnetic field intensity, discharge current, and plasma temperature and density.

The concept of fusion reactors utilizing deuterium as a fuel offers the possibility of an attractive solution to the worlds' energy needs. It is generally accepted that most magnetic confinement schemes for a fusion reactor (e.g., Tokamak-type devices) will require supplemental heating energy in order to bring the plasma up to thermonuclear fusion ignition temperatures. One method of supplying this energy is through radio-frequency plasma heating. In this scheme, the energy in the electromagnetic fields of an rf wave is converted into kinetic energy of the plasma particles through the excitation of one of the many natural modes of oscillation in a magnetized plasma.

The problem of coupling this rf energy into the plasma has received considerable interest and investigation. Most configurations have involved azimuthally-symmetric ($m = 0$) couplers between the rf energy source and the magnetoplasma.

The experimental investigation reported here concerns a new coupler¹ which launches Alfvén waves which vary as $\exp[j(m\theta + \omega t - kz)]$, where $m = \pm 1$. Although this investigation involves linear geometry, there is experimental evidence² which suggests that these azimuthally-asymmetric modes, where $|m| \neq 0$, are of fundamental importance when considering toroidal geometry. Also, $m = \pm 1$ waves have been used successfully in rf heating experiments on the ST Tokamak³. The transverse accelerating fields for such waves are a maximum on the plasma axis, whereas all other values of m result in waves with zero on-axis fields. This can be seen by considering Eq. (1), where the expressions for the transverse electric fields, $\underline{E}_r$ and $\underline{E}_\theta$, have been combined and then regrouped into left- and right-circularly polarized components¹:

$$\begin{aligned} \underline{E}_\perp &= \underline{E}_r + \underline{E}_\theta \\ &= (-j\hat{a}_r + \hat{a}_\theta) \left[C_1 p_1/2(j\ell_2 + L_2)J_{m-1}(p_1 r) \right. \\ &\quad \left. + C_2 p_2/2(j\ell_1 + L_1)J_{m-1}(p_2 r) \right] e^{j(m\theta + \omega t - kz)} \\ &\quad + (j\hat{a}_r + \hat{a}_\theta) \left[C_1 p_1/2(j\ell_2 - L_2)J_{m+1}(p_1 r) \right. \\ &\quad \left. + C_2 p_2/2(j\ell_1 - L_1)J_{m+1}(p_2 r) \right] e^{j(m\theta + \omega t - kz)} \end{aligned} \quad (1)$$

where k is the axial wave number, $p_{1,2}$ are the two transverse wave numbers associated with k which satisfy the cold-plasma dispersion relation and boundary conditions, $\ell_{1,2}$ and $L_{1,2}$ are functions of the magnetoplasma parameters, and $C_{1,2}$ are constants.

Only the integer values $m = +1$ and $m = -1$ will result in non-zero fields at $r = 0$. Having a maximum accelerating field on the plasma axis correlates quite well with the spatial distributions of plasma temperature and density, which are typically maximum on-axis also. Thus, using $m = \pm 1$ modes, one can deposit more energy in the core of the plasma, in effect heating it "from the inside out".

It can also be seen from Eq. (1) that the choice of m provides a direct control of the wave polarization on-axis. The left-circularly polarized component of an $m = +1$ wave varies radially as a zero-order Bessel function, while the corresponding right-circularly component varies radially as a second-order Bessel function. The opposite variation occur for an $m = -1$ wave. Thus, an $m = +1$ wave is left-hand polarized on-axis, the field rotating in the same direction as the gyrating ions, while the $m = -1$ wave is right-hand polarized on-axis, which is the rotation direction for the electrons. These relationships for the rotation of $\vec{E}_\perp$ relative to $\vec{B}_0$ and $\vec{k}$ are shown in Fig. 1.

LEFT-HAND POLARIZATION ON-AXIS
(ROTATION DIRECTION FOR IONS)

RIGHT-HAND POLARIZATION ON-AXIS
(ROTATION DIRECTION FOR ELECTRONS)

Fig. 1. On-axis polarization of transverse electric field for $m = \pm 1$ waves.

From considerations of the dispersion relation, it is known that, for each value of m , two modes of propagation exist. The waves associated with these modes are commonly known as the fast wave and the slow wave, according to the magnitude of their phase velocities. Theoretical plots of the phase velocities as a function of frequency for the $m = +1$ and $m = -1$ waves are given in Fig. 2. These plots were made assuming a uniform plasma which completely filled a perfectly conducting metal cylinder. It can be seen that the phase velocity goes to zero at a resonance and goes unbounded at a cutoff. The so-called "waveguide cutoff" experienced by the fast waves is directly analogous to the cutoff experienced by electromagnetic waves propagating in a conventional waveguide. Therefore, for the region of interest here, ($\omega \ll \omega_{ce}, \omega_{pe}$) one expects the existence of four waves: the $m = +1$ fast and slow waves, and the $m = -1$ fast and slow waves.

The coupler, as shown in Fig. 3, consists of four symmetric helical straps wound around the outside of the vacuum vessel. It is analogous to an ensemble of four helical antennas, spaced 90° apart around the polar axis. It differs from a conventional helical antenna, however, in that the current distribution along a particular helical strap is essentially a constant, since, for the frequencies of interest here, the coupler wavelength is

Fig. 2. Dispersion relation for $m = \pm 1$ waves propagating in a conducting cylinder.

Fig. 3. The helical coupler and feed sequence.

much less than the free-space wavelength. Azimuthal field rotation is accomplished by using four helices with a proper time phasing of the driving currents. The azimuthal direction of field rotation for a particular coupler is therefore determined by the feed sequence. Power is supplied to the device from a four-phase rf source, i.e., an rf source at 0° , 90° , 180° , and 270° . Referring to Fig. 3, advantage is taken of the fact that current fed in at terminal 1 (0° source) will undergo a 180° phase shift if allowed to return along terminal 3 (180° source). Thus, by shorting terminal 1 to 3, and 2 to 4 at the far end of the antenna, only two sources in phase quadrature are needed.

As is characteristic of helical antennas, the coupler is expected to exhibit a strong propagation directionality. To a first approximation, the device may be considered to be a travelling-wave current sheet⁴ of radius R_c , length $2\lambda_c$, varying sinusoidally with t , z and θ . The surface current density vector is given by

$$\vec{K} = K_{\max} [\hat{a}_\theta \sin\psi + \hat{a}_z \cos\psi] e^{j(m\theta + \omega t - k_c z)} \quad (2)$$

where $k_c = \frac{2\pi}{\lambda_c}$ = axial wavelength of each helix and

$$\psi = \tan^{-1} \left(\frac{\text{circumference of helix}}{\text{spacing of turns}} \right) = \tan^{-1}(k_c R_c).$$

Consequently, the waves excited by the helical coupler are expected to propagate primarily in the positive z direction. When the coupler is fed in the sequence shown in Fig. 3, the wave energy is directed predominantly to the right. Thus, the positive z direction (for the fields of the coupler) for this particular coupler and feed sequence is to the right in Fig. 3, and the azimuthal field rotation is in the left-hand sense. Reversing the feed sequence for the same coupler reverses both the m -number and the propagation direction of the travelling-wave current sheet. The coupler energy is then projected primarily to the left in Fig. 3.

The fields of the coupler are independent of the direction of $\underline{B}_0$, the static background magnetic field. The plasma wave fields are, of course, not independent of $\underline{B}_0$. However, one can choose the mode of operation of the coupler (either $m = +1$ or $m = -1$) with respect to the plasma wave modes, without changing the pitch of the helices or the feed sequence, by the choice of direction for $\underline{B}_0$. Thus, for the coupler and feed sequence of Fig. 3, the $m = +1$ mode is established by directing $\underline{B}_0$ to the right, the azimuthal field rotating in the left-hand sense with respect to $\underline{B}_0$. Reversing the direction of $\underline{B}_0$ establishes the azimuthal field rotation of the coupler in a right hand sense with respect to $\underline{B}_0$, thus placing the coupler in the $m = -1$ mode of operation.

As is also characteristic of helical antennas operating in the beam or axial mode⁵, the coupler is expected to be broadband, possessing desirable radiation patterns and polarization characteristics over a relatively wide (2 to 1) range in frequency. This comes about from the ability of the phase velocity to adjust itself along a particular helical strap so that the fields from each strap add nearly in phase in the axial direction.

CHAPTER II

EXPERIMENTAL ARRANGEMENT

A schematic diagram of the experimental apparatus is shown in Fig. 4. The discharge region between the molybdenum electrodes is 75 cm in length, 3.8 cm in radius and is immersed in a variable axial magnetic field (Appendix A). Field uniformity along the discharge region is $\pm 1\%$, with a maximum field strength of 5.5 kG. The vacuum pumping system consists of a mechanically-backed oil-diffusion pump with a peak pumping rate of 150 liters/sec. It is equipped with a liquid nitrogen-cooled baffle to minimize pump-oil backstreaming. Base pressure in the vessel is 2×10^{-7} Torr. Deuterium gas is bled in continuously during the experiment, providing neutral filling pressures of 0.5 - 5 mTorr.

The plasma is prepared by a highly damped capacitor discharge current ($I_{\text{peak}} > 15$ kA) between molybdenum electrodes. A 40 kV preionizing pulse is applied between a main discharge electrode and a third internal electrode to ensure reliable gas breakdown below 2 mTorr. Average density measurements made across the plasma column with an 8 mm microwave interferometer (Appendix B) indicate that, for the filling pressures used here, the plasma is essentially fully ionized at a time of 60 μsec after gas

Fig. 4. Schematic diagram of experimental arrangement.

breakdown. Transverse profiles of electron temperature and density made on a similar plasma prepared by this method⁶ indicate that, at this same time, the plasma is uniform over 85% of the vessel diameter with an electron temperature of 1.5 - 2 eV.

The helical coupler used in this experiment is two wavelengths long ($2\lambda_c = .30$ m), since the directivity (or directionality) is expected to increase linearly with axial length⁵. Each strap makes 2π radians azimuthal travel per axial wavelength. The pitch angle ψ is 61.1° , giving numerical values in Eq. (2) of

$$\vec{K} = K_{\max} [.875 \hat{a}_\theta + .484 \hat{a}_z] e^{j(m\theta + \omega t - \frac{2\pi}{.15} z)} \quad (5)$$

The inductance of the coupler in vacuum, with all straps included, is 1.3 μ H. The coupler was positioned as shown in Fig. 4, the positive z direction for the coupler being to the right.

As described earlier, a minimum of two rf sources in phase quadrature are needed to power the coupler. A simplified schematic of the circuit used to provide these two sources is shown in Fig. 5. Basically, the arrangement consists of a Class C amplifier and a $\lambda/4$ delay line to provide the necessary 90° phase shift. Capacitor C and resistor R are necessary to compensate for antenna

Fig. 5. Schematic diagram of four-phase rf source.

inductance and to terminate the delay line in its characteristic impedance, respectively. The currents at the two ends of the delay line will therefore be equal in amplitude, but will differ in phase by 90° . The coupler is powered continuously and wave measurements are made at a time of 60 μsec after gas breakdown.

Because of the presence of the background magnetic field coils, a plasma-vacuum boundary condition on the wave field components is not strictly applicable. Therefore, copper foil was wrapped around the region of wave propagation to establish a more definite boundary condition; namely that of a plasma-dielectric-conducting wall.

The waves are detected by a small, center-tapped magnetic probe (Appendix C) in a re-entrant, movable quartz tube. For both the $m = +1$ and $m = -1$ waves, b_r and b_θ have a maximum amplitude on-axis and the probe was therefore positioned in the center of the discharge vessel and oriented in such a manner as to detect $\dot{b}_r$.

CHAPTER III

EXPERIMENTAL RESULTS

As predicted, the coupler exhibits a strong propagation directionality. By reversing the magnetic field and the sequence in which the helices are driven, the wave with the same m -number reverses propagation direction. This directionality is shown by the differences in the wave envelope amplitudes for $\dot{b}_r$ in Fig. 6, where the feed sequence and magnetic field have been reversed between oscillograms, but the probe was kept in the same position. (The upper trace in these and the following oscillograms is $\dot{b}_r$, and the lower trace is an excitation current plus a timing pulse for reference.) Thus, the $m = -1$ wave is still launched in both cases but in the opposite axial direction in the second case. The probe received the large amplitude forward propagating wave in one case and the small amplitude reverse propagating wave in the other case. The directionality for the $m = -1$ wave was better than 4:1 at a frequency of 4 MHz.

That the coupler was indeed producing a wave with an $m = +1$ or an $m = -1$ azimuthal dependence can be confirmed by rotating the $\dot{b}_r$ probe at a fixed axial position. To distinguish a $m = +1$ wave or a $m = -1$ wave, one observes the increase or decrease in the phase of the probe signal

20 μsec /major division

Fig. 6. Directionality of coupler for $m = -1$ wave ($f_{ci} = 3.1 \text{ MHz}$, $\Omega = 1.2$, $n_i = 2 \times 10^{14} \text{ cm}^{-3}$).

relative to the direction of probe rotation. This, together with the known direction of the background magnetic field, establishes the sign of m . The amplitude of the probe signal, however, should not change for $|m| = 1$ waves.

The two oscillograms of Fig. 7 demonstrate these properties, where the probe was rotated 90° between shots. The coupler was operated in the $m = +1$ mode (see Fig. 3), and the probe was rotated in the right-hand sense with respect to $\vec{B}_0$. Consequently, the resulting phase shift after this rotation should be such that a point of constant phase would be seen sooner in time. The upper trace in each oscillogram of Fig. 7 is $\dot{b}_r$ and the lower trace is an excitation current to establish a phase reference. Referring to the lower oscillogram, a point of constant phase on $\dot{b}_r$ has been shifted to the left after the rotation (i.e., it appears sooner in time) with respect to the reference signal. This is characteristic of an $m = +1$ wave. The procedure for determining an $m = -1$ azimuthal variation is essentially the same.

As described previously, changing from $m = -1$ to $m = +1$ waves is accomplished by simply reversing the direction of the axial magnetic field. Figure 8 shows the observed wave amplitudes ($\omega > \omega_{ci}$, $\omega_{ci}/2\pi = 3.1$ MHz) for a reversal of $\vec{B}_0$, all other parameters being the same. It is seen

Fig. 7. Effect of probe rotation at constant axial position. Shots were made 100 μsec after gas breakdown. ($m = +1$, $f_{ci} = 4 \text{ MHz}$, $\Omega = 0.5$, $n_i = 1 \times 10^{14} \text{ cm}^{-3}$).

$m = +1$ fast wave

$m = -1$ fast wave

20 μsec /major division

Fig. 8. Effect of reversing direction of magnetic field; all other conditions same as Fig. 6.

from the amplitudes of b_r that the coupling to the $m = -1$ wave is much better than to the $m = +1$ wave. This is expected from theoretical calculations⁷ of transverse field profiles for the fast wave, as shown in Fig. 9. It should be emphasized that these calculations are for the profiles of the wave that can exist in the magnetoplasma and are exclusive of the fields of the coupler. The theoretical calculations show that the $m = -1$ fast wave is right-hand polarized for all radii while the $m = +1$ fast wave is LHP on axis but changes to RHP beyond a certain radius. This is a result of the radial variation of the transverse wave numbers p_1 and p_2 in Eq. (1). The antenna, however, produces a left- or right-hand polarized field (depending on the sign of m), regardless of radius, and cannot generate a radially-dependent phase reversal. It is therefore not surprising that the antenna does not couple into the $m = +1$ fast wave as efficiently as to the $m = -1$ fast wave.

As was expected, the coupler is broadband in that it is capable of launching waves whose axial wavelengths differ considerably from its own. The measured wavelengths of the $m = -1$ wave ranged from 12 to 40 cm depending upon the excitation frequency (2 - 5 MHz), the magnetic field strength, and the plasma density. A comparison was made of wave probe signal amplitudes at a fixed excitation

$m = -1$ fast wave

$m = +1$ fast wave

Fig. 9. Theoretical transverse fields for fast waves ($\Omega = 2.6$, $n_i = 3.2 \times 10^{14} \text{ cm}^{-3}$).

frequency ($\Omega \equiv \omega/\omega_{ci} = 1.2$, $m = -1$) and varying plasma densities. The axial wavelength varied from 16-28 cm, but the probe signal amplitude was almost constant, varying by less than 10% from the norm. Thus the coupler has only a broad "coupling resonance".

These measurements of axial wavelength for the $m = -1$ wave are in fair agreement (within 15%) with the theoretical wavelengths for the $m = -1$ fast wave. This wave was also observed to propagate well below ω_{ci} without undergoing any appreciable change in wave character, not even the characteristic fast wave cutoff. Some recent calculations on boundary conditions⁷ indicate that this is possible for the $m = -1$ fast wave if a vacuum layer of sufficient thickness is included between the plasma and the conducting wall. With this vacuum layer, a plasma/free-space boundary condition is sufficiently approximated such that the cutoff frequency is lowered, which allows for the existence of the $m = -1$ fast wave even down to very low frequencies.

The $m = +1$ fast wave was not observed for $\omega > \omega_{ci}$. Calculations of the dispersion relation for these plasma conditions indicate that this wave is in cutoff. Attempts to propagate this wave at higher frequencies where the wave should be out of cutoff were also unsuccessful. This discrepancy for the $m = +1$ fast wave has been observed by

other authors^{8,9}. However, there is evidence of its presence in a toroidal geometry¹⁰ where it was found that the onset of the $m = +1$ fast mode required a higher plasma density with $2.4 \leq \Omega \leq 3$. This corresponds to an excitation frequency which is 10% to 20% above the calculated cutoff frequency.

Measurements have also been made below the ion cyclotron frequency. An $m = +1$ slow wave was found to propagate with a phase velocity within 5% of the calculated Alfvén velocity for $\Omega < 0.8$ and was not observed above ω_{ci} . The $m = -1$ wave below ω_{ci} was observed to propagate with almost constant amplitude and phase velocity ($v_p \approx V_A$) for $0.6 \leq \Omega \leq 1$. This wave was identical in all characteristics to the $m = -1$ wave with $1 \leq \Omega \leq 2$. It is therefore believed that the $m = -1$ wave observed below ω_{ci} is essentially the fast wave. The $m = -1$ slow wave makes a negligible contribution to the wave character because of poor coupling efficiency to this wave, as can be seen by referring to Fig. 10. As before, these are theoretical calculations of the profiles of the wave in the plasma, exclusive of the field of the coupler. The $m = -1$ slow wave is RHP on axis but becomes LHP after a certain radius. This point of phase reversal moves inward as $\omega \rightarrow \omega_{ci}$ until the wave is entirely LHP at $\omega = \omega_{ci}$. Since the coupler generates a RHP field when operating in the $m = -1$ mode,

$$\Omega = .32$$

$$\Omega = .84$$

Fig. 10. Theoretical transverse fields for $m = -1$ slow wave ($n_i = 3.2 \times 10^{14} \text{ cm}^{-3}$, $f_{ci} = 3.1 \text{ MHz}$).

the coupling efficiency to the $m = -1$ slow wave is expected to be low.

A comparison was made of voltage and current characteristics of the coupler in vacuum and with a plasma present. Coil loading is deduced from the change in these V/I characteristics (amplitude and phase) due to the presence of the plasma. The voltage across each pair of straps was measured differentially with high impedance (10 Mohm - 2 pf) probes and the current was measured with a calibrated inductive probe which had a negligible insertion impedance. The minimum detectable change in impedance is 0.05 ohms. The impedance of the coupler without plasma present (R_{vacuum}) was typically 1.2 ohms.

The measurements indicate that, for all waves excited, the loading effect due to the presence of plasma is primarily resistive. The change in phase between coupler voltage and current with plasma present was always less than 10° . (The sensitivity of the phase detection system is 7° .) Since a phase change of this size results in a very small reactive change in impedance, and, in addition, this change was such as to reduce the phase angle between the voltage and the current, it is clear that the increase in antenna impedance with plasma present is best described as a resistive increase. For the $m = +1$ wave well below ω_{ci} ($\Omega \leq 0.7$), there was very little change in the

impedance of the antenna when the plasma was present and no increased loading was observed. For $\Omega > 0.7$, loading was observed for the $m = +1$ slow wave; e.g., at $\Omega = 0.9$, the loading resistance was ≈ 2.2 ohm. Increased loading is expected for the $m = +1$ slow wave as $\omega \rightarrow \omega_{ci}$, as can be explained from Fig. 11. Like the $m = -1$ slow wave, the $m = +1$ slow wave polarization is radially dependent; the $m = +1$ slow wave is LHP on axis, but changes to RHP after a certain radius. Unlike the $m = -1$ slow wave, however, the point of phase reversal moves outward as $\omega \rightarrow \omega_{ci}$, until the wave is entirely LHP at $\omega = \omega_{ci}$. Thus, one would expect increased coupling efficiency (and hence loading) as $\omega \rightarrow \omega_{ci}$, as predicted¹. The loading for the $m = -1$ wave for all frequencies below ω_{ci} was negligible.

With excitation frequencies above the ion cyclotron frequency, the loading effect was greatest when launching the $m = -1$ wave. The loading for this wave with $\Omega = 1.2$ was typically 3.4 ohms at $n_e = 6 \times 10^{13} \text{ cm}^{-3}$. Density variations (within a factor of 2) did not affect this resistance enough to be measureable. The loading for the $m = +1$ wave under the same conditions gave a loading resistance typically 50% less than that for the $m = -1$ fast wave. Since no propagating wave was observed, however, when operating under the conditions to produce $m = +1$ fast wave, the rf energy is apparently deposited in $m = +1$

$$\Omega = .32$$

$$\Omega = .84$$

Fig. 11. Theoretical transverse fields for $m = +1$ slow wave ($n_i = 3.2 \times 10^{14} \text{ cm}^{-3}$, $f_{ci} = 3.1 \text{ MHz}$).

evanescent modes. (These evanescent modes can have a component of real power flow when particle collisions are included.)

The above results for the coupler are briefly summarized in Table I.

TABLE I
SUMMARY OF RESULTS

m-number	PHASE VELOCITY		HELICAL COUPLER LOADING RESISTANCE	
	SLOW WAVE	FAST WAVE	$\omega < \omega_{ci}$	$\omega > \omega_{ci}$
+1	$\approx V_{\text{Alfvén}}$	Wave not observed	≈ 2.2 ohms	1.5 - 1.7 ohms
-1	Wave not observed	$\approx 2V_{\text{Alfvén}}$	$\approx R_{\text{vacuum}}$ (≈ 1.2 ohms)	3 - 4 ohms

CHAPTER IV

CONCLUSION

The helical coupler was found to have a directivity ratio (forward wave amplitude/reverse wave amplitude) of typically 4:1. This property can be used, for example, on a linear machine to propagate wave energy into a magnetic beach. The coupler was also found to be highly selective in m-number. Thus, instead of launching a variety of wave modes simultaneously, energy can be transferred selectively to the ions at $\omega = \omega_{ci}$ with the $m = +1$ slow wave, or at $\omega = 2\omega_{ci}$ with the $m = -1$ fast wave. In addition, the broadband capability of the antenna will not require maintaining plasma properties at a narrow "coupling resonance" in order to have efficient energy transfer to the plasma. Loading impedance measurements of the coupler indicated a resistive increase in impedance of typically 1 - 3 ohms with plasma present as compared to 1.2 ohms with no plasma. Future experiments are planned which include high power (0.5 - 1 MW) heating measurements and comparative coupling efficiency measurements with other coupling devices, such as the Stix coil and 1/2-turn loops.

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APPENDIX

- A. Magnetic Field Description
- B. Microwave Interferometer^{B1, B2}
- C. Wave-Magnetic Field Probe

APPENDIX A

MAGNETIC FIELD DESCRIPTION

The background magnetic field is generated by seven water-cooled coil modules which are energized by a 120 kW motor-generator set. It is advantageous to have the field region established by these modules to be as long as possible, and, at the same time, to maintain certain limits on uniformity and maximum field strength. The magnetic flux generated by these modules must of necessity decrease at the ends in order to satisfy Gauss' Law for the magnetic field. However, one can increase the uniform region by supplying a greater driving current to the end modules than to the center modules. In order to do this, a water-cooled resistor was constructed with a resistance ($r_r = .028$ ohms) greater than that of a coil module ($r_c = .021$ ohms). This resistor can dissipate greater than 10 kW and still maintain its resistance within 1% over a 30 second period. (The magnets are energized less than 30 sec per shot.) The coil modules and resistor are interconnected as shown in Fig. A1, such that the current through the four end modules (1, 2, 6, 7) is greater than through the three center modules (3, 4, 5). Thus the magnetic field strength per coil is greater on the ends, which effectively increases the length of the uniform magnetic field.

Fig. A1. Connection diagram for magnetic field modules.

A feedback control system (Fig. A2) is used to establish the magnetic field amplitude within a 1% shot-to-shot variation (as measured with an incremental gaussmeter) of the reference value. The simplified arrangement, as shown in Fig. A2(a), consists of a reference source, a difference amplifier, a DC amplifier, and the magnetic field coils. The feedback loop is closed by the gaussmeter. Basically, the circuit functions as follows: the error signal, derived from the algebraic addition of the reference signal and the feedback signal, is amplified ($A_I = 5 \times 10^5$) and drives the magnetic field coils. The resulting field is monitored by a Hall-effect probe, which provides the feedback signal to the difference amplifier. The reference field amplitude is set by simply adjusting the variable DC power supply at the input to the difference amplifier. The magnetic field is brought up and down between shots by supplying or removing the driving signal to the programmable DC power supply.

2(a). Block diagram.

2(b). Difference amplifier.

2(c). DC amplifier ($A_I = 5 \times 10^5$)

Fig. A2. Magnetic field feedback control system.

APPENDIX B

MICROWAVE INTERFEROMETER^{B1, B2}

For the temperature ($T_e < 3$ eV) and density ($n_e < 3 \times 10^{14} \text{ cm}^{-3}$) of the plasma used in this experiment, a microwave interferometer^{B3, B4} is one of the better methods for determining electron density. The relative dielectric constant of a plasma for oscillations of frequency ω in the absence of a background magnetic field is given by

$$K = 1 - \left(\frac{\omega_{pe}}{\omega} \right)^2, \quad (B1)$$

where the electron plasma frequency ω_{pe} is defined by

$$\omega_{pe} = \left(\frac{n_e e^2}{m_e \epsilon_0} \right)^{1/2} \text{ radians/sec.} \quad (B2)$$

For $\omega > \omega_{pe}$, so that K is positive, the oscillations are transmitted through the plasma, but if $\omega < \omega_{pe}$, essentially complete reflection occurs. The so-called "cutoff" condition, between transmission and reflection, is that for which $\omega = \omega_{pe}$; it is related to the electron density by

$$f_{\text{cutoff}} = 9.0 \times 10^3 n_e^{1/2} \text{ Hz.} \quad (B3)$$

In the propagation of a plane wave through a dielectric medium, the electric field strength can be expressed as

$$\underline{E} = \underline{E}_0 \exp(\gamma x - j\omega t) \quad (\text{B4})$$

where γ is the propagation constant. This propagation constant is, in general, complex and can be written as

$$\gamma = \alpha + j\beta, \quad (\text{B5})$$

where α is the attenuation coefficient of the wave, and β is the phase coefficient in radians per unit length. It has been shown^{B1} that, for an operating frequency much greater than the electron collision frequency, β has the form

$$\beta = \frac{\omega}{c} \left[1 - \left(\frac{\omega_{pe}}{\omega} \right)^2 \right]^{1/2}. \quad (\text{B6})$$

Consequently, if the phase shift of a known-frequency wave through a certain length of plasma is found, it is possible to determine ω_{pe} .

The quantity that can be determined experimentally is the difference between the phase shift with the plasma in the path and that for the same path length without the plasma. Since the plasma is seldom uniform, the phase shift can be expressed as

$$\Delta\phi = \frac{\omega d}{c} - \int_0^d \beta dx, \quad (B7)$$

where d is the transmission length. Substituting for β from Eq. (B6),

$$\begin{aligned} \Delta\phi &= \frac{\omega d}{c} - \int_0^d \frac{\omega}{c} \left[1 - \left(\frac{\omega_{pe}}{\omega} \right)^2 \right]^{1/2} dx \\ &= \frac{\omega}{c} \left[d - \int_0^d \left[1 - \frac{e^2}{m_e \epsilon_0 \omega^2} n_e(x) \right]^{1/2} dx \right], \end{aligned} \quad (B8)$$

where $n_e(x)$ represents the electron density distribution along the transmission path.

Since this is a difficult integral to evaluate with precise expressions for $n_e(x)$, a reasonably good approximation when ω_{pe}/ω is less than 0.6 results from expressing the integrand as a binomial series and neglecting all terms beyond the second:

$$\begin{aligned} \Delta\phi &\approx \frac{\omega}{c} \left[d - \int_0^d \left[1 - \frac{1}{2} \cdot \frac{e^2 n_e(x)}{m_e \epsilon_0 \omega^2} \right] dx \right] \\ &\approx \frac{\omega}{c} \left(\frac{e^2 N_e}{m_e \epsilon_0 \omega^2} \right), \end{aligned} \quad (B9)$$

where N_e is the total number of electrons per square centimeter of cross-sectional area of the plasma between the microwave horns. Note that convergence of the series requires that $\omega_{pe} < \omega$. The observed phase shift, then, is proportional to the average electron density.

The basic arrangement for the interferometer is shown in Fig. B1. The microwaves are split into two paths after leaving the source isolator. One path, called the transmission path, is through the plasma and contains the transmission and receiving horns. The other path, called the reference or null path, contains the frequency meter, an attenuator, and a 0° - 180° phase shifter. For a transient plasma, the latter two devices are adjusted such that the two signals, when recombined and detected, interfere destructively, producing a null output in the absence of plasma. When a plasma of sufficient density is present in the transmission path, a phase shift occurs which imbalances the circuit and the resulting output signal is recorded on the oscilloscope.

The transmission and receiving horns are aligned such that the electric field vector is parallel to the background magnetic field. Otherwise the expression for β in Eq. B6 is considerably altered, and must include the cyclotron frequency of the electrons.

Fig. B1. Schematic diagram of 8 mm microwave interferometer.

An idealized interferometer response, for changes in phase only, is shown in Fig. B2. The upper portion represents the variation with time of the electron density and the lower portion indicates the corresponding interferometer response. Each interference "fringe" corresponds to a phase shift of 2π radians. The crowding of the fringes before cutoff, as compared to those after, shows that the plasma was building up at a faster rate than it decayed.

At the time t_0 in Fig. B2 the density is just the cutoff value which is known from the microwave frequency. During the decay to essentially zero density, the total phase shift corresponds to five fringes, i.e., to $5 \times 2\pi$. Since the phase shift has been taken to be proportional to the electron density, by Eq. (B9), each fringe represents a change of $0.2 n_c$ in density. Hence, from the locations of successive minima (or null points) on the oscillogram, it is possible to plot a curve showing the rate of decay of the electron density. No information, other than that $n_e > n_c$, is obtained during the time the microwaves are cut off.

A more realistic interferometer response, which includes attenuation, is shown in Fig. B3. The attenuation of the microwave signal is evidenced by the amplitude envelope of the fringes. In cases where the collision rate

Fig. B2. Interpretation of microwave interferometer response.

Fig. B3. An example of a realistic fringe pattern obtained when cutoff is encountered and wave damping occurs.

is high, the signal cutoff is reached at a density somewhat below n_c (see section 1.4 of reference B2).

Shown in Fig. B4 are oscillograms of interferometer responses obtained with the experimental arrangement of Fig. B1. These were made at the same neutral filling pressure (1 mTorr D_2), but with and without the confinement field, B_0 . (Note the difference in horizontal sweep rates.) The data from these interferograms have been plotted by the techniques described here and are shown in Fig. B5. The effect of the confinement field is to increase the time of decay to n_c by a factor of more than 6. The decay pattern for $B_0 = 0$ appears to be exponential. Note that an extrapolation back to $6.4 \times 10^{13} \text{ cm}^{-3}$ (1 mTorr D_2 fully ionized) for the data with $B_0 = 0$ corresponds to a time of $\approx 58 \text{ } \mu\text{sec}$ after gas breakdown.

$$B_0 = 0$$

(20 μ sec/major division)

$$B_0 = 4\text{kG}$$

(200 μ sec/major division)

Fig. B4. Interferometer response with and without magnetic field ($p_0 = 1$ mTorr; upper trace: detector voltage; bottom trace: plasma preparation current).

Fig. B5. Plot of microwave interferometer data
 ($f = 36.3 \text{ GHz}$, $n_c = 1.63 \times 10^{13} \text{ cm}^{-3}$, $p_0 = 1 \text{ mTorr D}_2$).

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APPENDIX C

WAVE-MAGNETIC FIELD PROBE

The doubly-shielded probe (Fig. C1) used for all measurements of the plasma wave fields is a cylinder, 2 mm in diameter and 5 mm in length. The small size permits good spatial resolution in the 75 mm diameter plasma. It is encased in a quartz tube which is movable along a 0.5 m length of the machine axis. The probe consists of two coils wrapped simultaneously in opposite directions, such that complementary turns are immediately adjacent. The two currents induced will be opposite in phase, and any voltages induced due to capacitive (electrostatic) pickup will be in phase. Thus, when the two signals are added with a differential amplifier, the signal voltages will add together, but common-mode voltages will cancel. The resulting signal will be almost entirely that due to the time-varying magnetic fields of the wave in the plasma. Table C-1 gives the performance of the probe in the field of a Helmholtz coil.

Fig: C1. Details of wave probe.

TABLE C-1
PERFORMANCE OF MAGNETIC PROBE IN
FIELD OF HELMHOLTZ COIL

f (MHz)	COMMON-MODE REJECTION RATIO	DIRECTIONALITY FACTOR
4	13:1	> 68
10	7:1	68

$$\text{C-MRR} \equiv \frac{\text{Amplitude with coil signals subtracted}}{\text{Amplitude with coil signals added}}$$

$$\text{DF} \equiv \frac{\text{Amplitude at } 0^\circ \text{ to field}}{\text{Amplitude at } 90^\circ \text{ to field}}$$