FAST WAVE RESONANCES NEAR THE
ION CYCLOTRON FREQUENCY

by

RICHARD E. DOLLINGER, B.S. in E.E., M.S. in E.E.

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CHAPTER I

INTRODUCTION

The heating of a plasma by the damping of the fast (compressional) wave at the second harmonic of the ion cyclotron frequency ($2_i\Omega$) is an attractive method for supplementary heating of a fusion reactor. This method is attractive for three reasons. First, the ions are heated directly. Secondly, the fast wave can be effectively launched into high density plasmas. Thirdly, for frequencies near $2_i\Omega$, commercial rf generators are already available which can produce the high powers needed to heat a fusion reactor.

Our previous research on this damping method produced a large data spread.^{1,2} There was also concern that the plasma might have been too cold to observe the damping of the fast wave clearly at $2_i\Omega$, since this damping is a hot plasma effect. In an attempt to improve the repeatability, a large amplitude rf source was used.¹ The results were only improved slightly. Nonetheless, the fast wave was observed to damp at $2_i\Omega$. In addition, an "anomalous damping" of the fast wave was found which heated the plasma strongly and efficiently. This "anomalous damping" was

tentatively identified as a magneto-sonic column resonance.

The original goal of the research reported here was to use "turbulent heating" to obtain a hotter plasma so that the damping of the fast wave at $2_i\Omega$ would be enhanced. The advantages and the disadvantages of this plasma source are described in Ch. II along with the experimental arrangement. After much trial and error, the repeatability was finally improved by careful pre-ionization of the plasma. (This is also discussed in Ch. II.)

The experimental results described in Ch. II show damping of the fast wave near $2_i\Omega$ in a hydrogen plasma and damping near $2_i\Omega$ and $3_i\Omega$ in a deuterium plasma. Indications of damping were also found near $4_i\Omega$ and $5_i\Omega$ in a deuterium plasma. In all cases, the regions of greatest damping occurred at a frequency approximately 10% greater than the frequency of each harmonic of $_i\Omega$! This "shifting" of the resonance is due to collisions as explained in Ch's. III, IV and V. The magneto-sonic column resonance was also observed up through the second radial mode in a deuterium plasma.

The system of units used will be MKS with one exception. All temperatures in the equations are in °K, but the text and figures will use eV for units of temperature.

CHAPTER II

EXPERIMENT

Experimental Arrangement

Fig. II-1 shows the experimental arrangement. The vacuum vessel is a Pyrex tube with an internal diameter of 10 cm and a length of 1 m, which is immersed in an axial magnetic field, B_0 . The electrodes are located at both ends of the vessel just outside the peaks of the 150% magnetic mirrors. The plasma is prepared in hydrogen or deuterium gas at 5 mTorr pressure ($\sim 1.7 \times 10^{20}$ molecules/ m^3) by a special pre-ionization that is followed by an axial current from a ringing capacitive discharge ($Q \approx 5$, $f \approx 400$ kHz). At the end of this axial discharge the waves were monitored.

The waves were launched by a one turn loop around the cylindrical plasma column and damped in a 0.8% magnetic beach. The current in this one turn loop came from a 1 kW, 5.2 MHz oscillator. The waves were monitored by another one turn loop which was axially displaced 10 cm from the first loop. Table II-1 shows the measured plasma densities, n , and temperatures, T , that were present when the waves were monitored.

TABLE II-1

EXPERIMENTAL VALUES OF PLASMA TEMPERATURE AND DENSITY

Approximate Temperature	Approximate Density	Method Used	Comments
2 eV	$2.5 \times 10^{20}/\text{m}^3$	Spectroscopy (H_β to Continuum)	Radial Average
		Spectroscopy (Stark Broadening)	Radial Average
	$2.9 \times 10^{20}/\text{m}^3$	12 pass He-Ne Laser, Interferometry	Axial Average
	$2.2 \times 10^{20}/\text{m}^3$	Fast Wave Propagation (See Figs. II-6 and -7)	Momentum averaged over the paths of the ions in wave motion
0.8 eV		Damping Length of the Fast Wave	Near the outer edge of the plasma column (See Ch. V)
		Upper limit set by the initial filling pressure	Hydrogen gas at 5 mTorr if 100% ionized
2 eV	$\approx 4.8 \times 10^{20} \text{ eV}/\text{m}^3$	The loss of particles out the ends of the magnetic bottle as determined by the average speed of the ions	The end loss rate of the plasma must be known in order to know the average ion energy
		Integrated Diamagnetic Loop Signal	Volume Average

Second Harmonic Resonances

Fig. II-2 (for a hydrogen plasma) and Figs. II-3 and -4 (for a deuterium plasma) show the amplitude of the fast wave versus B_0 . In these figures, one can see a large region of damping about 10% below $2_i\Omega$ or $3_i\Omega$ and a relatively smaller region almost exactly at $2_i\Omega$ or $3_i\Omega$. The solid line in these figures is calculated using polynomial regression by the least squares method. The two regions just described can be represented by a "w" pattern in the dashed curve. Based on the "w" patterns observed near $2_i\Omega$ in hydrogen and $2_i\Omega$ and $3_i\Omega$ in deuterium, one can sketch in a similarly shaped curve near $4_i\Omega$ and $5_i\Omega$ in deuterium. The damping is not very clear in these regions, but one can see an indication of damping near $4_i\Omega$ and $5_i\Omega$. The location and the widths of the dips in the "w" pattern near $2_i\Omega$ in hydrogen will be discussed in Ch. V.

It should be noted that one can use an average density and average temperature to calculate an ion-ion (Eq. IV-10c) and an ion-electron (Eq. IV-3f) collision frequency. The first collision frequency is so large (i.e. $\omega < \langle v_{ii} \rangle$) that the second harmonic resonance should not even be observed. Furthermore, the ion-electron collision frequency is much too small to account for the damping of the fast wave far from any resonance. Since neither of these collision frequencies fits the observed data, it appears that the average

Fig. II-2. Wave amplitude versus B_o in a hydrogen plasma at $z = 0.10 \text{ m}$.

Fig. II-3. Wave amplitude versus B_0 in a deuterium plasma at $z = 0.10$ m.

Fig. II-4. Wave amplitude versus B_0 in a deuterium plasma at $z = 0.10 \text{ m}$.

density and temperature are not the correct parameters to use. Chapter V will show which density and temperature should have been used and it will also be shown that the ion-ion collision frequency calculated above is not the correct collision frequency to use for the second harmonic resonance.

One may question whether or not the resonance near $2_i\Omega$ in a deuterium plasma is really the ion cyclotron resonance of impurity hydrogen ions, but this appears unlikely. In order to avoid hydrogen from entering the deuterium plasma, the base vacuum was kept at least three orders of magnitude below the initial filling pressure of deuterium. In addition, a flowing gas supply was used so that any impurities which are evolved from the electrodes or glass wall will be swept out of the system. Also the wave amplitudes were measured about 20 μ s after the plasma was formed. Thus, impurities released from the electrodes by the axial discharge would not have time to enter the plasma before the waves were monitored. In order to prevent the plasma from touching the pyrex vacuum vessel and releasing impurity hydrogen, the gas was pre-ionized so that the plasma would not touch the wall (see the Appendix, steps 6 and 8). Furthermore, the resonance was also seen near $2_i\Omega$ in a hydrogen plasma (i.e. a frequency four times higher than the ion-ion (hydrogen-

impurity deuterium hybrid resonance). The fact that the resonance was seen in hydrogen also shows that it cannot be an impurity cyclotron resonance (i.e. no simple ion exists with a charge to mass ratio of $2e/m_p$, where m_p is the mass of a proton). The end result is that the damping near $2\Omega_i$ in a deuterium plasma is caused by the second harmonic resonance of the fast wave in deuterium and is not caused by ion cyclotron resonance of impurity hydrogen ions. This reasoning also excludes the possibility that an ion-ion (deuterium and impurity hydrogen ions) hybrid resonance is responsible for the observed damping.

Pre-ionization and Turbulent Heating

Historically, turbulent heating has been used to heat "low" density plasmas ($n < 7 \times 10^{19}/\text{m}^3$) to "high" temperatures (from ~ 10 eV to as high as ~ 10 keV). (The reader is referred to the literature for more details.)³⁻³⁶ Basically, the electron drift velocity, v_d , of the axial current discharge determines the plasma temperature, T as given by

$$v_d \approx \sqrt{\frac{3KT}{m_i}} \triangleq \text{acoustic speed} \quad (\text{II-1a})$$

$$T = \frac{i^m v_d^2}{3 K} \quad . \quad (II-1b)$$

It was originally decided to use turbulent heating to obtain a hotter plasma for the reasons given in the Introduction.

One of the main difficulties was to obtain the large values of v_d required for the desired temperature (~ 5 eV). This was a difficulty because of the many "trade-offs." Some examples are discussed. For the higher desired densities ($n \sim 3 \times 10^{20}/m^3$), a higher axial discharge current was needed in order to obtain the same v_d . This could be done by increasing the capacitance or decreasing the inductance of the axial discharge circuit. If the capacitance was increased, then the time taken for the axial discharge to die out was also increased and the launching of the waves was similarly delayed. This delay was undesirable because during the delay the density and temperature would have fallen significantly from their peak, initial values. These density and temperature decays would be even faster in a hot plasma. Thus, it was more desirable to lower the system inductance instead of increasing the capacitance. However, the minimum value of inductance was limited by the geometry of the plasma tube and the minimum inductance of the available capacitors, spark gaps, ignitrons and feed systems.

Both ringing and an over-damped discharge were tried. The ringing discharge worked better than an over-damped discharge for a plasma which was not pre-ionized because non-uniformities could be evened out during the current zeroes.³⁷ No comparison was made between an AC or a DC discharge when the plasma was pre-ionized.

In order to obtain sufficient energy with a small capacitor, a high voltage was needed. There was, however, a practical limit to the voltage ratings of capacitors (~ 75 kV), switches and system insulation.

The axial discharge current, if too high, can cause instabilities if the magnetic field, B_θ , from the current is a large fraction of B_0 .³⁸ Based on the increase in the scatter of the received wave signals indications were that the threshold for instability ranged from $B_\theta \approx 10\% B_0$ for low values of B_0 (~ 0.17 T) to $B_\theta \approx 30\% B_0$ for high values of B_0 (~ 0.5 T). Thus, the axial discharge current must not be too large.

One way to obtain the desired v_d without the high axial currents was to use an electrode with a small cross section, such as a thin, hollow cylinder. This worked quite well for initial pressures less than 1 mTorr. Above this pressure, however, the plasma current arced from the sides, edges and supports of the cylinder so that an undesirably large cross-section resulted. A disk with a hole in it,

which was placed just outside the magnetic mirror worked satisfactorily. The hole decreased the current cross-section while the rapidly diverging magnetic field outside the mirror shielded the support and the back side of the disc from the plasma arc which resulted with an even lower cross-section electrode.

In order to avoid the time consuming re-tuning of the rf system, it was desirable to hold the frequency of the waves, ω , constant and vary B_0 instead of keeping B_0 constant and varying ω . This worked reasonably well provided that the preparation of the plasma was the same for all values of B_0 .

It was found that some form of pre-ionization was needed to guarantee the same plasma preparation for all values of B_0 . Originally, a small trigger spark was initiated prior to the axial discharge. This procedure was not too successful because at the desired densities the timing of the spark and the physical position of the trigger electrode (see Fig. II-1) were extremely critical. This timing and the physical position were also strong functions of B_0 .

After trying many different combinations of the many different schemes presented in the literature, the only successful scheme found was the one shown in Fig. II-1. This scheme provided for a pre-ionizing axial DC discharge current of at least 40 mA which must last for at least 100

ms. This DC current limit was needed to insure a uniformly bright and extremely steady discharge for all values of B_0 . Furthermore, the trigger arc must be operated continuously and the pre-ionization power supply for this experiment must furnish at least 14 kVDC in order for a very steady pre-ionization discharge to be formed. A DC pre-ionization discharge which lasted less than 100 ms or an AC pre-ionization discharge did not work because they produced too many fluctuations. The fact that the pre-ionization discharge had to be uniformly bright and very steady cannot be over emphasized. The 28 ohm resistor, the 0.1 μ f capacitor, the negative polarity of the ignitron cathode and the positive polarity of the pre-ionization supply insured that the ignitron (or spark gap) ignited well and stayed ignited until the plasma broke down. (See the Appendix for the sequence of operations used in this experiment).

This turbulent heating and pre-ionization scheme produced plasmas as hot as ~ 15 eV. For four reasons, however, this scheme was used to produce only a ~ 2 eV plasma for this experimental investigation. The first reason for using a ~ 2 eV plasma was that it was reproducible over a wide range of B_0 . The second reason was that only a low charging voltage ($\sim$ H.V. ≈ 15 kV) was needed to produce a ~ 2 eV plasma and an ignitron could be used instead of a spark gap. Thus, the constant fiddling required with a spark gap was avoided.

The third reason was that the ~ 15 eV plasma was not suitable as a propagation medium for the fast wave. This plasma was so "turbulent" after the end of the axial discharge that the waves were heavily absorbed and essentially did not propagate.³⁷ The waves did start propagating at very late times (400 ms after the end of the axial discharge). By then, the density and the temperature were too low for the plasma to be of interest. After turbulent heating of the plasma to ~ 15 eV, the particles in the "high energy tail" had gyro-radii which were comparable to the physical dimension of the experiment.³⁷ Because of their large gyro-orbits, these particles not only hit the pyrex vacuum vessel and released impurities but they also under-went charge exchange with neutrals surrounding the plasma which introduced unwanted neutrals into the plasma.³⁷

There was a narrow "range" of operation where a plasma hotter than 2 eV could be produced without the difficulties discussed in the previous paragraph. This plasma had a temperature of ~ 5 eV and densities up to $\sim 7 \times 10^{20}/\text{m}^3$. In this plasma, the fast wave had "damping lengths" in excess of 30 cm (about three wavelengths) and would have been ideal for this experiment. The disadvantage was that this "range" varied with B_0 . This was the fourth reason why the 2 eV plasma was used for this experiment.

In order to avoid varying B_0 , it would be nice to have a wide band (~ 400 kHz to ~ 80 MHz), high power (≥ 300 Watt) rf signal source which did not require tuning. With such a source many interesting experiments could be performed with this "hot" (~ 5 eV), "dense" ($\lesssim 7 \times 10^{20}/\text{m}^3$) plasma.

Magneto-sonic Column Resonances

Figures II-2, -3 and -4 also show the regions, the slashed areas, where magneto-sonic column resonances occurred. In the previous work, these regions were labeled "anomalous" and tentatively identified as the result of a magneto-sonic column resonance.¹ Their locations in the present, as well as the previous, work were found to scale as magneto-sonic column resonances should. Therefore, the regions must indeed be the result of a magneto-sonic column resonance. From the previous work, it was found that the width of the resonance is approximately equal to the percent depth of the magnetic beach.¹ For this experiment, the 0.8% beach was measured with a relative accuracy of $\pm 0.1\%$ and an absolute accuracy of $\pm 1.5\%$. Therefore, the width of this resonance should be about 0.8% of B_0 . However, the values of B_0 in Figs. II-2, II-3 and II-4 were measured with a relative and absolute accuracy of $\pm 1.5\%$. (The horizontal error bars which mark the location of the harmonics of the ion cyclotron frequency represent the error in measuring B_0 and ω .) This 0.8% width is smaller

than the 1.5% resolution of B_0 . Therefore, there is a large scatter in the data points in these regions.

Even if the wide band signal source, which was described in the previous section, were unavailable, a "hot" (~ 5 eV), dense ($\sim 3 \times 10^{20}/\text{m}^3$) plasma might still be produced from the ~ 2 eV plasma which was used in this experiment. The magneto-sonic column resonance could be used to further heat the 2 eV plasma to the desired 5 eV. Previous work has already shown that this resonance is a very efficient means of heating a plasma substantially.¹ If the heating coils were located on the rising inside edge of the magnetic mirror, then the rf generator would not require tuning as long as the resonance occurred somewhere on the slope of the magnetic mirror. With a CW, rf source one might even be able to produce a continuous, 5 eV plasma.

It is not necessary for the work presented here, but the reader can refer to the previous work for more details on the observed magneto-sonic column resonance.¹ Figures II-5, II-6 and II-7, for this experiment, are presented as up-dated figures for those given in the previous work (i.e. Figs. III-12 and III-13).¹ The only differences between the previous and the present figures are that a different beach, density and wave frequency were used and that the magneto-sonic column resonance was seen through the second

Fig. II-5. Graph of α versus plasma radius for constant k_+ ($= \frac{\alpha}{r} = \frac{\omega}{V_A}$), for either a hydrogen or a deuterium plasma.

Fig. II-6. Phase velocity versus Ω (or B_0 or V_A or $k_{\perp}$) in a hydrogen plasma.

Fig. II-7. Phase velocity versus Ω (or B_0 or V_A or $k_{\perp}$) in a deuterium plasma.

radial mode in this experiment. No explanations or interpretations are given here for Figs. II-5, II-6 and II-7, as this was done in the previous work.¹

CHAPTER III

THEORY OF FUNDAMENTAL AND SECOND HARMONIC RESONANCES

In this chapter, a conductivity tensor is derived which includes both collisions and the resonances at ω_i and ω_e (the fundamental cyclotron frequencies) and at $2\omega_i$ and $2\omega_e$ (the second harmonic frequencies). The integrals used in the conductivity tensor are difficult to evaluate if the collision frequencies are functions of velocity. In order to avoid this difficulty, a simpler form of the conductivity tensor is derived which assumes that all collision frequencies are not functions of velocity. The actual expression of these collision frequencies will be derived in Ch. IV and the numerical evaluation is shown in Ch. V.

The notation used in this report generally follows the notation in the cited literature. Two exceptions should be noted. The first exception is that subscripts before the variables are used, especially for denoting the species (ion or electron). The second exception is that vectors are represented by a single arrow and tensors of order two or greater by a double arrow. The reader is

referred to the cited literature for more details.

The Boltzmann equation for the α species,

$$\begin{aligned} \frac{\partial_{\alpha} f(\vec{r}, \vec{v}, t)}{\partial t} + \alpha f \nabla_{\vec{r}} \cdot \vec{v} + \vec{v} \cdot \nabla_{\vec{r}} \alpha f + \alpha f \nabla_{\vec{v}} \cdot \vec{h} \\ + \alpha \vec{h} \cdot \nabla_{\vec{v}} \alpha f = \frac{\delta_{\alpha} f}{\delta t} \end{aligned} \quad (III-1)$$

is the phase space continuity equation for the distribution function $\alpha f(\vec{r}, \vec{v}, t)$.³⁹ (Sect. 2-1) The $\frac{\delta_{\alpha} f}{\delta t}$ term includes the short-range particle interactions, "collisions." Since only electromagnetic forces are considered, then

$$\alpha \vec{h} = \frac{\alpha q}{\alpha m} [\vec{E} + \vec{v} \times \vec{B}] = \alpha \vec{a} + \vec{v} \times \alpha \vec{\Omega} \quad (III-2)$$

The $\alpha f \nabla_{\vec{r}} \cdot \vec{v}$ term is zero since $\vec{r}$, $\vec{v}$ and t are considered to be independent.

The distribution function may be written in an expansion of spherical harmonics ($Y_{\ell m s}$) which is equivalent to a Cartesian tensor scalar product expansion.³⁹ (Sect. 3-3), 40

$$\alpha f(\vec{r}, \vec{v}, t) = \sum_{\substack{\ell=0 \rightarrow \infty \\ m=0 \rightarrow \ell \\ s=0 \rightarrow 1 \\ s \leq \ell}} \alpha f_{\ell m s}(\vec{r}, v^2, t) Y_{\ell m s}(\theta, \phi) \quad (III-3a)$$

$${}_a \vec{f}(\vec{r}, {}_a \vec{v}, t) = \sum_l \frac{{}_a \vec{f}_l (f_{lms}) \cdot (.) {}_a \vec{v}_l}{{}_a v^l}, \quad (\text{III-3b})$$

where $(.)$ is the l^{th} order dot product of the l^{th} order tensors ${}_a \vec{f}_l$ and ${}_a \vec{v}_l$. The elements of the ${}_a \vec{f}_l$ tensor are defined so that the l^{th} terms of Eqs. (III-3a) and (III-3b) are equal as shown below

$$\sum_{m,s} {}_a f_{lms} Y_{lms} = \frac{{}_a \vec{f}_l \cdot (.) {}_a \vec{v}_l}{{}_a v^l} \quad (l^{\text{th}} \text{ term}) \quad (\text{III-4})$$

Only tensors of order two or less are kept because only fundamental and second harmonic resonances will be considered. The following variables are assumed to vary in time and space as indicated:⁴⁰

$${}_a \vec{a}(\vec{r}, t) \rightarrow {}_a \vec{a} e^{j(\omega t - \vec{k} \cdot \vec{r})} \quad (\text{III-5a})$$

$${}_a f_0(\vec{r}, {}_a v, t) \rightarrow {}_a f_0^0({}_a v) + {}_a f_0^1(v) e^{j(\omega t - \vec{k} \cdot \vec{r})} \quad (\text{III-5b})$$

$${}_a \vec{f}_1(\vec{r}, {}_a v, t) \rightarrow {}_a \vec{f}_1^1({}_a v) e^{j(\omega t - \vec{k} \cdot \vec{r})} \quad (\text{III-5c})$$

$${}_a \vec{f}_2(\vec{r}, {}_a v, t) \rightarrow {}_a \vec{f}_2^1({}_a v) e^{j(\omega t - \vec{k} \cdot \vec{r})} \quad (\text{III-5d})$$

$$\frac{\delta {}_a \vec{f}_l}{\delta t} \rightarrow {}_a v_l({}_a v) {}_a \vec{f}_l \quad (\text{III-5e})$$

$$\vec{B} \rightarrow B_0 \hat{z} \quad . \quad (\text{III-5f})$$

The f_0 is the isotropic part of the distribution function (typically in Maxwellian equilibrium). The small perturbations to f_0 are $\vec{f}_1$ and $\vec{f}_2$. The $\vec{f}_1$ is the lowest order term which is used to calculate the average velocity of the species. It also gives the fundamental cyclotron resonance. The $\vec{f}_2$ is an anisotropic velocity distribution which is responsible for the second harmonic resonances at $2_i\Omega$ and $2_e\Omega$. The ν_0 is the collision frequency which determines the relaxation rate of f_0 to an equilibrium distribution. (If f_0 is assumed to be a Maxwellian then $\nu_0 \triangleq 0$.) Similarly, the ν_1 and the ν_2 are used to determine, respectively, the rate at which $\vec{f}_1$ and $\vec{f}_2$ relax to zero. In general, the ν_l 's are complex and are functions of velocity. (This is discussed in more detail in the last two sections of this chapter.) Only many small angle Coulomb collisions will be considered in calculating the ν_l 's.

The Boltzmann equation can be written in three parts after neglecting non-linear terms.⁴⁰

$$f_0 = \frac{\nu}{3} \frac{\vec{k} \cdot \vec{f}_1^1}{\omega_0} \quad (\text{a scalar Equation}) \quad (\text{III-6a})$$

$$j_{\alpha} \omega_l \vec{f}_l^1 - j_{\alpha} \nu \vec{k} \cdot \vec{f}_0^1 + \vec{\Omega} \times \vec{f}_1^1 - j \frac{2}{5} \nu \vec{k} \cdot \vec{f}_2^1$$

$$= - {}_{\alpha} \vec{a} \frac{\partial {}_{\alpha} f_O^O}{\partial {}_{\alpha} v} \quad (\text{a vector Equation}) \quad (\text{III-6b})$$

and

$$\begin{aligned} j_{\alpha} \omega_2 {}_{\alpha} \vec{f}_2^1 + 2[{}_{\alpha} \Omega \times {}_{\alpha} \vec{f}_2^1]_2 \\ = j_{\alpha} v [\vec{k} {}_{\alpha} \vec{f}_1^1 - \frac{1}{3}(\vec{k} \cdot {}_{\alpha} \vec{f}_1^1) \vec{I}_2]_2 \quad (\text{a tensor Equation}) \end{aligned} \quad (\text{III-6c})$$

where $j_{\alpha} \omega_l \triangleq j\omega + {}_{\alpha} v_l$. The $\vec{I}_2$ is a unit diagonal tensor and $[\]_2$ is a symmetrization operator.

Equation (III-6a) and (III-6c) can be used to solve for ${}_{\alpha} f_O^O$ and ${}_{\alpha} \vec{f}_2^1$, respectively, in terms of ${}_{\alpha} \vec{f}_1^1$. These values of ${}_{\alpha} f_O^O$ and ${}_{\alpha} \vec{f}_2^1$ are then substituted into Eq. (III-6b) to obtain

$${}_{\alpha} \vec{S}_2^{-1} \cdot j_{\alpha} \vec{f}_1^1 = - {}_{\alpha} \vec{a} \frac{\partial {}_{\alpha} f_O^O}{\partial {}_{\alpha} v}, \quad (\text{III-7})$$

where the $\vec{S}_2^{-1}$ is suitably defined (see Eq. III-9). Upon inversion, this results in

$${}_{\alpha} \vec{f}_1^1 = j_{\alpha} \vec{S}_2 \cdot {}_{\alpha} \vec{a} \frac{\partial {}_{\alpha} f_O^O}{\partial {}_{\alpha} v}. \quad (\text{III-8})$$

Without loss in generality $\vec{k}$ can be assumed to lie in

the x-z plane (i.e. $k_y \triangleq 0$). Then the elements of $\vec{\alpha S}_2$ are found to be:⁴⁰

$$\begin{aligned} \alpha S_{xx} = & - \frac{\alpha^{\omega_1}}{(\alpha^{\Omega^2} - \alpha^{\omega_1^2})} \left\{ 1 - \frac{\alpha^v^2}{3} \left[\left(\frac{5(\alpha^{\omega_1}/\alpha^{\omega_0}) + (\alpha^{\omega_1}/\alpha^{\omega_2})}{5(\alpha^{\Omega^2} - \alpha^{\omega_1^2})} \right. \right. \right. \\ & - \frac{3}{5} \frac{(\alpha^{\omega_2}/\alpha^{\omega_1})}{(4\alpha^{\Omega^2} - \alpha^{\omega_2^2})} - \frac{6}{5} \frac{(2\alpha^{\Omega^2} + \alpha^{\omega_2}\alpha^{\omega_1})}{(4\alpha^{\Omega^2} - \alpha^{\omega_2^2})(\alpha^{\Omega} - \alpha^{\omega_1^2})} \Big) k_x^2 \\ & \left. \left. - \left(\frac{3}{5} \frac{\alpha^{\Omega^2}(2 + \alpha^{\omega_2}/\alpha^{\omega_1}) + \alpha^{\omega_1}\alpha^{\omega_2}}{(\alpha^{\Omega^2} - \alpha^{\omega_2^2})(\alpha^{\Omega^2} - \alpha^{\omega_1^2})} \right) k_z^2 \right] \right\} , \quad (\text{III-9a}) \end{aligned}$$

$$\begin{aligned} \alpha S_{yy} = & + \frac{\alpha^{\omega_1}}{(\alpha^{\Omega^2} - \alpha^{\omega_1^2})} \times \frac{\alpha^v^2}{3} \left[\frac{1}{\alpha^{\omega_1}} \left(\frac{1}{\alpha^{\omega_0}} + \frac{1}{5\alpha^{\omega_2}} \right) k_x^2 \right] \\ & + \alpha S_{xx} , \quad (\text{III-9b}) \end{aligned}$$

$$\begin{aligned} \alpha S_{xy} = & \frac{-j\alpha^{\Omega}}{(\alpha^{\Omega^2} - \alpha^{\omega_1^2})} \left\{ 1 - \frac{\alpha^v^2}{3} \left[\frac{3}{5} \frac{1}{(\alpha^{\Omega^2} - \alpha^{\omega_2^2})} \left(1 - 2 \frac{(\alpha^{\Omega^2} + \alpha^{\omega_1}\alpha^{\omega_2})}{(\alpha^{\Omega^2} - \alpha^{\omega_1^2})} \right) k_z^2 \right. \right. \\ & + \left(\frac{5(\alpha^{\omega_1}/\alpha^{\omega_0}) + (\alpha^{\omega_1}/\alpha^{\omega_2})}{5(\alpha^{\Omega^2} - \alpha^{\omega_1^2})} \right. \\ & \left. \left. + \frac{6}{5(4\alpha^{\Omega^2} - \alpha^{\omega_2^2})} \left(1 - 2 \frac{(\alpha^{\Omega^2} + \alpha^{\omega_2}\alpha^{\omega_2})}{(\alpha^{\Omega^2} - \alpha^{\omega_1^2})} \right) \right) k_x^2 \right] \right\} , \quad (\text{III-9c}) \end{aligned}$$

and

$$\alpha S_{yx} = -\alpha S_{xy} \quad . \quad (\text{III-9d})$$

It can be shown that αS_{xz} , αS_{zx} , αS_{yz} and αS_{zy} are of little interest for the plasma wave theory that we are considering here.

The conductivity tensor, $\vec{\sigma}_2$ is defined by

$$\vec{J} = \vec{\sigma}_2 \cdot \vec{E} \quad . \quad (\text{III-10})$$

By definition, the current density for an electron-ion plasma can be written as

$$\vec{J} \triangleq i n e \langle \vec{v} \rangle - e n e \langle \vec{v} \rangle \quad , \quad (\text{III-11})$$

where charge neutrality is usually assumed so that

$$n = i n = e n \quad . \quad (\text{III-12})$$

The average velocities can be found by averaging³⁹ (Sect. 3-3)

$$\langle \vec{v} \rangle = \frac{\int \int \int \alpha f(\vec{r}, \alpha \vec{v}, t) \alpha \vec{v} [d^3 \alpha v]}{\int \int \int \alpha f [d^3 \alpha v]} \quad , \quad (\text{III-13})$$

where

$${}_{\alpha}\vec{v} = {}_{\alpha}v(\sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z}) \quad (\text{III-14a})$$

$${}_{\alpha}\vec{v} = {}_{\alpha}v(Y_{110} \hat{x} + Y_{111} \hat{y} + Y_{100} \hat{z}) \quad , \quad (\text{III-14b})$$

and

$${}_{\alpha}\vec{v} d^3 {}_{\alpha}v \triangleq [({}_{\alpha}v \sin \theta d \phi) ({}_{\alpha}v d \theta) (d {}_{\alpha}v)] \quad . \quad (\text{III-14c})$$

Substitution of Eqs. (III-3) and (III-14) into (III-13) together with the orthogonality principle for the Y_{lms} yields

$$\begin{aligned} \langle {}_{\alpha}\vec{v} \rangle = & \int_{{}_{\alpha}v=0}^{\infty} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \{ ({}_{\alpha}f_{110} Y_{110}) ({}_{\alpha}v Y_{110} \hat{x}) \\ & + ({}_{\alpha}f_{111} Y_{111}) ({}_{\alpha}v Y_{111} \hat{y}) + ({}_{\alpha}f_{100} Y_{100}) ({}_{\alpha}v Y_{100} \hat{z}) \} \\ & \times [({}_{\alpha}v \sin \theta d \phi) ({}_{\alpha}v d \theta) (d {}_{\alpha}v)] \quad (\text{III-15a}) \end{aligned}$$

which reduces to

$$\langle {}_{\alpha}\vec{v} \rangle = \frac{4\pi}{3} \int_0^{\infty} {}_{\alpha}f_1 {}_{\alpha}v^3 d {}_{\alpha}v \quad . \quad (\text{III-15b})$$

Equations (III-2), (III-8), (III-9), (III-11) and (III-15b)

yield

$$\begin{aligned} \vec{J} = ne \left[\frac{4\pi}{3} \int_0^\infty j_i \vec{S}_2 \cdot \vec{a}_i \frac{\partial f_i^0}{\partial v} v^3 dv \right. \\ \left. - \frac{4\pi}{3} \int_0^\infty j_e \vec{S}_2 \cdot \vec{a}_e \frac{\partial f_e^0}{\partial v} v^3 dv \right] \end{aligned} \quad (\text{III-16a})$$

$$\begin{aligned} \vec{J} = \left[j \frac{4\pi e^2}{3} \left(\int \vec{S}_2 \frac{\partial f_i^0}{\partial v} \frac{v^3}{v_m} dv \right. \right. \\ \left. \left. - \int \vec{S}_2 \frac{\partial f_e^0}{\partial v} \frac{v^3}{v_m} dv \right) \cdot \vec{E} \right] \end{aligned} \quad (\text{III-16b})$$

Therefore by Eq. (III-10), the conductivity tensor is the quantity in the square brackets in the last equation.

If f_α^0 "vanishes sufficiently strongly at the upper limit," Eq. (III-16b) can be integrated by parts to yield⁴⁰

$$\begin{aligned} \vec{\sigma}_2 = \left[- j e^2 \left(\frac{4\pi}{3} \right) \left(\int \frac{f_i^0}{v_m} \frac{\partial}{\partial v} (\vec{S}_2 v^3) dv \right. \right. \\ \left. \left. - \int \frac{f_e^0}{v_m} \frac{\partial}{\partial v} (\vec{S}_2 v^3) dv \right) \right] \end{aligned} \quad (\text{III-17})$$

If f_α^0 is assumed to be Maxwellian, then

$$f_\alpha^0 = n \left(\frac{m}{2\pi K T} \right)^{3/2} e^{-\left(\frac{m}{2 K T} v^2 \right)} \quad (\text{III-18})$$

where α^T is defined by

$$\frac{3 K \alpha^T}{\alpha^m} \underline{\Delta} \langle \alpha v^2 \rangle = \frac{4\pi}{n} \int \alpha f_O^O \alpha v^2 (\alpha v^2 d \alpha v) . \quad (\text{III-19})$$

" αv_ℓ " Independent of Velocity

If αv_0 , αv_1 , and αv_2 are independent of velocity, then each ij element of $\vec{\sigma}_2$ has the general form of $\alpha A_{ij} + \alpha B_{ij} (\alpha \vec{v} \cdot \alpha \vec{v})$, where αA_{ij} and αB_{ij} are appropriate constants. Equation (III-17) may then be written as⁴⁰

$$\begin{aligned} (\vec{\sigma}_2)_{ij} &= \frac{4\pi}{3} \int (3 {}_i A_{ij} {}_i v^2 + 5 {}_i B_{ij} {}_i v^4) d {}_i v \\ &\quad - \frac{4\pi}{3} \int (3 {}_e A_{ij} {}_e v^2 + 5 {}_e B_{ij} {}_e v^4) d {}_e v \quad (\text{III-20a}) \end{aligned}$$

$$\begin{aligned} (\vec{\sigma}_2)_{ij} &= 4\pi \int ({}_i A_{ij} + \frac{5}{3} {}_e B_{ij} {}_i v^2) {}_i f_O^O {}_i v^2 d {}_i v \\ &\quad - 4\pi \int ({}_e A_{ij} + \frac{5}{3} {}_e B_{ij} {}_e v^2) {}_e f_O^O {}_e v^2 d {}_e v . \quad (\text{III-20b}) \end{aligned}$$

The integral in Eq. (III-19) appears twice in Eq. (III-20b). Therefore, these integrals can be evaluated by means of Eq. (III-19) to get

$$(\vec{\sigma}_2)_{ij} = n({}_i A_{ij} + \frac{5 K {}_i^T}{{}_i^m} {}_i B_{ij}) - n({}_e A_{ij} + \frac{5 K {}_e^T}{{}_e^m} {}_e B_{ij}) . \quad (\text{III-21})$$

Equation (III-21) gives the same result as Eq. (III-17) only when ω is much greater than the dominant collision frequency, ν_{dominant} .³⁹ (Sect. 4-1) Outside of this range, the respective real and imaginary parts of the denominators of Eqs. (III-17) and (III-21) are no longer identical. This difference is a result of the velocity dependent collision frequencies which causes the real and the imaginary parts of the denominator of Eq. (III-17) to have different velocity dependences. (For future reference, the " g_{π} " and " h_{π} " factors used in Case II of Ch. IV multiply the real and the imaginary parts of the denominator of Eq. (III-21) so that Eq. (III-21) with the " g_{π} " and " h_{π} " gives the same result as Eq. (III-17) for any value of ω .)

" ν_{ℓ} " Dependent on Velocity

The derivation through Eq. (III-19) is valid for any velocity dependence of ν_{ℓ} . However, the integrals in Eq. (III-17) are not easily solved analytically unless the ν_{ℓ} 's are constant. Thus, it is advantageous to "judiciously" define constant ν_{ℓ} 's such that Eq. (III-21) gives the same numerical "result" as Eq. (III-17) evaluated with ν_{ℓ} as the correct functions of velocity. In general, this cannot be done because it is impossible for both the real and the imaginary parts of Eqs. (III-21) and (III-17) to be equal by one "judicious" definition of constant ν_{ℓ} 's.

The reason for this is given in the paragraph following Eq. (III-21).

The primary goal of the theory presented here is to explain why the second harmonic resonance is located $\sim 10\%$ away from $2_i\Omega$. Therefore, we are more interested in the imaginary part of the denominator of $\vec{\sigma}_2$ (the location of resonances) than in the real part of the denominator (the rate at which the wave is damped at resonances). Thus, one should always be able to find "judicious" v_ℓ 's that are independent of velocity such that only the imaginary part of the denominator of Eq. (III-21) is identical to Eq. (III-17) evaluated with the ${}_a v_\ell$'s as the correct functions of velocity. In the next chapter, some of these "judicious" ${}_a v_\ell$'s will be determined.

CHAPTER IV

THEORY OF COLLISION FREQUENCIES

The determination of collision frequencies is very difficult since intuitive misconceptions and mathematical artifices abound. These problems are further complicated by the non-uniform plasmas utilized in most experiments. This is especially true when the non-uniformities are not exactly known. Because of these difficulties, it is much wiser to find the bounds within which the important collision frequency must lie than it is to try finding the exact collision frequency itself.

The following discussion assumes a familiarity with most of the literature on collisions as there is insufficient room here for a review of collisional theory.³⁹⁻⁴⁷ Only many small angle, Coulomb collisions will be considered. Several different collision frequencies will be discussed in order to provide bounds for the collision frequencies considered important in this experiment.

The theory of collision frequencies will be described in two parts. Case I will neglect collisions between like particles while Case II will be concerned with collisions between ions only. In addition, Case I will neglect the

energy lost during a collision, whereas Case II will include the energy lost during a collision. Furthermore, all cases will include second harmonic resonances. Only Case I will include fundamental resonance effects. All cases will include the effects of deflections in collisions.

Even though the results of this chapter will be evaluated in Ch. V, it is advantageous to preview some of the findings to be given in Ch. V. The value of $i v_2$ from Case Ia will give the observed shift of the second harmonic resonance. The value of $i v_1$ from Case Ia will be shown to give the observed width of the second harmonic resonance and the observed damping length of the fast wave.

Even though the results of Case Ib will not give the observed shift of the second harmonic resonance, this case is presented for completeness. The results of Case II will also yield the observed shift.

Case I, Collisions Between Ions and Electrons

Two effects which are essentially restatements of the law of conservation of momentum should be noted at this point. First, the momentum gained (lost) by an ion (electron) in an i-e or e-i collision is exactly equal and opposite to the momentum lost (gained) by the other collision partner. Secondly, there is no momentum transfer (on the average) between like particle collisions as long as each species is in "equilibrium."³⁹ (Sects. 7-7 and 8-6), 42 (Ch. 5)

The second effect is based on "inverse collisions."
 42 (p. 227) This is illustrated by first assuming that
 there is a collision (with impact parameter b) such that

$$m \vec{v} + m_1 \vec{v}_1 \rightarrow m \vec{v}' + m_1 \vec{v}_1' \quad . \quad (\text{IV-1})$$

Statistically, when equilibrium is present, there exists
 another collision (with the same impact parameter b) such
 that

$$m \vec{v}' + m_1 \vec{v}_1' \rightarrow m \vec{v} + m_1 \vec{v}_1 \quad . \quad (\text{IV-2})$$

This is just the inverse collision of Eq. (IV-1). Thus, in
 one collision (Eq. IV-1) a particle is scattered from its
 initial momentum (i.e. $m \vec{v}$) but in the inverse collision
 (Eq. IV-2) there is another particle scattered into that
 same momentum (i.e. $m \vec{v}$). Equilibrium is required so that
 Eqs. (IV-1) and (IV-2) proceed at the same rate. Therefore,
 on a momentum basis, effectively no collision took place.
 Thus, the collisions between like particles in equilibrium
 maybe neglected for problems concerned with momentum trans-
 port.

The net result of these two effects is that a "sea"
 of electrons in equilibrium and a "sea" of ions in equi-
 librium experience a mutual drag (i.e. an exchange of

momentum) which is proportional to the velocity difference between the two "seas" as they move through each other. This type of electron-ion motion is present in those types of plasma waves which have a long wavelength, provided that each species is in equilibrium with itself. Examples of such plasma waves are small amplitude fast or slow waves far from resonances (i.e. they have a long wavelength). Examples of waves not fitting such a description are any wave whose amplitude is large enough to drive a species out of equilibrium or any wave whose species motion is too anisotropic (i.e. strictly speaking, the fast or slow wave at any resonance, including resonances at the harmonics of the cyclotron frequencies because at resonance the wavelengths are too short for the "sea" analogy to be accurate). In practice however, these results are often quite valid at resonances.³⁹ (Ch. 4), 46 (Ch.5) (For example, it is common practice to measure the shift and width of cyclotron resonances and then to calculate the collision frequency.)³⁹,
⁴⁶ In many experiments, the short wavelengths which are theoretically predicted at resonances do not occur.

Case Ia, Constant " α_{ℓ} ;" Deflections

The easiest and perhaps the most often used collision frequencies are the ones based on the assumption that the energy lost by a particle during a collision is negligible.

This assumption is reasonably valid for light electrons being slightly deflected by collisions with slower, heavier ions with negligible energy transfer. Thus, the magnitude of the electron's velocity remains unchanged but the direction is changed.

This assumption of negligible energy loss is often applied to ions colliding with electrons. This is a questionable practice because the ion in an ion-electron collision is slowed down (i.e. loses energy) but is only slightly deflected because the ion is much heavier than the electron. Nonetheless, this practice often leads to correct results for many types of plasma waves. (See the discussion just before Case Ia.)

The derivation of the collision frequencies which neglect the loss of energy during a collision is found in the literature.⁴⁰ (p. 1222) The αv_l 's used in Ch. III are, for electrons,

$$e^v_o = 0, \quad e^v_1 = e^v_{ei}, \quad e^v_2 = 3 e^v_{ei} \quad , \quad (IV-3a)$$

and similarly, for ions,

$$i^v_o = 0, \quad i^v_1 = i^v_{ie}, \quad i^v_2 = 3 i^v_{ie} \quad , \quad (IV-3b)$$

where

$$e^v_{ei} = \frac{4\pi n}{e^v_3} \left(\frac{e^2}{4\pi\epsilon_o e^m} \right)^2 \ln \left(\frac{3(4\pi\epsilon_o KT)^{3/2}}{2_e (\pi n)^{1/2}} \right) \quad (\text{IV-3c})$$

and

$$i^v_{ie} = \frac{4\pi n}{i^v_3} \left(\frac{e^2}{4\pi\epsilon_o i^m} \right)^2 \ln \left(\frac{3(4\pi\epsilon_o KT)^{3/2}}{2_e (\pi n)^{1/2}} \right) \quad (\text{IV-3d})$$

These ${}_a v_\ell$'s, the e^v_{ei} and the i^v_{ie} are velocity dependent.

In order to obtain velocity independent ${}_a v_\ell$'s, one can evaluate Eqs. (IV-3c) and (IV-3d) using the most probable speed, ${}_a v_o$, of the Maxwellian distribution of each species in place of ${}_a v$:

$$\langle e^v_{ei} \rangle \triangleq e^v_{ei} \Big|_{e^v} = e^v_o \triangleq 2 K e^T / e^m \quad (\text{IV-3e})$$

and

$$\langle i^v_{ie} \rangle \triangleq i^v_{ie} \Big|_{i^v} = i^v_o \triangleq 2 K i^T / i^m \quad (\text{IV-3f})$$

This makes ${}_a v_\ell$ independent of velocity so that the simpler Eq. (III-21) maybe used instead of Eq. (III-17).

With the value for i^v_2 from Eqs. (IV-3f) and (IV-3b), the location of the second harmonic resonance can be calculated from Eqs. (III-21) and (III-9) and the definitions of

$\alpha A_{ij} + \alpha B_{ij} (\alpha \vec{v} \cdot \alpha \vec{v})$. The resultant location is at

$$\omega = \sqrt{4_i \Omega^2 + i v_2^2}, \quad \omega \gg i v_2, \quad (IV-4a)$$

with a width of

$$\Delta\omega = i v_2, \quad \omega \gg i v_2, \quad (IV-4b)$$

where $i v_2 \triangleq 3 \langle i v_{ie} \rangle$. This result will be used in Ch. V.

Case Ib, Constant " $i v_1 = i v_2$ "

If a small signal model is assumed, then the thermal motion of the particles should predominate over the wave motion of the particles. In other words, the ions spiral around the magnetic field lines with a gyro-frequency of $i \Omega$ and with a gyro-radius which is determined by the ion thermal velocity. Furthermore, this spiralling ion motion is essentially unaffected by either fundamental or second harmonic resonances. Because of "inverse collisions," these ions experience collisions with electrons only and the dominant collision frequency is $\langle i v_{ie} \rangle$ (Eq. (IV-3f)). If the wave fields are sufficiently small so that thermal "equilibrium" is not disturbed, then $\langle i v_{ie} \rangle$ is the dominant collision for both the fundamental and the second harmonic resonances. Thus, the thermal motion and not the type of

resonance determines the dominant collision frequency.

Therefore, $i\nu_2$ is equal to $\langle i\nu_{ie} \rangle$ and the location of the second harmonic resonance from Eqs. (III-21) and (III-9) is

$$\omega = \sqrt{4i\Omega^2 + i\nu_2^2}, \quad \omega \gg i\nu_2, \quad (IV-5a)$$

and the width is

$$\Delta\omega = i\nu_2, \quad (IV-5b)$$

where

$$i\nu_2 \triangleq \langle i\nu_{ie} \rangle. \quad (IV-5c)$$

Case II, Ion-Ion Collisions

In Case I, it was possible to ignore like particles collisions because each species was assumed to be in "equilibrium." In Case II, it is assumed that the ions are sufficiently anisotropic that the effects of collisions between like particles predominate over the effects of collisions between species (which will be neglected).

In the literature, there exists a method for calculating the traceless ion pressure tensor, $\vec{\pi}_2$, that includes second harmonic effects and the energy lost during a collision.³⁹ (Sect. 8-8) This $\vec{\pi}_2$ and a $\vec{\sigma}_2$ which includes only

ion second harmonic effects are "equivalent" because both are based on $i\vec{f}_2$.

The approach, then, is to calculate the collision frequency, ν_π , responsible for the relaxation of $\vec{\pi}_2$. This collision frequency, which is independent of velocity is used as $i\nu_2$ and Eq. (III-21) is used to calculate a $\vec{\sigma}_2$ which includes both ion and electron inertia and both the fundamental and the second harmonic effects.

Normally, Eq. (III-21) is not very accurate because the dependence of the collision frequency on velocity was neglected in the derivation of this equation. This statement is not strictly true in this case because the derivation of $\vec{\pi}_2$ does include the dependence of like particle collisions on velocity. On the other hand when calculating $\vec{\sigma}_2$, the use of ν_π as $i\nu_2$ in Eq. (III-21) is still not as accurate as the use of the velocity dependent ion-ion collision frequency $i\nu_2$ was in Eq. (III-17). (See the discussion starting after Eq. III-21 through the end of Ch. III.)

Because of space limitations, the reader is referred to the literature for the finer details of the derivation.³⁹ (Sect. 8-8) The distribution function for ions, expanded in "intrinsic velocity space," is³⁹ (Sect. 8-8)

$$i^f = i^f_0 + i\vec{f}_1 \cdot \frac{\vec{w}_+}{w_+} + i\vec{f}_2 : \frac{\vec{w}_+ \vec{w}_+}{w_+^2} , \quad (\text{IV-6a})$$

where w_+ is the "intrinsic" velocity of the ion,

$$\vec{w}_+ \triangleq \vec{i} \vec{v} - \vec{c} \quad , \quad (\text{IV-6b})$$

and where $\vec{c}$ is the average velocity of the plasma,

$$\vec{c} \triangleq \frac{e^{mn} e^{\vec{v}} + i^{mn} i^{\vec{v}}}{e^{mn} + i^{mn}} \quad . \quad (\text{IV-6c})$$

The tensor relation for $i^{\vec{f}}_2$ is³⁹ (Sect. 8-8)

$$\frac{\partial i^{\vec{f}}_2}{\partial t} + i^{\vec{\Omega}} \times i^{\vec{f}}_2 - i^{\vec{f}}_2 \times i^{\vec{\Omega}} - \frac{w_+}{2} \frac{d i^f_o}{dw_+} \vec{U} = \frac{\delta i^{\vec{f}}_2}{\delta t} \quad , \quad (\text{IV-7a})$$

where

$$\vec{U} = \nabla \vec{c} + (\nabla \vec{c})^T - \frac{2}{3} I_2 \nabla \cdot \vec{c} \quad . \quad (\text{IV-7b})$$

The $i^{\vec{f}}_2$ and $\vec{U}$ elements are written in an equivalent "dyadic polarization form" of³⁹ (Sect. 8-8)

$$F_a = i^f_{xx} + i^f_{yy} = - i^f_{zz} \quad , \quad (\text{IV-8a})$$

$$U_a = U_{xx} + U_{yy} = - U_{zz} \quad , \quad (\text{IV-8b})$$

$$F_b = i^f_{yz} \pm j i^f_{xz} \quad , \quad (\text{IV-8c})$$

$$U_b = U_{yz} \pm j U_{xz} \quad , \quad (\text{IV-8d})$$

$$F_c = i f_{xx} - i f_{yy} \pm 2 j i f_{xy} \quad , \quad (\text{IV-8e})$$

and

$$U_c = U_{xx} - U_{yy} \pm 2 j U_{xy} \quad , \quad (\text{IV-8f})$$

where a, b and c represent the three "dyadic polarizations." Each of the F_n , $n = a, b, \text{ or } c$, are expanded in a series of Laguerre polynomials of order 5/2. Thus,³⁹ (Sect. 8-8)

$$i F_n = w_+^2 i F_0 \sum_{r=0}^{\infty} R_r L_r^{5/2} \left(\frac{i^m w_+^2}{2 K i^T} \right) \quad , \quad (\text{IV-9a})$$

where $i F_0$ is a Maxwellian

$$i F_0 = n \left(\frac{i^m}{2\pi K i^T} \right)^{3/2} e^{(- \frac{i^m w_+^2}{2 K i^T})} \quad , \quad (\text{IV-9b})$$

and where R_r are the coefficients, to be determined, of the Laguerre polynomials, $L_r^{5/2}$, with a functional argument $(\frac{i^m w_+^2}{2 K i^T})$. Equation (IV-7b) is then solved using the expansion given in Eq. (IV-9b). The result is³⁹ (Sect. 8-8)

$$\pi_n = \frac{p_+ U_n}{g\pi \langle v_{ii} \rangle + j h\pi (\omega \pm \eta i \Omega)} \quad , \quad (\text{IV-10a})$$

where $\eta = 0, 1$ or 2 ,

$$p_{+} \approx n K_i T \quad (\text{IV-10b})$$

and

$$\begin{aligned} \langle v_{ii} \rangle &\approx \frac{4\pi n}{i v^3} \left(\frac{e^2}{4\pi \epsilon_0 i m} \right)^2 \\ &\times \ln \left(\frac{3(4\pi \epsilon_0 K_i T)^{3/2}}{2 e (\pi n)^{1/2}} \right) \quad \Bigg| \quad i v = i v_0 \approx \frac{2 K_i T}{i m} \end{aligned} \quad (\text{IV-10c})$$

or

$$\langle v_{ii} \rangle = \left(\frac{e^m}{i m} \right)^{1/2} \langle e^v e_i \rangle \quad (\text{IV-10d})$$

The g_π and h_π are functions of $|\omega \pm \eta i \Omega| / \langle v_{ii} \rangle$ whose values vary smoothly between the limits³⁹ (Sect. 8-8) given in Table IV-1 below.³⁹ (Sect. 8-8)

TABLE IV-1
LIMITS OF " g_π " and " h_π "

$\frac{ \omega \pm \eta i \Omega }{\langle v_{ii} \rangle}$	g_π	h_π	Comments
$\gg 1$	≈ 0.85	≈ 1.00	Small collision limit
$\ll 1$	≈ 0.73	≈ 1.14	Large collision limit

One can obtain the characteristic collision frequency for this case by dividing Eq. (IV-9a) by h_π to obtain

$$\pi_n = \frac{-p + U_n/h_\pi}{\frac{g_\pi}{h_\pi} \langle v_{ii} \rangle + j(\omega + \eta_i \Omega)} \quad . \quad (\text{IV-11})$$

The collision frequency, ν_π , responsible for the relaxation of $\vec{\pi}$, and hence for $i\vec{f}_2$ is

$$\nu_\pi = \frac{g_\pi}{h_\pi} \langle v_{ii} \rangle \quad . \quad (\text{IV-12})$$

From Eqs. (IV-12) and (III-21), the second harmonic ion resonance occurs at

$$\omega = \sqrt{4i\Omega^2 + \nu_\pi^2}, \quad \omega \geq \nu_\pi \quad , \quad (\text{IV-13a})$$

with a width of

$$\Delta\omega = \nu_\pi \quad , \quad (\text{IV-13b})$$

where

$$i\nu_2 \triangleq \nu_\pi \quad . \quad (\text{IV-13c})$$

The results of this chapter are summarized in Table IV-2. The numerical calculations of all of these collision frequencies and the application of this theory to the experiment is given in Ch. V.

TABLE IV-2

SUMMARY OF THE RESULTS OF CHAPTER IV

Location of Second Harmonic Resonance	Width of Second Harmonic Resonance	Comments on Model Used and Reference Eqs.
$\omega = \sqrt{4_i \Omega^2 + i^{\nu 2}}$	$\Delta\omega = i^{\nu 2} \triangleq 3 \langle i^{\nu} i^e \rangle$	Case Ia, ion-electron collisions, neglects energy loss during a collision, $i^{\nu 2}$ is velocity independent, Eq. (IV-4).
$\omega = \sqrt{4_i \Omega^2 + i^{\nu 2}}$	$\Delta\omega = i^{\nu 2} \triangleq \langle i^{\nu} i^e \rangle$	Case Ib (and Ia), ion electron collisions, neglect energy loss during a collision, $i^{\nu 2}$ is velocity independent, small signal model (i.e. thermal equilibrium and not the wave motion determines the collision frequency), Eq. (IV-5)
$\omega = \sqrt{4_i \Omega^2 + i^{\nu 2}}$	$\Delta\omega = i^{\nu 2} \triangleq \nu_{\pi} \triangleq \frac{g_{\pi}}{h_{\pi}} \langle \nu_{i i} \rangle$	Case II, ion-ion collisions, based on $i^{\nu 2}$ only, Eq. (IV-13).

CHAPTER V

APPLICATION OF THEORY TO THE EXPERIMENT

Chapter IV discussed the effects of collisions either between ion and electrons (Case I) or between ion and ions (Case II). That discussion was also limited to long wavelength waves in an infinite uniform plasma.

In this experiment, the plasma is non-uniform and bounded. In addition, both electron-ion and ion-ion collisions, as well as, both fundamental and second harmonic effects are all present. Thus, the experimental conditions and the theoretical assumptions are quite different from each other.

These differences between the experimental conditions and the theoretical assumptions make it quite difficult to apply the theory to the experiment because one does not know, in advance, which theoretical assumptions are experimentally applicable. It is possible however to use the experimental observations to determine which theoretical assumptions are applicable. In other words, the theory can be used as a "confidence check" for the experimental observations.

The discussion in this chapter is limited to a hydro-

gen plasma. The results for a deuterium plasma are not presented because they are similar to the results presented here for a hydrogen plasma.

The Shift of the Second Harmonic

Resonance Away from $2_i\Omega$

The various collision frequencies derived in Ch. IV are plotted versus the plasma density, n , and the plasma temperature, T , in Fig. V-1. Table V-1 correlates these collision frequencies to the theoretically calculated "percent shifts" of the second harmonic resonance away from the second harmonic of the ion cyclotron frequency, $2_i\Omega$. The experimentally observed "percent shift" from Fig. II-2 is used with Table V-1 to determine a "collision frequency." This "collision frequency" and Fig. V-1 are used to determine the possible combinations of densities and temperatures which are required to give that "collision frequency." In Table V-2, these combinations of n and T are then compared to the values of n and T given in Table II-1 to see if they can exist experimentally somewhere in the radial profile of the plasma.

Table V-1 is essentially a summary of the relationships between the percent shift of the second harmonic resonance from $2_i\Omega$, the value of B_0 at the center of the resonance and the value of $i v_2$ which is theoretically

Fig. V-1. Curves of constant collision frequency versus the density and temperature of a hydrogen plasma.

TABLE V-1

THE THEORETICAL LOCATION OF THE SECOND HARMONIC RESONANCE IN RELATION TO " ν_2 "

Percent shift of the resonance center of the second harmonic resonance	B_0 at the resonance center of the second harmonic resonance	$\nu_2 = \sqrt{\omega^2 - 4\left(\frac{e}{i} B_0\right)^2}$ at resonance center from Eqs. (IV-4), (IV-5) or (IV-13)
$\left(\omega - \frac{2eB_0}{i}\right)$ at resonance center $\frac{\omega - (\text{percent shift}) \times \frac{\omega}{100\%}}{2e/i}$	$\sqrt{\frac{\omega^2 - \nu_2^2}{4e/i}}$ from Eqs. (IV-4), (IV-5) or (IV-13)	
$= \frac{\omega - (\text{percent shift}) \times \frac{\omega}{100\%}}{2e/i}$		
0 %	~ 0.17 T	$\sim 0.0 \times 10^7$ rad/sec
5 %	~ 0.16 T	$\sim 1.1 \times$
10 %	~ 0.15 T	$\sim 1.4 \times$
15 %	~ 0.14 T	$\sim 1.8 \times$

TABLE V-2

COMPARISON OF THEORY AND THE EXPERIMENT

Curve Number. Assumption. Ref. Equation. ($i^v_2 \triangleq 1.4$ 10^7 rad/sec)	Approximate Range of Values of the Ordered Pairs of n and T for Each Curve Number	Possible Locations of Each Ordered Pair in the Experimental Plasma	Reason Why the Location is Possi- ble. The Percent Ionization. The Validity of the Theory Based on the Percent Ionization
1 $i^v_2 \triangleq \frac{g_\pi}{h_\pi} \langle v_{ii} \rangle$ Eq. (III-13)	$n \approx 3.3 \times 10^{20}/m^3$ $T \approx 4$ eV to $n \approx 2.0 \times 10^{20}/m^3$ $T \approx 3$ eV	Near the center of the plasma column.	The plasma is denser and hotter in the center of the column than anywhere else. The plasma is 100% ionized. The theory is valid.
8 $i^v_2 \triangleq 3 \langle i^v_{ie} \rangle$ Eq. (III-4)	$n \approx 3.5 \times 10^{20}$ $T \approx 1.3$ eV to $n \approx 2.6 \times 10^{20}/m^3$ $T \approx 0.8$ eV	Near the edge of the plasma column (be- tween the locations given for curves 1 and 10).	The ion density exceeds the neu- tral density and the tempera- ture is consistent with the percent ionization. The plasma is at least 50% ionized but not 100% ionized. The theory is valid.
Curve below number 10 $i^v_2 \triangleq \langle i^v_{ie} \rangle$ Eq. (IV-5)	$n \approx 3.5 \times 10^{20}/m^3$ $T \approx 0.2$ eV	Extreme outer edge of plasma column.	The location is just inside the un-ionized background gas which surrounds the plasma column. The percent ionization is $\approx 0\%$. The theory is invalid.

required to give that percent shift and location. The entries in Table V-1 are derived theoretically from the equations listed in this table. The last equation in columns one and two is merely the relation between these two columns.

From Fig. II-2, it can be seen that the second harmonic resonance is shifted about 10% (i.e. $B_0 \approx 0.15$ T) from where it would be if there were no collisions (i.e. $B_0 \approx 0.17$ T). Table V-1 shows that $i\nu_2 \approx 1.4 \times 10^7$ rad/sec is theoretically required to give the observed 10% shift. This value of $i\nu_2$ will be used with Fig. V-1 to determine the ordered pairs of density and temperature which give $i\nu_2 \approx 1.4 \times 10^7$ rad/sec.

Figure V-1 is a graph of plasma density, n , versus plasma temperature, T , with curves of constant $\langle \nu_{ei} \rangle$ or constant $i\nu_2$. The figure represents a theoretical plasma with a density of n and a temperature of T . Each of the arbitrarily numbered curves corresponds to only one number on each of the four axes on the right of the figure. It can be shown from Eqs. (IV-5), (IV-4) and (IV-13) that $i\nu_2$ can be equal to $\langle \nu_{ie} \rangle$, $3\langle \nu_{ie} \rangle$ or $g_\pi \langle \nu_{ii} \rangle / h_\pi$, respectively, depending on what collisional model is chosen. From the original definitions of the collision frequencies, it can be shown that $\langle \nu_{ei} \rangle \triangleq \sqrt{i m / e m} \langle \nu_{ii} \rangle \triangleq i m / e m \langle \nu_{ie} \rangle$. Further-

more, h_π and g_π are defined by ω and $\langle v_{ii} \rangle$. Thus, any one chosen number on any one chosen axis of the five axes corresponds to one number on the other four axes. For example, curve 10 defines ordered pairs of n and T such that $\langle v_{ei} \rangle \approx 16. \times 10^9$ rad/sec or $i v_2 \triangleq \langle i v_{ie} \rangle \approx 9 \times 10^6$ rad/sec or $i v_2 \triangleq 3 \langle i v_{ie} \rangle \approx 27. \times 10^6$ rad/sec or $i v_2 \triangleq g_\pi \langle v_{ii} \rangle / h_\pi \approx 24. \times 10^7$ rad/sec.

It should be noted that the n in Fig. V-1 does not include neutrals because the collision frequencies which are also given in Fig. V-1 were derived by assuming that no neutrals were present. However, in the following discussion, n can be taken to include neutrals. Because of collisions, the neutrals are dragged along by the ions as the ions respond to the electric field of the wave. Thus, the ions and neutrals move together with the same wave motion. This coupling between the ions and neutrals is not perfect. Nonetheless, including the neutral density in n is a simple means of including the effects of neutrals on wave motion. Also the collisions between neutrals and electrons or ions are often about the same as the collisions between ions and electrons or other ions. The net result, is that n can include the neutral density. This is not too bad an assumption if the plasma is at least 50% ionized. Thus, any application of theory presented in Chs. III and IV is considered invalid if the plasma less than 50%

ionized. This criteria is used in Table V-2.

From Fig. II-1 and Table V-2 it was found that $i\nu_2$ had to be equal to 1.4×10^7 rad/sec in order to give the observed shift of the second harmonic resonance. Depending on what model is used, there are three choices for $i\nu_2$. First, $i\nu_2$ can equal $g_\pi \langle \nu_{ii} \rangle / h_\pi$ (i.e. Eq. IV-13). Curve number 1 corresponds to this case where $g_\pi \langle \nu_{ii} \rangle / h_\pi \triangleq i\nu_2 \approx 1.4 \times 10^7$ rad/sec. Secondly, $i\nu_2$ can equal $3 \langle \nu_{ie} \rangle$ (i.e. Eq. IV-4) and curve 8 represents this case. Thirdly, $i\nu_2$ can equal $\langle \nu_{ie} \rangle$ (i.e. Eq. IV-5) and a curve below number 10 must be used.

Each of these three curves represents a set of ordered pairs of n and T of a theoretical plasma. It is now necessary to determine which of these ordered pairs represent a plasma which could possibly exist somewhere in the experimental arrangement. This determination is made with the help of Table II-1 and a knowledge of the probable radial profiles of densities and temperatures in the experimental plasma. In this experiment one expects that the center of the plasma column is fully ionized with a higher temperature and ion density than the outer edge of the plasma which is cold and only partially ionized. The higher temperature and ion density in the center must balance the lower temperature and ion density at the edge in such a way that the average temperature and ion density

agree with the observations in Table II-1.

Also the ion and neutral densities must equal the total density so that the percent ionization is consistent with the ordered pair of n and T from Fig. V-1. In other words, the ordered pair of n and T must be consistent with the ion and neutral density and the percent ionization. The consistency is determined by the Saha equation. In Table V-2, the ordered pairs of n and T from Fig. V-1 are compared to the densities and temperatures found in the experimental plasma.

In Table V-2, the three curve numbers from Fig. V-1, the three assumptions used for $i v_2$ and the three reference equations for each assumption are given in the first column. Column two gives the approximate range of values from Fig. V-1 of the ordered pairs of n and T for each of the three curves considered. In the third column, the probable location of the plasma represented by the ordered pairs in the second column is given. The last column explains why the locations given in the third column were chosen. The percent ionization of the plasma at the location given in column three is also given in column four. The percent ionization is used to determine if the theory is valid.

From an examination of Table V-2, one can see that the plasmas represented by all three curves could exist in this

experiment. However, only curves 1 and 8 give a shift which is theoretically valid (i.e. the other curve represents a plasma which is less than 50% ionized). Curve 1 was derived from a model using ion-ion collisions; while curve 8 was based on ion-electron collisions. Both collision frequencies are smaller in the hot center of the plasma column than in the colder periphery. Only near the edge is $3\langle v_{ie} \rangle$ large enough to account for the observed shift. In contrast, $g_{\pi}\langle v_{ii} \rangle/h_{\pi}$ is sufficiently small only in the center to give the shift.

A possible scenario in which both $3\langle v_{ie} \rangle$ and $g_{\pi}\langle v_{ii} \rangle/h_{\pi}$ could simultaneously play a role in the plasma in this experiment is as follows. In the hot center of the plasma, the ion-electron collision frequency is too small to limit the ion motion at the second harmonic resonance. As a result, the ion motion becomes sufficiently anisotropic that ion-ion collisions limit the resonance (i.e. give the observed shift). Near the cold edge of the plasma, the ion-electron collisions account for the observed shift. Because the ion-electron collisions limit the resonance, the ion motion does not become anisotropic and ion-ion collisions can be ignored.

It can be seen from Fig. II-2 that there is a region of damping located much closer to $2_i\Omega$ than the second harmonic resonance just discussed. This region is also not

as wide or as deep as the damping located $\sim 10\%$ below $2_i\Omega$. This region is believed to be the result of damping at the second harmonic resonance by the particles in the "high energy tail" which were produced by turbulent heating. The relative narrowness of this region could be because the particles in the high energy tail have a relatively small collision frequency (i.e. $i\nu_2 \approx 0$) as a result of their relatively high velocities (i.e. $\Delta\omega = i\nu_2 \approx 0$). This relatively small collision frequency could also explain why this region is essentially unshifted from $2_i\Omega$ (i.e. $\omega = \sqrt{4_i\Omega^2 + i\nu_2^2} \approx 2_i\Omega$). The relatively shallow depth of this region could be the result of the relatively few number of particles in the high energy tail.

Damping Length of the Fast Wave Far from any Resonances

It has been shown in previous work that the ion and electron velocities which result from the wave motion are about the same for the fast wave when resonances are not present.¹ (Fig. II-3) However, the ions have much more of the particle momentum than the electrons have because the ions are much more massive than the electrons. Therefore, $i\nu_1$, and not $e\nu_1$, should determine the decay rate for the fast wave as it propagates. From Eqs. (IV-3b) and (IV-3f) $i\nu_1$ is equal to $\langle i\nu_{ie} \rangle$. Experimentally, it was found that the amplitude of the fast wave decays to $1/e$ of its

original value in about one time period of the 5.2 MHz wave frequency. Therefore, one can calculate $i\nu_1$ from

$$\frac{1}{5.2 \text{ MHz}} \times i\nu_1 \approx 1 \quad . \quad (V-1a)$$

Thus,

$$i\nu_1 \approx 5.2 \times 10^6 \text{ rad/sec} \quad . \quad (V-1b)$$

From Eqs. (IV-3f), (IV-3b) and (V-1b), one can further calculate $\langle i\nu_{ie} \rangle$ as

$$i\nu_1 \approx \langle i\nu_{ie} \rangle \quad . \quad (V-2a)$$

Therefore,

$$\langle i\nu_{ie} \rangle \approx 5.2 \times 10^6 \text{ rad/sec} \quad . \quad (V-2b)$$

This value of $\langle i\nu_{ie} \rangle$ corresponds to curve 8 in Fig. V-1, which is one of the curves which theoretically explain the shift of the second harmonic resonance. This indicates that the ion-electron collision frequency (i.e. Eq. IV-4) maybe the dominant collision frequency.

Width of the Second Harmonic Resonance

From Table V-1, it was already found that $i\nu_2$ had to be $1.4 \times 10^7 \text{ rad/sec}$ in order for the second harmonic

resonance to be shifted 10% from $2_i\Omega$. If this value of $_i v_2$ is substituted into Eq. (IV-4b), the width of the second harmonic resonance is found to be about 0.08 T. This is about four times the observed width of about 0.02 T.

From Eq. (V-1), it was shown that $_i v_1$ had to be equal to about 5.2×10^6 rad/sec in order for the fast wave to have the observed damping length. If this value of $_i v_1$ is used as $_i v_2$, then Eq. (IV-5b) gives the width of the second harmonic resonance as approximately 0.025 T. This agrees fairly well with the experimental observations.

The reason why $_i v_2$ gives the observed shift but not the observed width, while $_i v_1$ gives the observed width but not the observed shift, is as follows. It was pointed out after Eq. (III-21) that the real and the imaginary parts of the denominator were affected differently because of the velocity dependence of the collision frequencies. With this and the discussion at the end of Ch. III, one can deduce that one constant collision frequency is needed so that the real part of the denominators of Eqs. (III-17) and (III-21) are equal. Furthermore, another constant collision frequency is needed so that the imaginary parts of the denominators of Eqs. (III-17) and (III-21) are equal. This is essentially the reason why 1.4×10^7 rad/sec is needed to give the shift of the second harmonic resonance and 5.2×10^6 rad/sec is needed to give the width of the resonance.

CHAPTER VI

CONCLUSIONS

Wave damping has been observed for the fast (compressional) wave near harmonics of the ion cyclotron frequency, ω_i . The damping occurred about 10% above $2\omega_i$ in a hydrogen plasma and $\sim 10\%$ above $2\omega_i$ and $3\omega_i$ in a deuterium plasma. After examining several collisional models, it was shown theoretically that this 10% frequency shift was the result of collisions. It was deduced that a collision frequency of 1.4×10^7 rad/sec was needed to produce the 10% shift. It was shown that this collision frequency could be produced by ion-ion collisions (i.e. $g_\pi/h_\pi \langle v_{ii} \rangle = 1.4 \times 10^7$ rad/sec) on the axis of the plasma column or by electron-ion collisions (i.e. $3\langle v_{ie} \rangle = 1.4 \times 10^7$ rad/sec) near the edge of the plasma column. The collision frequency deduced from the damping length of the fast wave away from resonances did give the correct width of the resonance, however. The reason why different collision frequencies were needed to calculate the shift and the width of the second harmonic resonance is that velocity independent collision frequencies were used in the calculations.

Several magneto-sonic column resonances were also

observed. The first radial mode of this resonance was observed in a hydrogen plasma and the first and second radial modes were observed in a deuterium plasma. The frequencies of each of these resonances scaled according to magneto-sonic column resonance theory. Because of this correct scaling, this resonance was conclusively identified as a magneto-sonic column resonance. A scheme was postulated whereby a magneto-sonic column resonance could be used to heat plasmas.

An investigation was also made of "turbulent heating" for preparing a hot (~ 5 eV), dense ($\sim 3 \times 10^{20}/\text{m}^3$) plasma. Only one method was found which gave the desired plasma properties. In this plasma, the fast wave would have a damping length about three times longer than a wavelength. Even though this was desirable, this 5 eV plasma was not used because it could only be produced over a relatively narrow range of B_0 .

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APPENDIX

EXPERIMENTAL OPERATING SEQUENCE

The background material for the following sequence of events is given in Ch. II and Fig. II-1. The operating sequence of the experiment is as follows:

1. The magnet power supply is turned on. (As a safety precaution, an automatic timer will turn off the supply in about 30 seconds in case the operator forgets to turn it off.)

2. A timer is activated that turns on the 23 k VAC_{o-p} supply. (This allows the 0.1 μ f capacitor to charge before the relay is closed. The current from the 23 kV AC_{o-p} supply is set by series resistors in the primary so that the secondary short circuit current is $\gtrsim$ 50 mA.)

3. This timer also turns on the "-H.V." supply in order to charge the 0.8 μ f capacitor. (The magnitude of the "-H.V." and the charging time is used to set the voltage which the 0.8 μ f capacitor will have just prior to being discharged. This takes about 10 to 30 seconds.)

4. The operator adjusts the variac control to the magnet power supply slightly while the 0.8 μ f capacitor

is being charged so that the desired value of B_0 is obtained.

5. About 1/2 second before the end of the charging cycle, the -H.V. timer automatically keys the 1 kw oscillator and closes the relay. (The 0.1 μ f capacitor and the 50 k Ω resistor are used to decrease the A.C. ripple. At this time, there is about a 3 mA arc current between the pre-ionization electrode and the disk electrode. This current is set by the 6 M Ω resistor to ground. The IR drop of 18 kV DC across the 6 M Ω resistor is also the voltage on the left disk electrode. It is this 18 kV DC voltage between the left disk electrode and the grounded, right disk electrode which is used to break the gas down.)

6. An axial, small diameter arc appears between the disk electrodes because of the 18 kV DC between the disks and because of the trigger arc between the pre-ionization electrode and the left disk.

7. The plasma breaks down. (The plasma current rises to about 40 mA DC and the voltage on the left disk electrode falls to about 7 kV DC. At least 4 kV DC is needed between the disk electrodes and a plasma current of 40 mA DC is needed to sustain the arc with the required uniform brightness and the required steadiness. The current in the arc between the pre-ionization electrode and the disk electrode also rises to 40 mA DC because this current

flows from the recessed pre-ionization electrode to the left disk and then down the plasma column to the right disk. A negligible amount of current stills flows through the 6 M Ω resistor. The 40 mA DC arc current between the pre-ionization electrode and the left disk provides a continuous trigger arc that is required for an extremely steady pre-ionization discharge.)

8. The small diameter plasma arc mentioned in step (6) dies away and is replaced by a larger diameter arc that is uniformly bright and very steady. (This takes at least 100 ms. The diameter of the larger arc determines the diameter of the plasma column.)

9. About 300 ms after a H.V. probe has showed that the relay has closed, the ignitron is triggered and the oscilloscopes are triggered. (The 300 ms delay insures that the pre-ionized plasma is uniformly bright and extremely steady. This delay is also short enough to insure that any impurities generated by the plasma arcs do not have time to contaminate the plasma. During the 300 ms delay the 0.1 μ f capacitor is being charged positively through the 28 Ω resistor.).

10. The ignitron ignites. (Because the 0.8 μ f and the 0.1 μ f capacitors are charged to opposite polarities, there is a large surge current through the 28 Ω resistor. The surge current insures that the ignitron ignites. The

28 Ω resistor limits the maximum of the surge current. The opposite polarities on the 0.8 and 0.1 μf capacitors insures that there is sufficient voltage across the ignitron for reliable ignition. The polarity of the 0.8 μf capacitor was chosen negative so that the ignitron would fire reliably.)

11. There is a 1/2 to 10 μsec delay between the time the ignitron ignites and the time the axial discharge current from the 0.8 μf capacitor starts to flow. (The delay is a function of the voltage on the 0.8 μf capacitor, the magnitude of B_0 and the gas and the pressure used. During this delay the ignitron is kept ignited by the charging current from the 0.8 to the 0.1 μf capacitor through the 28 Ω resistor. The 28 ohm resistor is also used to limit the maximum charging current and to determine the RC charging time (i.e. the time that the ignitron stays ignited).)

12. The axial discharge current from the 0.8 μf capacitor flows and rings out. (The plasma is now ready for monitoring plasma waves.)

13. The waves which are launched are monitored.

14. The timer turns off the 1 kw oscillator, the relay, the -H.V. supply and the 23 kV AC_{o-p} supply.

15. The operator turns off the magnet power supply.