ENDPOINT DETECTION OF SILICON WAFER ETCHING

by

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ABSTRACT

This thesis describes the results of an investigation into the use of plane polarized light for performing in situ endpoint monitoring of oxide thickness during the etching of silicon wafers. The continuous monitoring of the oxide thickness can be accomplished by analysis of the reflection characteristics of the plane polarized light. This information then allows the etching to be terminated when the oxide thickness approaches zero, thus, providing a means to control the etching process.

To accomplish on line monitoring of the oxide thickness, plane polarized light from an argon ion laser passing through a rotating half-wave plate which alternately switches the polarization from transverse electric (TE) to transverse magnetic (TM). The modulated laser light is specularly reflected from the oxide-coated silicon wafer and is sensed by a silicon photo-detector. A.C. detection techniques are employed to measure the normalized reflectance intensity $\frac{I_{TE} - I_{TM}}{I_{TE} + I_{TM}}$ which is related to the thickness of the oxide coating. Oxide thicknesses on the order of 1000 Å can be measured with the device to an accuracy of about 0.4%. Oxide thicknesses less than 50 Å cannot be measured with accuracy.

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CHAPTER I

INTRODUCTION

1.1 From the Transistor to the Integrated Circuit

What has been referred to as the major invention of the century occurred at the Bell Laboratories in New Jersey in 1947 ⁽¹⁾. The transistor, which is a device that makes use of a semiconducting material to amplify or switch an electrical signal, was developed by Shockley, Bardeen and Brattain. This device was responsible for beginning a new technology which experienced tremendous growth during the four decades since its creation. In its early years the semiconductor industry was guided by a desire to develop new and more reliable techniques of manufacturing semiconductor devices with little or no emphasis on building better, more efficient devices.

As the size of semiconductor devices continued to decrease, it became extremely difficult to interconnect the various electrical components in an economical fashion to form complete circuits. This problem quickly became the limiting factor in circuit designs as the complexity of the system continuously grew. In simple and relatively small circuits, electrical engineers usually only had to be concerned with possible electromagnetic interference effects, induced by components such as transformers. Otherwise, they could position the devices quite freely. In larger and more complicated electrical networks, the complexity in wiring the circuits became too

cost-prohibitive and time-consuming. The major difficulty lay in the fact that the greater speed of operation, made possible by the transistor, was being impeded by the speed with which the electronic signals could travel along the interconnecting wires. As transistor technology developed, the switching rate of the circuits rapidly increased. By the mid-fifties, there was an obvious need to reduce the size of the circuit components as well as their physical separation.

In 1959, the problem was solved by Jack Kilby's development of the integrated circuit. This technological breakthrough made possible the fabrication of complete circuits on a single chip of silicon. These circuits consisted of transistors, diodes, resistors and capacitors as well. The first integrated circuits contained a dozen or so components on a single chip of silicon which measured only a few millimeters on a side. Today, many of the mass-produced integrated circuits consist of more than 10,000 individual components on chips of similar size.

1.2 The Manufacturing of Integrated Circuits

The economical production of integrated circuits is made possible only by batch fabrication of silicon wafers. These circuits are made by concurrently processing many single crystal silicon wafers, each containing thousands of integrated circuits. The physical properties required by the various circuit elements is accomplished by selectively diffusing minute traces of impurities into the silicon or

by oxidizing them directly into the silicon oxide coating. To make an integrated circuit, one starts with a wafer of single crystal silicon which is coated with an oxide film. These wafers are usually circular with a diameter of approximately 10 cm and a section of the circumference flattened along a chord to indicate the crystal orientation. A photoresist layer is then applied to the wafer. The photoresist is an organic photosensitive material that polymerizes due to the action of light, thereby creating a barrier which protects selected regions of the wafer from the etching substance. The photoresist allows specific regions of the wafer to be exposed to the diffusion and etching process, hence producing the desired circuit pattern on the wafer.

Etching refers to the process of removing unwanted layers of the oxide to form the integrated circuits and is the major process in the manufacturing of semiconductor devices. Care must be taken to avoid disturbing the underlying regions of the photoresist, or undesirable changes in the physical dimensions and operating characteristics of the circuit will result.

To control the accuracy of the etching process, it is desirable to have a device that is capable of continuously monitoring the oxide thickness as it decreases, or one which can at least determine when the oxide has been completely etched. This thesis describes such a device which can yield both types of information, i.e., continuous oxide monitoring and endpoint detection. To perform these measurements, we make use of the reflection characteristics of plane polarized light incident to the oxide coated silicon. In this sense

it is a modification of a standard ellipsometric arrangement to provide real time data on the oxide thickness. Before discussing the theory and construction of this device, an overview of the various etching techniques shall be presented.

1.3 Silicon Wafer Etching⁽²⁾

Etching of silicon wafers is usually performed by one or a combination of the following methods.

1. Chemical Etching
2. Plasma Etching
3. Ion Beam Milling
4. Reaction Ion Etching

Each method has inherent advantages but varies in degree of precision and uniformity of the etch. A factor that requires consideration when choosing an etching process is undercutting, which is also referred to as isotropic etching. Isotropic etching refers to etching along the sides of the structures as defined by the photoresist and etching directly down into the silicon oxide coating. Selectivity of the etch is another factor requiring attention when selecting an etching process. Selectivity refers to the extent of the removal of the silicon underlying the oxide coating. Thus, the greater the selectivity, the less the underlying silicon is disturbed.

1.3.1 Chemical Etching

In chemical etching, the silicon wafer, after the application of the photoresist, is placed in the etching chamber. The chamber is usually constructed of teflon or some similar material that is unaffected by the etching material. In liquid etching, the chamber is totally submerged in an acid bath which reacts with the outer layers of the silicon wafer through the openings in the photoresist, dissolving the oxide layer and leaving the underlying regions of the wafer intact. Regulation of the etching process is accomplished by controlling the temperature of the acid and the time of its chemical activity. The acidity of the etchant can be readily controlled by decreasing its concentration to provide a reasonable etching time. Etching rates usually last from one to seven minutes. Times greater than ten minutes cause the photoresist to detach from the wafer, allowing acute undercutting to occur.

In liquid etching, the process is terminated by removing the wafer from the etchant chamber and rinsing immediately with distilled deionized water. This process not only terminates the etching but also removes the etchant from the wafer, preventing any possible contamination.

Oxide etching can also be accomplished by dry etching techniques. Instead of using a liquid etchant, hydrogen fluoride gas (HF) is used. The fluorine in the gas combines with silicon and oxygen of the silicon oxide (SiO_2) layer forming a variety of compounds which are liberated from the wafer surface and evacuated from the chamber. The major disadvantage of this method is that the hydrogen fluoride gas etches

the silicon oxide at a high rate, making the process difficult to control. In general, HF etching times are less than one minute. Dilution of the HF gas does not significantly decrease the rate of etching. The controllability may be increased by preparing a mixture of hydrogen fluoride and ammonium fluoride (NH_3F) gas. The ammonium fluoride behaves as a buffer to the HF-SiO_2 reaction and decreases the rate of chemical activity. This process is terminated by pumping out the etchant gas and replacing it with an inert gas.

1.3.2 Plasma Etching

In this technique, a plasma is created in the etchant chamber, called the reactor, by the introduction of a highly reactive gas into a high energy RF-field. The energy of the plasma excites the gas molecules in the reactor to a highly energetic state, thereby facilitating a reaction with the outer layers of the wafer. After sufficient time for the reaction to occur, the reactive gas along with the etched particles of the wafer are removed by evacuation of the chamber.

The advantage of plasma etching is that the degree of undercutting is relatively small and the etchant is free to penetrate the smallest orifices of the photoresist, which adds an additional degree of dimensional control. Unfortunately this method has poor selectivity. To increase the selectivity, one can: (a) dilute the etchant gas near the end of the etch to decrease the etching rate or (b) place a detector in the reactor chamber to detect when the exciting gas is free of etched particles.

Since the plasma etch requires high temperatures within the chamber, the photoresist can and often does polymerize making the removal of the photoresist difficult. The high cost of this method is offset by its greater dimensional control and sensitivity, thus making it a desirable etching technique.

1.3.3 Ion Beam Milling

As the title might suggest, this method makes use of an ion beam to perform the etching. In this method, the wafer to be etched is placed in the etching chamber and electrically grounded. Argon atoms are ionized and then directed into the chamber through a portal. The argon ions are accelerated toward the wafer by means of an electric field and upon impact with the wafer they literally blast away the regions of the wafer not covered by the photoresist. If the ion beam is at normal incidence to the wafer, there is a limited amount of undercutting with a high degree of selectivity.

1.3.4 Reactive Ion Etching

The final technique to be discussed is reactive ion etching, which combines ion beam milling and plasma etching methods previously discussed. The gas to execute the etching enters the etching chamber and is then ionized. As the molecules of the gas collide with the wafer, the outer layers are removed. The combination of these two methods allows for enhanced sensitivity and control. For this reason, reactive ion etching is the preferred dry etching method and is the

most reliable production technique for manufacturing integrated circuits with surface structures on the order of a few microns. This method, however, is expensive and used only in special applications.

1.4 Summary

Each of the four methods used in the etching of integrated circuits into single crystal silicon wafers, though varying in strategy, has the same purpose: To remove unwanted layers of silicon oxide from the surface of the wafer in a pattern determined by the photoresist. To accomplish a uniform etch, a number of parameters need to be considered: The pattern density of the wafer, temperature and concentration of the etchant, adhesion of the photoresist and the uniformity of the reactive etchant on the surface of the wafer.

The quality of the etch can vary greatly from one batch of wafers to another. In an attempt to increase the consistency of processing, the usual procedure is to prepare a test wafer which is processed and carefully tested. After this is accomplished, the remaining wafers of the batch are processed in the same manner.

We now have a basic concept of the standard etching techniques and may state the requirements necessary to design a device that will permit in situ endpoint detection of the etching process or provide for continuous monitoring of the oxide thickness. A current method for endpoint detection was mentioned in the section on plasma etching. This method employs a detector in the reactor chamber which monitors the presence of the etched particles in the gas removed from the

chamber. When such particles are no longer present, the etching process is terminated. The detector does not yield information on the thickness of the oxide layer. It only provides information on the liberation of trace particles from the etched material. When the oxide coating is completely etched, the signal from the detector goes abruptly to zero. Since this method only determines the presence of oxide particles in the exiting gas, the possibility of over-etching is distinctly real and allows damage to the underlying regions of the wafer.

A device which can perform in situ measurements of oxide thickness yielding real time data is needed. This information will allow the operator to decrease the etching rate during the last few hundred angstroms of oxide thickness and stop the process at the moment the oxide coating goes to zero. With the use of a computer, the entire etching process may be automated to increase the control and sensitivity of the device.

Continuous monitoring of an oxide thickness which changes in time may be accomplished by analysis of the characteristics of plane polarized light reflected from the wafer. To develop the design of this device we must investigate the optical properties of a stratified medium, which, in our case, consists of three layers: Air or etchant gas, silicon oxide and single crystal silicon.

CHAPTER II

OPTICAL PROPERTIES OF A STRATIFIED MEDIUM

2.1 Reflection of Plane Polarized Light From a Transparent Film on Absorbing Substrate

As mentioned in the previous chapter, we wish to utilize the reflection properties of plane polarized light from oxide-coated silicon wafers as a means of determining the oxide thickness. In this chapter, the pertinent equations shall be presented along with a discussion of their physical significance.

The theory concerning the optical properties of a stratified medium is thoroughly discussed in the scientific literature.^(3,4) In particular, the reader is referred to the optics text by Born and Wolf,⁽³⁾ where the theory is developed in detail (chapters 1 and 13). The derivation of the equations presented in this chapter can be found in Appendix I.

The geometry used in discussion of the reflection of plane polarized light from a transparent film on an absorbing substrate is shown in Figure 1. Consider a plane oxide layer of thickness h in the z direction and of infinite extent in the xy -plane characterized by the real index of refraction n_2 . We assume the region of negative x to be a semi-infinite dielectric space with a real index of refraction n_1 , and the region $z > h$ be a semi-infinite conductor characterized by a

Figure 1. Geometry used when discussing oxide-coated silicon.

complex refractive index given by $\hat{n}_3 = n_3(1 + ik_3)$. All interfaces are considered to be perfectly smooth and the optical constants of each medium are assumed to have the same value throughout the film and substrate with discontinuities at each boundary.

We consider a plane electromagnetic wave propagating into region 2, incident from the first medium, restricting our analysis to having the incident light linearly polarized. By linearly polarized light, it is meant that the electric field vector maintains a constant direction of oscillation as the wave evolves in space and time. The direction of the electric field vector is used to describe the state of polarization of the wave, for example if the field vector is vertical, then the wave is referred to as vertically polarized, or alternately, the plane of polarization is in the vertical plane. The plane formed by the normal to the surface of the media and the direction of incidence of the wave is defined as the plane of incidence.

In the case where a linearly polarized wave is incident to the medium with its electric field vector orientated perpendicular to the plane of incidence, we refer to it as a transverse electric wave and denote it by TE. Alternately, when the electric field vector is parallel to the plane of incidence it is said to be transverse magnetic and is denoted by TM. We note that the TE and TM polarizations are defined only for non-normal angles of incidence since the plane of incidence is undefined for normal incidence. A light wave having arbitrary polarization may be resolved into two waves with polarizations along orthogonal axis as defined by the plane of incidence. Moreover, since the boundary conditions at each

interface for the TE and TM waves, as imposed by Maxwell's equations, are mutually independent, any set of equations developed for the TE wave can be used immediately to deduce the equations for a TM wave.

At the first interface (i.e., air-SiO₂ boundary), we need the ratios of the reflected and transmitted electric fields for the TE and TM waves. These are given by the Fresnel coefficients, which are developed from electromagnetic theory.

For the TE wave, the Fresnel coefficients at the first interface are given by⁽⁵⁾

$$r_{12}^{TE} = \frac{|E'_1|}{|E_1|} = \frac{2n_1 \cos \theta_1 - n_2 \cos \theta_2}{n_1 \cos \theta_1 + n_2 \cos \theta_2} \quad (2.1.1)$$

and

$$t_{12}^{TE} = \frac{|E_2|}{|E_1|} = \frac{2n_1 \cos \theta_1}{n_1 \cos \theta_1 + n_2 \cos \theta_2} . \quad (2.1.2)$$

The subscripts 12 refers to the air-SiO₂ interface, θ_1 and θ_2 are the angles of incidence and refraction, respectively, r and t refer to reflection and transmission. The angles θ_1 and θ_2 are related by Snell's law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 . \quad (2.1.3)$$

Similarly, for the TM wave at the air-SiO₂ interface the Fresnel coefficients are given by the ratios of the reflected and transmitted magnetic fields. That is

$$r_{12}^{TM} = \frac{|B'_1|}{|B_1|} = \frac{n_2 \cos \theta_1 - n_1 \cos \theta_2}{n_2 \cos \theta_1 + n_1 \cos \theta_2} \quad (2.1.4)$$

and

$$t_{12}^{TM} = \frac{n_1}{n_2} \frac{|B_2'|}{|B_1|} = \frac{2n_1 \cos \theta_1}{n_2 \cos \theta_1 + n_1 \cos \theta_2}. \quad (2.1.5)$$

With the Fresnel coefficients for both the TE and TM waves we have the complete solutions of the boundary value problem at the first interface. We note that by definition θ_1 is zero for normal incidence and hence by Snell's law θ_2 is also zero. Equations (1.1.1) and (2.1.4) reduce to equivalent forms for normal incidence provided that $r_{12}^{TE} = -r_{12}^{TM}$.

At the 23 interface ($\text{SiO}_2 - \text{Si}$) the normalized amplitude of the reflected TE wave is given by

$$r_{23}^{TE} = \rho_{23} e^{i\theta_{23}} = \frac{n_2 \cos \theta_2 - (u_3 + iv_3)}{n_2 \cos \theta_2 + (u_3 + iv_3)}, \quad (2.1.6)$$

where

$$\rho_{23}^2 = \frac{(n_2 \cos \theta_2 - u_3)^2 + v_3^2}{(n_2 \cos \theta_2 + u_3)^2 + v_3^2} \quad (2.1.7)$$

$$\tan \phi_{23} = \frac{2v_3 n_2 \cos \theta_2}{u_3^2 + v_3^2 - n_2^2 \cos^2 \theta_2} \quad (2.1.8)$$

$$2u_3^2 = R - n_2^2 \sin^2 \theta_2 + [(R - n_2^2 \sin^2 \theta_2)^2 + I^2]^{\frac{1}{2}} \quad (2.1.9)$$

$$2v_3^2 = - (R - n_2^2 \sin^2 \theta_2) + [(R - n_2^2 \sin^2 \theta_2)^2 + I^2]^{\frac{1}{2}} \quad (2.1.10)$$

$$R = n_3^2 - n_3^2 k_3^2 \quad (2.1.11)$$

$$I = 2n_3^2 k_3 \cdot \quad (2.1.12)$$

For the TM wave

$$\rho_{23}^2 = \frac{(R \cos \theta_2 - n_2 u_3)^2 + (I \cos \theta_2 - n_2 v_3)^2}{(R \cos \theta_2 + n_2 u_3)^2 + (I \cos \theta_2 + n_2 v_3)^2} \quad (2.1.13)$$

$$\tan \phi_{23} = \frac{2n_2 \cos \theta_2 (I u_3 - R v_3)}{R^2 \cos^2 \theta_2 + I^2 \cos^2 \theta_2 - n_2^2 (u_3^2 + v_3^2)} \cdot \quad (2.1.14)$$

We note that the expressions for r_{12} , ρ_{23} and ϕ_{23} are implicitly dependent on the polarization of the incident light (TE or TM).

2.2 Reflection Properties of the Thin Film

We now possess the complete solutions of the boundary value problem at each interface of the air-SiO₂-Si system. With these

solutions at our disposal we can develop an expression for the reflectivity. When the conductor and dielectric are combined, most of the light incident on the oxide layer is transmitted to the conducting surface (Si), reflected and then transmitted back through the oxide layer. However, there is a small component of light which is reflected back and forth many times, decreasing exponentially in amplitude and changing in phase. Since the Fresnel coefficients give the fraction of transmitted and reflected light at each interface, we have only to add up the contributions of each of these partial waves to the net wave reflected from the first interface.

The reflectance and transmittance of each interface is given by the square of the Fresnel coefficients, That is

$$\begin{aligned} R_{12} &= |r_{12}|^2 & R_{23} &= |\rho_{23}|^2 \\ T_{12} &= 1 - R_{12} & T_{23} &= 1 - R_{23} \end{aligned} \quad (2.2.1)$$

T_{23} will be a real quantity only if the silicon layer is on the order of a few hundred angstroms thick. For our purposes, however, $R_{23} = 1$ and $T_{23} = 0$. Summing up the partial waves we find the total reflection coefficient to be given by $\text{Re}^{i\delta r}$, where

$$R = \left[\frac{r_{12}^2 + \rho_{23}^2 + 2r_{12}\rho_{23}\cos(\phi_{23} + 2\delta)}{1 + r_{12}^2 + \rho_{23}^2 + 2r_{12}\rho_{23}\cos(\phi_{23} + 2\delta)} \right]^{\frac{1}{2}} \quad (2.2.2)$$

and

$$\tan r = \frac{\rho_{23}(1-r_{12}^2)\sin(\phi_{23}+2\beta)}{r_{12}(1+\rho_{23}^2) + \rho_{23}(1+r_{12}^2)\cos(\phi_{23}+2\beta)}, \quad (2.2.3)$$

with

$$\beta = \frac{2\pi}{\lambda} n_2 h \cos\theta_2. \quad (2.2.4)$$

The expressions for r_{12} , ρ_{23} and ϕ_{23} are implicitly dependent on the state of polarization of the incident wave. Analysis of equation (2.2.2) which governs the reflection of plane polarized light from a stratified medium clearly indicates that the reflected amplitudes and hence the intensities will be different for the TE and TM waves. It is this fact that we wish to exploit in designing a device to yield real time data on a changing oxide thickness of silicon wafers which are being etched.

To deduce the implication of the formulas we have developed, we begin by noting that equation (2.2.2) remains unchanged when β is replaced by $\beta + \pi$ (that is when h is replaced by $h + \Delta h$), where

$$\Delta h = \frac{\lambda}{2n_2 \cos\theta_2}. \quad (2.2.5)$$

Therefore, the reflectivities of dielectric films on an absorbing substrate, in which the oxide thicknesses differ by an integral multiple of $\lambda / (2n_2 \cos\theta_2)$ are the same. If a plot of the reflectance

intensity of the TE and TM waves is made versus the oxide thickness, we expect the graph to be periodic in oxide thickness. The periodicity of the reflected TE and TM waves is due to the interference effects produced by the SiO_2 film.

The physical reason for the periodicity of the reflected TE and TM waves is easily seen by examining the interference effects produced by a thin film (i.e., the SiO_2 layer). From Appendix II, which gives a detailed analysis of thin film interference, we find that if conditions are such that

$$m\lambda = 2n_2h\cos\theta_2, \quad m = 0,1,2,\dots, \quad (2.2.6)$$

then destructive interference occurs and a minimum in the reflected intensity is found. If, however, we find

$$(m + 1/2)\lambda = 2n_2h\cos\theta_2, \quad m = 0,1,2,\dots, \quad (2.2.7)$$

then constructive interference results and the reflected intensity is a maximum.

Equation (2.2.2) and the equations leading up to it were programmed into an Apple IIe Computer and the reflectance intensities for both the TE and TM waves were calculated (see Appendix III). A check of the program was made by recalculating data given by Burge and Bennett⁽⁶⁾. Figure 2 shows a plot of the calculated reflectance and normalized intensities versus oxide thickness for $\lambda = 4880 \text{ \AA}$ for $\theta = 80^\circ$.

Figure 2. Reflectance intensity and normalized intensity for the TE and TM waves as a function of oxide thickness for $\lambda = 4880 \text{ Å}$, $\theta = 80^\circ$. Optical constants $n_1 = 1.00$, $n_2 = 1.46$, $n_3 = 4.27$ and $k_3 = 0.0253$.

Having completed our discussion on the reflection of plane polarized light from a silicon wafer with an oxide coating we now proceed to discuss the standard technique of thin film thickness determination and then discuss a modification of this device which allows real-time continuous monitoring of oxide thickness during the etching process.

CHAPTER III

EXPERIMENTAL DESIGN

3.1 Standard Ellipsometry

Ellipsometry^(7,8) is defined as the measurement of the state of polarization of a polarized vector wave and is generally conducted in order to obtain information about an optical system which, through some interaction, modifies the state of polarization. When a light wave is reflected or refracted at an interface between two optically dissimilar media, the state of polarization is changed abruptly. Such a change is caused by the difference in the Fresnel reflection or transmission coefficients for the two linear polarizations parallel (TM) and perpendicular (TE) to the plane of incidence. For the case of transmission, the state of polarization changes continuously as the light progresses through a medium which exhibits optical anisotropy (refractive, absorptive or both).

Reflection ellipsometry has long been recognized as an important tool for the study of surfaces of thin films. This type of ellipsometry is the most widely used method for determining the optical constants of both dielectrics and conductors.

The principles and techniques of ellipsometry have been described by a number of authors.^(7,8,9) In using ellipsometry in the study of thin films, one is concerned with two physical parameters of the substrate, namely, its index of refraction n and thickness h .

Ellipsometry operates on the principle that if plane polarized light is incident to a medium, it is reflected as elliptically polarized light. The amount of ellipticity of the reflected light can be directly related to the index of refraction and thickness of the film and substrate.

In Figure 3, two arrangements for measuring n and h are shown. In arrangement A the ellipticity of the light incident on the sample is adjusted via the quarter wave plate such that the reflected light is plane polarized. From the degree of ellipticity of the incident light the thickness and index of refraction can be calculated. In arrangement B the situation is reversed. Plane polarized light (at 45° with respect to the plane of incidence) is incident on the sample and the reflected light is elliptically polarized. The degree of ellipticity of the reflected light is determined with a quarter wave plate, rotated to a position such that the transmitted light becomes plane polarized. In both arrangements, the analyzing polarizer has its optic axis oriented so that it blocks the incident plane polarized light. Therefore, both arrangements utilize null detection. The arrangements of Figure 3 are not suited to on-line monitoring of silicon wafer etching. The main reason is that the film (SiO_2) thickness changes as the etching progresses. Thus, one would have to continuously change the orientation of the quarter wave plate in the ellipsometer to maintain the null condition in detection. This orientation would have to be matched to the etching speed. Moreover, the ellipticity of the incident (A) or reflected light (B) in Figure 3 is not a linear function of the SiO_2 film thickness.

Figure 3. Standard ellipsometric arrangements with incident elliptically polarized light (A) and incident, 45° plane polarized light (B).

The measured quantities which characterize the ellipticity of the reflected beam are ψ and Δ . Tangent ψ is the ratio of the reflectance amplitudes and Δ is the phase difference of the two components of the electric field vector measured parallel and perpendicular to the plane of incidence. Their magnitudes depend upon the optical constants of the substrate and film as well as the thickness of the film. If the film thickness and optical constants of the substrate and film are known, the resultant ψ and Δ for the film-substrate combination may be calculated. The equations relating ψ and Δ to n and h are complicated and require the use of a computer to solve. Because of the nature of the exact equations, they cannot be explicitly solved for the film thickness, index of refraction, etc. Several approximations have been derived to relate changes in the reflectance amplitude and phase change to the film thickness.^(10,11)

Ellipsometric measurements on a film-free surface yield values of the optical constants n and k for a conducting substrate, which, according to theory, do not vary with the angle of incidence, but are highly dependent upon the wavelength of the illuminating light. However, if a film is present, pseudo-index values will be obtained from the experimental measurements rather than values for the substrate alone.⁽⁵⁾ The refractive index will thus be a combination of the refractive index for the film-substrate combinations.

3.2 Ellipsometry for Semiconductor Process Control

In recent years, semiconductor manufacturers have been increasingly utilizing the ellipsometer to measure thin dielectric (SiO_2) layer thickness. Ellipsometric monitoring of the oxide layer is usually made on blank areas of the device wafer or on separate control wafers. Since monolithic device fabrication requires that the SiO_2 layer be redeposited as many as a dozen times, ellipsometric measurements are usually performed each time the dielectric is grown. Typical thicknesses of the dielectric layers are: 5,000 Å for the SiO_2 first layer, 600 to 1,200 Å for the SiO_2 gate layers and a 5,000 to 12,000 Å SiO_2 or silicon nitride (Si_3N_4) passivation layer.⁽¹²⁾ Commercial ellipsometers are not designed for in situ oxide measurements on the silicon wafer. By this, it is meant that the oxide thickness cannot be monitored during etching.

Most ellipsometers manufactured within the last ten years contain a low power helium neon laser which produces a measuring light beam wavelength of $\lambda = 6328$ Å. For this wavelength, films with a thickness greater than the first complete cycle (approximately 2,000 Å) need to have the thickness range approximated by some other method, such as color interference or process time before ellipsometry can give the exact result. Measurements at two different angles of incidence is often successful in establishing the thickness range so that the exact thickness may be determined ellipsometrically. It is therefore desirable to modify ellipsometry in such a fashion to allow

for monitoring of the dielectric layer thickness during the etching process.

3.3 Modification of Ellipsometry for Continuous Film Thickness Monitoring

Consider the plane formed by a beam of plane polarized light and the specularly reflected light in the arrangement shown in Figure 4. The light incident on the sample is modulated such that its electric field vector is alternately transverse (TE) to this plane or parallel to this plane (TM, because the magnetic field vector is transverse to the plane of incidence). If the initial intensities of the TE and TM configurations are equal, one finds that for the specularly reflected light the two waves differ in intensity as determined by the equations of Chapter II. For small oxide thickness, the TE wave will be reflected with greater intensity than the TM wave. These intensities, normalized with respect to the incident light, are dependent upon the following parameters; the angle of incidence, the wavelength of the illuminating light, the optical constants of the media (air, SiO_2 and silicon), and the thickness of the film. If the optical constants are known, the angle of incidence and the wavelength of light may be fixed, thus allowing the thickness of the film to be calculated from theory. Alternately, we may determine the optical constants of silicon to a good approximation if the thickness and refractive index of the film are known.

If we now consider the case where the oxide thickness changes as a

Figure 4. Alternately TE and TM polarized light incident and reflected from an oxide coated surface. With equal polarization intensities in incidence, the reflected TE intensity is higher than the reflected TM intensity for thin films.

function of time, we note that the intensities of the reflected TE and TM waves will change accordingly. These changes in intensities can be directly related to the change in film thickness. By monitoring the intensities of the TE and TM waves we can continuously measure the thickness of the SiO_2 layer.

In order to monitor the silicon oxide layer on silicon wafers during etching, it is necessary to design a device which possesses the following capabilities:

- a) Modulate the polarization such that the incident light on the sample is alternately TE or TM.
- b) Measure the intensities of the specularly reflected TE and TM light.
- c) Electronically process the information provided by the reflected light in such a manner that it will provide real time data on the oxide thickness.

The action of the device is as follows: Light incident from a collimated light source passes through a polarization modulator which alternately switches the polarization from TE to TM as defined by the plane of incidence. The reflected light from the sample is then passed through a collimator allowing only specularly reflected light to enter the detector. The signal from the detector then goes to the signal processing electronics. In the remainder of this chapter, we discuss each of these components in detail.

3.3.1 Collimated Light Source

From our discussion of the interference effects in section 2.2 we found that the reflectance intensities of the TE and TM waves are wavelength-dependent (see equations 2.2.2, and 2.2.4). Theory predicts that light waves of high frequency (ultraviolet) can yield better information on the film thickness. Ultraviolet light, while being the optimum wavelength, has an adverse effect on certain materials used in the etching chamber. For example, lexan, which is a material commonly used in the windows of an HF gas etching chamber, will polymerize when exposed to U.V. radiation, causing severe degradation of the material. To circumvent this problem, we must use light of longer wavelength.

We chose as a light source an air-cooled argon laser (Spectra-Physics Model 162). This particular laser emits plane polarized light with a power output of 10 mW at 4880 Å or 5 mW at 5145 Å. The desired wavelength can be selected by replacing the rear optics with a prism and a flat broadband high reflector mirror. The prism serves as a wavelength selector by bending the various spectral lines through different angles. An alternate method of wavelength selection is to place a narrow bandpass interference filter in the path of the laser beam which will transmit the desired wavelength of light. The Spectra-Physics Model 162 argon laser has a beam diameter at $1/e^2$ points of 0.65 mm with a beam divergence of 1.0 milliradians at full angle. The polarization is better than 100 to 1 with the E vector vertical.

Since the laser is air-cooled and its power supply (Model 262) operates from 10 amps at 117 VAC $\pm$ 10%, 50 or 60 Hz, no special

electrical or cooling systems are required.

3.3.2 Polarization Modulation: Methods of TE-TM Polarization Switching

The output of the argon laser used in our experiments is vertically polarized. If the plane of the wafer is vertical, the electric field vector is parallel to the plane of incidence and is hence transverse electric. To alternately switch from TE to TM, keeping the plane of the wafer constant, we first employed a transverse field electro-optic light modulator (Laser Metrics AF3). The effect of an EOM on polarized light is similar to the effect obtained with optical retarders such as $\frac{1}{4}$ and $\frac{1}{2}$ wave retardation plates. A retardation plate operating with light of a single wavelength, introduces a fixed phase shift between the ordinary and extraordinary light waves passing through the plate. The electric field vector of the wave undergoes a corresponding change, that is a rotation fixed by the thickness of the plate and the birefringence of the material. A more detailed explanation of the half-wave plate will be presented later. Electro-optic modulators operate on light in a similar manner except that the value of birefringence is controlled electronically to produce a desired retardation.

Due to the temperature sensitivity of the modulator's electronics, the EOM proved to be unsuitable for our purposes. We, therefore, switched to a rotating half-wave plate to alternate between TE and TM polarizations. Sufficiently stable rotation speeds were attained by use of a small 12 V.D.C. motor (Figure 5). There are no sources of

Figure 5. Schematic of the rotating half-wave plate assembly.

instability since the polarization switching is a result of the optical anisotropy of the material of which the retarder is constructed.

The action of the retarder may be easily understood from the following. Consider a material (e.g., crystalline quartz) which has two perpendicular optic axes such that light with polarization (E vector) directed along one of them is transmitted with greater speed than light with polarization along the other. If plane polarized light incident on a plate of such a material (perpendicular to the surface) has its E vector along one of the material's optic axes then the emerging light will have unchanged polarization. Otherwise, the emerging light will generally be elliptically polarized. If the half wave plate is positioned such that the electric field vector of the incident light is oriented at 45° with respect to the optical axis of the half-wave plate the input beam is resolved into two overlapping beams with orthogonal polarizations, one being parallel to, and one orthogonal to the optical axis of the retarder with the sum of these beams equaling the original. The retarder serves to delay one of these beams with respect to the other. The phase of one of the electric field components is delayed by 180° , with respect to the other, thus giving rise to a path difference of a half a wavelength. The sum of the two emergent beams is then a wave with the polarization rotated by 90° . Clearly, if the input is considered to be a wave with TE polarization then the output has TM polarization.

A half-wave retarder placed in linearly polarized light and orientated such that the active axis makes an angle θ with respect to

the plane of polarization, rotates this plane by 2θ . Hence, if $\theta = 45^\circ$ then the plane is rotated by 90° , i.e., TE $\rightarrow$ TM, that is for every 45° rotation, the polarization direction switches. At angles intermediate angles the emerging polarization is elliptical. Since the greatest reflectivity of the wafer occurs for TE and the lowest for TM, at small oxide thickness, the rotating half-wave plate switches periodically from the highest to the lowest reflectivity, and this difference is a measure of the SiO_2 film thickness.

3.3.3 Detection Electronics

The detector initially used was a Spectra radiometer (Model 301) with a silicon photovoltaic cell. This solid state detector has good spectral response at $\lambda = 4880 \text{ \AA}$ and exhibits stability over long periods of time. With this instrument light intensities as low as 0.01 microwatts are accurately measured and the device allows for the elimination of ambient light from the signal.

Initially, data points were taken by manually rotating the half-wave plate to produce either TE or TM polarizations and reflected intensities were measured and normalized with respect to the incident beam. To collect data points in this manner is a tedious affair and does not take into account fluctuations in intensity of the incident laser light. To compensate for these fluctuations of incident light it is desirable to perform an A.C. ratio measurement which will increase the sensitivity of the oxide thickness measurement.

Though the radiometer initially used had good sensitivity, its

response time was too slow to be used for A.C. detection. We therefore switched to a silicon photodiode. Silicon photodiodes are fabricated especially to allow light to enter the PN junction. We require that the output voltage of the detector be a linear function of the intensity of the light entering the detector. To accomplish this, consider circuit A of Figure 6. When R is equal to zero, the photodiode operates along the negative I axis. This is known as the short circuit current which is linear with increasing light. No voltage is sensed in this linear photovoltaic mode. If, however, the diode is connected to an op amp, as is shown by circuit B, the linear short circuit current is transformed into a voltage. The op amp in this configuration is known as a transimpedance amplifier and the gain has the dimensions of ohms since it transforms an input current to an output voltage. The output voltage is given by

$$V_o = i_p R_f \quad (3.3.4)$$

where i_p is the photocurrent generated and R_f is the op amp feedback resistor.

3.3.4 Triggering Electronics for the Lock-in Amplifier

The A.C. signal from the detector was processed with a Princeton Applied Research Model 5101 lock-in amplifier. For the lock-in amplifier to work, a reference signal which provides information on the position of the rotating half-wave plate must be supplied it. The

CIRCUIT A

CIRCUIT B

Figure 6. Detection electronics

triggering signal was provided by an optoelectronic circuit (Figure 7). The diode emission was incident upon the circumference of the cylinder containing the half-wave plate. One eighth sections of the circumference were alternately painted black to correspond to the change of TE $\rightarrow$ TM for every 45° rotation of the half-wave plate. Reflected light incident on the phototransistor base generated a current which transformed the transistor from a nonconducting to a conducting state. A current limiting resistor isolated the emitter from the ground potential. The emitter potential was then passed through a Schmitt trigger circuit to produce a square wave with little or no noise which was then used to trigger the lock-in amplifier. Small changes in frequency did not effect the measurements since the lock-in amplifier automatically tracks the reference signal.

3.3.5 A.C. Detection of the Normalized Reflected Intensity

Utilization of an A.C. detection technique will allow us to measure $\frac{I_{TE} - I_{TM}}{I_{TE} + I_{TM}}$, which is simply the normalized reflected intensities. To visualize this, consider a DC signal with intensity I which has superimposed upon it an AC signal of amplitude A (Figure 8). The use of a lock-in amplifier, which is a common instrument used for A.C. detection, yields the rms value of A , while the use of a DC measuring device such as a voltmeter will measure the value of I . I_{TE} is then given by $I + A$ and I_{TM} is given by $I - A$. Therefore,

Figure 7. Triggering circuitry

Figure 8. Sketch of an AC signal of amplitude A superimposed on a DC signal of intensity I .

$$\frac{I_{TE} - I_{TM}}{I_{TE} + I_{TM}} = \frac{(I + A) - (I - A)}{(I + A) + (I - A)} \quad (3.3.5.1)$$

hence

$$\frac{I_{TE} - I_{TM}}{I_{TE} + I_{TM}} = \frac{A}{I} \quad (3.3.5.2)$$

The ratio A/I is easily measured by taking the output of the lock-in amplifier and voltmeter then taking the ratio by means of a ratiometer (Hewlett Packard D.C. Differential Voltmeter/Ratiometer Model 3420A).

Figure 9 is a schematic of the final experimental configuration for endpoint detection and continuous monitoring of oxide thickness on silicon wafers.

Figure 9. Schematic of end-point detection system for monitoring oxide thickness on silicon wafers.

CHAPTER IV

EXPERIMENTAL RESULTS AND CONCLUSIONS

4.1 Test Wafers

In the last chapter we developed the operational characteristics necessary for measuring oxide thicknesses that change in time on silicon wafers. We have modified standard ellipsometric techniques to yield the information on the oxide thickness by sacrificing irrelevant information unnecessary for our current interests. To clarify, we have lost any experimental information about the phase relation of the incident and reflected light.

In the remainder of this chapter we shall discuss the results of laboratory experiments and their implications. Experimentally, it was necessary to make an analysis of the effect of wavelengths on the accuracy of the oxide measurements and to determine the most effective angle of incidence.

To determine the operating characteristics of the endpoint detection system discussed in the previous chapter it was necessary to have silicon wafers with known oxide thickness. Therefore, F.S.I., Inc., provided 40 unpatterned single crystal silicon wafers with oxide coatings ranging from 33 to 2423 Å. These wafers were circular, with a diameter of 10 cm and a thickness of 0.5 mm. A section of the circumference was flattened along a chord to show the crystal orientation.

The oxide thickness of each test wafer was measured at Texas Instruments using a commercial ellipsometer with an accuracy of ± 1 Å. By using samples with known oxide coating it was possible to obtain a direct correlation between the experimental and theoretical reflectance values for the TE and TM polarization.

Three patterned wafers of unknown dielectric thickness and composition were also provided by F.S.I., Inc., to determine the feasibility of continuous monitoring of patterned wafers during etching.

4.2 Optical Constants of Silicon Dioxide and Silicon

The optical properties of a dielectric can be completely characterized at a particular wavelength by a single parameter, namely its index of refraction. For a conductor another parameter (k) is required which is a measure of the absorption of light as it enters the conductor. For a dielectric the index of refraction n is easily determined from refraction measurements. For conductors this method of determining the optical constants is not always practical due to their strong absorption characteristics.

To perform the theoretical calculations of the reflectance intensity of the TE and TM wave it is necessary to know the optical constants for silicon oxide and silicon. For the wavelengths of interest ($\lambda = 4880$ Å and 6328 Å) the refractive index of SiO_2 is readily found from the literature to be $n_2 = 1.45$ and 1.46 ,

respectively⁽¹⁵⁾.

The complex refractive index of silicon is known for $\lambda = 6328 \text{ \AA}$ ($n_3 = 3.85$, $k_3 = 0.02$) from ellipsometric studies of oxide coated silicon. No values for n_3 and k_3 could be found for the argon laser ($\lambda = 4880 \text{ \AA}$). The A.I.P. Handbook⁽¹⁵⁾, however, gives values for n_3 and k_3 for wavelengths near 4880 $\text{\AA}$. The values of n_3 and k_3 change significantly for a small change in wavelength. From the measured intensities of the reflected TE and TM waves and with the known value of $n_2 = 1.45$ it was possible to curve fit theory to experiment by varying the values of n_3 and k_3 . This was done for a wide range of oxide thickness and angles of incidence in an effort to minimize any possible error. The experimental values obtained were $n_3 = 4.27$ and $k_3 = 0.253$. Comparison with the values given by reference 15 (which gives for $\lambda = 4750 \text{ \AA}$, $n_3 = 4.33$, $k_3 = 0.0254$ and for $\lambda = 5150 \text{ \AA}$, $n_3 = 4.16$ and $k_3 = 0.0240$) indicate these to be valid numbers.

4.3 Oxide Measurements on Unpatterned Wafers

Initially, data points on the unpatterned wafers were taken by manually rotating the half-wave plate to produce either TE or TM polarizations, then measuring the reflected intensities with a Spectra radiometer (see section 3.3.3). These intensities were then normalized with respect to the incident beam. Data were collected using the 4880 $\text{\AA}$ line of the argon laser and the 6328 $\text{\AA}$ line from a helium-neon laser incident on the samples at a variety of angles.

From the equations presented in Chapter II and derived in Appendix

It is clear that there are two major parameters which determine the magnitude of the normalized reflected intensity, namely the wavelength of the illuminating light and the angle of incidence. These two factors also determine the ultimate resolution of the oxide thickness which is determined by the slope of the curve generated by plotting the normalized reflected intensity versus the oxide thickness. From the data in collaboration with theory that the best sensitivity is obtained using the argon laser ($\lambda = 4880 \text{ \AA}$). The data collected at $\lambda = 4880 \text{ \AA}$ for several angles of incidence are shown graphically in Figure 10.

To determine the optimum or principal angle of incidence, which corresponds to the Brewster angle for reflection from dielectrics, we need to determine at what angle the TM intensity goes to a minimum value for a given thickness. Figure 11A is a plot of the reflectance intensity of the TE and TM waves as a function of the incident angle for an oxide thickness of $300 \text{ \AA}$ and Figure 11B shows the normalized reflected intensity vs. angle of incidence for various oxide coating thicknesses.

From these data plots, and in agreement with theoretical calculations, it was determined that the intensity of the reflected TM wave is a minimum at 75° . Figure 12 shows for $\lambda = 4880 \text{ \AA}$ the normalized reflectance as a function of the oxide thickness at the principal angle of incidence ($\theta = 75^\circ$).

To this point, our discussion has been limited to unpatterned wafers. We have found that the experimental data are in good agreement with theory. As was previously mentioned, the sensitivity

Figure 10. Normalized reflectance intensity as a function of the oxide thickness for various angles of incidence for $\lambda = 4880 \text{ Å}$.

Figure 11. (A) Reflectance intensity of TE and TM waves as a function of the angle of incidence and (B) Normalized reflectance intensity as a function of oxide thickness for oxide coatings.

Figure 12. Normalized reflectance intensity at the principal angle of incidence ($\theta = 75^\circ$) for $\lambda = 4880 \text{ Å}$.

of the device is determined by the slope of $(I_{TE} - I_{TM})/(I_{TE} + I_{TM})$. The greater the slope the better the sensitivity. For $\lambda = 4880 \text{ \AA}$ incident at an angle of 75° the error in the oxide thickness measurement is found to be approximately $\pm 0.4\%$ of the total oxide thickness. For a silicon wafer with an oxide thickness of, say, $800 \text{ \AA}$ the error in measurement would be approximately $\pm 3.2 \text{ \AA}$.

4.4 Oxide Measurements on Patterned Wafers

For the device we have developed to measure oxide thickness to be used in the production of integrated circuits, it must be extended to patterned wafers. The patterned wafers obtained from F.S.I., Inc., have a dielectric coating of unknown composition and thickness, making it impossible to compare experimental measurements with theoretical calculations.

Several experiments were conducted to determine the proper beam diameter of the incident beam. On plain unpatterned wafers the beam diameter was not critical provided the expanded beam did not diverge. For patterned wafers the reflected laser light formed complicated diffraction patterns which diverged quickly. It was found that reproducible data could be obtained with beam diameters ranging from 0.1 to 0.5 cm . Larger beam diameters resulted in data which were not reproducible. Since we measure the normalized reflected intensity, the diffracted light has no influence on data collection provided only the intensity of the specularly reflected light is measured.

4.5 Discussion

- This thesis has shown the feasibility of utilizing plane polarized light for performing on-line monitoring of oxide thickness during the etching process. The device described in this thesis is relatively inexpensive to construct and requires very little maintenance after the initial set-up and calibration. Since this device provides continuous information on the oxide thickness it is capable of becoming a valuable diagnostic aid in the analysis of etching rates and techniques.

At the time of writing the device has been on-line to monitor the oxide thickness of single crystal silicon wafers being etched with HF gas. Currently, only the relative intensities of the specularly reflected TE and TM waves are being measured. The quality of the signal is quite good since the zero points (where the TE and TM intensities are equal) show little or no drift.

The diagnostic value of the device became clear when it was found that an additional 800 Å of oxide was etched after the flow of HF was terminated.

LIST OF REFERENCES

1. E. Braun and S. MacDonald, Revolution in Miniature, Cambridge Univ. Press, Cambridge, 1982.
2. P. V. Zant, Microchip Fabrication, Semiconductor Services, San Jose, 1984.
3. M. Born and E. Wolf, Principles of Optics, 6th ed., Pergamon, N.Y., 1970.
4. O. S. Heavens, Optics of Thin Solid Films, Butterworths Scientific Publications, London, 1955.
5. M. D. Williams, Appl. Optics 21, 747 (1982).
6. D. K. Burge and H. E. Bennet, J. Opt. Soc. Am. 54, 1428 (1964).
7. R. J. Archer, J. Opt. Soc. Am. 52, 970 (1962).
8. F. Partovi, J. Opt. Soc. Am. 52, 918 (1962).
9. R. M. A. Azzam and N. M. Bashara, Ellipsometry and Polarized Light, 2nd. ed., North-Holland, N.Y., 1979.
10. A. B. Winterbottom, J. Opt. Soc. Am. 38, 1074 (1948).
11. L. Tronstad, and T. Hoverstad, Trans. Faraday Soc. 30, 349 (1934).
12. R. J. Kutko, Solid State Tech. 21, 43 (1978).
13. J. R. Reitz, F. J. Milford and R. W. Christy, Foundations of Electromagnetic Theory, Addison-Wesley, Reading, MA, 2nd ed., 1980.
14. F. A. Jenkins and H. E. White, Fundamentals of Optics, McGraw-Hill, N.Y., 4th ed., 1976.
15. D. E. Gray ed., American Institute of Physics Handbook, McGraw-Hill, N.Y., 3rd ed., 1972.

APPENDIX I

REFLECTION OF PLANE POLARIZED LIGHT FROM OXIDE COATED SILICON - DERIVATION OF REFLECTANCE EQUATION

The theoretical model assumed in this paper consists of a plane, semi-infinite substrate covered by a thin nonabsorbing film and surrounded by air. All interfaces are considered to be perfectly smooth and the optical constants of each medium are assumed to have the same value throughout the film or substrate, with discontinuities at each interface.

The geometry used in the discussion of oxide-coated silicon was shown in Figure 1. The x-axis is directed vertically along the first interface with the z-axis passing through the media. We may resolve the field vectors E and B into their x-components in the plane of incidence and y-components perpendicular to the plane of incidence.

The first medium (air or etchant) will be characterized by a propagation constant $\kappa_1 = 2\pi n_1/\lambda$; the second medium by $\kappa_2 = 2\pi n_2/\lambda$; and the third medium by $\kappa_3 = 2\pi n_3(1 + ik_3)/\lambda$. Let a plane polarized electromagnetic wave from medium 1 be incident upon the left side of the second medium, and let the wave form and angle θ_1 with the normal to the x-axis. At the first interface, the expressions for the electric fields of the incident, reflected and transmitted waves are, respectively

$$E_1 = E_1 e^{i(\kappa_1 \cdot r - \omega t)} \quad (I.1)$$

$$E'_1 = E_1 e^{i(\kappa'_1 \cdot r - \omega t)} \quad (I.2)$$

$$E_2 = E_2 e^{i(\kappa_2 \cdot r - \omega t)}. \quad (I.3)$$

For oblique incidence, the reflected and transmitted waves must have the same frequency as the incident wave, with the phases matching everywhere on the boundary,

$$\kappa_1 \cdot r = \kappa'_1 \cdot r = \kappa_2 \cdot r. \quad (I.4)$$

We now derive the expressions which will give the ratios of the field amplitudes which are called the Fresnel coefficients. To do this, we must examine the boundary conditions of the field vectors at each interface. The boundary condition which must be satisfied is that the tangential components of E and B must be continuous across all boundaries. For the tangential component of the electric field vector, we need the following vector identity:

$$n \times (n \times E) = (n \cdot E) n - E \quad (I.5)$$

or, upon rearranging and solving for E

$$E = (n \cdot E) n - n \times (n \times E). \quad (I.6)$$

where $\mathbf{n}$ is the unit normal vector at the boundary. Noting that $(\mathbf{n} \cdot \mathbf{E})\mathbf{n}$ is the normal component of $\mathbf{E}$, the remaining terms in (2.1.6) must be the tangential component for any vector $\mathbf{E}$. The boundary condition on the electric field vector is then

$$\mathbf{n} \times (\mathbf{E}_1 + \mathbf{E}'_1) = \mathbf{n} \times \mathbf{E}_2. \quad (1.7)$$

The continuity of the tangential component of the magnetic field vector $\mathbf{B}$ can be expressed in a similar form, that is,

$$\mathbf{n} \times (\mathbf{B}_1 + \mathbf{B}'_1) = \mathbf{n} \times \mathbf{B}_2. \quad (1.8)$$

From Maxwell's equations, the electric and magnetic fields are related to each other by

$$\mathbf{B} = \frac{n}{c} \mathbf{u} \times \mathbf{E} \quad (1.9)$$

or, solving for $\mathbf{E}$,

$$\mathbf{E} = -\frac{c}{n} \mathbf{u} \times \mathbf{B}, \quad (1.10)$$

where $\mathbf{\kappa} = \kappa \mathbf{u}$ with the direction of $\mathbf{u}$ being completely arbitrary. The magnitude of κ for a given frequency ω is determined by the refractive index of the material by

$$|\kappa_1| = n_1 \frac{\omega}{c} = \frac{2\pi n_1}{\lambda} \quad (1.11)$$

Substitution of equation (I.9) into (I.8) yields, for the first interface

$$n_1 \mathbf{n} \times (\mathbf{u}_1 \times \mathbf{E}_1 + \mathbf{u}_1 \times \mathbf{E}_1') = n_2 \mathbf{n} \times (\mathbf{u}_2 \times \mathbf{E}_2). \quad (\text{I.12})$$

Solving (I.7) and (I.12) simultaneously for $\mathbf{E}_1$ and $\mathbf{E}_2$ in terms of $\mathbf{E}_1$, we find

$$\mathbf{n} \times (\mathbf{u}_1 \times \mathbf{E}_1) = (\mathbf{n} \cdot \mathbf{E}) \mathbf{u}_1 - (\mathbf{n} \cdot \mathbf{u}_1) \mathbf{E}_1 \quad (\text{I.13})$$

By noting that $\mathbf{n} \cdot \mathbf{u}_1 = \cos \theta_1$ and that for a TE wave $\mathbf{n} \cdot \mathbf{E}_1 = 0$, (I.13) becomes

$$n_1 (\cos \theta_1 \mathbf{E}_1 - \cos \theta_1 \mathbf{E}_1') = n_2 \cos \theta_2 \mathbf{E}_2 \quad (\text{I.14})$$

and since the angle of incidence is equal to the angle of reflection, we have, $\theta_1 = \theta_1'$. Therefore (I.14) reduces to

$$n_1 \cos \theta_1 (\mathbf{E}_1 - \mathbf{E}_1') = n_2 \cos \theta_2 \mathbf{E}_2. \quad (\text{I.15})$$

Finally, after performing the cross product of (I.7) for the TE case, we have

$$\mathbf{E}_1 + \mathbf{E}_1' = \mathbf{E}_2, \quad (\text{I.16})$$

No such simplification occurs for the TM component. Therefore, we must consider the cases for the TE and TM waves separately to develop the Fresnel coefficients at the first interface.

Electric Vector Perpendicular to the Plane of Incidence: TE Waves

To find the ratios of the reflected and transmitted electric fields at the first interface, we solve equations (I.13) and (I.14) simultaneously. Upon doing so we find,

$$r_{12} = \frac{|E'_1|}{|E_1|} = \frac{n_1 \cos \theta_1 - n_2 \cos \theta_2}{n_1 \cos \theta_1 + n_2 \cos \theta_2} \quad (I.17a)$$

and

$$t_{12} = \frac{|E_2|}{|E_1|} = \frac{2n_1 \cos \theta_1}{n_1 \cos \theta_1 + n_2 \cos \theta_2} \quad (I.17b)$$

Electric Vector Parallel to the Plane of Incidence: TM Wave

For this case, we solve equations (I.10) and (I.17) simultaneously and note that $\mathbf{n} \cdot \mathbf{B}_1 = \mathbf{n} \cdot \mathbf{B}_2 = \mathbf{n} \cdot \mathbf{B}'_2 = 0$. Hence, we find

$$\frac{1}{n_1} \cos \theta_1 (B_1 - B_1') = \frac{1}{n_2} \cos \theta_2 B_2 \quad (I.18)$$

Substitution of this equation into (I.18) and performing the indicated cross products, equation (I.18) reduces to

$$B_1 + B_1' = B_2. \quad (I.19)$$

The Fresnel coefficient for the magnetic field vector B may be obtained by simultaneously solving (I.18) and (I.19). Doing this we find

$$r_{12} = \frac{|B_1'|}{|B_1|} = \frac{n_2 \cos \theta_1 - n_1 \cos \theta_2}{n_2 \cos \theta_1 + n_1 \cos \theta_2} \quad (I.20)$$

$$t_{12} = \frac{n_1}{n_2} \frac{|B_2|}{|B_1|} = \frac{2n_1 \cos \theta_1}{n_2 \cos \theta_1 + n_1 \cos \theta_2} \quad (I.21)$$

With the Fresnel coefficients developed for both the TE and TM cases, we now have the complete solutions for the boundary value problem at the first interface. We note that by definition θ_1 is zero for the normal incidence and hence by Snell's law θ_2 is also zero. Equations (I.17a) and (I.20a) reduce to equivalent forms for normal incidence provided that $r_{12}^{TE} \equiv -r_{12}^{TM}$.

The Boundary Value Problem at the Second Interface

To solve the boundary value problem at the second interface, we introduce Snell's law in a modified form, which takes into account the complex index of refraction of the third medium. Thus, since for the third medium we have $\hat{n}_3 = n_3(1 + ik_3)$, we may write Snell's law as

$$n_2 \sin \theta_2 = \hat{n}_3 \sin \theta_3. \quad (I.22)$$

For later convenience, we define

$$\hat{n}_3 \cos \theta_3 \equiv u_3 + iv_3 \quad (I.23)$$

where u_3 and v_3 are real. To express u_3 and v_3 in terms of the optical constants one of the second and third media, we square equation (I.23) and with the use of (I.22) we find

$$\begin{aligned} (u_3 + iv_3)^2 &= \hat{n}_3^2 \cos^2 \theta_3 \\ &= \hat{n}_3^2 (1 - \sin^2 \theta_3) \\ &= \hat{n}_3^2 \left[1 - \left(\frac{n_2}{\hat{n}_3} \right)^2 \sin^2 \theta_2 \right]. \end{aligned} \quad (I.24)$$

Upon expanding the left side of this equation we are left with

$$u_3^2 + 2iu_3v_3 + v_3^2 = \hat{n}_3^2 - n_2^2 \sin^2 \theta_2. \quad (I.25)$$

Equating the real and imaginary components of (I.25), we have

$$u_3^2 + v_3^2 = n_3^2 - n_2^2 \sin^2 \theta_2 \quad (I.26)$$

$$u_3 v_3 = n_3^2 k_3. \quad (I.27)$$

Solving (I.26) and (I.27) simultaneously for u_3 and v_3

$$2u_3^2 = n_3^2(1 - k_3^2) - n_2^2 \sin^2 \theta_2 + \left[\left[n_3^2(1 - k_3^2) - n_2^2 \sin^2 \theta_2 \right]^2 + 4n_2^4 k_3^2 \right]^{1/2} \quad (I.28)$$

and

$$2v_3^2 = - \left[n_3^2(1 - k_3^2) - n_2^2 \sin^2 \theta_2 \right] + \left[\left[n_3^2(1 - k_3^2) - n_2^2 \sin^2 \theta_2 \right]^2 + 4n_2^4 k_3^2 \right]^{1/2}. \quad (I.29)$$

We next evaluate the reflection coefficients for the 2-3 interface. As before, we consider separately the cases when the electric field vector of the incident wave is perpendicular or parallel to the plane of incidence.

Transverse Electric Field:
The 2-3 Interface

For the TE case, we may use equation (I.17a) and with modifications apply it to the second interface. Replacing n_1, θ_1 by n_2, θ_2 and $n_2 \cos \theta_2$ by $u_3 + iv_3$ we have

$$r_{23} = \rho_{23} e^{i\phi_{23}} = \frac{n_2 \cos \theta_2 - (u_3 + iv_3)}{n_2 \cos \theta_2 + (u_3 + iv_3)}, \quad (I.30)$$

where $\rho_{23} e^{i\phi_{23}}$ is the normalized amplitude of the reflected TE wave. Equating the real and imaginary parts of (I.30), we immediately find explicit expressions for the amplitude ρ_{23} and the phase change ϕ_{23} . We find

$$\rho_{23}^2 = \frac{(n_2 \cos \theta_2 - u_3)^2 + v_3^2}{(n_2 \cos \theta_2 + u_3)^2 + v_3^2} \quad (I.31)$$

and

$$\tan \phi_{23} = \frac{2v_3 n_2 \cos \theta_2}{u_3^2 + v_3^2 - n_2^2 \cos^2 \theta_2} \quad (I.32)$$

Transverse Magnetic Field: The 2-3 Interface

As was previously mentioned, the formula pertaining to the reflection and transmission coefficients for the TM wave may be deduced from the equations developed for the TE wave provided the appropriate substitutions are made.

The quantity r_{23} now refers to the ratios of the magnetic field vector and not the electric field vector. In particular, making the substitution $\frac{1}{n_j} \cos \theta_j$ for $n_j \cos \theta_j$ in equation (I.30), we find for the TM case at the 2-3 interface

$$r_{23} = \rho_{23} e^{i\phi_{23}} = \frac{\hat{n}_3^2 \cos \theta_2 - n_2 \hat{n}_3 \cos \theta_2}{\hat{n}_3^2 \cos \theta_2 + n_2 \hat{n}_3 \cos \theta_2} \quad (I.33)$$

Substitution of $\hat{n}_3 \cos \theta_2 = u_3 + iv_3$ into equation (I.33), we find

$$r_{23} = \rho_{23} e^{i\phi_{23}} = \frac{[n_3^2(1-k_3)^2 + 2in_3^2k_3] \cos \theta_2 - n_2(u_3 + iv_3)}{[n_3^2(1-k_3)^2 + 2in_3^2k_3] \cos \theta_2 + n_2(u_3 + iv_3)} \quad (I.34)$$

Once again, if we equate the real and imaginary components of (I.34) and solving for the normalized amplitude ρ_{23} and the phase change ϕ_{23} , we obtain

$$\tan \phi_{23} = 2n_1 n_2^2 \cos \theta_2 \frac{2k_3 u_3 - (1 - k_3^2) v_3}{n_3^4 (1 + k_3^2)^2 \cos^2 \theta_2 - n_2^2 (u_3^2 + v_3^2)} \quad (1.35)$$

and

$$\rho_{23}^2 = \frac{[n_3^2 (1 - k_3^2) \cos \theta_2 - n_2 u_3]^2 + [2n_3^2 k_3 \cos \theta_2 - n_3 v_3]^2}{[n_3^2 (1 - k_3^2) \cos \theta_2 + n_2 u_3]^2 + [2n_3^2 k_3 \cos \theta_2 + n_3 v_3]^2} \quad (1.36)$$

When the conductor and dielectric are combined, there is a small component of light which is reflected back and forth many times. To find the total reflectance amplitude we need to add up the contribution of each of these partial waves to the net wave reflected from the first interface. To do this we utilize the Fresnel coefficients for each boundary together with the phase shift δ . The Fresnel coefficients are different for the TE and TM polarizations, but we will do a general treatment of the problem remembering that the two polarizations must be treated independently. We have then

$$r = r_{12} + t_{12} \rho_{23} t_{21} e^{i(\phi_{23} + 2\delta)} + t_{12} \rho_{23} r_{12} \rho_{23} t_{21} e^{i(\phi_{23} + 2\delta)} + \dots \quad (1.37)$$

$$r = r_{12} + t_{12} \rho_{23} t_{21} e^{i(\phi_{23} + 2\beta)} [1 + r_{12} \rho_{23} e^{i(\phi_{23} + 2\beta)} + (r_{12} \rho_{23} e^{i(\phi_{23} + 2\beta)})^2 + \dots] \quad (I.38)$$

Since $1 + z + z^2 + \dots = \frac{1}{1-z}$, (I.39)

then (I.40)

$$r = r_{12} + \frac{t_{12} t_{21} \rho_{23} e^{i(\phi_{23} + 2\beta)}}{1 - r_{12} \rho_{23} e^{i(\phi_{23} + 2\beta)}} .$$

Recalling $r_{12} = -r_{21}$ (I.41)

and noting that $r_{12} + t_{21} t_{12} = 1$ (I.42)

we get

$$r = \frac{r_{12} + \rho_{23} e^{i(\phi_{23} + 2\beta)}}{1 + r_{12} \rho_{23} e^{i(\phi_{23} + 2\beta)}} , \quad (I.43)$$

where β is given by

$$\beta = \frac{2\pi}{\lambda} n_2 h \cos \theta_2 . \quad (I.44)$$

h is the thickness of the oxide (medium 2) and λ is the wavelength of light. The total reflectance amplitude is given by $\text{Re}^{i\delta r}$, where

$$R = \left[\frac{r_{12}^2 + \rho_{23}^2 + 2r_{12}\rho_{23}\cos(\phi_{23} + 2\beta)}{1 + r_{12}^2 + \rho_{23}^2 + 2r_{12}\rho_{23}\cos(\phi_{23} + 2\beta)} \right]^{\frac{1}{2}} \quad (I.45)$$

$$\tan \delta r = \frac{\rho_{23}(1 - r_{12}^2)\sin(\phi_{23} + 2\beta)}{r_{12}(1 + \rho_{23}^2) + \rho_{23}(1 + r_{12}^2)\cos(\phi_{23} + 2\beta)} \quad (I.46)$$

and

$$\beta = \frac{2\pi n_2 h \cos \theta_2}{\lambda} \quad (I.47)$$

APPENDIX II

INTERFERENCE INVOLVING MULTIPLE REFLECTIONS IN THIN FILMS

To determine the optical thickness of the thin film for which the reflection coefficient has a maximum or minimum, we shall investigate the interference produced by a non-absorbing film of thickness h .⁽¹³⁾ We consider a beam of plane polarized light incident to the film at some point A from a source S, with the incident beam forming an angle θ_1 with the normal to the first interface (as shown in Figure 13). This wave will be partially reflected and refracted at point A. The reflected wave will travel along path 1 while the refracted wave travels along the line segment AD. When the refracted wave reaches D it will be reflected along DB. After arriving at B, the wave is partially refracted along path 2 while the rest of the wave is reflected along a path defined by the line segment BE. This process of reflection and refraction continues. The amplitude of each of the waves emerging from the thin film decreases exponentially from the preceding wave. If we now collect each of the waves with a lens and focus them at point P, we find that each of these partial waves has traveled different optical path lengths. The phase relations of each of the waves will contribute to either constructive or destructive interference as viewed at point P. The path length d , traveled by a ray of light in any medium is given by the product of velocity with time: $d = vt$. By definition, we know the index of refraction, n , of

Figure 13. Multiple reflections in a plane parallel dielectric film of thickness h .

a medium is given by $n = c/v$. Solving for v and substituting into the expression for the path length we find

$$d = (c/n) \text{ or } nd = dt. \quad (\text{II.1})$$

The product nd is defined as the optical path Δ :

$$\Delta = nd. \quad (\text{II.2})$$

The optical path clearly represents the distance that light travels in a vacuum in the same time it travels a distance Δ in a medium of refractive index n . Therefore, if the light wave travels through a series of optical media of a given thickness $d, d', d'',$ and refractive indices $n, n', n'', \dots$, the total optical path is given by the sum of the separate values:

$$\Delta = nd + n'd' = n''d'' + \dots. \quad (\text{II.3})$$

To calculate the phase difference of the reflected waves which are collected by the lens and focused at P we first calculate the optical path traveled by any two successive waves, where h denotes the thickness of the film, n_2 its index of refraction, λ the wavelength of light, and θ_1 and θ_2 the angles of incidence and refraction (Figure 14). We may construct the line segment BD so that it is perpendicular to path 1. Therefore, the optical paths of waves 1 and 2 from D to B are equal. Beginning at point A, the wave traveling along the second path travels along AFB in the film and the wave

Figure 14. Optical path difference between two consecutive rays from a thin film giving multiple reflections.

traveling along path 1 has the path AD in air. The difference in the optical paths traveled by 1 and 2 is given by

$$\Delta = n_2(\text{AFB}) - \text{AD} . \quad (\text{II.4})$$

If we extend the line segment BF to intersect the perpendicular line AE at G, then the line segments AF and GF are equal since the angles of incidence and reflection are equal at the lower boundary of the film. We may now write the optical path differences as:

$$\Delta = n_2(\text{GB}) - \text{AD} = n_2(\text{GC} + \text{CB}) - \text{AD} . \quad (\text{II.5})$$

Constructing AC perpendicular to FB, the line segments AC and DB represent the position of two successive wave fronts which have undergone a reflection from the lower surface of the film. The optical paths of any two successive wave fronts in the film must be equal so that

$$n_2(\text{CB}) = \text{AD} . \quad (\text{II.6})$$

The optical path difference Δ , reduces to

$$\Delta = n_2(\text{GC} + \text{CB}) - n_2\text{CB}$$

$$\Delta = n_2\text{GC} = n_2(2d\cos \theta_2) . \quad (\text{II.7})$$

The wave that is reflected from the upper surface and takes path 1 experience a phase change of π radians upon reflection, while the wave traveling along path 2 remains unchanged since it is internally reflected. We, therefore, find a minimum in the reflected intensity, at point P, when the optical path difference of any two successive waves is an integer. The condition for destructive interference is then

$$m \lambda = 2n_2 h \cos \theta_2, \text{ where } m = 0, 1, 2, \dots \quad (\text{II.8})$$

To calculate the path difference which will result in constructive interference we begin by noting that the path difference between any successive wave fronts, for example 3 and 2, will be identical to equation (II.8). These waves also have the same phase since only internal reflections are involved. Under these conditions, waves 1 and 2 must be out of phase but waves 2, 3, 4, ..., will be in phase. If conditions are such that

$$(m + \frac{1}{2}) \lambda = 2n_2 h \cos \theta_2, \quad m = 0, 1, 2, \dots, \quad (\text{II.9})$$

then wave 2 is in phase with 1 but waves 3, 5, 7, ..., are out of phase with 2, 4, 6, ..., . However, since wave 2 has greater intensity than 3, and 4 is more intense than 5, etc., these pairs do not add destructively but rather will constructively add with 1 to produce an intensity maximum at point P. To summarize, if the film thickness h is

$$h = \frac{m \lambda}{2n_2 \cos \theta_2} \quad , \quad m = 0, 1, 2, \dots, \quad (\text{II.10})$$

then the reflected light intensity is a minimum. If, on the other hand,

$$h = \frac{(2m + 1)\lambda}{4n_2 \cos \theta_2} \quad , \quad m = 0, 1, 2, \dots, \quad (\text{II.11})$$

then constructive interference occurs and a maximum in the reflected intensity results.

APPENDIX III

COMPUTER PROGRAMS

Reflectance Intensity Calculations

```

10 DIM RE(2,2),RM(2,2),R1(3,3),R2(3,3),R3(1000)
20 DIM R4(1000),P1(3,3),P2(3,3),B(1000),G(1000),F(1000)
25 DIM CN(1000)
30 REM RE = R12 OF TE & RM = R12 OF TM
40 REM R1 = RHO-23 OF TE & R2 = RHO-23 OF TM
50 REM R3 = R^2 OF TE & R4 = R^2 OF TM
60 REM P1 = PHI-23 OF TE & P2 = PHI-23 OF TM
70 PI = 3.1415926
80 D$ = CHR$(4)
90 HOME : PRINT D$;"PR#3"
100 INPUT "THETA 1 (IN RADIANS) ";T1
110 N1 = 1.000:N2 = 1.45:N3 = 4.25
120 INPUT "MAX. OXIDE THICKNESS (ANGSTROMS) ";HMAX
130 LAMBDA = 4880
140 K3 = .05
150 REM CALC. THETA2
160 REM T2 = COS (THETA2) & T3 = SIN (THETA2)
170 T3 = (N1 / N2) * SIN (T1)
180 T2 = SQR (1 - (T3 * T3))
185 PRINT T2
200 RE(1,2) = N1 * COS (T1) - N2 * T2
201 RE(1,2) = - RE(1,2) / (N1 * COS (T1) + N2 * T2)
202 RM(1,2) = N2 * COS (T1) - N1 * T2
203 RM(1,2) = RM(1,2) / (N2 * COS (T1) + N1 * T2)
210 RR = (N3 * N3) - (N3 * N3 * K3 * K3)
220 IC = 2 * N3 * N3 * K3
230 UU = RR - (N2 * N2 * T3 * T3)
240 VV = SQR (((RR - (N2 * N2 * T3 * T3)) ^ 2) + IC * IC)
250 U3 = SQR (.5 * (UU + VV))
260 V3 = SQR (.5 * (-UU + VV))
270 REM FOR TE WAVE & EN = NUMERATOR, ED = DENOMINAT
OR
280 EN = (N2 * T2 - U3) ^ 2 + V3 * V3
290 ED = (N2 * T2 + U3) ^ 2 + V3 * V3
300 R1(2,3) = SQR (EN / ED)
310 P1(2,3) = ATN ((2 * V3 * N2 * T2) / (U3 * U3 + V3 *
* V3 - (N2 * N2 * T2 * T2)))
320 REM DO TM CALC
330 MN = (RR * T2 - N2 * U3) ^ 2 + (IC * T2 - N2 * V3)
^ 2
340 MD = (RR * T2 + N2 * U3) ^ 2 + (IC * T2 + N2 * V3)
^ 2
350 R2(2,3) = SQR (MN / MD)
360 P2(2,3) = ATN ((2 * N2 * T2 * (IC * U3 - RR * V3))
/ (RR * RR * T2 * T2 + IC * IC * T2 * T2 - (N2 * N2 * (U3 * U3 + V3 *
V3))))
370 REM NOW READY TO CALCULATE BETA & R-SQUARED
380 N = INT (HMAX / 50)
385 PRINT "N = ";N
390 REM N = TOTAL # OF POINTS
400 FOR I = 0 TO N

```

```

410 REM ALL_M = TM & ALL_E = TE
420 B(I) = ((2 * PI * N2 * T2) / LAMBDA) * (I * 50)
430 REM GOES TO TE AND F'S GO TO TM
440 G(I) = 2 * RE(1,2) * R1(2,3) * COS (P1(2,3) + 2 *
B(I))
450 F(I) = 2 * RM(1,2) * R2(2,3) * COS (P2(2,3) + 2 *
B(I))
460 R3(I) = (((RE(1,2) * RE(1,2)) + (R1(2,3) * R1(2,3))
+ G(I)) / (1 + (
(RE(1,2) * R1(2,3)) ^ 2) + G(I)))
470 R4(I) = (((RM(1,2) * RM(1,2)) + (R2(2,3) * R2(2,3))
+ F(I)) / (1 + (
(RM(1,2) * R2(2,3)) ^ 2) + F(I)))
480 CN(I) = (R3(I) - R4(I)) / (R3(I) + R4(I))
490 NEXT I
500 FOR I = 0 TO N
510 PRINT I * 50, R3(I), R4(I), CN(I)
520 NEXT I
530 REM STORE TO DISK
540 INPUT "Name of disk file ( type NONE for no disk s
ave) "; N$
545 IF N$ = "NONE" THEN 850
550 PRINT D$; "OPEN"; N$; ".ITE"
560 PRINT D$; "WRITE"; N$; ".ITE"
570 PRINT N + 1; REM NUMBER OF POINTS
580 FOR I = 0 TO N
590 REM STORE H & ITE (RE)
700 PRINT (I * 50); NEXT
710 FOR I = 0 TO N: PRINT R3(I) * 100; NEXT
720 PRINT D$; "CLOSE"; N$; ".ITE"
730 PRINT D$; "OPEN"; N$; ".ITM"
740 PRINT D$; "WRITE"; N$; ".ITM"
750 PRINT N + 1
760 FOR I = 0 TO N: PRINT I * 50; NEXT
770 FOR I = 0 TO N: PRINT R4(I) * 100; NEXT
780 PRINT D$; "CLOSE"; N$; ".ITM"
790 PRINT D$; "OPEN"; N$; ".CON"
800 PRINT D$; "WRITE"; N$; ".CON"
810 PRINT N + 1
820 FOR I = 0 TO N: PRINT I * 50; NEXT
830 FOR I = 0 TO N: PRINT CN(I) * 100; NEXT
840 PRINT D$; "CLOSE"; N$; ".CON"
850 PRINT : PRINT "HARDCOPY? Y OR N"
860 GET A$: IF A$ = "" THEN 860
870 IF A$ < > "N" AND A$ < > "Y" THEN 860
880 IF A$ = "N" THEN END
885 PRINT CHR$ (4); "PR#3"
890 PRINT D$; "PR#7"
900 PRINT "H ITE ITM CON"
910 FOR I = 0 TO N
920 PRINT I * 50, R3(I), R4(I), CN(I)
930 NEXT
940 PRINT D$; "PR#3"
950 END

```

Plotting Program

```

10 DIM X1(500),Y1(500),X(500),Y(500)
20 E$ = CHR$(3)
30 MD = 0.0
40 GOSUB 610
50 GOSUB 1210
60 PRINT CHR$(4);"PR#3"
70 INPUT "X-AXIS NAME = ";XAXIS$
80 INPUT "Y-AXIS NAME = ";YAXIS$
90 GOSUB 1960
100 PRINT CHR$(4);"PR#3"
110 PRINT CHR$(4);"PR#1"
120 PRINT "IN;SP1;IP1500,1325,9000,6825;"
130 PRINT "SC";XN;"",XX;"",YN;"",YX;"",
140 PRINT "SI 0.3,0.3;TL 1,0;"
150 PRINT "PU";XN;"",YN
160 PRINT "PD";XX;"",YN;"",XX;"",YX;"",XN;"",YX;"",
";XN;"",YN;" PU
170 IF MD = 1.0 THEN GOTO 430
180 FOR X = XN TO XX STEP (XX - XN) / 5
190 PRINT "PA";(X / 1);";";YN;"";XT;"
200 X$ = LEFT$(STR$(X / MULT),4)
210 PRINT "DI"
220 PRINT "CP-2 , -1;LB";X$;E$
230 NEXT X
240 PRINT "PA ";(XX + XN) / 2;"",YN;"";
250 PRINT "CP-";(LEN (XAXIS$) / 2);",-2.5;LB";XAXIS$;
E$
260 FOR Y = YN TO YX STEP (YX - YN) / 5
270 PRINT "PA";XN;"",Y / 1.0;"";YT;"
280 Y$ = LEFT$(STR$(Y / MULT),4)
290 PRINT "CP-4 ,-.25;LB";Y$;E$
300 NEXT Y
310 PRINT "PA ";XN;"",YX + YN) / 2;"";
320 PRINT "DIO,1"
330 PRINT "CP-";(LEN (YAXIS$)) / 2;"5;LB";YAXIS$;E$
340 PRINT "PA ";(XX + XN) / 2;"",YX;"";
350 PRINT "DI"
360 PRINT "CP-";(LEN (TITLE$(1))) / 2;"1;LB";TITLE$
(1);E$
370 FOR I = 2 TO 5
380 PRINT "PA ";(XX + XN) / 2;"",YX;"";
390 PRINT "DI"
400 IF TITLE$(I) = "END" THEN 430
410 PRINT "CP-";(LEN (TITLE$(I))) / 2;"",-(I - 1);
";LB";TITLE$(I);E$
420 NEXT
430 PRINT "PU"
440 PRINT "IW 1500,1325,9000,6825;"
450 IF LINE$ = "L" THEN GOTO 540
460 PRINT "SM";CHR$(C);";"
470 PRINT "PU"
480 PRINT "SI ";SIZE;"",SIZE;"";
490 FOR I = 1 TO N
500 PRINT "PA";X(I);";";Y(I);"PD;"
510 PRINT "PU"
520 NEXT I
530 GOTO 580
540 PRINT "PU; LT ;"
550 FOR I = 1 TO N
560 PRINT "PA";X(I);";";Y(I);"PD;"
570 NEXT I
580 PRINT CHR$(4);"PR#3"
590 GOSUB 2290

```

```

620 REM
630 REM
640 REM
650 REM *****
*
660 HOME
670 PRINT TAB( 32);: INVERSE : PRINT "P L O T T E R":
NORMAL
680 PRINT
690 PRINT TAB( 25);: INVERSE : PRINT "D A T A
I N P U T": NORMAL

700 PRINT : PRINT : PRINT
710 PRINT TAB( 23)"1. Type in the data at the keyboar
d."
720 PRINT
730 PRINT TAB( 23)"2. Input the data from a disk file
."
740 PRINT : PRINT :
750 PRINT TAB( 25)"Please type a 1 or a 2.": PRINT
760 GET A$: IF A$ = "" THEN 760
770 IF A$ < > "1" AND A$ < > "2" THEN 760
780 PRINT : PRINT : PRINT TAB( 25)"Thank you.": FOR I
= 1 TO 1000: NEXT

790 IF A$ = "1" THEN GOTO 920
800 IF A$ = "2" THEN GOTO 810
810 HOME
820 INPUT "Name of disk file where data is stored: ";N
$
830 PRINT CHR$( 4);"OPEN ";N$
840 PRINT CHR$( 4);"READ ";N$
850 INPUT N1
860 FOR I = 1 TO N1: INPUT X(I): NEXT
870 FOR I = 1 TO N1: INPUT Y(I): NEXT
880 PRINT CHR$( 4);"CLOSE"
890 PRINT
900 GOSUB 1090
910 RETURN
920 HOME
930 INPUT "Number of points: ";N1
940 PRINT
950 FOR I = 1 TO N1: PRINT "POINT # ";I;: INPUT " X,
Y = ";X(I),Y(I): NEXT

960 PRINT : PRINT :
970 PRINT : PRINT "Do you want to save this data to a
disk file? Y or N
"
980 GET A$: IF A$ = "" THEN 980
990 IF A$ < > "N" AND A$ < > "Y" THEN 980
1000 IF A$ = "N" THEN RETURN
1010 PRINT : INPUT "Name for disk storage file: ";N$
1020 PRINT CHR$( 4);"OPEN ";N$
1030 PRINT CHR$( 4);"WRITE ";N$
1040 PRINT N1
1050 FOR I = 1 TO N1: PRINT X(I): NEXT
1060 FOR I = 1 TO N1: PRINT Y(I): NEXT
1070 PRINT CHR$( 4);"CLOSE"
1080 RETURN
1090 REM *****
1100 REM LIST INPUTTED DATA FROM DISK
1110 REM
1120 PRINT : PRINT "List data from disk ? Y or N ";
1130 GET A$: IF A$ = "" THEN 1120

```



```

1140 IF A$ < > "N" AND A$ < > "Y" THEN 1130
1150 IF A$ = "N" THEN RETURN
1160 PRINT : PRINT
1170 FOR I = 1 TO N1
1180 PRINT X(I),Y(I)
1190 NEXT
1200 RETURN
1210 REM *****
***
1220 REM
1230 REM MIN - MAX ROUTINES
1240 REM
1250 REM *****
***
1260 N = N1
1270 BIG = Y(1)
1280 FOR I = 1 TO N
1290 IF BIG - Y(I) > 0 THEN NEXT
1300 IF I > N THEN 1340
1310 IF BIG - Y(I) < 0 THEN BIG = Y(I)
1320 IF BIG - Y(I) = 0 THEN BIG = Y(I)
1330 NEXT
1340 YX = BIG
1350 N = N1
1360 BBIG = X(1)
1370 FOR I = 1 TO N
1380 IF BBIG - X(I) > 0 THEN NEXT
1390 IF I > N THEN 1430
1400 IF BBIG - X(I) < 0 THEN BBIG = X(I)
1410 IF BBIG - X(I) = 0 THEN BBIG = X(I)
1420 NEXT
1430 XX = BBIG
1440 SYM = Y(1)
1450 FOR I = 1 TO N
1460 IF SYM - Y(I) < 0 THEN NEXT
1470 IF I > N THEN 1510
1480 IF SYM - Y(I) > 0 THEN SYM = Y(I)
1490 IF SYM - Y(I) = 0 THEN SYM = Y(I)
1500 NEXT
1510 YN = SYM
1520 SXM = X(1)
1530 FOR I = 1 TO N
1540 IF SXM - X(I) < 0 THEN NEXT
1550 IF I > N THEN 1590
1560 IF SXM - X(I) > 0 THEN SXM = X(I)
1570 IF SXM - X(I) = 0 THEN SXM = X(I)
1580 NEXT
1590 XN = SXM
1600 PRINT
1610 PRINT "Xmin, Xmax = ";XN;"",";XX
1620 PRINT
1630 PRINT "Ymin, Ymax = ";YN;"",";YX
1640 PRINT
1650 GOSUB 1670
1660 RETURN
1670 CH(1) = XN - INT (XN):CH(2) = XX - INT (XX)
1680 CH(3) = YN - INT (YN):CH(4) = YX - INT (YX)
1690 IF CH(1) + CH(2) + CH(3) + CH(4) = 0.0 THEN MULT
= 1.0
1700 IF MULT = 1. THEN RETURN
1710 PRINT
1720 PRINT "These Max-Min values MUST be integers. You
may now either:"
; PRINT
1730 PRINT "(1) Input your own scale factors, or": PRI

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NT
1740 PRINT "(2) Input a multiplicative offset factor (
usually 10 or 100
etc.) "; PRINT
1750 PRINT "Type either 1 or 2.";
1760 GET A$; IF A$ = "" THEN 1760
1770 IF A$ < > "1" AND A$ < > "2" THEN 1760
1780 IF A$ = "2" THEN 1850
1790 PRINT : PRINT
1800 INPUT " Xmin & Xmax = "; XN, XX
1810 PRINT
1820 INPUT " Ymin & Ymax = "; YN, YX
1830 MULT = 1.
1840 RETURN
1850 PRINT : INPUT "Multiplication offset factor = "; M
ULT
1860 FOR I = 1 TO N
1870 X1(I) = (X(I) * MULT); Y1(I) = (Y(I) * MULT)
1880 NEXT
1890 FOR I = 1 TO N: X(I) = X1(I); Y(I) = Y1(I); NEXT
1900 XN = INT (XN * MULT); XX = INT (XX * MULT)
1910 YN = INT (YN * MULT); YX = INT (YX * MULT)
1920 RETURN
1930 REM *****
1940 REM PLOTTING CHARACTER
1950 REM *****
1960 PRINT "Do you want to plot DATA or draw a LINE ?
(D or L)";
1970 GET A$: IF A$ = "" THEN 1970
1980 IF A$ < > "D" AND A$ < > "L" THEN 1970
1990 IF A$ = "D" THEN LINE$ = "D"
2000 IF A$ = "L" THEN LINE$ = "L"
2010 IF LINE$ = "L" THEN 2210
2020 PRINT
2030 PRINT "Input the ASCII decimal value of the chara
cter you wish to
use for plotting"
2040 PRINT : PRINT "NOTE: 33 < NUMBER < 127 (except 59
). See any table
of ASCII codes";
2050 PRINT : PRINT " or page C-9 in plotter manual."
2060 PRINT : PRINT "Default character is *"
2070 PRINT : PRINT "Some other characters are: Char
ASCII"
2080 PRINT TAB( 32);"! 33"
2090 PRINT TAB( 32);"% 37"
2100 PRINT TAB( 32);"+ 43"
2110 PRINT TAB( 32);"X 88"
2120 PRINT TAB( 32);"x 120"
2130 PRINT : INPUT "Input character code:"; C$
2140 IF LEN (C$) < 1 THEN C = 42
2150 IF C < > 42 THEN C = VAL (C$)
2160 PRINT : PRINT "What size, in % of a cm, do you wa
nt the character
size to be ?
2170 PRINT : INPUT "Input size: (0.3 is default value)
"; SIZE$
2180 IF LEN (SIZE$) < 1 THEN SIZE = 0.3
2190 IF SIZE = 0.3 THEN 2210
2200 SIZE = VAL (SIZE$)
2210 IF MD = 1.0 THEN RETURN
2220 PRINT : PRINT "You may have up to 5 lines of titl
es."
2230 FOR I = 1 TO 5
2240 INPUT "Title ? <CR> for no line, END to end: "; TI

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TLE$(I)
2250 IF TITLE$(I) = "END" THEN RETURN
2260 NEXT
2270 RETURN
2280 REM *****
2290 REM PLOT MORE DATA
2300 REM *****
2310 PRINT "Plot more data ? Y or N";
2320 GET A$: IF A$ = "" THEN 2320
2330 IF A$ < > "Y" AND A$ < > "N" THEN 2320
2340 IF A$ = "N" THEN END
2350 MD = 1.0
2360 GOSUB 610
2370 N = N1
2380 FOR I = 1 TO N
2390 X1(I) = X(I) * MULT: Y1(I) = Y(I) * MULT: NEXT
2400 FOR I = 1 TO N: X(I) = X1(I): Y(I) = Y1(I): NEXT
2410 GOSUB 1960
2420 PRINT CHR$(4); "PR#3"
2430 GOTO 100

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