A BIOMECHANICAL MODEL FOR THE UPPER EXTREMITY
USING OPTIMIZATION TECHNIQUES

by

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TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS	ii
LIST OF TABLES	v
LIST OF ILLUSTRATIONS	vi
I. INTRODUCTION	1
Biomechanics	1
Techniques of Human Motion Analysis	4
Statement of the Problem	12
Purpose and Scope	14
II. THE MODEL	16
Assumptions	16
Notation	21
Dynamic Analysis	24
Performance Criteria	33
The Model	39
III. MODEL SOLUTION	43
Suboptimization	43
Dynamic Programming	48
Simulation	54
IV. MODEL IMPLEMENTATION	58
Planar Motion Problem	58
Dynamics of the Arm	60

Model Algorithms	71
Choice of Model Algorithm	96
V. MODEL TESTING	105
The Task	105
Experimental Variables	106
Equipment	113
Experimental Procedure	116
Hand Path of Motion	117
Results and Interpretations	122
VI. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS . . .	148
Summary	148
Conclusions	150
Recommendations for Further Research . . .	153
LIST OF REFERENCES	156
APPENDIX	169

LIST OF TABLES

Table		Page
1.	Anthropometric Characteristics	97
2.	Anthropometric Characteristics for the Subjects	111
3.	Correlation Coefficients between Experimental Paths and the Theoretical Ones	132
4.	Percentage Differences in Areas between Experimental Paths and the Theoretical Ones . .	133
5.	Percentage Differences of Motion Path Coordinates	134

LIST OF ILLUSTRATIONS

Figure	Page
1. Biomechanics	2
2. Techniques of Human Motion Analysis	5
3. Structural Equivalent of Muscles' Actions	18
4. Displacement-Time Curve	20
5. Reactive Forces and Moments Acting upon a Link at Any Instant (t) during Its Motion	25
6. Reactive Forces and Moments Acting upon the i th Cross Section	36
7. Feasible Region for the Hand Path of Motion under Suboptimization Approach	45
8. Arm Motion for Dynamic Programming Approach	50
9. Stages of Dynamic Programming Approach	53
10. Arm Motion under Simulation Approach, Assuming Sine, Ellipse, Parabola As Possible Shapes for the Hand Path of Motion	56
11. Arm Motion under Simulation Approach, Using Enumeration	57
12. Arm Configuration at an Instant t during the Motion	61
13. Free Body Diagram of the Forearm-Hand Link	67
14. Free Body Diagram of the Upper Arm	68
15. Stages of Dynamic Programming Approach	81
16. Dynamic Programming Iteration Scheme	86
17. Simulation Iteration Scheme	88

Figure	Page
18. Simulation--Sine and Ellipse Functions	90
19. Simulation--Parabola	92
20. Task Configurations	98
21. Hand Path of Motion--Task I	100
22. Hand Path of Motion--Task II	101
23. Hand Path of Motion--Task III	102
24. Hand Path of Motion--Task IV	103
25. Task Configuration	106
26. Arm Segment Masses and Their Locations Expressed as Percentages of Arm Lengths and Total Body Mass	110
27. Typical Effect of 40% Variations in the Anthropometric Coefficients upon the Optimum Path of Motion	112
28. Experimental Equipment	114
29. Typical Examples of the Photographic Records Obtained during the Experiment	118
30. Correlation Analysis for Two Paths of Motion	120
31. Motion Performed at Elbow Height of 6 Inches above the Work Surface and Distance of 9 Inches	123
32. Motion Performed at Elbow Height of 6 Inches above the Work Surface and Distance of 12 Inches	124
33. Motion Performed at Elbow Height of 6 Inches above the Work Surface and Distance of 15 Inches	125
34. Motion Performed at Elbow Height of 3 Inches above the Work Surface and Distance of 9 Inches	126

Figure	Page
35. Motion Performed at Elbow Height of 3 Inches above the Work Surface and Distance of 12 Inches	127
36. Motion Performed at Elbow Height of 3 Inches above the Work Surface and Distance of 15 Inches	128
37. Motion Performed at Elbow Height of 0 Inches above the Work Surface and Distance of 9 Inches	129
38. Motion Performed at Elbow Height of 0 Inches above the Work Surface and Distance of 12 Inches	130
39. Motion Performed at Elbow Height of 0 Inches above the Work Surface and Distance of 15 Inches	131
40. Effect of Work Surface Height upon Accuracy of Prediction	142
41. Effect of Motion Distance upon Accuracy of Prediction	143
42. Effect of Subject upon Accuracy of Prediction . .	144

CHAPTER I

INTRODUCTION

Biomechanics

In a society which has witnessed space missions and highly sophisticated automatic systems, it still is man who has to fly supersonic jets, guide spacecraft and operate high speed computers. Without man, the decision-maker, the prime mover, the controller, many of our sophisticated systems would not function. Over the years, the advancements in science and technology have made it possible to learn about both man's capacity and limitations under various environments. Biomechanics (Figure 1), utilizing the findings of the basic engineering sciences, anatomy, physiology and psychology, is one of several scientific disciplines which studies man within his environment. Biomechanics is concerned with the scientific study of the interaction between the human body and the external forces resulting from the surrounding working environment. The term environment includes all working conditions external to the human body whether such are on earth under normal or stressful situations, or on the lunar surface under subgravitational effects.

Fig. 1.--Biomechanics [adopted from Contini, 1963]

Basically, biomechanics measures and assesses the mechanical and physiological characteristics of the human body in motion and rest. The main objective of biomechanics is to increase the efficiency of human performance by minimizing or economizing the effort required to perform the motor activities. It analyzes and specifies the functional capabilities and limitations of the human body under different environments.

Applications of biomechanics appear in almost every area where man performs an activity. Significant improvement of the human body utilization in various scientific fields has been accomplished through biomechanical analysis. Some of these are as follows: (1) industry [Taylor, 1912; Gilbreth, 1917, 1919; Darcus, 1954; Dempster, 1955; Ayoub, 1966; Tichauer, 1965; Chaffin, et al., 1967]; (2) medicine and medical rehabilitation [Contini, et al., 1949, 1953, 1954; Eberhard and Inman, 1947, 1951; Fletcher and Leonard, 1955; Gavagna, et al., 1963; Stone, 1963]; (3) sports [Furusawa, 1928; Tarrant, 1938; Morton, 1952; Hopper, 1951; Carlsöö, 1960, Idai and Asami, 1961; Lloyd, 1965; Cooper, 1968; Elizabeth, et al., 1968]; (4) music [Hodgson, 1934; Polnauer and Marks, 1965]; (5) traffic and motor vehicles [Domey and McFarland, 1953, 1955, 1963; Severy, et al., 1954, 1955, 1956]; and (6) space research [Kuehnegger, 1964].

Techniques of Human Motion Analysis

In biomechanics, the determination of human motion characteristics, such as displacement, velocity, and acceleration, is a prerequisite before any objective analysis can be performed. The scientific analysis of human motion has been recorded for more than a century. Leonardo da Vinci is generally credited with conducting the first systematic observations on human motion. Human motion analysis, in general, can be carried out either experimentally or theoretically as shown in Figure 2.

Experimental Analysis

Experimental motion analysis is the technique of obtaining the motion characteristics from the physical records obtained for the motion. In most of the experimental analyses, either displacement, velocity or acceleration, or their analogues versus time are measured as an output from the recording system. In any case, motion characteristics are obtained by either differentiating or integrating the measured data. Experimental analysis includes three basic techniques.

1. Photography.--Three methods are available for photographic analysis of human motion: (a) cyclography, chronocyclography, and interrupted light photography [Marey, 1895, 1902; Gilbreth, 1917; Popova, 1934; Polnauer,

Fig. 2.--Techniques of human motion analysis

et al., 1952]; (b) gliding cyclograms [Bernstein, 1928; Drillis, 1930, 1959]; and (c) motion pictures [Eberhart and Inman, 1947; Taylor and Blaschke, 1951].

2. Electronic and/or electromechanical methods.--

Motion analysis by electromechanical techniques is based upon the principle of converting the physical motion into electrical signals which can be related to the motion characteristics. These techniques can be classified into three basic systems: (a) potentiometric systems [Karpovich, 1960; Reheja, 1966; Ramsey, 1968]; (b) radar-like systems [Goldman, 1955; Covert, 1965]; and (c) accelerometers [Liberson, 1936; Karger, 1958; Ayoub, 1966].

3. Stereophotogrammetry.--Recently, human motion analysis has been pursued by means of stereogrammetrical techniques which include three basic methods: (a) stereophotography, (b) stereotelevision, and (c) stereoradar. Although the three systems vary in technical principles, all of them make use of the very basic concepts of binocular vision. Yet, stereophotography is the leading technique over the other two as far as the principles, methodology, and accuracy are concerned. Stereophotography has been used by Zeller [1953], Brewer [1962], Guetwart [1967], Preston [1967], and Ayoub [1969] for subjective recording of human motion.

Theoretical Analysis

Theoretical analysis of human motion was introduced several decades ago. The use of theoretical mechanics for human motion analysis has been the subject of several investigations. The classical work of Braune [1895], Fischer [1906], Amar [1920], and Bernstein [1926] is notable in this respect. Theoretical mechanics alone, however, has failed to be an efficient technique for human motion analysis. Nubar and Contini note ". . . the fact that the equations of theoretical mechanics are by themselves incapable of determining completely the unknowns in human motion, a matter largely of free choice on the part of the individual" [1961].

Aside from theoretical mechanics, human motion analysis based upon optimization approaches has received attention. It has been accepted for years that the human body--a machine which thinks, learns, and shows a high degree of adaptability to the external environment--selects an optimum performance according to a certain criterion in given circumstances. That is to say, the human body will perform according to the hypothesis of minimal principles. Milsum stresses the applicability of the minimal principles hypothesis to the human body as:

An attractive parallel arises in the nonliving physical world, namely in the "minimal" principles by which structural, electrical, hydraulic, and other networks

reach equilibrium when either the stored energy or dissipated power is minimized. While, therefore, we must beware of guessing rashly how nature operates, on the basis of being "logical," nevertheless living systems are also constrained to operate within physical laws and hence probably must use some of the same criteria. . . . There are many combinations of muscle tensions which could achieve any given desired posture, each requiring, in general, a different metabolic rate to sustain it. This condition may be compared with that of a statically indeterminate engineering structure, and, as is well known, such a problem is solved, at least in principle, by writing a stored-energy expression for the structure and by differentiating for a minimum to solve for the equations specifying the forces. It would seem plausible that the body's equilibrium posture should be deducible by a similar approach, at least in principle . . . [1968].

Cotes and Meade [1960] have verified experimentally that the human, indeed, follows an optimizing criterion in walking. For a subject walking naturally on the flat, Cotes and Meade express the power consumed as:

$$P_{O_2} = b + aV^2 \quad (1.1)$$

where

P_{O_2} = oxygen consumption rate,

V = speed of walking, and

a, b = numerical constants.

Using the above formula, and their empirically determined constants, Cotes and Meade predict the optimum walking speed equal to 2.25 miles/hr which closely approximates the walking speed of the average individual. On the other

hand, Milsum [1968] postulated a simplified model for walking in which the legs are considered as cylinders, swinging as simple pendulums with slight difference so that each leg comes to rest on the ground at the end of each swing. Combining the natural frequency of the cylindrical form leg--77 paces/min--with a reasonably normal pace length of 30 inches, the optimum walking speed for the idealized simplified walking model is obtained as 2.6 miles/hr. Comparing the two optimum speeds, i.e., the one obtained by the simple pendulums assumption against the experimentally determined one, should demonstrate the applicability of the minimal principle to the human body.

Respiration studies by Christie [1953] and Meade [1960] have proven without doubt that under normal conditions the human being does optimize his breathing frequencies.

Nubar and Contini have attempted to develop a theoretical model for human locomotion by using an optimization approach. They postulate the minimal principle in biomechanics as follows: "A mentally normal individual will, in all likelihood, move (or adjust his posture) in such a way as to reduce his total muscular effort to minimum, consistent with the constraints" [1961]. Based upon their minimal principle, Nubar and Contini propose the following mathematical model for human motion analysis.

has never been tested or actually applied to human motion analysis. That is, the model did not exceed the mathematical formulation phase. However, Nubar and Contini are considered the first to present a somewhat full mathematical treatment to the problem of human motion.

Rashevsky [1962] has attempted to develop functional relationships between the optimal speed of walking, the optimal step size and the amount of metabolic energy for human locomotion. He proposes the following two expressions:

$$V_m = 2 \sqrt[4]{\frac{mMg}{l} W^*{}^2} \quad (1.4)$$

and

$$S_0 = 2 \sqrt{\frac{m^3 l}{Mg} W^*{}^2} \quad (1.5)$$

where

V_m = optimal walking speed,

S_0 = optimal length step,

m = mass of legs,

M = body mass,

l = leg length, and

W^* = metabolic energy available.

The above two expressions are obtained under extremely simplifying assumptions. Nevertheless, Rashevsky states

that "in view of the crudeness of the approximation used, it is noteworthy that we obtain plausible orders of magnitude for V_m and S for average human walk."

Statement of the Problem

So far, since the time of Leonardo da Vinci, a fund of knowledge has been gained from the previous studies concerning human motion analysis. However, there is no general model available for describing and predicting human motion characteristics in their general form, e.g., three-dimensional motion or even under planar motions. It is very obvious that the nature of the experimental analysis of human motion eliminates the possibility of developing a generalized model based upon experiments alone. Most of the existing motion analysis techniques require a considerable amount of time for both recording and data reduction phases which is, undoubtedly, beyond the capabilities of most research activities. Furthermore, in the previous attempts which have been made to describe human motion theoretically, instead of a complete analysis of the problem, either a definition is given or a proposed solution procedure is presented.

It seems, therefore, that there is a real need for developing a generalized biomechanical model which can be used to simulate all possible classes of human motion theoretically.

Unfortunately, developing such a model is not an easy task, if not impossible at the present state of the art. Furthermore, it seems very difficult to even agree confidently upon optimization criteria which can be used to describe human performance. The complexities multiply at a fast rate when one discovers that the criteria are not simple or necessarily always the same under different tasks. For instance, in some tasks the efficiency of performance is the main concern. On the other hand, maximization of the human output effort, power say, might be the objective of the task. It is evident that there is a need for two optimization criteria for the two stated objectives.

Considering the previously mentioned difficulties, an attempt to develop a biomechanical model to study a special class of transport movements will be of value, both in the ultimate objective of developing a generalized model for the human body and in developing some practical applications. Indeed, developing such a model would clarify some aspects of model building in connection with the human body. For example, answers to questions concerning the optimization criteria, assumptions, mechanics, and possible algorithms for the model might be obtained. This is to say, however, that a simplified model completely developed would eventually lead to the development of a complete model as our knowledge about the human body increases and perfects.

Applications of such a simplified model are envisioned in two distinct fields. First, application could be made in the design of a work place associated with light manual activities. A typical example of such application would be in designing cockpits for aircraft and spacecraft. Second, application could be made in the field of medicine and in medical rehabilitation for which the model could serve as a basis for designing and evaluating artificial limbs. Also, the model could be used to simulate the performance of the disabled and patients with severe deformities; that is, application of the model under the restrictive conditions of those people would permit an assessment or prediction of their performance without actually experimenting with them.

Purpose and Scope

The primary objective of this investigation was to develop a biomechanical model for predicting the path of motion of the arm articulation joints which would minimize a measure of the physical effort necessary to perform the actual motion. The underlying principle of the model is that the human does follow an optimizing criterion in performing his tasks. The use of the model is restricted to tasks which are to be performed under normal environmental conditions and require the maximization of performance

efficiency rather than the maximum possible effort output from the body.

Basically, the model utilizes both theoretical mechanics and an optimization approach for the analysis of arm motions. Model assumptions, mechanics, and formulation are presented for three-dimensional motions. Different possible algorithms for the model solution were investigated. These are linear and geometric programmings, dynamic programming, and simulation analysis.

Principles of the model with its associated algorithms are applied in detail to analyze planar motions of the arm. Under planar motion conditions, the adequacy as well as the accuracy of the model was investigated.

CHAPTER II

THE MODEL

This chapter, dealing with the formulation of a biomechanical model for the upper extremity, is presented in four major sections. First, model assumptions are discussed and their validity is supported. Second, the general features of the model dynamics are explained. Third, the model's possible performance criteria and the rationale for selecting a specific criterion are discussed. Finally, formulation of the optimization model in terms of the selected objective function and constraint equations is outlined.

Assumptions

The human body can be viewed as a structure composed of several links hinged together about the articulation joints. The stability of the structure is provided by the action of several muscles connecting the different links. In order to simplify the dynamic analysis and the subsequent calculations of the model, the following assumptions are adopted.

1. The human body can be approximated structurally by rigid links of uniform geometrical shapes and densities.

Further, the anthropometric characteristics of the links will not be affected by changes in body configurations. According to this assumption, the arm is considered as a system of two links; the first is the upper arm and the second is the forearm and hand.

2. The muscles' actions are represented by several tie rods which can withstand tension. A typical joint of the human body and its structural equivalent is shown in Figure 3. In the structural equivalent, the hinge joint and the muscles' actions are combined to give the effect of a rigid joint, that is, a joint capable of withstanding moments and reactive forces as well. It should be understood that the rigid joint effect is instantaneously such that variations in the joints' links orientation during motion are permitted. This concept can be extended to the other articulation joints considered in this model.

3. Rotational motions of the links around their longitudinal axes are not permitted. That is, the class of motions considered in this study can be performed with translatory motions only. For example, pronations or supinations of the hand are not permitted.

4. There are no velocity or acceleration components due to coriolis motion between the moving links. This assumption is validated by the findings of Pearson,

et al., [1963] in which they show that the displacement between the adjacent bones is negligible.

- a. triceps
- b. biceps
- c. brachialis
- d. brachioradialis
- e. extensor carpi radialis longus and brevis
- f. flexor carpi radialis and palmaris longus
- g. pronator, teres

Fig. 3.--Structural equivalent of muscles' actions

5. Space motion of different body members can be treated as a two-dimensional motion performed in a plane oriented in accordance with the direction of motion.

Kattan and Nadler support this assumption by stating:

It is possible to treat a motion of a body member in space as essentially two-dimensional, one dimension

¹Adopted from Steindler [1964].

along the line of motion and the other along the height axis. The maximum value of depth dimension, Z, of motion path for any experimental condition is less than 1% of the linear movement distance. This indicates that the subject moves more or less on the X-Y plane within the experimental region, thus optimizing the motion with respect to the Z dimension [1969].

Based on this assumption, the displacement-time curve of the free-joint, the hand in most tasks, of the body in at least two dimensions is as shown in Figure 4. Slote and Stone [1963] describe such a displacement-time curve by the following functional equation:

$$x_t = \frac{x_T}{2\pi} \left\{ \frac{2\pi t}{T} - \sin\left(\frac{2\pi t}{T}\right) \right\}$$

where

x_t = displacement of time t ,

x_T = max displacement; the distance between initial and terminal points measured in that direction,

T = total motion time.

6. Motion time is assumed for the given task or predetermined from similar studies or standard motion time tables.

7. Due to the restriction on the task duration (less than 5 sec), performance criterion is taken to be related to the mechanical characteristics of the skeletal muscular system rather than the physiological indices of the supporting respiratory and cardiovascular systems.

Karvonen and Ronnholm indicate in their studies: "Purely

Fig. 4.--Displacement-time curve

mechanistic concepts (e.g., ventilation, heart rate, and oxygen consumption) have only a limited application to the problems of light manual work" [1964].

Notation

X, Y, Z = global frame of reference

x, y, z = principal axes

$\bar{I}, \bar{J}, \bar{K}$ = unit vectors for the XYZ frame

$\bar{I}_0, \bar{J}_0, \bar{K}_0$ = unit vectors for the xyz frame

m_{ij} = mass of link ij

L_{ij} = length of link ij

r_{ij} = distance of link ij center of mass from end i

x_{ij}, y_{ij} = dimensions of link ij cross section

$(Ac)_{ij}$ = cross section area of link ij

$(I_{ij})_x, (I_{ij})_y, (I_{ij})_z$ = principal moments of inertia of link ij at point i

θ_{ij}, ϕ_{ij} = Euler's angles of link ij at time t

$\dot{\theta}_{ij}, \ddot{\theta}_{ij}, \dot{\phi}_{ij}, \ddot{\phi}_{ij}$ = first and second derivatives of Euler's angles

$\bar{\omega}_{ij}$ = angular velocity vector of XYZ frame attached to link ij at time t

$(\omega_{ij})_{X;Y;Z}$ = components of $\bar{\omega}_{ij}$ along X, Y, Z axes

$\bar{\Omega}_{ij}$ = angular velocity vector of xyz frame of link ij at time t

$(\Omega_{ij})_{x;y;z}$ = components of $\bar{\Omega}_{ij}$ along x,y,z axes

$\bar{V}_i$ = velocity of joint i w.r.t. XYZ frame at time t

$\bar{a}_i$ = acceleration vector of joint i w.r.t. XYZ frame at time t

$(\bar{a}_{ij})_G$ = acceleration vector of C G of link ij w.r.t. XYZ frame at time t

$(V_i)_{X;Y;Z}, (a_i)_{X;Y;Z}$ = components of velocity and acceleration of joint i at time t along X,Y,Z axes

$\bar{L}_{ij}$ = position vector of joint j at time t w.r.t. XYZ frame oriented at joint i

$\bar{r}_{ij}$ = position vector of link ij center of mass at time t w.r.t. XYZ frame oriented at joint i

$\bar{F}_i$ = reactive force vector acting at joint i at time t

$(F_i)_{X;Y;Z}, (F_i)_{x;y;z}$ = components of $\bar{F}_i$ at time t along X,Y,Z and x,y,z axes respectively

$(\bar{M}_i)_i$ = moment of a reactive force or a moment vector acting upon link ij about joint i

$(M_i)_{X;Y;Z}, (M_i)_{x;y;z}$ = components of $(\bar{M}_i)_i$ at time t along X,Y,Z and x,y,z axes respectively

$(\bar{S}_{ij})_i$ = first moment vector of link ij at time t about joint i

$(\bar{H}_{ij})_i$ = angular momentum of link ij at time t about joint i w.r.t. xyz frame

$\frac{d(\bar{H}_{ij})_i}{dt}$ = first derivative $(\bar{H}_{ij})_i$

$$\frac{d'(\bar{H}_{ij})_i}{dt} = \text{first derivative of } (\bar{H}_{ij})_i, \\ \text{treating xyz frame fixed}$$

A_{ij} = matrix of transformation from
the XYZ frame to xyz frame at
time t

$(\sigma_i)_k$ = normal stresses at point k of
the i^{th} cross section at time t

x_k, y_k = coordinates of point k

AI = angular impulse

LI = linear impulse

Units: The following system of units (MKS) is
adopted

1. mass--kilograms
2. moment of inertia--kilogram-meters
squared
3. angular velocity--radians per second
(rad/sec)
4. angular acceleration--radians per second
squared
5. linear velocity--meters per second
6. linear acceleration--meters per second
squared
7. force--newtons
8. moment--newton-meter
9. work--newton-meter
10. power--newtons per second
11. linear impulse--newtons per second
12. angular impulse--newton-meter per second

13. stress--newtons per meter squared, and
14. stress rate--newtons per meter squared per second.

Dynamic Analysis

Consider an intermediate link ij of a system of moving links. Let X, Y, Z be a right-hand set of orthogonal axes rigidly attached to joint i as shown in Figure 5. Also, let x, y, z represent the principal axes of the link such that the z -axis is chosen along the longitudinal axis of the link; the x -axis is perpendicular to the z -axis and it is always in the vertical plane through the z -axis; and the y -axis is perpendicular to the x and z axes. At any time t during the motion, the link orientation in space is specified by the two Euler's angles θ_{ij} and ϕ_{ij} relative to the coordinate system X, Y, Z . It is assumed that the link motion is influenced only by the reactive forces and moments at its ends and its weight.

Velocity and Acceleration

The angular velocity of the XYZ frame can be represented as follows:

$$\bar{\omega}_{ij} = (\omega_{ij})_X \bar{i} + (\omega_{ij})_Y \bar{j} + (\omega_{ij})_Z \bar{k} \quad (2.1)$$

where

$$\bar{\omega}_{ij} = \text{angular velocity vector at time } t, \text{ and}$$

Fig. 5.--Reactive forces and moments acting upon a link at any instant (t) during its motion.

$(\omega_{ij})_X, (\omega_{ij})_Y, (\omega_{ij})_Z$ = components of the angular velocity along X,Y,Z axes, respectively

From Figure 5 the components of the angular velocity $\bar{\omega}_{ij}$ can be expressed as

$$\begin{aligned}(\omega_{ij})_X &= -\dot{\theta}_{ij} \cos \phi_{ij} , \\(\omega_{ij})_Y &= \dot{\theta}_{ij} \sin \phi_{ij} , \text{ and} \\(\omega_{ij})_Z &= \dot{\phi}_{ij} .\end{aligned}$$

Substituting the above expressions in equation (2.1) yields

$$\bar{\omega}_{ij} = -\dot{\theta}_{ij} \cos \phi_{ij} \bar{i} + \dot{\theta}_{ij} \sin \phi_{ij} \bar{j} + \dot{\phi}_{ij} \bar{k} . \quad (2.2)$$

Similarly, the angular velocity $\bar{\Omega}_{ij}$ of the xyz frame can be written as

$$\bar{\Omega}_{ij} = \dot{\phi}_{ij} \sin \theta_{ij} \bar{i}_0 + \dot{\theta}_{ij} \bar{j}_0 + \dot{\phi}_{ij} \cos \theta_{ij} \bar{k}_0 . \quad (2.3)$$

Linear velocity of joint j is given by

$$\bar{V}_j = \bar{V}_i + \bar{\omega}_{ij} \times \bar{L}_{ij} \quad (2.4)$$

where

$\bar{V}_j$ = velocity of joint j w.r.t. the XYZ frame,

$\bar{V}_i$ = velocity of joint i w.r.t. the XYZ frame,

$\bar{\omega}_{ij}$ = angular velocity of XYZ frame, and /

$\bar{L}_{ij}$ = position vector of joint j

$$= L_{ij} \sin \theta_{ij} \sin \phi_{ij} \bar{i} + L_{ij} \sin \theta_{ij} \cos \phi_{ij} \bar{j} + L_{ij} \cos \theta_{ij} \bar{k} .$$

Linear acceleration $\bar{a}_j$ of joint j is written as follows:

$$\bar{a}_j = \bar{a}_i + \dot{\bar{\omega}}_{ij} \times \bar{L}_{ij} + \bar{\omega}_{ij} \times (\bar{\omega}_{ij} \times \bar{L}_{ij}) \quad (2.5)$$

where

$\bar{a}_i$ = linear acceleration vector of joint i
w.r.t. the XYZ frame,

$\bar{\omega}_{ij}, \bar{L}_{ij}$ = as defined before, and

$\dot{\bar{\omega}}_{ij}$ = first derivative of the angular acceleration $\bar{\omega}_{ij}$.

Linear acceleration $(\bar{a}_{ij})_G$ of the center of mass of link ij can be deduced from equation (2.5) upon replacing $\bar{L}_{ij}$ by the position vector of the center of mass $\bar{r}_{ij}$. Thus,

$$(\bar{a}_{ij})_G = \bar{a}_i + \dot{\bar{\omega}}_{ij} \times \bar{r}_{ij} + \bar{\omega}_{ij} \times (\bar{\omega}_{ij} \times \bar{r}_{ij}) . \quad (2.6)$$

Equations of Motion

During the motion, the link is always in a dynamic stability under the effect of the external forces and moments as well as the inertia forces generated by the link motion. The equilibrium equations for the link at any instant t are written as follows [Hagerty and Plass, 1967; Thomson, 1961; Nelson and Loft, 1962]:

$$\sum_{i=1}^j \bar{F}_i = m_{ij} \cdot (\bar{a}_{ij})_G \quad (2.7)$$

and

$$A_{ij} \cdot \left\{ \sum_{i=1}^j (\bar{M}_i)_i - (\bar{S}_{ij})_i \bar{a}_i \right\} = \frac{d(\bar{H}_{ij})_i}{dt} \quad (2.8)$$

where

$\bar{F}_i$ = reactive force vector acting about joint i,

m_{ij} = mass of link ij,

$(\bar{a}_{ij})_G$ = acceleration vector of center of mass of link ij,

$(\bar{M}_i)_i$ = moment of a reactive force or moment vector acting upon link ij about joint i,

$(\bar{S}_{ij})_i$ = first moment vector of link ij about joint i,

$\bar{a}_i$ = acceleration vector of joint i,

A_{ij} = matrix of transformation from the XYZ system to xyz system, and

$\frac{d(\bar{H}_{ij})_i}{dt}$ = first derivative of the angular momentum vector for link ij written w.r.t. joint i.

By examining Figure 5 (page 25), the above term can be written as follows:

$$\begin{aligned}
& \underline{\sum_{i=1}^j \bar{F}_i} \\
&= \bar{F}_i + \bar{F}_j + m_{ij} \cdot \bar{g} \\
&= ((F_i)_X \bar{i} + (F_i)_Y \bar{j} + (F_i)_Z \bar{k}) + ((F_j)_X \bar{i} + (F_j)_Y \bar{j} \\
&\quad + (F_j)_Z \bar{k}) - m_{ij} g \bar{k}
\end{aligned}$$

where

$\bar{F}_i, \bar{F}_j$ = reactive forces vectors at joints i and j respectively,

$(F_i)_X, (F_i)_Y, (F_i)_Z, (F_j)_X, (F_j)_Y, (F_j)_Z$ = components of the reactive forces along XYZ axes, and

g = gravitational force.

$$\underline{m_{ij} \cdot (\bar{a}_{ij})_G}$$

Using equation (2.6), the above term can be written as follows:

$$m_{ij} \cdot (\bar{a}_{ij})_G = m_{ij} \cdot \bar{a}_i + m_{ij} \cdot \dot{\bar{\omega}}_{ij} \times \bar{r}_{ij} + m_{ij} \cdot \bar{\omega}_{ij} \times (\bar{\omega}_{ij} \times \bar{r}_{ij})$$

$$\underline{\sum_{i=1}^j (M_i)_i}$$

$$\begin{aligned}
&= \bar{M}_i + \bar{M}_j + m_{ij} \cdot \bar{g} \times \bar{r}_{ij} + \bar{F}_j \times \bar{L}_{ij} \\
&= ((M_i)_X \bar{i} + (M_i)_Y \bar{j} + (M_i)_Z \bar{k}) + ((M_j)_X \bar{i} + (M_j)_Y \bar{j} + (M_j)_Z \bar{k}) \\
&\quad + m_{ij} \cdot \bar{g} \times \bar{r}_{ij} + \bar{F}_j \times \bar{L}_{ij}
\end{aligned}$$

where

$\bar{M}_i, \bar{M}_j$ = moment vectors at joints i and j respectively,

$(M_i)_X, (M_i)_Y, (M_i)_Z, (M_j)_X, (M_j)_Y, (M_j)_Z$ = components of the moment vectors along the XYZ axes, and

$m_{ij}, \bar{r}_{ij}, \bar{L}_{ij}, \bar{F}_j, \bar{g}$ = as defined before.

$$\underline{(\bar{S}_{ij})_G X \bar{a}_i}$$

$$= m_{ij} \cdot \bar{r}_{ij} X \bar{a}_i$$

where all the terms are as defined before.

$$\underline{A_{ij}}$$

In all the previous expressions, all the terms are written with respect to the XYZ coordinate system. In order to use equation (2.8), these terms have to be written with respect to the xyz axes. The transformation from the XYZ to the xyz frame is obtained by using the appropriate matrix of transformation A_{ij} . The matrix A_{ij} is the result of two rotations: ϕ_{ij} rotation about the Z axis, followed by θ_{ij} rotation about the resulting y axis. By examining Figure 5 (page 25), the matrix of transformation A_{ij} can be expressed as:

$$A_{ij} = \begin{bmatrix} -\cos\theta_{ij}\sin\phi_{ij} & -\cos\theta_{ij}\cos\phi_{ij} & -\sin\theta_{ij} \\ -\cos\phi_{ij} & \sin\phi_{ij} & 0 \\ -\sin\theta_{ij}\sin\phi_{ij} & -\sin\theta_{ij}\cos\phi_{ij} & \cos\theta_{ij} \end{bmatrix}$$

The relation between the two coordinate systems is written as:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\frac{d(\bar{H}_{ij})_i}{dt}$$

$$= \frac{d'(\bar{H}_{ij})_i}{dt} + \bar{\omega}_{ij} X(\bar{H}_{ij})_i \quad (2.9)$$

where

$(\bar{H}_{ij})_i$ = angular momentum of the link ij about joint i w.r.t. xyz axes,

$\frac{d'(\bar{H}_{ij})_i}{dt}$ = first derivative of the angular momentum, treating the xyz axes fixed,

$\bar{\omega}_{ij}$ = angular velocity of xyz frame.

The scalar components of equation (2.9) are given by:

$$\left(\sum_1^j (M_i)_i\right)_x = \{ (I_{ij})_x (\dot{\bar{\Omega}}_{ij})_x - ((I_{ij})_y - (I_{ij})_z) (\Omega_{ij})_y (\Omega_{ij})_z \}$$

$$\left(\sum_1^j (M_i)_i\right)_y = \{ (I_{ij})_y (\dot{\bar{\Omega}}_{ij})_y - ((I_{ij})_z - (I_{ij})_x) (\Omega_{ij})_z (\Omega_{ij})_x \}$$

$$\left(\sum_1^j (M_i)_i\right)_z = \{ (I_{ij})_z (\dot{\bar{\Omega}}_{ij})_z - ((I_{ij})_x - (I_{ij})_y) (\Omega_{ij})_x (\Omega_{ij})_y \}$$

where

$$\left(\sum_1^j (M_k)_i\right)_x, \left(\sum_1^j (M_i)_i\right)_y, \left(\sum_1^j (M_i)_i\right)_z = \text{components of the external moments joint } i \text{ at time } t \text{ along the link principal axes } xyz,$$

$$(I_{ij})_x, (I_{ij})_y, (I_{ij})_z = \text{principal moments of inertia of link } ij.$$

The above scalar components are the well-known Euler's equations of motion.

Using equations (2.7) and (2.8), the reactive forces and moments at the link's joint i , say, can be expressed in terms of the external forces and moments at joint j as well as the link Euler's angles and their derivatives. The analysis can easily be extended to determine the reactive forces and moments for the different joints of a link system. The procedure is best summarized in the following steps:

1. Using equations (2.1) through (2.6), velocity and acceleration for each joint in the system can be

obtained. It is important to start the analysis with a joint of known motion or with a support and proceed from it to the other joints until all the unconnected links of the system are reached, considering one joint at a time.

2. Using the concepts of the free body diagram and equations (2.7) and (2.8), the reactive forces and moments at the different joints of the system can be obtained.

Following the above mentioned procedure, it can be shown that the moments and the reactive forces at any joint such as i of a link system of the arm are expressed as follows:

$$(M_i)_{x,y,z} = M(\theta_{ij}, \dot{\theta}_{ij}, \ddot{\theta}_{ij}; \phi_{ij}, \dot{\phi}_{ij}, \ddot{\phi}_{ij}) \quad i,j=1,2 \quad (2.10)$$

$$(F_i)_{x,y,z} = F(\theta_{ij}, \dot{\theta}_{ij}, \ddot{\theta}_{ij}; \phi_{ij}, \dot{\phi}_{ij}, \ddot{\phi}_{ij}) \quad i,j=1,2 \quad (2.11)$$

Performance Criteria

Describing human effort during motion based upon mechanical criteria has long been in use. Some of the most frequently used criteria are work, power, angular impulse, linear impulse, and stress at the articulation joints. Generally, it is assumed that the external mechanical function at any articulation joint is related to the forces developed in muscles during motion.

Expressions for calculating some mechanical criteria are given below.

1. Work.--The work of a couple moment M acting on a rigid body during a finite rotation is given by:

$$\text{Work} = \int_{\theta_1}^{\theta_2} \bar{M} d\theta \quad (2.12)$$

where

$\bar{M}$ = a moment vector acting on the body,

$d\theta$ = small angles expressed in radians through which the body rotates,

θ_1, θ_2 = the initial and the final values of the angle of rotations.

2. Power.--It is defined as the rate of performing work. Power is expressed as:

$$\text{Power} = \int_{\dot{\theta}_1}^{\dot{\theta}_2} \bar{M} d\dot{\theta} \quad (2.13)$$

where

$d\dot{\theta}$ = small angular velocity of the body,

$\dot{\theta}_1, \dot{\theta}_2$ = initial and final velocities of the body.

3. Angular Impulse.--Angular impulse is defined as:

$$AI = \int_0^T \bar{M} dt \quad (2.14)$$

where

$\bar{M}$ = moment vector acting on the body,

T = total motion time.

4. Linear Impulse.

$$LI = \int_0^T \bar{F} dt \quad (2.15)$$

where

$\bar{F}$ = resultant force vector acting on the body.

5. Normal Stress.--The stress at a cross section of a moving link of the human body, taken at the articulation joint, results from the moments and reactive forces acting upon the moving link during the instant considered. Basically, there are two types of stresses acting upon any cross section of the body: normal stress and shear stress. In this study only normal stress is considered.

Normal stress is due to normal compression forces acting along the longitudinal axis of the limb and bending moments acting along the axes of the cross section (Figure 6). The normal stress at any instant t is given as:

$$(\sigma_i)_k = \pm \frac{(F_i)_z}{(Ac)_{ij}} \pm \frac{(M_i)_x}{(I_{ij})_x} \cdot y_k \pm \frac{(M_i)_y}{(I_{ij})_y} \cdot x_k \quad (2.16)$$

where

$(\sigma_i)_k$ = normal stress at point k of the i^{th} cross section of link ij,

$(F_i)_z$ = normal compression force at the i^{th} cross section,

$(M_i)_x, (M_i)_y$ = bending moment components along the x and y axes of the i^{th} cross section,

$(Ac)_{ij}$ = area of the i^{th} cross section,

x_k, y_k = coordinates of point k.

During the entire motion, the resulting total stress is given by:

$$\text{Stress} = \int_0^T (\sigma_i)_k dt . \quad (2.17)$$

Fig. 6.--Reactive forces and moments acting upon the i^{th} cross section.

6. Rate of Stress.--Rate of applying stress is obtained by:

$$\text{Stress Rate} = \int_{\dot{\theta}_1}^{\dot{\theta}_2} (\sigma_1)_k \cdot d\dot{\theta} . \quad (2.18)$$

Which mechanical criterion from the ones mentioned above would best describe human effort? In other words, which mechanical criterion is related to the muscular forces? Extensive studies by Hill, Fenn and their associates [Hill, 1960] as well as several others have revealed that power developed at the different articulation joints is the mechanical function which best describes human effort.

Harrison states:

For a muscle the force is independent of displacement but dependent on the instantaneous speed. From this it may be inferred that two muscles with identical properties could contract the same distance in the same time, but with different speed-time relations, and give rise to differing amounts of work done. Further, since the contraction times are the same, the power outputs will be different in the same ratio as the work outputs [1963].

Power and work, as any other mechanical functions, can be either positive or negative. For instance, a positive work will result if a motion is performed against gravity, and a negative work will occur for a motion with gravity. As far as the human body is concerned, doing either a positive or a negative work is an energy consumed.

Therefore, it seems very appropriate to take the absolute values of work in order to estimate human effort [Starr, 1951].

However, Hill [1960]; Abbott, et al., [1952]; and Abbott and Bigland [1953] present an interesting argument about the contribution of positive and negative work to human effort. Abbott, et al., state:

When an active muscle exerts a force P and shortens a distance x it does an amount of work Px ; on the other hand, if it is stretched a distance x while exerting this force, it absorbs work and is said to do an amount Px of negative work. For example, when a man climbs a vertical ladder, his leg extensors shorten and do positive work against gravity; when he descends, the same muscles are stretched while actively resisting the gravitational pull and may be said to do negative work [1952].

What is the physiological cost of such negative work? It has been shown by Abbott and his associates that doing a negative work is either as or more beneficial for the body than doing a positive one.

In this study the performance criteria are estimated by using the absolute values. However, the use of positive and negative values, as will be explained later on, will be applied in the simulation approach for solving the model.

For the purposes of this investigation, all the previously mentioned criteria were investigated in some trial runs. From the results obtained, it was evident that power is the most suitable criterion for describing

the path of motion and its characteristics. Therefore, power was chosen to be a performance criterion for the model.

The Model

The proposed biomechanical model, as previously mentioned, seeks the path of motion which will minimize a certain performance criterion and will satisfy the physical constraints imposed upon the motion. This is a typical optimization model in which one seeks the optimization of an objective function subject to some constraints. Mathematically the model can be stated as follows:

minimize:

$$J = \int_0^T f(\theta_{ij}, \dot{\theta}_{ij}, \ddot{\theta}_{ij}; \phi_{ij}, \dot{\phi}_{ij}, \ddot{\phi}_{ij}) dt \quad (2.19)$$

$i, j = 1, 2$

subject to:

$$f_k = f(\theta_{ij}, \dot{\theta}_{ij}, \ddot{\theta}_{ij}; \phi_{ij}, \dot{\phi}_{ij}, \ddot{\phi}_{ij}) = 0 \quad (2.20)$$

$k = 1, \dots, r$

where:

J = performance criterion,

f_k = constraint function,

r = integer denoting the number of constraint functions.

Solving the above model would yield the necessary information about the motion and its characteristics such as the path of motion, velocity and acceleration profiles, reactive forces and moments for each articulation joint.

Model solution has to satisfy three different classes of constraints: (a) physical constraints, (b) task constraints, and (c) stress constraints. The nature of these constraints is discussed in some detail below.

a. Physical Constraints.--The maximum and minimum values of the Euler's angles for each link (segment) of the human body are, more or less, fixed by the structure of the body and its ligaments. Therefore, the purpose of this class of constraints is to assure that the optimum solution of the model at any time t is possible to be assumed by the human body. The maximum and minimum values for the angles are determined from the previous work concerning anthropometry of the human body. Generally, the physical constraints are written as follows:

$$(\theta_{ij})_{\min} \leq \theta_{ij} \leq (\theta_{ij})_{\max} \quad (2.21)$$

$$i, j = 1, 2$$

and

$$(\phi_{ij})_{\min} \leq \phi_{ij} \leq (\phi_{ij})_{\max}$$

$$i, j = 1, 2 .$$

b. Task Constraints.--The nature of the task to be performed introduces some restrictions on the feasible solutions of the model. For example, in a certain task the weight has to be moved between two specific points. This necessitates that any feasible solution should be constrained in such a way that the resulting path of motion of the hand will lead the weight to the desired point. That is, the solution should satisfy certain boundary conditions given as:

$$XH_0 = X_1; YH_0 = Y_1; ZH_0 = Z_1$$

and

(2.22)

$$XH_T = X_2; YH_T = Y_2; ZH_T = Z_2$$

where

$XH_0, YH_0, ZH_0, XH_T, YH_T, ZH_T$ = space coordinates of the hand at the initial and terminal points.

For another example, consider the task of moving a control stick during motion. In this class of motion, the hand path of motion is restricted to that of the control stick. It can be seen that in moving a stick, and in similar tasks, the task constraints may vary considerably from the previous case.

c. Stress Constraints.--Stress constraints provide a means to exclude all possible feasible solutions of the

model which might lead to excessive stresses at the joints other than that joint at which the performance criterion is minimized. It follows that moments at the articulation joints, taken to be equivalent to stress, should be less than or equal to certain maximum values. These maximum values are obtained from the previous moment analyses concerning the human body. The nature of each stress constraint is similar to that of the objective function except that it is written in the inequality form.

CHAPTER III

MODEL SOLUTION

This chapter introduces some algorithms for solving the proposed upper extremity biomechanical model presented in Chapter II. In the interest of obtaining a suitable as well as an accurate algorithm for solving the model, three different solution approaches were investigated:

1. a suboptimization approach, ✓
2. a dynamic programming approach, ✓
3. a simulation approach.

Applications of the above three approaches for the model solution are presented in some detail in the following discussions.

Suboptimization

Suboptimization as a technique for solving the model was chosen in the hope that some of the well-developed algorithms such as linear programming could be used. In the suboptimization approach, the motion path is divided into several points in time.

For each point in time, minimizing the performance criterion subject to the model constraints yields the necessary motion characteristics as well as the associated

arm configuration at the instant considered. By repeatedly solving the model several times at different time points with different task constraints each time, the path of motion as well as motion characteristics for each articulation joint can be obtained. The task constraints of the model, page 41, have to be modified somewhat. At any instant, the hand Z-coordinate is expressed by two constraint equations as follows:

$$\begin{aligned} ZH_t &\leq ZH_{\max}(t) \\ ZH_t &\leq ZH_{\min}(t) \end{aligned} \tag{3.1}$$

where

ZH_t = hand Z-coordinate, and

$ZH_{\max,\min}$ = maximum and minimum values for ZH_t , expressed in percentage of the total motion distance.

The above two equations present a feasible region for the hand position which would force the optimal solution to take the hand to the desired terminal point. By adopting this concept for ZH_t , the task constraints can now be written as follows:

$$\begin{aligned} XH_t &= \frac{XH}{2\pi} \left\{ \frac{2\pi t}{T} - \sin\left(\frac{2\pi t}{T}\right) \right\} \\ YH_t &= \frac{YH}{2\pi} \left\{ \frac{2\pi t}{T} - \sin\left(\frac{2\pi t}{T}\right) \right\} \end{aligned} \tag{3.2}$$

Solving the model as a suboptimization problem becomes an easy task upon adopting one of the well-developed computerized algorithms for this kind of problem. It was decided to use linear and geometric programming for solving the suboptimization model.

Linear Programming

The nature of the model, as expected, is a non-linear one which eliminates direct application of linear programming to it. The model, however, can be linearized by adopting the small angles assumption. Under this assumption the following approximations can be made:

$$\sin\theta_{ij} \approx \theta_{ij},$$

$$\cos\theta_{ij} \approx 1,$$

$$\ddot{\theta}_{ij}^2 \approx 0, \text{ and}$$

$$\text{all cross products} \approx 0.$$

Similar relations can be obtained for ϕ_{ij} . By applying the above approximation, the model can be written in a linear form as follows:

minimize:

$$J = \sum_{i,j=1}^2 \{C_{1ij}\theta_{ij} + C_{2ij}\ddot{\theta}_{ij} + C_{3ij}\phi_{ij} + C_{4ij}\ddot{\phi}_{ij}\} \quad (3.3)$$

subject to:

$$f_k = \sum_{i,j=1}^2 \{C_{1ik}\theta_{ij} + C_{2ik}\ddot{\theta}_{ij} + C_{3ik}\phi_{ij} + C_{4ik}\ddot{\phi}_{ij}\} \quad (3.4)$$

$k=1, \dots, r$

where

$$C_{1i}, \dots, C_{4ik} = \text{numerical constants.}$$

Geometric Programming

A geometric programming model may be written as follows: (A complete discussion of the model may be found in Teske [1970].)

minimize:

$$g_0(X) = \sum_{t=1}^T \sigma_t C_t \prod_{n=1}^N X_n^{a_{nt}}, \quad t=1, \dots, T \quad (3.5)$$

where

$$\sigma_t = \pm 1$$

such that

$$C_t > 0$$

subject to:

$$g_m(X) \leq \sigma_m (\equiv \pm 1), \quad m=1, \dots, M \quad (3.6)$$

where

$$g_m(X) = \sum_{t=1}^T \sigma_{mt} C_{mt} \prod_{n=1}^N X_n^{a_{mnt}}, \quad m=0, 1, \dots, M$$

which

$$c_{mt} = \pm 1, m=0,1,\dots,M; t=1,\dots,T_m$$

such that

$$C_{mt} > 0, m=0,1,\dots,M; t=1,\dots,T_m$$

and

$$X_n > 0, n=1,\dots,N. \quad (3.7)$$

The optimization model can be transformed very easily to a geometric programming model and solved by a special computer program written by Blau [1969] and modified by Teske [1970]. The only disadvantage of geometric programming is that the optimum solution is obtained for one form of constraints, i.e., either equality or inequality but not both. This disadvantage, however, can be removed to some extent by applying some heuristic rules stated by both Blau and Teske.

Dynamic Programming

In the dynamic programming approach, the model under consideration has four state variables, namely, θ_{ij} and ϕ_{ij} , for $i,j=1,2$. As can be seen, it would be a very difficult task to solve such a model under this large number of state variables. The model dimensions, however, can be reduced to just one state variable by adopting the following principles:

1. Angular velocity and acceleration can be obtained by numerically differentiating angular displacements. That is,

$$\dot{\theta}_i = \frac{\theta_{i+1} - \theta_{i-1}}{2\Delta t}$$

$$\ddot{\theta}_i = \frac{\theta_{i+1} + \theta_{i-1} - 2\theta_i}{(\Delta t)^2}.$$

2. The space configuration of the arm at any instant during the motion would be completely described upon knowing the four Euler's angles for the arm linkages. Referring to Figure 8, the space coordinates of the hand with respect to the shoulder joint (origin) can be written as:

$$XH_t = L_1 \sin \theta_1 \cdot \sin \phi_1 + L_2 \sin \theta_2 \cdot \sin \phi_2 \quad (3.8)$$

$$YH_t = L_1 \sin \theta_1 \cdot \cos \phi_1 + L_2 \sin \theta_2 \cdot \cos \phi_2 \quad (3.9)$$

$$ZH_t = L_1 \cos \theta_1 + L_2 \cos \theta_2 \quad (3.10)$$

where

$\theta_1, \phi_1, \theta_2, \phi_2$ = Euler's angles for the two links of the arm at time t ,

XH_t, YH_t, ZH_t = space coordinates of the hand at time t ,

L_1, L_2 = lengths of the arm's two links.

Fig. 8.--Arm motion for dynamic programming approach

The XH_t and YH_t coordinates for equations (3.8) and (3.9) can be obtained by using assumption 5, page 18, as:

$$XH_t = \frac{XH}{2} \left(\frac{2\pi t}{T} - \sin\left(\frac{2\pi t}{T}\right) \right) \quad (3.11)$$

$$YH_t = \frac{YH}{2} \left(\frac{2\pi t}{T} - \sin\left(\frac{2\pi t}{T}\right) \right) \quad (3.12)$$

where

XH_t, YH_t = hand displacement in the X and Y directions at time t,

XH, YH = maximum displacements, and

T = motion time.

Furthermore, it is assumed that the angular displacement-time relationship for the elbow joint in the XY plane (Figure 8, page 50) can be obtained by using the function equation of assumption 5, page 18. That is,

$$\phi_{2t} = \frac{\phi_{2\max}}{2\pi} \left\{ \frac{2\pi t}{T} - \sin\left(\frac{2\pi t}{T}\right) \right\} \quad (3.13)$$

where

ϕ_{2t} = angular displacement at time t,

$\phi_{2\max}$ = maximum displacement for the elbow joint in the XY plane,

T = motion time.

By using equations (3.8) through (3.13) for a given value

of ZH, one can obtain three nonlinear equations in three unknowns, namely ϕ_2 , θ_1 , θ_2 .

By using the Newton iteration scheme, a solution for the nonlinear equations can be obtained which results in defining the Euler's angles for the arm during the instant considered.

Therefore, the hand Z-coordinate is the only state variable left to be determined in order to specify the arm configuration in space. The hand Z-coordinate should be determined in such a way as to minimize the performance criterion consistent with the constraints.

3. The model is transformed from the continuous case to a discrete one. In doing so, the plane of motion will be divided into a fine grid. Its horizontal intervals represent all possible stages and the vertical intervals represent the possible Z-coordinate of the hand. The stage intervals are taken to be equivalent to time intervals, Δt . On the other hand, the vertical interval is expressed as a percentage of the total motion distance.

4. Between consecutive stages, i.e., the small time interval considered, the velocity and acceleration as well as other motion characteristics are assumed to be constant. In other words, the changes in those characteristics are negligible.

5. The performance criterion J during the motion is computed by step integration over all stages.

6. A very fine grid is essential in order to minimize the errors inherent in both numerical differentiation and integration. However, to reduce the core requirement for the digital computer, a somewhat coarse grid can be used, and smoothing by regression analysis should be applied to the resulting optimum path of motion.

By adopting the above principles, one can write the recursive equation for the dynamic programming approach as follows (see Figure 9):

Fig. 9.--Stages of dynamic programming approach

$$f_n(X_n, X_{n-1}) = \{ | \int C_{X_n, X_{n-1}} | + f_{n-1}^*(X_{n-1}) \} \quad (3.14)$$

and

$$f_n^*(X_n) = \min_{X_n} \{ | \int C_{X_n, X_{n-1}} | + f_{n-1}^*(X_{n-1}) \} \quad (3.15)$$

where

$f_n(X_n, X_{n-1})$ = the total power of best over-all policy for the first n stages, given that the hand is in state X_n and X_{n-1} was the previous state it occupied,

$C_{X_n, X_{n-1}}$ = the cost of going from position X_{n-1} to X_n ,

$f_n^*(X_n)$ = min value of $f_n(X_n, X_{n-1})$.

A computer program was written to solve the dynamic programming formulation based upon the previous discussion.

Simulation

Aside from the optimization approaches proposed to solve the model, it was decided to investigate the possibility of using simulation for obtaining the path of motion and the associated characteristics for the arm.

In the simulation analysis, it is assumed that the hand path of motion can be approximated by certain geometrical shapes. Some of these possible shapes are:

1. a portion of a sine curve,
2. a portion of an ellipse,
3. a parabola, and

4. a polynomial regression curve fitted to a number of points of a grid.

Using any geometric shape from the above-mentioned curves along with the displacement equations presented on pages 44 and 45 would yield the necessary information about the motion characteristics as well as the arm configurations during the entire motion.

Generally, the equations for sine, ellipse, and parabola curves are written as a function of motion distance and the maximum height of the hand above the work surface as shown in Figure 10.

Varying the maximum height (H) over discrete points and keeping the motion distance constant would yield a set of different curves. It is possible to compute the performance criterion for each curve in this set. Of the curves investigated, the one which yielded the minimum criterion was considered to be the path for the best motion.

For the grid approach, a grid similar to the one shown in Figure 11 was constructed. The vertical and horizontal intervals are expressed as percentages of the motion distance. An enumeration procedure was used to generate a set of points among all the grid points. A polynomial regression function fitted to any selected set of points was obtained. The resulting polynomial function provided a path of motion and the associated motion

characteristics. The generated motion which yielded the minimum performance criterion was selected.

$$DH_t = \sqrt{(XH_t - XH_0)^2 + (YH_t - YH_0)^2}$$

$$ZH_t = H \cdot \sin\left(\frac{DH_t}{DH}\right) \text{---sine curve}$$

$$ZH_t = H \cdot \sqrt{1 - \left(\frac{DH_t}{DH}\right)^2} \text{---ellipse}$$

$$ZH_t = B_0 + B_1 \cdot DH_t + B_2 \cdot DH_t^2 \text{---parabola}$$

Fig. 10.--Arm motion under simulation approach, assuming sine, ellipse, parabola as possible shapes for the hand path of motion.

Fig. 11.--Arm motion under simulation approach, using enumeration.

CHAPTER IV

MODEL IMPLEMENTATION

This chapter presents and discusses the implementation of the model and the algorithms presented in Chapters II and III to study the arm under planar motions. Applications of the model to analyze arm motion under different tasks are presented. During the course of the description of these applications, the feasibility of the model and the algorithms--linear and geometric programmings, dynamic programmings, and simulation--were tested and evaluated.

Planar Motion Problem

The planar motion problem can be stated in general as follows:

A subject of known anthropometric characteristics is required to move his hand between two previously prescribed points carrying a known weight following a certain pace. It is assumed that the motion is restricted to arm movement, with the trunk remaining fixed. The problem is to find the path of motion which would be assumed by the subject in performing the actual motion under the previously mentioned conditions.

Model solution of the above problem can be summarized in the following steps.

Step 1

- a. Define the arm anthropometric characteristics for a chosen subject. These characteristics for each arm segment include: mass, length, distance of the center of mass from the proximal joint, moment of inertia, and cross sectional area.
- b. Define task parameters. These are motion distance, motion angle, external load, initial arm configuration, and motion time.

All the characteristics mentioned in a and b above constitute the input to the model.

Step 2

- a. In accordance with Step 1 and the dynamic analysis presented on page 24, Chapter II, calculate velocities and accelerations as well as reactive forces and moments for each arm segment.
- b. Based upon the principles presented on page 39, Chapter II, formulate the model.

Step 3

Solve the model by using one of the proposed algorithms discussed in Chapter III.

The resultant solution of Step 3 is the optimal or near optimal (depending upon the algorithm) solution of the problem.

Formulation of the optimization model and the use of the proposed algorithms are presented in some detail in the following sections.

Dynamics of the Arm

The motion of the arm is equivalent to a motion of two links: (1) the forearm and hand, and (2) the upper arm. Figure 12 shows the arm's two links with their Euler's angles, dimensions, and external forces acting upon them at a time instant t during motion.

Velocity and Acceleration

a. Link--12

Using equations (2.1) through (2.5) at $\phi_{12}=90^\circ$ ¹ and θ_{12} , the following expressions can be obtained for any instant t during motion:

¹For planar motion, the ϕ angle will remain constant throughout the motion, i.e., $\phi_{12}=90^\circ$.

L_{12} = length of the upper arm

r_{12} = distance of the upper arm center of mass from shoulder joint

L_{23} = length of the forearm-hand link

m_{12} = mass of the upper arm

m_{23} = mass of the forearm-hand link

θ_{12}, θ_{23} = arm Euler's angles at time t

W = weight carried by the hand

Fig. 12.--Arm configuration at an instant t during the motion

$$\bar{\omega}_{12} = \dot{\theta}_{12} \bar{J} \quad (4.1)$$

$$\dot{\bar{\omega}}_{12} = \ddot{\theta}_{12} \bar{J} \quad (4.2)$$

$$\bar{\eta}_{12} = \dot{\theta}_{12} J_0 , \quad (4.3)$$

$$\dot{\bar{\eta}}_{12} = \ddot{\theta}_{12} J_0 . \quad (4.4)$$

Linear velocity at elbow is given as:

$$\begin{aligned} \bar{V}_2 &= \bar{V}_1 + \bar{V}_{2/1} \\ &= \bar{V}_1 + \dot{\theta}_{12} \bar{J} X(L_{12} \cos \theta_{12} \bar{i} + L_{12} \sin \theta_{12} \bar{k}) . \end{aligned} \quad (4.5)$$

Taking $\bar{V}_1 = 0$ (shoulder joint is fixed) and expanding the second term, the above expression becomes:

$$\bar{V}_2 = L_{12} \sin \theta_{12} \dot{\theta}_{12} \bar{i} + L_{12} \cos \theta_{12} \dot{\theta}_{12} \bar{k} . \quad (4.6)$$

It follows that:

$$(V_2)_X = L_{12} \sin \theta_{12} \dot{\theta}_{12} , \quad (4.7)$$

$$(V_2)_Y = 0 , \quad (4.8)$$

$$(V_2)_Z = L_{12} \cos \theta_{12} \dot{\theta}_{12} , \quad (4.9)$$

where

L_{12} = upper arm length,

θ_{12} = upper arm angle at time t ,

$\dot{\theta}_{12}$ = angular displacement of the upper arm, and

$(V_2)_X, (V_2)_Y, (V_2)_Z$ = elbow velocity components with respect to XYZ frame.

The elbow acceleration vector can be written as:

$$\begin{aligned}
 \bar{a}_2 &= \bar{a}_1 + \bar{a}_{2/1} \\
 &= 0 + \ddot{\theta}_{12} \bar{j}_X (L_{12} \cos \theta_{12} \bar{i} + L_{12} \sin \theta_{12} \bar{k}) \\
 &\quad + \dot{\theta}_{12} \bar{j}_X (\dot{\theta}_{12} \bar{j}_X (L_{12} \cos \theta_{12} \bar{i} + L_{12} \sin \theta_{12} \bar{k})) \\
 &= (\ddot{\theta}_{12} L_{12} \sin \theta_{12} + \dot{\theta}_{12}^2 L_{12} \cos \theta_{12}) \bar{i} \\
 &\quad + (\ddot{\theta}_{12} L_{12} \cos \theta_{12} - \dot{\theta}_{12}^2 L_{12} \sin \theta_{12}) \bar{k} . \tag{4.10}
 \end{aligned}$$

From the above expression, the scalar components of the elbow acceleration become:

$$(a_2)_X = L_{12} \cdot \sin \theta_{12} \cdot \ddot{\theta}_{12} + L_{12} \cdot \cos \theta_{12} \cdot \dot{\theta}_{12}^2 \tag{4.11}$$

$$(a_2)_Y = 0 \tag{4.12}$$

$$(a_2)_Z = L_{12} \cdot \cos \theta_{12} \cdot \ddot{\theta}_{12} - L_{12} \cdot \sin \theta_{12} \cdot \dot{\theta}_{12}^2 . \tag{4.13}$$

Similarly, the acceleration components of upper arm (link 12) center of mass are obtained as:

$$(a_{12})_{G_X} = r_{12} \cdot \sin \theta_{12} \cdot \ddot{\theta}_{12} + r_{12} \cdot \cos \theta_{12} \cdot \dot{\theta}_{12}^2 \quad (4.14)$$

$$(a_{12})_{G_Y} = 0 \quad (4.15)$$

$$(a_{12})_{G_Z} = r_{12} \cdot \cos \theta_{12} \cdot \ddot{\theta}_{12} - r_{12} \cdot \sin \theta_{12} \cdot \dot{\theta}_{12}^2 \quad (4.16)$$

b. Link--23

Similar to link 12, the velocity and acceleration expressions are as follows:

$$\bar{\omega}_{23} = \dot{\theta}_{23} \bar{J} \quad (4.17)$$

$$\dot{\bar{\omega}}_{23} = \ddot{\theta}_{23} \bar{J} \quad (4.18)$$

$$\bar{\Omega}_{23} = \dot{\theta}_{23} \bar{J}_0 \quad (4.19)$$

$$\dot{\bar{\Omega}}_{23} = \ddot{\theta}_{23} \bar{J}_0 \quad (4.20)$$

Hand linear velocity vector is given as:

$$\begin{aligned} \bar{V}_3 &= \bar{V}_2 + \bar{V}_{3/2} \\ &= (L_{12} \cdot \sin \theta_{12} \cdot \dot{\theta}_{12} + L_{23} \cdot \sin \theta_{23} \cdot \dot{\theta}_{23}) \bar{i} \\ &\quad + (L_{12} \cdot \cos \theta_{12} \cdot \dot{\theta}_{12} + L_{23} \cdot \cos \theta_{23} \cdot \dot{\theta}_{23}) \bar{k} \quad (4.21) \end{aligned}$$

Its scalar components are:

$$(V_3)_X = L_{12} \cdot \sin \theta_{12} \cdot \dot{\theta}_{12} + L_{23} \cdot \sin \theta_{23} \cdot \dot{\theta}_{23} \quad (4.22)$$

$$(V_3)_Y = 0 \quad (4.23)$$

$$(V_3)_Z = L_{12} \cdot \cos \theta_{12} \cdot \dot{\theta}_{12} + L_{23} \cdot \cos \theta_{23} \cdot \dot{\theta}_{23} \quad (4.24)$$

where

L_{23} = forearm-hand length,

θ_{23} = forearm angle at time t ,

$\dot{\theta}_{23}$ = angular velocity of the forearm at time t .

Hand linear acceleration is obtained as:

$$\bar{a}_3 = \bar{a}_2 + \bar{a}_{3/2} \quad (4.25)$$

From the above expression, it follows immediately that:

$$(a_3)_X = (a_2)_X + L_{23} \cdot \sin \theta_{23} \cdot \ddot{\theta}_{23} + L_{23} \cdot \cos \theta_{23} \cdot \dot{\theta}_{23}^2 \quad (4.26)$$

$$(a_3)_Y = 0 \quad (4.27)$$

$$(a_3)_Z = (a_2)_Z + L_{23} \cdot \cos \theta_{23} \cdot \ddot{\theta}_{23} - L_{23} \cdot \sin \theta_{23} \cdot \dot{\theta}_{23}^2 \quad (4.28)$$

Also, in a similar fashion, the linear acceleration components of the forearm hand link center of mass can be written as:

$$(a_{23})_{G_X} = (a_2)_X + r_{23} \cdot \sin \theta_{23} \cdot \ddot{\theta}_{23} + r_{23} \cdot \cos \theta_{23} \cdot \dot{\theta}_{23}^2 \quad (4.29)$$

$$(a_{23})_{G_Y} = 0 \quad (4.30)$$

$$(a_{23})_{G_Z} = (a_2)_Z + r_{23} \cdot \cos \theta_{23} \cdot \ddot{\theta}_{23} - r_{23} \cdot \sin \theta_{23} \cdot \dot{\theta}_{23}^2 \quad (4.31)$$

Reactive Forces and Moments

By referring to the dynamical analysis presented in Chapter II, page 24, the reactive forces and moments of the arm links can be computed upon examining Figures 13 and 14.

a. Link--23

The equations of motion for link 23 can be written by using its free body diagram in Figure 13 as:

$$A_{23} \cdot \left\{ \sum_2^3 (\bar{M}_1)_2 - (\bar{S}_{23})_G \bar{x} \bar{a}_2 \right\} = \frac{d(\bar{H}_{23})_2}{dt} \quad (4.32)$$

where

$$\begin{aligned} (i) \quad \sum_{i=2}^3 (\bar{M}_1)_2 &= (\bar{M}_2)_2 + \bar{W} \bar{X} \bar{L}_{23} + m_{23} \cdot \bar{g} \bar{X} \bar{r}_{23} \\ &= (\bar{M}_2)_2 + (W \cdot L_{23} \cdot \cos \theta_{23} + m_{23} \cdot g \cdot r_{23} \cdot \cos \theta_{23}) \bar{j} \end{aligned}$$

$$(ii) \quad (\bar{S}_{23})_G \bar{x} \bar{a}_2 = m_{23} \cdot \bar{r}_{23} \bar{x} \bar{a}_2$$

$$\begin{bmatrix} m_{23} \cdot r_{23} \cdot \cos \theta_{23} \cdot (a_2)_X \\ + \\ m_{23} \cdot r_{23} \cdot \cos \theta_{23} \cdot (a_2)_Z \end{bmatrix} \cdot \bar{j}$$

$\bar{V}_3$ = hand velocity vector

$\bar{a}_3$ = hand linear acceleration vector

$(\bar{a}_{23})_G$ = forearm center of mass linear acceleration vector

$\bar{a}_2$ = elbow linear acceleration vector

$\bar{F}_2$ = reactive force vector at the elbow

$\bar{M}_2$ = moment vector at the elbow

$m_{23} \cdot g$ = weight of the forearm-hand link

W = weight carried by the hand

x, y, z = principal axes of the forearm

X, Y, Z = global axes

Fig. 13.--Free body diagram of the forearm-hand link

$\bar{M}_2, \bar{F}_2$ = moment and reactive force resulting from the forearm

$(\bar{a}_{12})_G$ = upper arm center of mass linear acceleration vector

$\bar{M}_1$ = moment vector at the shoulder joint

$\bar{F}_1$ = reactive force vector at the shoulder joint

$m_{12} \cdot g$ = weight of the upper arm

Fig. 14.--Free body diagram of the upper arm

$$(iii) A_{23} = \begin{bmatrix} \sin\theta_{23} & 0 & \cos\theta_{23} \\ 0 & 1 & 0 \\ \cos\theta_{23} & 0 & \sin\theta_{23} \end{bmatrix}$$

$$(iv) \frac{d(\bar{H}_{23})_2}{dt} = (I_{23})_y \cdot \ddot{\theta}_{23} \bar{J}_0 .$$

Using the above expressions, equation (4.32) reduces to:

$$(M_2)_X = 0 \quad (4.33)$$

$$\begin{aligned} (M_2)_Y &= (I_{23})_y \cdot \ddot{\theta}_{23} - m_{23} \cdot r_{23} \cdot \sin\theta_{23} \cdot (a_{23})_{G_X} \\ &\quad + m_{23} \cdot r_{23} \cdot \cos\theta_{23} \cdot (a_{23})_{G_Z} \\ &\quad - m_{23} \cdot g \cdot r_{23} \cdot \cos\theta_{23} \\ &\quad - W \cdot L_{23} \cdot \cos\theta_{23} \end{aligned} \quad (4.34)$$

$$(M_2)_Z = 0 .$$

Notice in this particular case of arm motions that the y and Y axes coincide with each other. Therefore all their components (velocity, acceleration, forces, and moment) are equivalent, i.e., $(M_2)_Y = (M_2)_y$.

Using equation (2.8), the reactive force vector at the elbow can be obtained as:

$$(F_2)_X = m_{23} \cdot (a_{23})_{G_X} \quad (4.35)$$

$$(F_2)_Y = 0 \quad (4.36)$$

$$(F_2)_Z = -W - m_{23} \cdot g + m_{23} \cdot (a_{23})_{G_Z} \quad (4.37)$$

Components of the elbow reactive force can be computed with respect to the link principal axes xyz by using the transformation matrix A_{23} . The resulting components are the normal and shearing forces at the elbow joint.

b. Link--12

Similar to link 23 and by using the free body diagram in Figure 14 (page 68), the following expressions for moment and reactive force at the shoulder joint can be obtained:

$$\begin{aligned} (M_1)_Y = (M_2)_Y + (I_{12})_Y \cdot \ddot{\theta}_{12} - m_{12} \cdot r_{12} \cdot \sin \theta_{12} \cdot (a_{12})_{G_X} \\ + m_{12} \cdot r_{12} \cdot \cos \theta_{12} \cdot (a_{12})_{G_Z} - m_{12} \cdot g \cdot r_{12} \cdot \cos \theta_{12} \\ - (F_2)_X \cdot L_{12} \cdot \sin \theta_{12} + (F_2)_Z \cdot L_{12} \cdot \cos \theta_{12} , \quad (4.38) \end{aligned}$$

and components of shoulder reactive force are

$$(F_1)_X = (F_2)_X + m_{12} \cdot (a_{12})_X \quad (4.39)$$

$$(F_1)_Z = (F_2)_Z + m_{12} \cdot (a_{12})_Z - m_{12} \cdot g \quad (4.40)$$

Model Algorithms

In accordance with the assumptions and principles of each one of the proposed algorithms, formulation and solution of the model can be achieved by using equations (4.1) through (4.40) and the discussions presented on page 43, Chapter III. Applications of the proposed algorithms--linear and geometric programmings, dynamic programming, and simulation--are presented below.

Suboptimization--Linear Programming

With reference to the principles presented on page 43, Chapter III, the formulation of the model in accordance with the linear programming algorithm should be a straightforward matter. However, writing the objective function might need some comments. It was decided to use power as the performance criterion for the model. Since under the suboptimization approach, power function should be optimized at a different point in time during motion, the objective function is written as:

$$J = (M_1)_Y \cdot \dot{\theta}_{12} \quad (4.41)$$

where

$(M_1)_Y$ = shoulder moment at time t ,

$\dot{\theta}_{12}$ = angular velocity of the upper arm at time t .

Linearization of the above expression is not possible by the small angles assumption. Therefore, it was decided to replace power by another performance criterion which can be linearized by the small angles assumption. Normal stress at the shoulder joint was chosen to be such criterion.

The linear programming model at any instant t during the motion can be formulated as

minimize:

$$g_0 = C_{01}X_{11} + C_{02}X_{12} \quad (4.42)$$

subject to:

a. Physical Constraints

$$g_1 = X_1 \leq C_{10}$$

$$g_2 = X_1 \geq C_{20}$$

$$g_3 = X_2 \leq C_{30}$$

$$g_4 = X_2 \geq C_{40}$$

(4.43)

b. Stress Constraints

$$g_5 = -C_{52}X_2 + C_{53}X_4 + C_{54}X_8 - X_9 = 0$$

$$g_6 = X_9 \leq C_{60}$$

$$g_7 = C_{71}X_4 - C_{72}X_8 - X_{10} = 0$$

(4.44)

$$g_8 = -C_{81}X_1 + C_{82}X_4 + X_9 - X_{11} - X_{10} = 0$$

$$g_9 = X_{11} \leq C_{91}$$

c. Position Constraints

$$g_{10} = C_{101}X_1 + C_{102}X_2 = C_{100}$$

where

$$X_1 = \theta_{12} \qquad X_9 = (M_2)_Y$$

$$X_2 = \theta_{23} \qquad X_{10} = (F_2)_X$$

$$X_4 = \ddot{\theta}_{12} \qquad X_{11} = (M_1)_Y$$

$$X_8 = \ddot{\theta}_{23} \qquad X_{12} = (F_1)_Z$$

$C_{10}, C_{20}, C_{30}, C_{40}$ = maximum and minimum values for the two Euler's angles of the arm

$$C_{52} = m_{23} \cdot g \cdot r_{23} + WL_{23}$$

$$C_{53} = m_{23} \cdot r_{23} \cdot L_{12}$$

$$C_{54} = (I_{23})_y$$

C_{60} = maximum allowable moment at the elbow joint

$$C_{71} = m_{12}L_{12}$$

$$C_{72} = m_{12}r_{23}$$

$$C_{81} = (M_{23}g + W) \cdot L_{12}$$

$$C_{92} = L_{12}$$

$$C_{93} = (I_{12})_y$$

C_1 = maximum allowable moment at the shoulder joint

$$C_{101} = L_{12}$$

$$C_{111} = L_{23}$$

$$C_{100} = \frac{d}{2\pi} \left\{ \frac{2\pi t}{T} - \sin\left(\frac{2\pi t}{T}\right) \right\} + XH_0; \quad d = \text{motion distance},$$

$T = \text{motion time}.$

Solving the above model at different points in time would yield the necessary information to generate the hand path of motion and its characteristics. The model objective function and constraints will remain the same for all points in time along the path of motion except for the position constraint which will vary in accordance with the instant considered.

Worthy of notice is that after linearizing the model many terms completely disappeared from both the objective function and the constraint equations. Further, there is no constraint equation written for the hand motion in the Z direction.

Suboptimization--Geometric Programming

Power at the shoulder joint was selected to be the criterion for the geometric programming formulation as mentioned on page 47, Chapter III.

Minimize:

$$G_0 = X_6 X_{12} \quad (4.45)$$

subject to:

(moment at 2)

$$g_1 = c_3 x_9 x_6^{-1} x_3^{-1} x_1^{-1} + c_4 x_8 x_6^{-1} x_3^{-1} x_1^{-1} - c_5 x_5 x_6^{-1} x_3^{-1} x_1^{-1} \\ + x_6^{-1} x_5 x_4^{-1} x_3^{-1} - x_6^{-1} x_5 x_2 x_1^{-1} + x_4 x_3^{-1} x_2 x_1^{-1} = 1$$

(X-component of reactive force at 2)

$$g_2 = c_6 x_{10} x_4^{-1} x_2^{-1} + x_4^{-1} x_3 x_2 x_1^{-1} + c_7 x_8 x_6 x_4^{-1} x_2^{-1} \\ + c_8 x_7 x_5 x_4^{-1} x_2^{-1} = 1 \quad (4.46)$$

(Z-component of reactive force at 2)

$$g_3 = c_9 x_{11} x_8^{-1} x_6^{-1} - c_{10} x_8^{-1} x_6^{-1} + c_{11} x_8^{-1} x_6^{-1} x_4 x_1 \\ + c_{12} x_8^{-1} x_6^{-1} x_3 x_2 + x_6^{-1} x_5 = 1$$

(moment at 1)

$$g_4 = c_{13} x_{12} x_1^{-1} - c_{14} x_4 x_1^{-1} - x_9 x_1^{-1} \\ + c_{15} x_{10} x_2 x_1^{-1} + c_{16} x_{11} = 1$$

(position constraints for the hand)

$$\begin{aligned}
 \text{(X-accel)} \quad g_5 = & -c_{17}x_4^{-1}x_2^{-1} - x_4^{-1}x_3x_2^{-1}x_1 - c_{18}x_8x_6x_4^{-1}x_2^{-1} \\
 & - x_2x_5x_4^{-1}x_2^{-1} = 1
 \end{aligned} \tag{4.47}$$

$$\text{(X-veloc)} \quad g_6 = -c_{22}x_3^{-.5}x_2^{-1} + c_{23}x_7^{.5}x_6x_3^{-.5}x_2^{-1} = 1$$

$$\text{(X-coord)} \quad g_7 = c_{19}x_1 + c_{20}x_5 = 1$$

(law of sines and cosines)

$$g_8 = x_1^2 + x_2^2 = 1$$

$$g_9 = x_5^2 + x_6^2 = 1$$

(physical constraints)

$$g_{10} = x_1 \leq (\sin\theta_{12})_{\max}$$

$$g_{11} = x_1 \geq (\sin\theta_{12})_{\min}$$

$$g_{12} = x_2 \leq (\sin\theta_{12})_{\max}$$

$$g_{13} = x_2 \geq (\cos\theta_{12})_{\max}$$

$$g_{14} = x_5 \geq (\sin\theta_{23})_{\max}$$

(4.48)

$$g_{15} = X_5 \leq (\cos \theta_{23})_{\min}$$

$$g_{16} = X_6 \geq (\sin \theta_{23})_{\max}$$

$$g_{17} = X_6 \leq (\sin \theta_{23})_{\min}$$

where

$$X_{1t} = \sin \theta_{12t}$$

$$X_{2t} = \cos \theta_{12t}$$

$$X_{3t} = \dot{\theta}_{12t}^2$$

$$X_{4t} = \ddot{\theta}_{12t}$$

$$X_{5t} = \sin \theta_{23t}$$

$$X_{6t} = \cos \theta_{23t}$$

$$X_{7t} = \dot{\theta}_{23t}^2$$

$$X_{8t} = \ddot{\theta}_{23t}$$

$$C_1 = r_0 / (I_{12})_y$$

$$C_2 = 1/A$$

$$C_3 = 1/m_{23} r_{23} L_{12}$$

$$C_4 = (I_{23})_y / m_{23} r_{23} L_{12}$$

$$C_5 = (M_{23} g r_{23} + W L_{23}) / m_{23} r_{23} L_{12}$$

$$C_6 = 1 / m_{12} L_{12}$$

$$C_7 = r_{23} / L_{12}$$

$$C_8 = r_{23} / L_{12}$$

$$C_9 = 1 / r_{23}$$

$$C_{10} = (m_{23} g + W) / r_{23}$$

$$C_{11} = L_{12} / r_{23}$$

$$C_{12} = L_{12} / r_{23}$$

$$C_{13} = 1 / m_{12} g r_{12}$$

$$C_{14} = (I_{12})_y$$

$$C_{15} = L_{12} / m_{12} g r_{12}$$

$$C_{16} = L_{12} / m_{12} g r_{12}$$

$$C_{17} = 1 / L_{12} \cdot (a_{3t})_x$$

$$C_{18} = L_{23}/L_{12}$$

$$C_{19} = L_{23}/L_{12}$$

$$C_{20} = 1/L_{12}(a_{3t})_z$$

$$C_{21} = L_{23}/L_{12}$$

$$C_{22} = L_{23}/L_{12}$$

$$C_{23} = \frac{1}{L_{12}} \cdot (V_{3t})_x$$

Upon solving the above geometric programming formulation at different points in time with different position constraints each time, the hand path of motion and its characteristics would be obtained.

Dynamic Programming Algorithm

Model solution by the dynamic programming approach is obtained by solving the following recursive equation in accordance with the principles presented on page 48, Chapter III.

$$f_n^*(X_n) = \min_{X_n} \{ | \int C_{X_n X_{n-1}} | + f_{n-1}^*(X_{n-1}) \} .$$

Application of the above recursive equation to determine the optimum path of motion can be demonstrated in the following discussion.

Consider an intermediate stage n at which the motion time is given as t_n . At this stage, the costs in terms of power required for the hand to move from the initial motion point to each possible position along the Z-axis (Figure 15) are assumed to have been computed previously. Now, the task is to determine the next Z-coordinate of the hand after small time interval Δt , that is, determining the hand Z-coordinate at the beginning of stage $n+1$ at time $t_{n+1}(t_n + \Delta t)$. Let us assume that for each possible Z-coordinate for the hand at stage $n+1$, there are m possible Z-coordinates at stage n for the hand to occupy. That is to say, there are m possible links for the hand to follow in moving from stage n to stage $n+1$. Next, for each possible coordinate $ZH_{n+1,i}, i=1, \dots, m$, at stage $n+1$, what is the corresponding Z-coordinate for the hand at stage n in order to minimize the total cost (power) necessary to move from the initial point to stage $n+1$? The answer to this question can be obtained by using the following steps.

Step 1

By using the two time values t_n and t_{n+1} , compute the hand X-coordinates at stages n and $n+1$ as follows:

Fig. 15.---Stages of dynamic programming approach

$$XH_n = \frac{X}{2\pi} \left\{ \frac{2\pi t_n}{T} - \sin\left(\frac{2\pi t_n}{T}\right) \right\} + XH_0 \quad (4.49)$$

$$XH_{n+1} = \frac{X}{2\pi} \left\{ \frac{2\pi t_{n+1}}{T} - \sin\left(\frac{2\pi t_{n+1}}{T}\right) \right\} + XH_0 \quad (4.50)$$

where

XH_n, XH_{n+1} = X-coordinates of the hand at stages n and $n+1$, i.e., at times t_n and t_{n+1} ,

XH_0 = X-coordinates of the hand at the initial point of motion, and

X = total displacement in the X direction.

Step 2

For a chosen position $i, i=1, \dots, m$, at stage $n+1$, choose a position $j, j=1, \dots, m$, at stage n . The two positions i and j define a possible motion link between stages n and $n+1$ for which the Z-coordinates ($ZH_{n,j}, ZH_{n+1,i}$) can be obtained by examining the motion grid.

Step 3

At stage n , by knowing the two coordinates XH_n and $ZH_{n,j}$, the arm Euler's angles can be obtained by solving two nonlinear equations written as:

$$f = L_{12} \cos(\theta_{12,n}) + L_{23} \cos(\theta_{23,n}) - XH_n \quad (4.51)$$

$$g = L_{12} \sin(\theta_{12,n}) + L_{23} \sin(\theta_{23,n}) - ZH_{n,j} \quad (4.52)$$

The above two equations can be solved by using Newton's iteration scheme in which the two angles are expressed as [Pennington, 1965]:

$$\theta_{12} = \theta_{12} + [g \cdot f_{\theta_{23}} - f \cdot g_{\theta_{23}}] / (f_{\theta_{12}} \cdot g_{\theta_{23}} - g_{\theta_{12}} \cdot f_{\theta_{23}}) \quad (4.53)$$

$$\theta_{23} = \theta_{23} + [f \cdot g_{\theta_{12}} - g \cdot f_{\theta_{12}}] / (f_{\theta_{12}} \cdot g_{\theta_{23}} - g_{\theta_{12}} \cdot f_{\theta_{23}}) \quad (4.54)$$

where

$$f_{\theta_{12}}, f_{\theta_{23}}, \dots, \text{etc.} = \text{partial derivatives.}$$

Similarly for stage $n+1$, the two Euler's angles θ_{12} and θ_{23} can be obtained.

Step 4

By knowing the arm Euler's angles at stages n and $n+1$, angular velocity and acceleration are obtained by numerical differentiations as follows (drop angle subscripts for simplicity, i.e., θ_{12} will be written as θ):

$$\begin{aligned} \dot{\theta}_{n+1} &= \frac{\theta_{n+1} - \theta_n}{(\Delta t)} \\ \ddot{\theta}_{n+1} &= \frac{\dot{\theta}_{n+1} - \dot{\theta}_n}{\Delta t} \end{aligned} \quad (4.55)$$

where

$$\theta_n, \theta_{n+1} = \text{angular displacement at stages } n \text{ and } n+1,$$

$\dot{\theta}_n, \dot{\theta}_{n+1}$ = angular velocities at stages n and n+1,

$\ddot{\theta}_{n+1}$ = angular acceleration at stage n+1.

Step 5

By using Euler's angles and their derivatives at stages n and n+1 as well as equations (4.1) through (4.40), reactive forces and moments for all joints can be obtained.

Step 6

Power required to move from the initial point to stage n+1 following link ij between stages n and n+1 is given by:

$$= 0.5\{|M_{n,j}| + |M_{n+1,i}|\} \cdot |\dot{\theta}_{n+1,i} - \dot{\theta}_{n,j}|$$

$$\sum_{n=1}^{n-1} \sum_{i=1}^m \sum_{j=1}^m 0.5\{|M_{n,j}| + |M_{n+1,i}|\} \cdot |\dot{\theta}_{n+1,i} - \dot{\theta}_{n,j}|$$

(4.56)

where

$M_{n,j}$ = shoulder moment at stage n when the hand is at position j along the Z axis,

$M_{n+1,i}$ = shoulder moment at stage n+1 when the hand is at position i along the Z axis,

$\dot{\theta}_{n+1,i}, \dot{\theta}_{n,j}$ = angular velocity of the upper arm at stages n and n+1 for motion along link ij.

Step 7

Repeating Steps 1 through 6 for all possible values of j and keeping i fixed should yield the cost for each possible motion from stage n to position i at stage $n+1$. Upon selecting the link ij which gives the minimum cost, the following parameters are defined for position i at stage $n+1$:

- a. cost necessary to move from the initial point to stage $n+1$,
- b. the hand position at stage n ,
- c. angular velocities, moments, forces.

Step 8

By varying i over all possible positions for the hand at stage $n+1$ and repeating Steps 1 through 7, the cost and associated parameters at all possible hand positions at stage $n+1$ can be obtained.

The above analysis for an intermediate stage can be easily extended to all stages considered between the initial and terminal points of the motion. An iteration scheme for the dynamic programming approach is as shown in Figure 16. A computer program was written in accordance with the dynamic programming principles presented above. The program is given in the Appendix.

Fig. 16.--Dynamic programming iteration scheme

Simulation

The basic assumption for the simulation approach, as previously discussed (page 54), is that the hand follows a certain geometrical shape of known function during its motion. That is, in the simulation's solution the shape of the hand path of motion is specified beforehand and the task is then to determine the function's parameters which would yield the minimum performance criterion (power) among a set of investigated criteria. Solution of the model in accordance with the simulation approach is as shown in Figure 17. A step-by-step description of the simulation algorithm is given as follows.

Step 1

Define the following characteristics for the algorithm:

- a. Anthropometric characteristics of the arm which include: masses (m_{12} , m_{23}), lengths (L_{12} , L_{23}), C.G. distances (r_{12} , r_{23}), and moments of inertia (I_{12} , I_{23}),
- b. Task characteristics which include: motion distance (d), motion angle (α), motion time (T), initial Euler's angles (θ_{12} , θ_{23}).

Fig. 17.--Simulation iteration scheme

Step 2

Calculate the hand X-coordinates at different points in time by using the following functional relationship:

$$XH_t = XH_0 + \frac{X}{2\pi} \left\{ \frac{2\pi t}{T} + \sin\left(\frac{2\pi t}{T}\right) \right\} \quad (4.57)$$

where

XH_t = hand X-coordinate at time t ,

XH_0 = hand X-coordinate at the initial point, and

X = displacement in the X direction = $d \cos \alpha$,

d, T = as defined before.

Step 3

Choose a geometrical function to describe the hand path of motion. There are four possible functions from which to choose. These are sine, ellipse, and parabola functions as well as a polynomial regression function fitted through a set of grid points constructed between the initial and terminal motion points. Each one of the four functions is uniquely determined by specifying some parameters pertinent to the hand path of motion.

For instance, the sine and ellipse functions are defined upon specifying the maximum height of the hand above the work surface (h) and motion distance (d). The two functions can be written as follows (see Figure 18):

d = Motion Distance

α = Motion Angle

h = Maximum Height of the Hand above the Work Surface

Fig. 18.--Simulation--sine and ellipse functions

(i) sine function

$$ZH_t = ZH_0 + h \cdot \sin(2\pi \cdot \frac{XH_t}{X}) + XH_t \cdot \sin(\alpha) \quad (4.58)$$

(ii)

$$ZH_t = ZH_0 + \sqrt{1 - (\frac{XH_t - X}{X})^2} + XH_t \cdot \sin(\alpha) \quad (4.59)$$

where

XH_t, ZH_t = X- and Z-coordinates of the hand at time t,

ZH_0 = hand Z-coordinate at time zero, and

X, h, d, α = as defined before.

The parabola function is defined by fitting a second order polynomial function through three points which include the initial and terminal points of motion. The second point is specified by two parameters: its position along the X-axis (P) and its maximum height above the work surface (h) as shown in Figure 19. By using these three points, the parabola function for the hand Z-coordinate is written as:

$$ZH_t = ZH_0 + \beta_0 + \beta_1 \cdot XH_t + \beta_2 \cdot XH_t^2 \quad (4.60)$$

where

$\beta_0, \beta_1, \beta_2$ = regression coefficients determined by using the three points, and

ZH_0, ZH_t, XH_t = as defined before.

P = Position of Point 2

h, d, α = As defined in Figure 18 (page 90)

1, 2, 3 = Parabola Points

Fig. 19.--Simulation--parabola

On the other hand, the polynomial function is obtained by fitting a fifth order function through a set of selected points between the initial and terminal points of motion. The function is given as:

$$ZH_t = ZH_0 + \beta_0 + \beta_1 \cdot XH_t + \beta_2 \cdot XH_t^2 + \beta_3 \cdot XH_t^3 + \beta_4 \cdot XH_t^4 + \beta_5 \cdot XH_t^5 . \quad (4.61)$$

Step 4

For the chosen function, define some initial values for its associated parameters. Using these values, define the function's equation as explained in Step 3.

Step 5

Using the hand X-coordinates generated in Step 2 above and the chosen function, determine the hand Z-coordinates at different points in time during the motion.

Step 6

Upon determining the X- and Z-coordinates of the hand during the entire motion, the corresponding Euler's angles for the arm segments at any instant t are obtained by solving the following two equations:

$$f = L_{12} \cos \theta_{12} + L_{23} \cos \theta_{23} - XH_t = 0 \quad (4.62)$$

$$g = L_{12} \sin \theta_{12} + L_{23} \sin \theta_{23} - ZH_t = 0 . \quad (4.63)$$

Solution of the above two equations can be obtained by employing Newton's iteration scheme as explained before on page 83. Repeatedly solving the above two equations for all different values of XH and ZH, Euler's angles for the arm segments can be determined for the entire motion.

Step 7

By knowing the arm Euler's angles for all time points during the motion, angular velocities and accelerations are obtained by numerical differentiations as follows:

$$\dot{\theta}_1 = \frac{\theta_{i+1} - \theta_i}{2\Delta t}$$

$$\ddot{\theta}_1 = \frac{\theta_{i+1} + \theta_{i-1} - 2\theta_i}{(\Delta t)^2}$$

By applying the above two expressions, angular velocities and accelerations for each arm segment can be obtained.

Step 8

By using Euler's angles, angular velocities, and accelerations determined in Steps 6 and 7 as well as equations (4.1) through (4.40), reactive forces and moments at the different joints can be obtained.

Step 9

Calculate total power, performance criterion, expended during performance of the motion as:

$$\text{power} = \sum_{t=0}^T 0.5(\dot{\theta}_{t+1} - \dot{\theta}_t) \cdot (M_{t+1} + M_t) \quad (4.64)$$

where

$\dot{\theta}_t, \dot{\theta}_{t+1}$ = angular velocities of the upper arm at times t and $t+1$, and

M_t, M_{t+1} = shoulder moment at times t and $t+1$.

Step 10

Increment the initial values of the chosen function's parameters by preselected values.

Step 11

For the new function's parameters, repeat Steps 2 through 9 above and obtain the corresponding power's value.

Step 12

After repeating Steps 2 through 11 over all possible choices of the chosen function's parameters, select the parameters which yield the minimum power value among the parameters investigated. Next, use these parameters to define the hand path of motion and its motion characteristics.

A computer program was written for the simulation algorithm in accordance with the above-mentioned steps. A full description of the program is given in the Appendix.

Choice of Model Algorithm

In order to test the feasibility of the previous algorithms, some examples of hypothetical motions were considered. For these examples, motions of a subject of known anthropometric characteristics (Table 1) were analyzed under different tasks. Four different tasks were investigated (Figure 20). Each of the tasks was defined by the initial position of the hand with respect to the shoulder joint, motion distance, motion angle, and motion time.

Using suboptimization algorithms--linear and geometric programmings--to solve the above-mentioned tasks was not a successful attempt. In the case of the linear programming approach, solution of the model became infeasible for all cases considered. The failure of linear programming to provide a feasible solution can be attributed to the gross approximations which were introduced in order to linearize the model. That is, the method of linearizing the model by the small angles assumption seems to be an invalid one. It is worth mentioning at this point that the small angles assumption was adopted by Nubar and Contini [1961] in formulating their model.

TABLE 1
ANTHROPOMETRIC CHARACTERISTICS^a

	Upper Arm	Forearm
Length-----meter	0.2550	0.3604
Distance of C.G.---meter	0.1137	0.1599
Mass-----kilogram(s)	1.2430	1.1170
Moment of Inertia---newton-meter squared	0.2345	0.3990

^aAdopted from Fischer [1906], cited by Hansen and Cornog [1958],
p. 240.

S--Shoulder Joint
E--Elbow Joint
H--Hand

Fig. 20.--Task configurations

The geometric programming approach to solve the model seems to be a valid one, at least from the theoretical point of view. However, the computer algorithm [Teske, 1970] which was used created the following difficulties.

1. Convergence of the primal and dual functions was in most cases impossible or obtained after a large number of iterations.

2. Computational times were extremely large. In some cases it took between 12 and 24 minutes of computer time before an optimum solution for one point was obtained.

In accordance with the previously mentioned difficulties and because the validity of suboptimization approaches as applied to human motion is questionable, no further attempt was made to consider other suboptimization algorithms.

Feasible solutions for the above examples, however, were achieved by both dynamic programming and simulation algorithms. Figures 21 through 24 show the hand paths of motion obtained by the two algorithms under the four different tasks considered. As can be seen, there is a large similarity between the predictions of some simulation approaches and dynamic programming. The closeness between the dynamic programming and the enumeration approaches are considerably good. However, the variability between dynamic

MOTION CHARACTERISTICS

Distance = 0.3048 meter

Angle = 0.0 deg.

Weight = 0.0 kg.

Time = .6 sec.

INITIAL ARM CONFIGURATION

Upper Arm = 225 deg.

Forearm = 321 deg.

HAND PATH OF MOTION

Key:

- Dynamic Programming
- Simulation--Enumeration
- .- Simulation--Parabola
- ... Simulation--Sine

Fig. 21.--Hand path of motion--task I

MOTION CHARACTERISTICS

Distance = 0.3048 meter

Angle = 0 deg.

Weight = 0 kg.

Time = 0.6 sec.

INITIAL ARM CONFIGURATION

Upper Arm = 225.0 deg.

Forearm = 351.0 deg.

HAND PATH OF MOTION

Key:

—— Dynamic Programming

--- Simulation--Enumeration

Fig. 22.--Hand path of motion--task II

MOTION CHARACTERISTICS

Distance = 0.3048 meter

Angle = 0.0 deg.

Weight = 0.0 kg.

Time = .6 sec.

INITIAL ARM CONFIGURATION

Upper Arm = 270 deg.

Forearm = 43 deg.

Key:

Straight line obtained by both
dynamic programming and simulation
approaches as the optimum path

Fig. 23.--Hand path of motion--task III

MOTION CHARACTERISTICS

Distance = 0.3048 meter

Angle = 0.0 deg.

Weight = 0.0 kg.

Time = 0.4 sec.

INITIAL ARM CONFIGURATION

Upper Arm = 294.0 deg.

Forearm = 73.0 deg.

HAND PATH OF MOTION

Key:

- Dynamic Programming
- — Simulation--Enumeration
- Simulation--Parabola
- . - . Simulation--Sine
- - - Simulation--Ellipse

Fig. 24. --Hand path of motion--task IV

programming and the rest of the simulation approaches (sine, ellipse, and parabola) is somewhat large.

The question which might be asked then is which algorithm should be adopted for the model? The most obvious answer, of course, would be the use of dynamic programming since it gives an optimum path. Next to dynamic programming comes simulation by enumerating through a grid point. For the other simulation approaches (sine, ellipse, and parabola functions) it seems there is a little evidence, based upon the tasks analyzed, of their validity to predict an optimum path of motion which would be the same as that of the human.

CHAPTER V

MODEL TESTING

An experiment was conducted to test the accuracy as well as the adequacy of the proposed model in predicting planar motions. It is obviously impossible to do a very thorough investigation or consider all possible parameters affecting human motion in a short study, and at best this can only serve as a test for the applicability of the model. This chapter includes a discussion of the task, the variables chosen for use in the experiment, the procedure, and a statistical analysis of the data.

The Task

The task chosen for this study was a simple transport movement. Examples of transport movements are plentiful in the military, industry, and everyday life. Many industrial production processes, including many types of assembly, require rapid movements from one position to another. This class of movements, therefore, is of some importance in practical situations. The motion variables of interest were motion distance and work surface height.

The subject participating in this experiment was standing at a table on the top of which a path of motion

for the various motion distances was marked (Figure 25). He was asked to move his right hand between preselected points without repetition, i.e., performing discrete motions. In the course of the experiment, the subject's motion was recorded by photographic means.

Experimental Variables

Motion Distance

It is well known that the motion distance has an important effect on the motion path and the associated motion characteristics such as velocity and acceleration profiles [Ayoub, 1966; Ramsey, 1968; Ayoub, 1969]. Greene emphasizes the necessity of including distance measurements as an important variable when he states:

. . . distance is a necessary ingredient to any determination of a quantity of work in the physical sense . . . the lack of the distance factor is not an insurmountable problem . . . But to measure work without the distance factor is impossible [1958].

Two levels of distance, 12 inches and 24 inches, are usually recommended. The shorter distance corresponds to reach from the edge of the work surface to the center of the work area, and the longer distance corresponds to reach from the edge of the work surface to the rear of the work area. For the purpose of this study, three levels of distance, 9, 12, and 15 inches, were selected. These were found to be very adequate to compensate for arm motion without causing bending of the subject's torso.

Fig. 25.--Task configuration

Work Surface Height

Work surface height is one of the important factors which affects the shape of the path of motion to a large extent. Several studies have been conducted in the interest of determining the best work surface height under different industrial tasks [Gilbreth, 1917; Warner, 1920; Knowle, 1946; Ellis, 1951]. It has been shown by Ellis [1951] and verified by Konz [1967] through extensive experimentation that the distance between the elbow and the work surface is the best criterion for determining the work surface height. Konz recommends a surface height one inch below the elbow as the optimum height in contrast to a three-inch height suggested by Ellis. For this study three surface heights were selected. They measure 6, 3, and 0 inches from the elbow joint.

Subjects

Ten male subjects were selected for this experiment. The only limiting factors concerning the subjects were that they be of similar age, race, with average physical build, i.e., excluding the athletic type individual, and with an average degree of performance skills.

The anthropometric characteristics for each subject were determined by using the method of coefficients introduced by Fischer [Drillis, et al., 1964]. By using this

method, the arm segments' masses, distances of center of masses, and radius of gyration are determined by using three coefficients C_1 , C_2 , C_3 respectively. The first coefficient represents the ratio of the segment mass to the total body mass. The second coefficient is the ratio of the distance of the center of mass from the proximal joint to the total segment length. The third coefficient is the ratio of the radius of gyration of the segment about the mediolateral axis to the total segment length.

Therefore, upon knowing the total body mass and the arm dimensions, the mass, distance of center of mass, and radius of gyration for each segment can be obtained. In this study the values for C_1 and C_2 were obtained from Dempster's estimates rather than from those of Fischer (Figure 26). The value proposed by Fischer for C_3 was adopted. It was equal to 0.3.

Table 2 shows subjects' anthropometric characteristics for the arm as obtained by the method of coefficients discussed above. It should be appreciated that the method of coefficients is an approximate method with some inherent errors. However, through sensitivity analysis, variations in the three coefficients C_1 , C_2 , and C_3 by as much as $\pm 20\%$ from the original values proposed by Dempster [1955] resulted in minor changes for the model's predictions. In most cases, variation in the total power

Fig. 26.--Arm segment masses and their locations expressed as percentages of arm lengths and total body mass [adopted from Dempster, 1955].

TABLE 2

ANTHROPOMETRIC CHARACTERISTICS FOR THE SUBJECTS

SUBJECT									
1	2	3	4	5	6	7	8	9	10
<u>Upper Arm</u>									
Length-----meter	.365	.360	.343	.381	.357	.371	.345	.361	.345
C.G. distance--meter	.160	.1569	.150	.166	.155	.163	.152	.155	.152
Mass-----kgs.	2.0	1.85	1.91	2.36	1.84	2.1	1.8	1.95	1.83
Radius of gyration-----meter	.109	.108	.103	.114	.102	.106	.103	.105	.103
<u>Forearm</u>									
Length-----meter	.460	.457	.445	.508	.449	.464	.45	.465	.452
C.G. distance--meter	.1878	.198	.191	.218	.1935	.1978	.193	.197	.194
Mass-----kgs.	1.7	1.5	1.55	1.91	1.6	1.85	1.6	1.75	1.45
Radius of gyration-----meter	.146	.143	.134	.152	.133	.138	.134	.140	.135

required to perform the motion was within $\pm 1.0\%$ of the total power computed with the original coefficients. On the other hand, varying the three coefficients by $\pm 40\%$ of the original values resulted in a definite change in the predictions of the model. For instance, total power changed by as much as 10%. Also, considerable change in the shape of the optimum path of motion occurred as shown in Figure 27.

Key:

- optimum path of motion obtained with the original values of the coefficients (C_1, C_2, C_3)
- optimum path of motion obtained with + 40% variations in the coefficient values

Fig. 27.--Typical effect of 40% variations in the anthropometric coefficients upon the optimum path of motion.

It can be concluded from the sensitivity analysis that variations in the three coefficients by as much as 40% from the original values are necessary before any appreciable effects upon the model's prediction can be observed. This phenomenon could be attributed to the small magnitude of the coefficients and consequently the values resulting from their use.

Equipment

Camera

A photographic camera was used to record the subjects' motions. The camera used in this study was a 4 x 5 Crown Graphic camera with a focal length $f=135$ mm. The Crown Graphic camera is designed to use Polaroid films as well as sheet films. The distortion curve and the principal points of the camera's lens were previously determined in another study [Ayoub, 1969]. The lens' distortion error was proved to be small and can be disregarded without affecting the accuracy of measurements. The camera was supported by a special adjustable base as shown in Figure 28. By means of this base the camera can be completely leveled and oriented parallel to the vertical plane of the work table. Polaroid films (Type 52-4x5) were used for all recordings.

Fig. 28.--Experimental equipment

Lighting during Recording

For motion recording a General Radio Strobotac type 1531 was used to record multiple exposures on the camera films. This necessitated performing the experiment under total darkness. The disadvantage in using the strobe for human motion recording is its effect under certain conditions of frequency and intensity upon the subject's performance. This did not constitute a serious drawback in this study, however, simply because of the very short duration of the experimental task and the relatively low frequency used. Also, during the recording process, the strobe was positioned behind the subject to eliminate direct exposure of subject to the strobe flashing light.

Adjustable Table

The table selected for this experiment is shown in Figure 28 (page 114). It was adjustable as to height, rotation, and tilting angle. This was accomplished by mounting a board on a converted base of a dental chair. A protractor was fixed to the table's board to indicate the tilting angle, thus making adjustment of the board particularly direct and simple. On top of the table, three scales were positioned in such a way that they formed a vertical plane. This plane was used to position the camera parallel to the subjects' sagittal planes. Also, six control

targets were attached to the three scales for the purpose of obtaining the photographic scales from the pictures.

Experimental Procedure

Each of the subjects participating in this study was instructed and trained thoroughly with regard to his duties in the experiment. This was done essentially to familiarize each subject with the equipment and with the motions he would be required to make.

Following the training phase, photographic records for all subjects' motions under all different experimental combinations were taken. From these records a path of motion for each subject under each experimental combination was generated.

At each experimental session the subject was asked to wear a black glove, and a light-reflective tape was placed at the arm segments. Through an adjustment in table height, the subject was standing at the work table in such a way that the predetermined distance between the elbow and work surface could be obtained. Each subject was instructed to move his hand between two preselected terminal points at his normal pace, i.e., a pace he could maintain for extended periods. After photographing each motion, the subject was given a two-minute rest period and he was allowed to move from his position. A total of 180 motions was recorded for the ten subjects under all levels

of distance and elbow heights considered in the experiment. Figure 29 shows typical examples of some of the motions obtained during the course of the experiment.

Hand Path of Motion

The hand paths of motion for all subjects under different motions investigated in this study were measured from the photographic records obtained for the motions. The measurements were obtained by retracing upon graph paper the hand paths of motion as well as the positions of the six control targets appearing on each photograph. Next, for each motion, the path of motion coordinates as well as three measurements for the photographic scale (distances between each pair of the control targets) was read from the graph paper. By computing the photographic scale and using the measured coordinates, the actual paths of motion for each subject under each experimental condition were obtained. The time of each motion was obtained directly by counting the number of exposures recorded on the photographic film.

For each experimental motion an optimum path of motion was generated by using the proposed model and the dynamic programming algorithm under the same conditions as those of the experimental one. That is, under the same motion distance, initial arm configuration, subject's anthropometric characteristics, and motion time an optimum

Fig. 29.--Typical examples of the photographic records obtained during the experiment.

path of motion as predicted by the model was obtained. Therefore, for each motion investigated in this study two paths were obtained: an experimental one and an optimum one predicted by the model. To test the relationship between the two paths the following quantitative analyses were used.

1. A correlation analysis was conducted to test the relationship between the two paths of motion, that is, to test for similarity between the paths as far as locations of the peak points and linearity are concerned. It should be understood that correlation does not test for the closeness between the two paths of motion. An example might be useful in explaining this point. Let us consider the two paths shown in Figure 30. The two paths are similar to each other in such a way that they increase and decrease together and the location of their peak points is the same. Computing the correlation coefficient for the two paths presented in Figure 30a would be equal to or very close to 1.0. On the other hand, shifting the two paths' peak points along the X-axis as shown in Figure 30b would result in lowering the correlation coefficient to approximately 0.60. It follows, therefore, that correlation can be used to measure the similarity between the two paths of motion but not the closeness between them. That is to say, a perfect correlation (coefficient=1) between the two paths

0.0	1.0	0.0	Correlation Coefficient = 1.0
0.0	.10	0.0	

a.--Paths with peak points at the same position along the X-axis and with different height along the Z-axis.

0.0	1.0	.5	0.0	Correlation Coefficient = .6
0.0	.667	1.0	0.0	

b.--Paths with peak points shifted from each other along the X-axis and with the same height at the Z-axis.

Fig. 30.--Correlation analysis for two paths of motion

of motion would indicate that they follow the same trend but do not necessarily coincide on each other or even come close to coinciding.

2. To test for the closeness between the two paths of motion, the following quantitative measures were used:

(i) The difference at each point in time between the coordinates of the experimental and the theoretical paths, expressed as:

$$E_t = \frac{ZH_{pt} - ZH_{mt}}{ZH_{mt}} \times 100.0$$

where

E_t = percentage error at time t ,

ZH_{pt} = predicted hand Z-coordinate at time t by the model, and

ZH_{mt} = measured hand Z-coordinate at time t by photographic means.

(ii) The difference between the total areas under the two paths of motion, expressed as:

$$E_A = \frac{A_p - A_m}{A_m} \times 100.0$$

where

E_A = percentage errors of the two areas,

A_p = area under predicted path by the model, and

A_m = area under the experimental path.

Results and Interpretations

Figures 31 through 39 show typical examples for some plots of all different paths of motion which were recorded experimentally versus those predicted by the model using the dynamic programming algorithm under the same task conditions. The sequence number given on each figure indicates a particular combination of the experimental variables under which the motion was recorded. The numbers in each sequence indicate the levels of subjects, replication, distance, and work surface height respectively. For example, the sequence 31096 indicates that a first trial for motion was performed by subject 3 at a distance of 9 inches with the elbow 6 inches above the work surface. It can be seen that the paths predicted by the model are very close to those obtained by experimental analysis for the subjects. Also, the plots of motion paths show little variation among different motions investigated for subjects' performances. This could be attributed to the training phase of the experiment.

Tables 3, 4, and 5 show the correlation coefficients and measures of difference for all subjects under different levels of distance and work surface height.

 MOTION SEQUENCE= 11096

PERCENT DIFFERENCE OF MOTION PATH COOR.

0.0 -2.94 -3.98 0.0 -0.21 0.0

AVERAGE= -1.18952
 STANDARD ERROR= 1.63631

DIFFERENCE BETWEEN CURVES AREAS IN PERCENT= 1.40068

PLOTS OF THE HAND PATH OF MOTION

* THEORETICAL X EXPERIMENTAL

.
 T .X
 I . X
 M . *X
 E . X
 . X
 .X

 MOTION SEQUENCE= 81096

PERCENT DIFFERENCE OF MOTION PATH COOR.

0.0 -0.64 -0.01 0.0

AVERAGE= -0.16356
 STANDARD ERROR= 0.27635

DIFFERENCE BETWEEN CURVES AREAS IN PERCENT= 0.21656

PLOTS OF THE HAND PATH OF MOTION

* THEORETICAL X EXPERIMENTAL

.
 T .X
 I . X
 M . X
 E .X

Fig. 31.--Motion performed at elbow height of 6 inches above the work surface and distance of 9 inches.

```

*****
MOTION SEQUENCE= 11126
-----
PERCENT DIFFERENCE OF MOTION PATH COOR.

0.0 -2.08 -3.32 -0.22 0.0 0.0

AVERAGE= -0.93719
STANDARD ERROR= 1.29950
-----
DIFFERENCE BETWEEN CURVES AREAS IN PERCENT= 1.10922
-----

```

PLOTS OF THE HAND PATH OF MOTION

* THEORETICAL X EXPERIMENTAL

```

. . . . .
T .X
I . X
M . *X
E . X
. X
.X

```

```

*****
MOTION SEQUENCE= 62126
-----
PERCENT DIFFERENCE OF MOTION PATH COOR.

0.0 -4.79 -4.56 -0.40 -1.53 -2.66 0.0

AVERAGE= -1.99335
STANDARD ERROR= 1.91184
-----
DIFFERENCE BETWEEN CURVES AREAS IN PERCENT= 0.91377
-----

```

PLOTS OF THE HAND PATH OF MOTION

* THEORETICAL X EXPERIMENTAL

```

. . . . .
T .X
I .*X
M . *X
E . X
. X
. X*
.X

```

Fig. 32.--Motion performed at elbow height of 6 inches above the work surface and distance of 12 inches.

 MOTION SEQUENCE= 92156

PERCENT DIFFERENCE OF MOTION PATH COOR.

0.0 -2.60 -1.82 -0.85 -2.98 0.0

AVERAGE= -1.37685
 STANDARD ERROR= 1.17991

DIFFERENCE BETWEEN CURVES AREAS IN PERCENT= 0.13055

PLOTS OF THE HAND PATH OF MOTION

* THEORETICAL X EXPERIMENTAL

.
 T .X
 I .X
 M . X
 E . X
 . X
 .X

 MOTION SEQUENCE= 52156

PERCENT DIFFERENCE OF MOTION PATH COOR.

0.0 -1.12 -3.04 -3.46 -1.09 -1.02 0.0

AVERAGE= -1.39015
 STANDARD ERROR= 1.26369

DIFFERENCE BETWEEN CURVES AREAS IN PERCENT= -1.24745

PLOTS OF THE HAND PATH OF MOTION

* THEORETICAL X EXPERIMENTAL

.
 T .X
 I .X
 M . X
 E . X
 . X
 . X
 .X

Fig. 33.--Motion performed at elbow height of 6 inches above the work surface and distance of 15 inches.

```

=====
MOTION SEQUENCE=  82093
-----

PERCENT DIFFERENCE OF MOTION PATH COOR.

    0.0   -0.25  -0.36  -0.26   0.0

      AVERAGE= -0.17391
    STANDARD ERROR=  0.14782
-----

DIFFERENCE BETWEEN CURVES AREAS IN PERCENT= -0.08937
-----

PLOTS OF THE HAND PATH OF MOTION

* THEORETICAL          X EXPERIMENTAL

      . . . . .

    T  .X
    I  .X
    M  . X
    E  . X
      .X

```

```

=====
MOTION SEQUENCE= 102123
-----

PERCENT DIFFERENCE OF MOTION PATH COOR.

    0.0   -1.84  -1.45  -0.18  -1.37  -2.45   0.0

      AVERAGE= -1.04239
    STANDARD ERROR=  0.91308
-----

DIFFERENCE BETWEEN CURVES AREAS IN PERCENT= -0.04349
-----

PLOTS OF THE HAND PATH OF MOTION

* THEORETICAL          X EXPERIMENTAL

      . . . . .

    T  .X
    I  .X
    M  . X
    E  . X
      . X
      . X
      . X

```

Fig. 34.--Motion performed at elbow height of 3 inches above the work surface and distance of 9 inches.

```

=====
MOTION SEQUENCE= 32123
-----

PERCENT DIFFERENCE OF MOTION PATH COOR.

    0.0   -0.97  -0.74  -1.55   0.0

      AVERAGE= -0.65182
    STANDARD ERROR= 0.59342
-----

DIFFERENCE BETWEEN CURVES AREAS IN PERCENT= -0.32187
-----

PLOTS OF THE HAND PATH OF MOTION

* THEORETICAL          X EXPERIMENTAL

      . . . . .

    T .X
    I .X
    M . X
    E . X

=====

MOTION SEQUENCE= 22123
-----

PERCENT DIFFERENCE OF MOTION PATH COOR.

    0.0    0.0    0.0    0.0

      AVERAGE= 0.0
    STANDARD ERROR= 0.0
-----

DIFFERENCE BETWEEN CURVES AREAS IN PERCENT= 0.0
-----

PLOTS OF THE HAND PATH OF MOTION

* THEORETICAL          X EXPERIMENTAL

      . . . . .

    T .X
    I . X
    M . X
    E .X

```

Fig. 35.--Motion performed at elbow height of 3 inches above the work surface and distance of 12 inches.

 MOTION SEQUENCE= 71153

PERCENT DIFFERENCE OF MOTION PATH COOR.

0.0 -0.99 -0.03 -1.54 -1.46 0.0

AVERAGE= -0.67170
 STANDARD ERROR= 0.68256

DIFFERENCE BETWEEN CURVES AREAS IN PERCENT= 0.20162

PLOTS OF THE HAND PATH OF MOTION

* THEORETICAL X EXPERIMENTAL

.

T .X

I .X

M . X

E . X

. X

.X

 MOTION SEQUENCE= 91153

PERCENT DIFFERENCE OF MOTION PATH COOR.

0.0 -3.72 -3.00 -0.66 -1.89 0.0

AVERAGE= -1.54564
 STANDARD ERROR= 1.44431

DIFFERENCE BETWEEN CURVES AREAS IN PERCENT= 0.83566

PLOTS OF THE HAND PATH OF MOTION

* THEORETICAL X EXPERIMENTAL

.

T .X

I .X

M . X

E . X

. X

.X

Fig. 36.--Motion performed at elbow height of
 3 inches above the work surface and distance of 15 inches.

```

*****
MOTION SEQUENCE= 51090
-----

PERCENT DIFFERENCE OF MOTION PATH COOR.

0.0 -5.08 -2.11 -0.04 0.0

      AVERAGE= -1.44532
      STANDARD ERROR= 1.99121
-----

DIFFERENCE BETWEEN CURVES AREAS IN PERCENT= 1.75584
-----

PLOTS OF THE HAND PATH OF MOTION

* THEORETICAL          X EXPERIMENTAL

      . . . . .
      T .X
      I . * X
      M . * X
      E . X
      .X

*****

MOTION SEQUENCE= 61090
-----

PERCENT DIFFERENCE OF MOTION PATH COOR.

0.0 -2.51 -0.51 -0.74 -1.42 0.0

      AVERAGE= -0.86317
      STANDARD ERROR= 0.87882
-----

DIFFERENCE BETWEEN CURVES AREAS IN PERCENT= 0.17890
-----

PLOTS OF THE HAND PATH OF MOTION

* THEORETICAL          X EXPERIMENTAL

      . . . . .
      T .X
      I . * X
      M . X
      E . X
      . X
      .X

```

Fig. 37.--Motion performed at elbow height of 0 inches above the work surface and distance of 9 inches.

```
=====
MOTION SEQUENCE=  92120
-----
```

```
PERCENT DIFFERENCE OF MOTION PATH COOR.
```

```
0.0  -3.07  -1.55  -2.00  -2.61  0.0
```

```
      AVERAGE=  -1.53707
      STANDARD ERROR=  1.18471
-----
```

```
DIFFERENCE BETWEEN CURVES AREAS IN PERCENT=  0.00952
-----
```

```
PLOTS OF THE HAND PATH OF MOTION
```

```
* THEORETICAL          X EXPERIMENTAL
```

```
      . . . . .
```

```
T  .X
```

```
I  .X
```

```
M  . X
```

```
E  . X
```

```
      . X
```

```
=====
MOTION SEQUENCE=  51120
-----
```

```
PERCENT DIFFERENCE OF MOTION PATH COOR.
```

```
0.0  -3.74  -1.69  -0.56  0.0
```

```
      AVERAGE=  -1.19712
      STANDARD ERROR=  1.41122
-----
```

```
DIFFERENCE BETWEEN CURVES AREAS IN PERCENT=  1.48145
-----
```

```
PLOTS OF THE HAND PATH OF MOTION
```

```
* THEORETICAL          X EXPERIMENTAL
```

```
      . . . . .
```

```
T  .X
```

```
I  .X
```

```
M  . X
```

```
E  . X
```

```
      .X
```

Fig. 38.--Motion performed at elbow height of 0 inches above the work surface and distance of 12 inches.

MOTION SEQUENCE= 81150

PERCENT DIFFERENCE OF MOTION PATH COOR.

0.0 -3.27 -1.34 -3.20 -2.79 0.0

AVERAGE= -1.76653
STANDARD ERROR= 1.40132

DIFFERENCE BETWEEN CURVES AREAS IN PERCENT= -0.25613

PLOTS OF THE HAND PATH OF MOTION

* THEORETICAL X EXPERIMENTAL

.

T .X

I .X

M . X

E . X

. X

.X

MOTION SEQUENCE= 42150

PERCENT DIFFERENCE OF MOTION PATH COOR.

0.0 -3.32 -1.25 -3.11 -2.96 0.0

AVERAGE= -1.77566
STANDARD ERROR= 1.42424

DIFFERENCE BETWEEN CURVES AREAS IN PERCENT= 2.09392

PLOTS OF THE HAND PATH OF MOTION

* THEORETICAL X EXPERIMENTAL

.

T .X

I .X

M . X

E . X

. X

.X

Fig. 39.--Motion performed at elbow height of 0 inches above the work surface and distance of 15 inches.

TABLE 3

CORRELATION COEFFICIENTS BETWEEN EXPERIMENTAL PATHS
AND THE THEORETICAL ONES

Elbow Height	Subject		1	2	3	4	5	6	7	8	9	10
	Distance											
6	9		.730	.954	.876	.851	.579	.873	.999	.994	.759	.531
			.899	.997	.985	.912	.783	.793	.543	.805	.934	.637
6	12		.925	.993	.762	.745	.745	.600	.976	.804	.792	.923
			.9087	.999	.887	.849	.751	.697	.907	.863	.713	.857
6	15		.914	.967	.884	.589	.747	.687	.984	.969	.883	.672
			.856	.998	.928	.832	.964	.766	.910	.979	.905	.738
3	9		.872	.893	.736	.788	.746	.873	.998	.917	.953	.828
			.930	.993	.789	.992	.869	.730	.747	.992	.853	.913
3	12		.895	.832	.934	.713	.937	.913	.922	.942	.912	.671
			.925	.895	.947	.825	.847	.892	.900	.932	.842	.899
3	15		.778	.887	.886	.893	.900	.876	.977	.822	.884	.691
			.915	.989	.896	.925	.876	.767	.892	.796	.805	.845
0	9		.957	.775	.786	.80	.813	.877	.987	.864	.915	.883
			.937	.939	.855	.876	.795	.894	.989	.781	.776	.944
0	12		.895	.765	.718	.788	.728	.794	.959	.913	.547	.816
			.903	.945	.792	.815	.776	.859	.998	.832	.819	.675
0	15		.861	.840	.677	.805	.787	.852	.971	.863	.911	.916
			.885	.869	.834	.845	.856	.618	.938	.814	.900	.714

TABLE 4

PERCENTAGE DIFFERENCES IN AREAS BETWEEN EXPERIMENTAL PATHS
AND THE THEORETICAL ONES

Elbow Height	Subject		1	2	3	4	5	6	7	8	9	10
	Distance											
6	9	1.46	0.14	2.03	0.92	0.27	0.82	1.60	0.22	2.00	0.73	
		1.07	0.64	0.22	1.09	2.60	0.65	1.04	0.04	1.01	0.95	
	12	1.10	0.49	0.92	0.47	0.93	0.35	0.63	1.49	0.71	0.50	
		2.11	2.59	0.74	0.69	0.39	0.91	0.45	0.07	1.09	1.24	
	15	0.59	1.15	0.78	1.83	0.63	0.19	1.73	2.16	0.76	2.27	
		2.26	1.40	1.33	0.40	1.24	0.96	1.35	1.98	0.13	2.80	
3	9	1.72	1.29	2.17	1.12	0.39	0.48	2.00	1.08	0.64	1.38	
		0.91	0.69	0.75	2.52	2.86	0.34	0.18	0.09	1.50	0.04	
	12	1.61	0.21	0.43	0.16	0.14	0.26	1.63	1.16	2.0	1.16	
		1.31	0.0	0.32	5.07	0.74	0.60	0.93	1.21	0.78	1.18	
	15	1.32	2.126	0.85	0.10	0.13	0.76	0.20	1.46	0.84	2.01	
		1.83	0.79	0.13	1.24	1.33	0.24	0.04	1.23	0.96	1.57	
0	9	2.49	0.95	2.12	2.75	1.76	1.23	1.70	0.09	0.83	1.42	
		0.80	1.10	1.76	2.75	2.12	0.18	0.40	0.79	0.43	0.50	
	12	2.02	0.67	0.42	1.91	1.48	0.3	2.32	2.03	0.01	0.73	
		2.02	1.19	1.48	1.91	1.61	0.97	2.12	1.30	0.89	1.32	
	15	2.79	2.00	3.56	0.35	1.66	0.06	0.97	0.26	0.23	0.93	
		2.79	0.95	0.149	2.09	3.28	0.09	2.91	1.74	0.46	1.09	

TABLE 5
PERCENTAGE DIFFERENCE OF MOTION PATH COORDINATES

Motion Sequence	Elapsed Time (seconds)							
	0	.1	.2	.3	.4	.5	.6	.7
11096	0.0	-2.18	-1.56	-1.54	-0.25	0.0		
12096	0.0	-2.94	-3.98	0.0	-0.21	0.0		
11126	0.0	-2.08	-3.32	-0.22	0.0	0.0		
12126	0.0	-1.01	-5.97	-3.19	-0.71	0.0		
11156	0.0	-2.08	-1.33	-2.43	-4.06	0.0		
12156	0.0	-3.25	-4.95	-1.20	0.0			
11093	0.0	-1.24	-2.13	-3.26	-2.13	0.0		
12093	0.0	-1.89	-3.81	-2.33	0.0			
11123	0.0	-2.13	-4.44	-1.77	0.0	0.0		
12123	0.0	-2.08	-2.17	-1.09	0.0			
11153	0.0	-1.03	-1.08	-3.33	0.0			
12153	0.0	-2.19	-4.23	-4.74	-1.51	0.0		
11090	0.0	-1.89	-4.08	-4.36	0.0			
12090	0.0	-0.43	-3.83	0.0	0.0			
11120	0.0	-3.03	-3.11	-2.13	0.0			
12120	0.0	-3.03	-3.11	-2.13	0.0			
11150	0.0	-2.80	-6.29	-3.80	-1.63	0.0		
12150	0.0	-2.80	-6.29	-3.80	-1.63	0.0		
21096	0.0	-0.63	-0.22	0.0				
22096	0.0	-0.79	-1.25	-2.12	0.0			
21126	0.0	0.0	-1.53	0.0				
22126	0.0	-3.69	-4.31	0.0				
21156	0.0	-4.55	-0.90	0.0				
22156	0.0	-2.13	-2.22	0.0				

TABLE 5--Continued

Motion Sequence	Elapsed Time (seconds)							
	0	.1	.2	.3	.4	.5	.6	.7
21093	0.0	-2.02	-1.95	0.0				
22093	0.0	0.0	-2.22	0.0				
21123	0.0	-0.53	-0.11	0.0				
22123	0.0	0.0	0.0	0.0				
21153	0.0	-2.91	-7.43	-2.14	0.0			
22153	0.0	-1.03	-1.10	-1.10	0.0			
21090	0.0	-3.12	-2.41	-2.05	0.0			
22090	0.0	-4.08	-2.11	-0.04	0.0			
21120	0.0	-3.75	-1.69	-0.89	0.0			
22120	0.0	-2.35	-1.90	-3.52	0.0			
21150	0.0	-3.32	-1.25	-3.11	0.0			
22150	0.0	-1.72	-3.38	-2.12	0.0			
31096	0.0	-4.14	-2.25	-1.82	0.0			
32096	0.0	-0.80	-0.13	0.0		0.0		
31126	0.0	-4.41	-1.01	-0.55	-0.32			
32126	0.0	-1.34	-1.97	-0.32	0.0	0.0		
31156	0.0	-2.79	-0.20	-2.94	-3.66			
32156	0.0	-2.06	-2.95	-0.40	0.0			
31093	0.0	-1.81	-2.74	-4.26	0.0			
32093	0.0	-1.93	-0.21	-2.18	-3.77	0.0		
31123	0.0	-2.35	-1.36	-1.93	-1.34	0.0		
32123	0.0	-0.97	-0.74	-1.55	0.0			
31153	0.0	-3.59	-1.24	-3.46	-3.34	0.0		
32153	0.0	-4.38	-1.38	-2.44	-2.75	0.0		
31090	0.0	-4.12	-2.41	-2.05	0.0			
32090	0.0	-5.08	-2.11	-0.04	0.0			
31120	0.0	-3.59	-2.03	-0.89	-2.63	0.0		

TABLE 5--Continued

Motion Sequence	Elapsed Time (seconds)							
	0	.1	.2	.3	.4	.5	.6	.7
32120	0.0	-3.74	-1.69	-0.56	0.0			
31150	0.0	-2.59	-6.76	-6.65	-2.09	0.0		
32150	0.0	-3.26	-2.61	-0.02	-1.33	-3.63	0.0	
41096	0.0	-2.88	-0.05	0.0				
42096	0.0	-3.81	-0.43	0.0				
41126	0.0	-2.58	-0.59	-1.27	0.0			
42126	0.0	-3.43	-1.26	-1.88	0.0			
41156	0.0	-3.43	-3.51	-0.49	0.0			
42156	0.0	-1.28	-3.21	-2.74	0.0			
41093	0.0	-2.10	-1.33	0.0				
42093	0.0	-3.08	-4.54	-2.56	0.0			
41123	0.0	-0.96	-1.22	-0.88	-2.26	0.0		
42123	0.0	-2.33	-8.47	-10.38	-4.91	0.0		
41153	0.0	-1.11	-1.28	-2.12	-0.84	-5.78	0.0	
42153	0.0	-3.51	-2.91	-4.61	-4.41	0.0		
41090	0.0	-1.72	-3.38	-6.13	0.0			
42090	0.0	-1.72	-3.38	-6.13	0.0			
41120	0.0	-3.52	-1.90	-2.35	0.0			
42120	0.0	-2.35	-1.90	-3.52	0.0			
41150	0.0	-5.49	-1.85	-2.48	0.0			
42150	0.0	-3.32	-1.25	-3.11	-2.96	0.0		
51096	0.0	-2.91	-0.17	-1.94	0.0			
52096	0.0	-4.68	-4.29	-1.60	0.0			
51126	0.0	-3.78	-3.51	-1.90	-0.66	0.0		
52126	0.0	-3.78	-1.61	-0.26	-3.12	0.0		
51156	0.0	-3.23	-1.48	-2.38	-0.35	-3.66	0.0	
52156	0.0	-1.12	-3.04	-3.46	-1.09	-1.02	0.0	

TABLE 5--Continued

Motion Sequence	Elapsed Time (seconds)									
	0	.1	.2	.3	.4	.5	.6	.7		
51093	0 0	-1.92	-1.72	-0.07	-1.61	0.0				
52093	0 0	-2.09	-4.85	-4.67	0.0					
51123	0 0	-3.26	-2.61	-0.02	-1.33	-3.63	0.0			
52123	0 0	-1.34	-1.97	-0.32	0.0					
51153	0.0	-4.38	-1.38	-2.44	-2.75	0.0				
52153	0.0	-2.06	-2.95	-0.40	0.0					
51090	0.0	-5.08	-2.11	-0.04	0.0					
52090	0.0	-4.12	-2.41	-2.05	0.0					
51120	0.0	-3.74	-1.69	-0.56	0.0	0.0				
52120	0.0	-2.59	-1.48	-2.24	-2.09	0.0				
51150	0.0	-5.34	-6.01	-3.88	-1.36	0.0				
52150	0.0	-2.13	-2.01	-3.06	-0.99	0.0				
61096	0.0	-2.32	-1.99	-0.80	-0.09	-1.32	0.0			
62096	0.0	-2.18	-2.53	-0.26	0.0					
61126	0.0	-2.60	-0.82	-2.34	-2.65	0.0				
62126	0.0	-2.78	-3.05	-1.35	-1.92	-3.10	0.0			
61156	0.0	-3.31	-1.52	-0.92	0.0					
62156	0.0	-2.51	-0.51	-0.74	-1.42	0.0				
61093	0.0	-6.05	-1.94	-3.20	-4.26	0.0				
62093	0.0	-2.59	-3.45	-1.16	-0.97	-4.77	0.0			
61123	0.0	-2.40	-0.71	-2.15	-2.65	0.0				
62123	0.0	-2.51	-0.51	-2.06	-2.70	0.0				
61153	0.0	-1.65	-1.49	-0.59	-2.68	-4.57	0.0			
62153	0.0	-2.22	-0.82	-1.77	-2.65	0.0				
61090	0.0	-2.51	-1.48	-0.55	-1.04	0.0				
62090	0.0	-2.93	-3.55	-2.06	-2.58	-5.36	-7.22	0.0		
61120	0.0	-4.79	-4.56	-0.40	-1.53	-2.66	0.0			
62120	0.0	-3.47	-2.06	-0.36	-1.04	0.0				

TABLE 5--Continued

Motion Sequence	Elapsed Time (seconds)							
	0	.1	.2	.3	.4	.5	.6	.7
61150	0.0	-2.65	-2.17	-0.02	-3.21	-3.76	0.0	
62150	0.0	-4.28	-4.75	-1.65	-2.03	-3.36	-4.03	0.0
71096	0.0	-4.63	-5.04	-4.13	-0.88	-0.19	-3.05	0.0
72096	0.0	-1.39	-0.75	-0.42	0.0			
71126	0.0	-1.80	-1.86	-3.46	0.0			
72126	0.0	-1.02	-2.18	0.0				
72156	0.0	-1.39	-1.52	-1.11	0.0			
72156	0.0	-2.56	-2.87	-0.05	0.0			
71093	0.0	-2.58	-3.60	0.0				
72093	0.0	-2.34	-3.52	-0.80	0.0			
71123	0.0	-0.99	-0.03	-1.54	-1.46	0.0		
72123	0.0	-1.80	-0.77	-1.79	0.0			
71153	0.0	-1.84	-2.56	-1.20	-2.83	0.0		
72153	0.0	-0.99	-3.76	-0.29	-3.57	0.0		
71090	0.0	-0.49	-2.33	-4.21	0.0			
72090	0.0	-2.92	-3.32	-3.20	0.0			
71120	0.0	-2.50	-1.54	-0.29	-1.09	0.0		
72120	0.0	-1.02	-0.20	0.0				
71150	0.0	-1.02	-3.52	-4.23	0.0			
72150	0.0	-4.44	-4.84	-4.09	-1.50	0.0		
81096	0.0	-3.31	-3.32	-1.59	0.0			
82096	0.0	-2.56	-1.54	-0.87	0.0			
81126	0.0	-3.31	-2.32	-0.35	0.0			
82126	0.0	-3.27	-1.34	-3.20	-2.79	0.0		
81156	0.0	-3.73	-2.87	-0.44	0.0			
82156	0.0	-0.64	-0.01	0.0				
81093	0.0	-3.70	-2.71	-0.35	0.0			

TABLE 5--Continued

Motion Sequence	Elapsed Time (seconds)									
	0	.1	.2	.3	.4	.5	.6	.7		
82093	0.0	-4.48	-2.20	0.0						
81123	0.0	-1.80	-0.92	-1.05	0.0					
82123	0.0	-2.15	-0.59	-1.29	0.0					
81153	0.0	-2.92	-3.14	0.0						
82153	0.0	-1.80	-2.33	-0.26	0.0					
81090	0.0	-2.34	-1.52	-0.80	0.0					
82090	0.0	-2.69	-0.80	-4.14	-5.16	0.0				
81120	0.0	-0.25	-0.36	-0.26	0.0					
82120	0.0	-2.53	-1.52	-0.80	0.0					
81150	0.0	-2.31	-0.22	-2.45	-5.86	0.0				
82150	0.0	-1.61	-0.18	-1.05	0.0					
91096	0.0	-3.91	-1.00	-2.54	-1.71	0.0				
92096	0.0	-3.07	-1.55	-2.00	-2.61	0.0				
91126	0.0	-3.72	-1.85	-0.20	-1.18	0.0				
92126	0.0	-2.60	-0.68	-2.54	-3.16	0.0				
91156	0.0	-4.50	-1.49	-2.32	0.0					
92156	0.0	-1.71	-0.06	-0.07	0.0					
91093	0.0	-3.72	-0.44	-4.70	-3.52	0.0				
92093	0.0	-3.63	-2.51	-0.52	-1.71	0.0				
91123	0.0	-2.98	-3.77	-1.12	-0.27	0.0				
92123	0.0	-3.72	-3.00	-0.66	-1.89	0.0				
91153	0.0	-3.54	-3.02	-1.90	0.0					
92153	0.0	-1.15	-0.55	-1.84	-2.07	0.0				
91090	0.0	-2.60	-1.82	-0.85	-2.98	0.0				
92090	0.0	-5.34	-1.55	-1.81	-1.53	0.0				
91120	0.0	-4.30	-1.88	0.0						
92120	0.0	-4.20	-2.65	-0.63	-0.49	-1.29	0.0			

TABLE 5--Continued

Motion Sequence	Elapsed Time (seconds)							
	0	.1	.2	.3	.4	.5	.6	.7
91150	0.0	-3.31	-3.72	-1.88	-0.07	-2.27	0.0	
92150	0.0	-4.10	-2.04	-0.75	-0.27	0.0		
101096	0.0	-2.71	-1.16	-1.18	-3.07	-6.25	0.0	
102096	0.0	-2.39	-1.45	-2.19	-4.21	-5.40	0.0	
101126	0.0	-1.60	-1.76	-3.96	-3.14	0.0		
102126	0.0	-2.60	-0.44	-3.81	-3.16	0.0		
101156	0.0	-1.84	-1.83	-1.65	-3.34	-3.16	0.0	
102156	0.0	-1.15	-0.36	-1.48	-2.60	0.0	0.0	
101093	0.0	-1.84	-1.45	-0.18	-1.37	-2.45	0.0	
102093	0.0	-1.44	-0.61	-2.45	-3.25	-5.91	0.0	
101123	0.0	-2.15	-1.40	-3.96	-2.79	0.0		
102123	0.0	-1.44	-0.79	-2.63	-4.84	-6.91	0.0	
101153	0.0	-1.55	-1.56	-1.30	-0.23	-5.04	-7.13	0.0
102153	0.0	-1.33	-1.44	-3.60	-3.29	0.0		
101090	0.0	-1.95	-0.78	-2.78	-5.58	-6.68	-7.45	0.0
102090	0.0	-1.19	-0.84	-0.54	-2.41	-2.78	-5.13	0.0
101120	0.0	-2.62	-3.74	-2.49	-0.26	-3.21	0.0	
102120	0.0	-2.53	-1.34	-3.33	-6.54	-7.57	0.0	
101150	0.0	-2.39	-1.64	-0.20	-1.18	-1.00	-2.52	
102150	0.0	-0.45	-0.24	-3.58	0.0			

Figures 40 through 42 present the correlation coefficients and measures of difference grouped by motion distances, elbow height and the over-all average for each subject.

Based upon the quantitative analyses conducted for all motions, the following remarks can be made:

1. There is a remarkable similarity between the experimental (actual) paths and the corresponding ones obtained by the model. Correlation coefficients as high as .92 and as low as .78 were obtained for different subjects. This high correlation implies that the paths of motion predicted by the model have the same trend as the experimental ones, i.e., decrease or increase in the same fashion. Furthermore, the closeness of the location of the paths' peak points along the X-axis does prove that the model predicts very closely the point at which the human starts to decelerate in order to terminate his motion.

2. The measures of difference both for the different points along the path of motion as well as for the area under it for all different motions investigated showed little discrepancy between the experimental and the predicted paths. In most cases the maximum difference for the motion points was obtained as 6.0 percent, which is reasonably good and falls within the variational limits of average individuals. Also, a similar statement can be made with regard to the difference between the areas.

Fig. 40.--Effect of work surface height upon accuracy of prediction.

Fig. 41.--Effect of motion distance upon accuracy of prediction.

Fig. 42.--Effect of subject upon accuracy of prediction

3. The model's consistency in predictions was not affected by the variations in the motion configurations. That is, for all combinations of motion distance, work surface height, and subject, the paths predicted by the model described very well the actual motions as obtained by experimental analysis.

4. As motion time decreases, the model's accuracy in prediction increases. That is, the discrepancy between the experimental path and the corresponding theoretical one decreases as the motion time decreases. This can be attributed to the fact that for short motion time, the initiation and execution of motion are accomplished with fewer decisions and feed-back corrections on the subject's part. On the other hand, for longer motion times, the subject would have time to evaluate his path and might introduce some corrections in order to terminate the motion at the desired point, i.e., more decision and more variations along the path of motion.

5. For most motions the model consistently underestimated the paths of motion followed by the subjects. That is, the experimental paths of motion occurred higher than the corresponding predicted ones. The large errors (as high as 5%) between the experimental paths and the predicted ones occurred very close to the initial motion point. This may have resulted from the uncertainty of

the subjects at the beginning of the motion. In other words, at the beginning of the motion the reaction time phase and the movement phase overlapped and resulted in some errors. The effect of the strobe and the lighting conditions during the course of the experiment might cause greater change in subject's reaction time than would be expected under normal lighting conditions.

In general, it should be appreciated that exact prediction of human motion by the proposed model is difficult to achieve. The reasons rest with human nature. It is known that human beings possess some variations in performing their daily activities. Therefore, a deterministic model such as the proposed one will show some discrepancy in predicting and quantifying a stochastic process such as human motion. The degree of discrepancy will depend to a large extent upon the nature of the motion to be performed and the familiarity of individuals with it. It seems reasonable to expect that the paths of motions would be made with minimum variations as far as well-learned body motions such as walking and running are concerned. This is exactly what happened in this investigation. All subjects showed little variations in performing their motions. The magnitude of variations was within reasonable values and could be attributed to individual variations and did not warrant any alteration of the model's formulation or assumption.

In short, it can be hypothesized, and the data substantiate, that the proposed biomechanical model does predict with a high degree of accuracy human motions which fall under its assumptions.

CHAPTER VI

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

This investigation was undertaken in an attempt to develop a biomechanical model for the upper extremity. A model for predicting paths of motion and the associated motion characteristics for the arm articulation joints was developed. The underlying principle of the model is that the average individual does follow an optimizing criterion in performing his tasks. The use of the model is restricted to the analysis of transport movements which are to be performed under normal environmental conditions and require the maximization of performance efficiency. Anthropometric characteristics such as links' masses, dimensions, moments of inertia as well as motion distance, angle, and time comprise the information required as an input to the model.

Basically, the model utilizes both theoretical mechanics and an optimization approach for the analysis of arm motions. A detailed description of the model assumptions, mechanics, and formulation is presented for three-dimensional motions. The model treats the arm as a system of two links: the first is the upper arm and the second is

the forearm and hand. A physical measure for performance is selected as the criterion for the model. Minimizing this performance criterion under physical and stress constraints imposed by the nature of the human body as well as under task characteristics is the key idea of the model. Three algorithms for the model solution are presented. These are linear and geometric programmings, dynamic programming, and simulation.

Principles of the model and its associated algorithms are applied in detail to analyze hypothetical examples of planar motions for the arm. Through the analysis of the hypothetical motions, the feasibility of the model algorithms was tested and evaluated.

The accuracy as well as the adequacy of the model in predicting human motion was tested. For testing the model's accuracy, an experiment was conducted to record the paths of motion for ten subjects under three levels of work surface height, 6, 3, 0 inches measured from the elbow, and motion distances, 9, 12, and 15 inches. Measurements from the photographic records obtained for different motions were used to generate the path of motion assumed by each subject for each motion considered. For each experimental motion, a corresponding motion determined by the model was obtained. A correlation analysis as well as two measures for difference was

conducted between each two paths of motion, the experimental one and the predicted one.

Conclusions

The conclusions which can be drawn from this investigation in regard to the model, its associated algorithms, and verification experiment will be summarized in the following points:

1. It is possible to construct a model for accurately analyzing and quantifying motions of the upper extremity. Paths of motion and the associated motion characteristics for the arm articulation joints predicted by the model were very close to those measured experimentally under the same task characteristics. Correlation coefficients as high as .98 and as low as .78 were obtained for some motions. The difference in percentages between the measured path of motion and the predicted one was below 5% and averaged below 2% for all motions investigated. Also, the differences between the coordinates of the measured paths of motion and the predicted ones were as high as 6% and as low as 0% for most motions.

2. As far as describing human effort is concerned, total power required for the motion has been found to be the most suitable mechanical criterion which is sensitive to motion characteristics. This may be due to the

fact that work, impulses and stress (pages 34 and 35) lack a velocity term in their expressions.

3. Linear and geometric programmings seem to be insufficient for providing a solution for the model. Linearization of the model by the small angles assumption resulted in an infeasible solution for the linear programming approach. On the other hand, in spite of large degrees of difficulty associated with the geometric programming formulation, it seems that it is a valid approach at least from the theoretical point of view. However, the geometric programming algorithm which was used failed to provide a solution in most cases. Therefore, the use of geometric programming with the model is still questionable.

Generally, it seems very appropriate to believe that the validity and the use of suboptimization approaches for model building in connection with human motion are not encouraging, if not infeasible. This is especially true when the dynamic forces generated during the motion are considerably larger than static forces.

4. Dynamic programming algorithms have yielded paths of motion very close to those measured experimentally for human subjects. Based upon the analysis, page 96, Chapter IV, it is evident that dynamic programming is the most favorable algorithm to be used in connection with

the model. This is simply true because it is an optimization approach and it is assumed that the human body optimizes its performance.

5. The simulation approach by means of enumeration has provided feasible solutions for the motions investigated for the hypothetical examples. In most cases, enumeration has yielded curves very close to those of dynamic programming. The assumption that the hand path of motion can be approximated by certain geometric shapes, namely sine, ellipse, and parabola, seems to be inaccurate. The sine, ellipse, and parabola functions yielded low paths of large discrepancies in comparison to those of dynamic programming in most of the motions investigated.

6. Although the model formulation and algorithms were tested only for two-dimensional motions, it is believed that the above remarks can be extended to apply to the three-dimensional motions.

In short, it can be concluded that the model developed in this study might be a useful tool for the ultimate objective of developing a generalized biomechanical model which can be used to simulate all possible classes of human motion theoretically. Indeed, the model developed in this investigation has clarified some aspects of model building in connection with the human body. For example, answers to the questions concerning the

optimization criteria, assumptions, mechanics, and suitable algorithms have been obtained.

Unquestionably, the proposed model in this investigation might open a new era in the theoretical analysis of human motion.

Recommendations for Further Research

Attempts at modeling the human body are worthy of careful consideration from researchers in the field of biomechanics and related areas. The following research pertaining to modifying and refining the proposed model in this study merit further investigation. The recommendations are presented chronologically according to their degrees of difficulty:

1. Modify the existing computer algorithms to handle three-dimensional motions in accordance with the model principles and assumptions.

2. Eliminate the restrictions which prevent the study of tasks with rotational motions about the longitudinal axes of body links. This could be accomplished by modifying the model dynamics in order to compensate for rotational motions.

3. Refine the model dynamics by using the actual forces developed by the muscles at the different articulation joints instead of grouping their actions under a single moment as presented on page 18. This could be

accomplished by treating the body as a statically indeterminate space structure which consists of several links and tie rods. Determining the forces and moments for this space structure might be possible through the use of advanced techniques such as structural analysis and optimization in conjunction with high-speed computers.

4. Examine the model performance criterion for possible usages of physiological indices with or without the addition of mechanical criteria to express human effort over an extended period of task duration. Theoretical expressions for physiological indices such as oxygen consumption and heart rate could be written in terms of motion characteristics such as displacement, velocity, and accelerations as well as muscle tensions. If developing such theoretical expressions would be difficult, and it seems so, empirical expressions might be used instead of the theoretical ones. These empirical expressions might be obtained through experimental analysis.

5. So far, the model developed in this study as well as the above suggested modifications is concerned only with the arm. However, once a somewhat general model for the arm is developed, extending it to cover the entire body would not be a difficult task. In modeling the entire body, careful consideration should be given to modeling the spinal column. With regard to the spine, the

following questions might be raised: Could the spine be considered as one link or two curved links? Would elasticity of the spine be considered, or would it be treated as a solid link?

6. Stochastic approaches to model building in connection with human motion are worthy of investigation. Probabilistic analysis of human motions seems to be very relevant, since man can be viewed as a machine, so to speak, which operates to some extent in accordance with the law of chance. That is, variations in human performance on a daily basis and even instantaneously are known to occur. Therefore, by using stochastic approaches in developing the models for human motion, variations for individuals as well as between individuals would be considered in the model.

It is hoped that modeling of human motion will attract wide interest among the specialists in biomechanics and that the future will produce more attempts at modeling the human body in general under a variety of tasks and environmental conditions.

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APPENDIX
COMPUTER PROGRAM DOCUMENTATION

```

C*****MAPC0010
C
C          MOTION ANALYSIS PROGRAM          MAPC002C
C
C
C          M A P          MAP00030
C
C          BY          MAP00040
C
C          M.A. AYOUB          MAP00050
C
C          TEXAS TECH UNIVERSITY          MAP00060
C
C          DECEMBER, 1970          MAP00070
C
C
C          MAP00080
C          MAP00090
C          MAPC0100
C          MAP00110
C          MAP00120
C          MAP00130
C          MAP00140
C          MAP00150
C          MAP00160
C          MAP00170
C          MAP00180
C          MAP00190
C          MAP00200
C          MAP00210
C          MAP00220
C          MAP00230
C          MAP00240
C          MAP00250
C          MAP00260
C          MAP00270
C          MAP00280
C          MAP00290
C          MAP00300
C          MAP00310
C          MAP00320
C          MAP00330
C          MAP00340
C          MAP00350
C          MAP00360
C          MAP00370
C          MAP00380
C          MAP00390
C          MAP00400
C          MAP00410
C          MAP00420
C          MAP00430
C          MAP00440
C          MAP00450
C          MAP00460
C          MAP00470
C          MAP00480
C          MAP00490
C          MAP00500
C          MAP00510
C          MAP00520
C          MAP00530
C          MAP00540
C          MAP00550
C          MAP00560
C          MAP00570
C          MAP00580
C          MAP00590
C          MAP00600

```

1. PROGRAMMER--M.A. AYOUB
 2. ADVISOR-- DR. M.M. AYOUB
 3. MACHINE-- IBM 360 MODEL 50
 4. LANGUAGE -- FORTRAN IV
 5. COMPILER -- FORTRAN G COMPILER, O/S OPERATING SYSTEM
 6. DATE COMPLETED -- DECEMBER 1970
 7. APPROXIMATE COMPILE TIME -- 2.0 MINUTES
 8. COMPUTATION TIME -- VARIABLE, DEPENDS UPON THE ANALYSIS CONDUCTED. IN MOST CASES, IT IS 5 MIN.
 9. LINES OF OUTPUT -- APPROXIMATELY= 1000 LINES/ MOTION
 10. APPROXIMATE CORE SPACE REQUIRED-- 150,000 BYTES

THE FOLLOWING CHANGES ARE SUGGESTED TO REDUCE CORE SPACE :
 - FOR SIMULATION ANALYSIS, REPLACE SUBROUTINE DYNAMIC BY STATEMENTS 15470, 17150, 17160 AND DELETE SUBROUTINES NEWTON AND MINIM
 - FOR DYNAMIC PROGRAMMING, REPLACE STATEMENTS 03610-03680 BY NEW STATEMENTS WHICH USE INTEGER 2 INSTEAD OF 400

PURPOSE
 THE PROGRAM IS WRITTEN TO ANALYZE PLANAR MOTIONS OF THE UPPER EXTERMITIES BY USING BOTH THEORETICAL AS WELL AS EXPERIMENTAL APPROACHES. THE PROGRAM USES THE FOLLOWING FEATURES TO DETERMINE THE PATH OF MOTION AND THE ASSOCIATED CHARACTERISTICS FOR A GIVEN TASK.

1. SIMULATION ANALYSIS
2. DYNAMIC PROGRAMMING
3. EXPERIMENTAL DATA

HISTORY AND BACKGROUND
 DISCUSSIONS CONCERNING THE USE OF SIMULATION AND DYNAMIC PROGRAMMING ARE PRESENTED IN M.A. AYOUB, PH.D. DISSERTATION TEXAS TECH, 1971.

RESTRICTIONS
 IF DIMENSIONS LARGER THAN THOSE APPEARING IN THE PROGRAM LISTING ARE REQUIRED, THE DIMENSION STATEMENTS MUST BE MODIFIED.

DEFINITIONS

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C      SEE PROGRAM LISTING FOR ALL DEFINITIONS. MAP00610
C      ..... MAP00620
C      ..... MAP00630
C      INPUT MAP00640
C      SEE PROGRAM LISTING MAP00650
C      ..... MAP00660
C      ..... MAP00670
C      PROGRAM FLOW CHARTS MAP00680
C      PAGE MAP00690
C      ..... MAP00700
C      PROGRAM LISTING MAP00710
C      ..... MAP00720
C      DISCRIPTION OF PARAMETERS: MAP00730
C      ..... MAP00740
C      ..... MAP00750
C      NUMBER--SUBJECT'S CODE NUMBER MAP00760
C      L12--LENGTH OF UPPERARM MAP00770
C      L23--LENGTH OF FOREARM MAP00780
C      R12--C.G. DISTANCE OF UPPERARM MAP00790
C      R23--C.G. DISTANCE OF FOREARM MAP00800
C      M12--MASS OF UPPERARM MAP00810
C      M23--MASS OF FOREARM MAP00820
C      K12--RADIUS OF GYRATION :UPPER ARM MAP00830
C      K23--RADIUS OF GYRATION: FOREARM MAP00840
C      I12,I23--MOMENTS OF INERTIA OF THE TWO ARM SEGMENTS MAP00850
C      A12--CROSS SECTION AREA AT THE SHOULDER MAP00860
C      R112--RADIUS OF THE CROSS SECTION AT THE SHOULDER MAP00870
C      ICODE--MOTION CODE NUMBER MAP00880
C      ITIME--MOTION TIME IN SECONDS MULTIPLIED BY 100 MAP00890
C      TIME--TIME INCREMENT MAP00900
C      WEIGHT--EXTERNAL LOAD TO BE CARRIED BY HAND MAP00910
C      MOTOIS--MOTION DISTANCE MAP00920
C      MOTANG--MOTION ANGLE MAP00930
C      XIT, YINT--INITIAL ANGLES OF THE ARM SEGMENTS AT TIME 0 MAP00940
C      N--NUMBER OF POINTS TO BE GENERATED FOR THE PATH OF MOTION MAP00950
C      USING TIME INCREMENT=.05 MAP00960
C      MASTER--CONTROL VARIABLE TO INDICATE THE TYPE OF ANALYSIS REQUIRED MAP00970
C      MASTER=0 EXPERIMENTAL DATA FOR THE ARM MAP00980
C      EULER ANGLES IS EXPECTED. MAP00990
C      MASTER>0 MOTION DATA HAS TO BE GENERATED THEORETICALLY. MAP01000
C      NOW1--CONTROL VARIABLE FOR PRINTING THE ANALYSIS FOR MAP01010
C      EACH ITERATION MAP01020
C      NOW1=1 ANALYSIS OF EACH ITERATION WILL BE PRINTED MAP01030
C      NOW1=0 NO ANALYSIS WILL BE PRINTED MAP01040
C      NOW--CONTROL VARIABLE FOR PLOTTING DATA FOR EACH ITERATION MAP01050
C      NOW=0 RESULTS WILL BE PLOTTED FOR EACH ITERATION MAP01060
C      NOW=1 NO GRAPHS. MAP01070
C      ITYPE--FUNCTION TYPE ASSUMED FOR THE HAND PATH OF MOTION MAP01080
C      ITYPE=0--NUMERICAL DATA FOR HAND PATH OF MOTION IS GIVEN MAP01090
C      ITYPE=1--SINE CURVE FUNCTION IS TO DESCRIBE HAND MAP01100
C      PATH OF MOTIONS MAP01110
C      ITYPE=2--ELLIPSE FUNCTION IS USED TO DESCRIBE HAND PATH MAP01120
C      OF MOTION MAP01130
C      ITYPE=3--PARABOLA FUNCTION IS USED TO DESCRIBE MAP01140
C      HAND PATH OF MOTION MAP01150
C      ITYPE=4--PATH OF MOTION IS GENERATED MAP01160
C      BY DIRECT ENUMERATIONS MAP01170
C      PERINT, PERMAX--INITIAL AND MAXIMUM VALUES FOR VERTICAL MAP01180
C      STEP SIZE--TO BE USED WITH SINE, ELLIPSE, MAP01190
C      PARABOLA FUNCTIONS MAP01200

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C PTINT--INITIAL POSITION FOR HIGHEST POINT OF THE PARABOLA          MAP01210
C   ALONG THE X-AXIS                                              MAP01220
C DELTA-- INCREMENTAL VALUE FOR PARABOLA'S HIGHEST POINT POSITION    MAP01230
C POINT--POSITION OF THE HIGHEST POINT OF THE PARABOLA              MAP01240
C P1N,P2N,P3N,P4N,P5N,P1M,P2M,P3M,P4M,P5M--INITIAL AND            MAP01250
C MAXIMUM VALUES FOR DEFINING GRID TO BE USED WITH THE UNUMRATION MAP01260
C POINTS                                                            MAP01270
C P1,P2,P3,P4,P5--GRID POINTS FOR ENUMERATION                     MAP01280
C KOUNT-- ITERATION NUMBER                                          MAP01290
C PERCNT--MAX HAND Z-COORD. IN PERCENT OF THE MOTION DISTANCE FOR AN MAP01300
C   ITERATION                                                       MAP01310
C *****MAP01320
C   VARIABLES TO BE USED WITH DYNAMIC ANALYSIS                     MAP01330
C *****MAP01340
C ANG1--UPPER ARM ANGLE                                             MAP01350
C ANG2--FOREARM ANGLE                                              MAP01360
C ANGV1--ANGULAR VELOCITY OF UPPER ARM                             MAP01370
C ANGV2--ANGULAR VELOCITY OF FOREARM                               MAP01380
C ANGA1--ANGULAR ACCELERATION OF UPPER ARM                         MAP01390
C ANGA2--ANGULAR ACCELERATION OF FOREARM                           MAP01400
C AV2MAX--MAXIMUM ANGULAR VELOCITY OF FOREARM                     MAP01410
C AV1MAX--MAXIMUM ANGULAR VELOCITY OF UPPERARM                    MAP01420
C AA1MAX--MAXIMUM ANGULAR ACCELERATION OF UPPERARM                MAP01430
C AA2MAX--MAXIMUM ANGULAR ACCELERATION OF FOREARM                  MAP01440
C XA2--ELBOW LINEAR ACCELERATION IN THE X DIRECTION                MAP01450
C ZA2--ELBOW LINEAR ACCELERATION IN THE Z DIRECTION                MAP01460
C XA3--HAND LINEAR ACCELERATION IN THE X DIRECTION                 MAP01470
C ZA3--HAND LINEAR ACCELERATION IN THE Z DIRECTION                 MAP01480
C XGA2--X LINEAR ACCELERATION OF THE UPPER ARM C.G.                MAP01490
C ZGA2--Z LINEAR ACCELERATION OF THE UPPER ARM C.G.                MAP01500
C XGA3--X LINEAR ACCELERATION OF THE FOREARM C.G.                  MAP01510
C ZGA3--Z LINEAR ACCELERATION OF THE FOREARM C.G.                  MAP01520
C YMOM2--ELBOW MOMENT                                              MAP01530
C XF2--X-REACTIVE FORCE AT THE ELBOW                                MAP01540
C ZG2--Z-REACTIVE FORCE AT THE ELBOW                                MAP01550
C F2--RESULTANT REACTIVE FORCE AT THE ELBOW                          MAP01560
C YMOM1--SHOULDER MOMENT                                           MAP01570
C XF1--X-REACTIVE FORCE AT THE SHOULDER                             MAP01580
C ZF1--Z-REACTIVE FORCE AT THE SHOULDER                             MAP01590
C F1--RESULTANT REACTIVE FORCE AT THE SHOULDER                      MAP01600
C TOR2--ELBOW MOMENT DUE TO STATIC FORCES                           MAP01610
C SXF1--X-COMPONENT OF STATIC REACTIVE FORCE AT ELBOW               MAP01620
C SZF2--Z-COMPONENT OF STATIC REACTIVE FORCE AT ELBOW               MAP01630
C SF2--RESULTANT STATIC REACTIVE FORCE AT ELBOW                     MAP01640
C TOR1--SHOULDER MOMENT DUE TO STATIC FORCES                       MAP01650
C SXF1--X COMPONENT OF STATIC REACTIVE FORCE AT SHOULDER            MAP01660
C SZF1--Z COMPONENT OF STATIC REACTIVE FORCE AT SHOULDER            MAP01670
C SF1--RESULTANT STATIC REACTIVE FORCE AT ELBOW                     MAP01680
C F2MAX--MAXIMUM RESULTANT FORCE AT THE ELBOW                      MAP01690
C F1MAX--MAXIMUM RESULTANT FORCE AT THE SHOULDER                   MAP01700
C YMOMM2--MAXIMUM MOMENT AT THE ELBOW                              MAP01710
C YMOMM1--MAXIMUM MOMENT AT THE SHOULDER                            MAP01720
C TOR2M--MAXIMUM STATIC MOMENT AT THE ELBOW                         MAP01730
C TOR1M--MAXIMUM STATIC MOMENT AT THE SHOULDER                     MAP01740
C SF2MAX--MAXIMUM STATIC REACTIVE FORCE AT ELBOW                   MAP01750
C SF1MAX--MAXIMUM STATICREACTIVE FORCE AT SHOULDER                  MAP01760
C RATIO1--RATIO OF MAXIMUM DYNAMIC MOMENT TO MAXIMUM STATIC        MAP01770
C   MOMENT AT ELBOW                                                MAP01780
C RATIO2--RATIO OF MAXIMUM DYNAMIC MOMENT TO MAXIMUM               MAP01790
C   STATIC MOMENT AT SHOULDER                                       MAP01800

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C  RATIO3--RATIO OF MAXIMUM DYNAMIC FORCE TO MAXIMUM STATIC      MAP01810
C  FORCE AT ELBOW                                                MAP01820
C  RATIO4--RATIO OF MAXIMUM DYNAMIC FORCE TO MAXIMUM STATIC      MAP01830
C  FORCE AT SHOULDER                                             MAP01840
C  STRESS--NORMAL STRESS AT SHOULDER                            MAP01850
C  TOTSTR--TOTAL STRESS                                         MAP01860
C  WORK--TOTAL WORK COMPUTED AT SHOULDER                        MAP01870
C  POWER--TOTAL POWER COMPUTED AT SHOULDER                     MAP01880
C  SSRATE--STRESS RATE                                          MAP01890
C  TANGIM--TOTAL ANGULAR IMPULSE                                MAP01900
C  TFIMP--TOTAL LINEAR IMPULSE                                  MAP01910
C  FCT1,FCT2,FCT3,FCT4,FCT5,FCT6,--CRITERION FUNCTIONS        MAP01920
C  REMARKS                                                       MAP01930
C  .....MAP01940
C  THE FOLLOWING SUBPROGRAMS ARE REQUIRED                          MAP01950
C  ---SOLVER                                                    MAP01960
C  ---PARAB                                                      MAP01970
C  ---SIMUL                                                       MAP01980
C  ---RGRSS                                                       MAP01990
C  ---MATEQS                                                      MAP02000
C  ---PLOT                                                         MAP02010
C  ---STICK                                                        MAP02020
C  ---MINTIR                                                       MAP02030
C  ---DYNAMC                                                       MAP02040
C  ---NEWTON                                                       MAP02050
C  ---MINIM                                                        MAP02060
C  ---BIG                                                         MAP02070
C  *****MAP02080
C  INPUT DATA                                                    MAP02090
C  *****MAP02100
C  FIRST CARD --PROBLEM TITLE (ALPHANUMERIC CHARACTERS COLUMN1-80)MAP02110
C  MAP02120
C  SECOND CARD--ANTHROPOMETRIC DATA FOR THE UPPER ARM;L12,R12,M12 MAP02130
C  ,K12,A12,R112 ( FORMAT 6F10.5)                                MAP02140
C  MAP02150
C  THIRD CARD--ANTHROPOMETRIC DATA FOR THE FOREARM;L23,R23,M23, MAP02160
C  K23 (FORMAT 5F10.5)                                           MAP02170
C  MAP02180
C  FOURTH CARD--MOTION CHARACTERISTICS;MOTION NUMBER,MOTION TIME, MAP02190
C  TIME INCRMENT,WEIGHT,MOTION DISTANCE,MOTION ANGLE,          MAP02200
C  INITIAL ARM ANGLES (FORMAT 2I6,6F10.5)                        MAP02210
C  MAP02220
C  FIFTH CARD--CONTROL CARD;MASTER,NOW1,NOW,ITYPE,PERINT,PERMAX  MAP02230
C  (FORMAT 4I2,2X,2F5.2)                                         MAP02240
C  MASTER=0 EXPERIMENTAL DATA                                   MAP02250
C  MASTER=2 THEORETICAL ANALYSIS                                 MAP02260
C  NOW1=1 NO INTERMEDIATE RESULTS WILL BE PRINTED              MAP02270
C  NOW1=0 ALL RESULTS WILL BE PRINTED                           MAP02280
C  NOW=0 NO GRAPHS WILL BE PRINTED                              MAP02290
C  NOW=1 GRAPHS WILL BE PRINTED                                 MAP02300
C  ITYPE=0,1,2,3,4 FUNCTION TYPE FOR THE HAND PATH            MAP02310
C  OF MOTION                                                     MAP02320
C  MAP02330
C  SIXTH CARD-- THIS CARD WILL BE VARIED IN ACCORDANCE WITH THE MAP02340
C  ANALYSIS REQUIRED                                              MAP02350
C  FOR SINE AND ELLIPSE IT IS NOT REQUIRED                        MAP02360
C  FOR PARABOLA;PTINT,DELTA (FORMAT 2F5.2)                      MAP02370
C  FOR ENUMERATION;P1N,....P5M (FORMAT 10F5.2)                 MAP02380
C  FOR GIVEN PATH OF MOTION;KONT=0 (FORMAT 16 )                 MAP02390
C  FOR DYNAMIC PROGRAMMING;KONT=1 (FORMAT 16)                   MAP02400

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C
C SEVENTH CARD--DELTA,Y2IINT,Y2IINC,Y1IINT,Y1IINC;STAGE MAP02410
C PARAMETERS FOR DYNAMIC PROGRAMMING (FORMAT 5F5.2) MAP02420
C THIS CARD IS REQUIRED ONLY FOR DYNAMIC PROG. MAP02430
C MAP02440
C MAP02450
C EIGHTTH CARD--ANGULAR DISPLACEMENT IN DEG.FOR THE ARM(ANG1,ANG2 MAP02460
C (FORMAT 2F10.5) MAP02470
C OR MAP02480
C X AND Z COORDINATES OF THE HAND (FORMAT 2F10.5) MAP02490
C ***** MAP02500
C REPEAT CARD 4 THROUGH 7 AS MANY TIMES AS REQUIRED FOR THE SAME MAP02510
C SUBJECT-- AT THE END USE BLANK CARD MAP02520
C IF THERE IS ANOTHER SUBJECT TO BE ANALYZED REPEAT CARD 2THROUGH 7 MAP02530
C OTHERWISE USE A BLANK CARD MAP02540
C ..... MAP02550
C INPUT DATA WILL BE GIVEN IN ACCORDANCE WITH MAP02560
C ALL POSSIBLE USAGES OF THE PROGRAM. MAP02570
C ..... MAP02580
C ..... MAP02590
C ***** BIOMECHANICAL ANALYSIS OF THE ARM ***** M.A.AYOUB** MAP02600
C MAP02610
C 1 MAP02620
C .255 .1137 1.243 .0795 .009 .06 MAP02630
C .3604 .1599 1.117 .1043 MAP02640
C MAP02650
C MAP02660
C MAP02670
C MAP02680
C EXAMPLE1.--EXPERIMENTAL DATA MAP02690
C ..... MAP02700
C 15600 40 0.06 0.3811 242.3 257.0 MAP02710
C MAP02720
C 0 0 1 2 0.0 40.0 MAP02730
C 9 MAP02740
C MAP02750
C 242.33 257.00 MAP02760
C 248.17 274.36 MAP02770
C 263.90 315.96 MAP02780
C 294.01 365.43 MAP02790
C 325.18 407.57 MAP02800
C 346.55 448.31 MAP02810
C 360.76 482.34 MAP02820
C 376.32 504.06 MAP02830
C 370.27 511.16 MAP02840
C MAP02850
C MAP02860
C MAP02870
C EXAMPLE2.--HAND PATH OF MOTION IS GIVEN MAP02880
C ..... MAP02890
C 13415 40 0.05 0.3048 294.0 73.5 MAP02900
C 2 0 1 0 0.0 40.0 MAP02910
C 0 MAP02920
C 0.2059347 0.1124793 MAP02930
C 0.2097289 0.1226177 MAP02940
C 0.2335996 0.1387476 MAP02950
C 0.2858742 0.1527008 MAP02960
C 0.3582572 0.1581993 MAP02970
C 0.4306856 0.1527185 MAP02980
C 0.4830694 0.1387477 MAP02990
C 0.5070494 0.1224728 MAP03000

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C0.5107347 0.1124793
C
C EXAMPLE 3.--SIMULATION--SINE CURVE
C .....
C13415 40 0.05 0.3048 294.0 73.5
C
C2 1 0 1 0.0 40.0
C
C EXAMPLE 5.--SIMULATION--ELLIPSE CURVE
C .....
C13415 40 0.05 0.3048 294.0 73.5
C
C2 1 0 2 0.0 40.0
C
C EXAMPLE 6.--SIMULATION--PARABOLA CURVE
C .....
C13415 40 0.05 0.3048 294.0 73.5
C
C2 1 0 3 0.0 40.0
C
C 0.0 10.0
C
C EXAMPLE 7.--SIMULATION--ENUMERATION APPROACHE
C .....
C13415 40 0.05 0.3048 294.0 73.5
C
C2 1 0 4 0.0 40.0
C
C 0.0 10.0 0.0 15.0 0.0 20.0 0.0 15.0 0.0 10.0
C
C EXAMPLE 8.-- DYNAMIC PROGRAMMING
C .....
C13415 40 0.05 0.3048 294.0 73.5
C
C2 0 1 0 0.0 40.0
C
C 1
C
C 5.0 -1.0 1.0 -1.0 1.0
C
C *****
C PROGRAM LISTING
C *****
REAL L12,L23,M12,M23,I12,I23,K12,K23
REAL MUTDIS,MOTANG,MOTIME
DATA ISTAR/IH*/
DIMENSION TITLE(80),LINE(80)
DIMENSION ANG1(20), ANG2(20), ANGV1(20), ANGV2(20), ANGA1(20),
LANGA2(20)
DIMENSION XA2(20), ZA2(20), XA3(20), ZA3(20), XGA2(20),ZGA2(20),
IXGA3(20), ZGA3(20)
DIMENSION YMON1(20), YMON2(20), XF2(20), ZF2(20), XF1(20),
IZF1(20),F1(20),F2(20)
DIMENSION STRESS(20), WORK(20), POWER(20)
DIMENSION SXF2(20),SZF2(20),SF2(20),TOR2(20)
DIMENSION SXF1(20),SZF1(20),SF1(20),TOR1(20)

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MAP03010
MAP03020
MAP03030
MAP03040
MAP03050
MAP03060
MAP03070
MAP03080
MAP03090
MAP03100
MAP03110
MAP03120
MAP03130
MAP03140
MAP03150
MAP03160
MAP03170
MAP03180
MAP03190
MAP03200
MAP03210
MAP03220
MAP03230
MAP03240
MAP03250
MAP03260
MAP03270
MAP03280
MAP03290
MAP03300
MAP03310
MAP03320
MAP03330
MAP03340
MAP03350
MAP03360
MAP03370
MAP03380
MAP03390
MAP03400
MAP03410
MAP03420
MAP03430
MAP03440
MAP03450
MAP03460
MAP03470
MAP03480
MAP03490
MAP03500
MAP03510
MAP03520
MAP03530
MAP03540
MAP03550
MAP03560
MAP03570
MAP03580
MAP03590
MAP03600

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WRITE(6,66)(LINE(I),I=1,22)
WRITE(6,71)
66 FORMAT(1H0,42X,22A1)
WRITE(6,67)
67 FORMAT(1H0,42X22H* DYNAMICAL ANALYSIS *)
WRITE(6,71)
WRITE(6,66)(LINE(I),I=1,22)
C PRINT MOTION PARAMETERS
WRITE(6,68) ICODE,MOTOIS,MOTANG,WEIGHT,MOTIME
68 FORMAT(1H0,15HMOTION NUMBER ,I6//2X,23HMOTION CHARACTERISTICS=//
125X,11HDISTANCE..=,F9.5,5HMETER//
225X,11HANGLE.....=,F9.5,4HDEG.//
325X,11HWEIGHT....=,F9.5,4HKGS.//
425X,11HTIME.....=,F9.5,4HSEC.)
C PRINT INITIAL ARM CONFIGURATION
WRITE(6,681)
681 FORMAT(1H0,1X,25HINITIAL ARM CONFIGURATION )
WRITE(6,682) XINT,YINT
682 FORMAT(1H0,25X,16HUPPER ARM ANGLE=,F10.5,5H DEG.///26X,
116HFOREARM ANGLE =,F10.5,5H DEG.)
71 FORMAT(1H ,42X,1H*)
KOUNT=1
C READ CONTROL NUMBERS FOR THE COMPUTATIONAL PROCEDURE IN THE PROGRAM
READ(5,5) MASTER,NOW1,NOW,ITYPE,PERINT,PERMAX
5 FORMAT(4I2,2X,2F5.2)
C INITIALIZE VARIABLES FOR PARABOLA AND ENUMERATION
DELTA=25.0
PTINT=0.0
P1=0.0
P2=0.0
P3=0.0
P4=0.0
P5=0.0
P1M=10.0
P2M=15.0
P3M=20.0
P4M=15.0
P5M=10.0
IF(MASTER.EQ.0) GO TO 1004
C READ PARABOLA PARAMETERS
IF(ITYPE.EQ.3) READ(5,999) PTINT,DELTA
C READ ENUMERATION PARAMETERS
IF(ITYPE.EQ.4) READ(5,999)P1N,P1M,P2N,P2M,P3N,P3M,P4N,P4M,P5N,P5M
999 FORMAT(10F5.2)
POINT=PTINT
2003 POINT=POINT+DELTA
PERCNT=PERINT
P1=P1N
1003 H=MOTOIS*PERCNT/100.0
2009 P1=P1+5.0
P2=P2N
2008 P2=P2+5.0
P3=P3N
2007 P3=P3+5.0
P4=P4N
2006 P4=P4+5.0
P5=P5N
2005 P5=P5+5.0
IF(ITYPE.EQ.0.OR.ITYPE.EQ.4) GO TO 1009
IF(NOW1.EQ.0) WRITE(6,74) KOUNT,PERCNT

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MAP04210
 MAP04220
 MAP04230
 MAP04240
 MAP04250
 MAP04260
 MAP04270
 MAP04280
 MAP04290
 MAP04300
 MAP04310
 MAP04320
 MAP04330
 MAP04340
 MAP04350
 MAP04360
 MAP04370
 MAP04380
 MAP04390
 MAP04400
 MAP04410
 MAP04420
 MAP04430
 MAP04440
 MAP04450
 MAP04460
 MAP04470
 MAP04480
 MAP04490
 MAP04500
 MAP04510
 MAP04520
 MAP04530
 MAP04540
 MAP04550
 MAP04560
 MAP04570
 MAP04580
 MAP04590
 MAP04600
 MAP04610
 MAP04620
 MAP04630
 MAP04640
 MAP04650
 MAP04660
 MAP04670
 MAP04680
 MAP04690
 MAP04700
 MAP04710
 MAP04720
 MAP04730
 MAP04740
 MAP04750
 MAP04760
 MAP04770
 MAP04780
 MAP04790
 MAP04800

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74 FORMAT(1H1,18HITERATION NUMBER ,15,5X,27HPERCENT OF MAXIMUM HEIGHT,10
1T ,F10.5)
C CALL SUBROUTINE SOLVER TO DETERMINE ANGULAR DISPLACEMENTS
C FOR BOTH UPPER ARM AND FOREARM.
1009 CALL SOLVER(L12,L23,MOTDIS,MOTANG,ITIME,H,N,ANG1,ANG2,ITYPE,
1XINT,YINT,POINT,NOW1,P1,P2,P3,P4,P5,M12,M23,I12,I23,R12,R23,
2WEIGHT)
IF(ITYPE.EQ.0) PERCNT=PERMAX
IF(ITYPE.EQ.0.OR.ITYPE.EQ.1.OR.ITYPE.EQ.2) POINT=100.0
GO TO 1005
1004 PERCNT=PERMAX
POINT=100.0
C READ EXPERIMENTAL DATA
READ(5,3) N
DO 10 I = 1, N
READ (5,2) ANG1(I), ANG2(I)
ANG1(I)=ANG1(I)*0.017453
ANG2(I)=ANG2(I)*0.017453
10 CONTINUE
C SMOOTH THE EXPERIMENTAL DATA WHICH HAS BEEN READ BY
C POLYNOMIAL REGRESSION
CALL SMOOTH(ANG1,ANG2,N,TIME)
GO TO 1005
1006 NUM=NUM+1
PERCNT=PERMAX
POINT=100.0
P1=P1M
P2=P2M
P3=P3M
P4=P4M
P5=P5M
KOUNT=1
C *****
C DYNAMIC ANALYSIS STATEMENTS 0510 THROUGH 6490
C *****
C I. VELOCITY AND ACCELERATION
C *****
C INITIALIZE ANGULAR VELOCITIES AND ACCELERATIONS
1005 ANGV1(1)=0.0
ANGV2(1) = 0.0
ANGA1(1) = 0.0
ANGA2(1)=0.0
ANGV1(N)=0.0
ANGV2(N)=0.0
ANGA1(N)=0.0
ANGA2(N)=0.0
NM1=N-1
C COMPUTE ANGULAR VELOCITY AND ACCELERATION ARRAYS
C NUMERICAL DIFFERENTIATION IS EMPLOYED
DO 20 I=2,NM1
ANGV1(I) = (ANG1(I+1) - ANG1(I - 1))/(2. * TIME)
ANGV2(I) = (ANG2(I + 1) - ANG2(I - 1))/(2. * TIME)
ANGA1(I)=(ANG1(I+1)+ANG1(I-1)-2.0*ANG1(I))/(TIME**2)
ANGA2(I)=(ANG2(I+1)+ANG2(I-1)-2.0*ANG2(I))/(TIME**2)
20 CONTINUE
C FIND MAXIMUM VELOCITY AND ACCELERATION FOR BOTH UPPER ARM AND
C FOREARM FOR THE CURRENT ITERATION
AV2MAX(KOUNT)=BIG(ANGV2,N)
AV1MAX(KOUNT)=BIG(ANGV1,N)
AA2MAX(KOUNT)=BIG(ANGA2,N)

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      AAIMAX(KOUNT)=BIG(ANGA1,N) MAP05410
C ***** MAP05420
C *****CALCULATION OF LINEAR ACCELERATIONS MAP05430
C ***** MAP05440
C INITIALIZE LINEAR ACCELERATIONS MAP05450
      XA2(1) = 0.0 MAP05460
      ZA2(1) = 0.0 MAP05470
      XA3(1) = 0.0 MAP05480
      ZA3(1) = 0.0 MAP05490
      XGA2(1)=0.0 MAP05500
      ZGA2(1)=0.0 MAP05510
      XGA3(1)=0.0 MAP05520
      ZGA3(1)=0.0 MAP05530
C COMPUTE LINEAR ACCELERATIONS FOR THE ARM JOINTS MAP05540
      DO 30 I = 2, N MAP05550
      XA2(I) = - L12*SIN(ANG1(I))*ANGA1(I)-L12*COS(ANG1(I))*(ANGV1(I)**2) MAP05560
      ZA2(I)=L12*COS(ANG1(I))*ANGA1(I)-L12*SIN(ANG1(I))*(ANGV1(I)**2) MAP05570
      XA3(I)=XA2(I)-L23*SIN(ANG2(I))*ANGA2(I)-L23*COS(ANG2(I))*(ANGV2(I)
1**2) MAP05580
      ZA3(I)=ZA2(I)+L23*COS(ANG2(I))*ANGA2(I)-L23*SIN(ANG2(I))*(ANGV2(I)
1**2) MAP05590
      XGA2(I)=R12*XA2(I)/L12 MAP05600
      ZGA2(I)=R12*ZA2(I)/L12 MAP05610
      XGA3(I)=XA2(I)-R23*SIN(ANG2(I))*ANGA2(I)-R23*COS(ANG2(I))*(ANGV2
1(I)**2) MAP05620
      ZGA3(I)=ZA2(I)+R23*COS(ANG2(I))*ANGA2(I)-R23*SIN(ANG2(I))*(ANGV2
1(I)**2) MAP05630
      30 CONTINUE MAP05640
C ***** MAP05650
C II. REACTIVE FORCES AND MOMENTS MAP05660
C ***** MAP05670
C CALCULATE DYNAMIC FORCES AND MOMENTS STARTING WITH ELBOW JOINT MAP05680
      DO 40 I=1,N MAP05690
      YMOM2(I)=I23*ANGA2(I)-M23*R23*SIN(ANG2(I))*XGA3(I)+ M23*R23*COS
1(ANG2(I))*ZGA3(I)-M23*9.8*R23*COS(ANG2(I))-WEIGHT*L23*COS(ANG2(I)) MAP05700
      XF2(I)=M23*XGA3(I) MAP05710
      ZF2(I)=-WEIGHT-M23*9.8+M23*ZGA3(I) MAP05720
      F2(I)=SQRT(XF2(I)**2+ZF2(I)**2) MAP05730
      YMOM1(I)=YMOM2(I)+I12*ANGA1(I)-M12*R12*SIN(ANG1(I))*XGA2(I)+M12*
[R12*COS(ANG1(I))*ZGA2(I)-M12*9.8*R12*COS(ANG1(I))-XF2(I)*L12*SIN
2(ANG1(I))+ZF2(I)*L12*COS(ANG1(I)) MAP05740
      XF1(I)=XF2(I)+M12*XGA2(I) MAP05750
      ZF1(I)=ZF2(I)+M12*ZGA2(I)-M12*9.8 MAP05760
      F1(I)=SQRT(XF1(I)**2+ZF1(I)**2) MAP05770
C CALCULATE STATIC FORCES AND MOMENTS DURING THE MOTION MAP05780
      TOR2(I)=-M23*9.8*R23*COS(ANG2(I))-WEIGHT*L23*COS(ANG2(I)) MAP05790
      SXF2(I)=0.0 MAP05800
      SZF2(I)=-WEIGHT-M23*9.8 MAP05810
      SF2(I)=SQRT(SXF2(I)**2+SZF2(I)**2) MAP05820
      TOR1(I)=TOR2(I)-M12*9.8*R12*COS(ANG1(I))+SZF2(I)*L12*COS(ANG1(I)) MAP05830
      SXF1(I)=0.0 MAP05840
      SZF1(I)=SZF2(I)-M12*9.8 MAP05850
      SF1(I)=SQRT(SXF1(I)**2+SZF1(I)**2) MAP05860
      40 CONTINUE MAP05870
C COMPUTE MAXIMUM VALUES FOR FORCES AND MOMENTS MAP05880
C BOTH FOR DYNAMIC AND STATIC VALUES MAP05890
      F2MAX(KOUNT)=BIG(F2,N) MAP05900
      F1MAX(KOUNT)=BIG(F1,N) MAP05910
      YMOMM2(KOUNT)=BIG(YMOM2,N) MAP05920
      YMOMM1(KOUNT)=BIG(YMOM1,N) MAP05930
      MAP05940
      MAP05950
      MAP05960
      MAP05970
      MAP05980
      MAP05990
      MAP06000

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TOR2M(KOUNT)=BIG(TOR2,N) MAP06010
TOR1M(KOUNT)=BIG(TOR1,N) MAP06020
SF2MAX(KOUNT)=BIG(SF2,N) MAP06030
SF1MAX(KOUNT)=BIG(SF1,N) MAP06040
RATIO1(KOUNT)=YMOMM2(KOUNT)/TOR2M(KOUNT) MAP06050
RATIO2(KOUNT)=YMOMM1(KOUNT)/TOR1M(KOUNT) MAP06060
RATIO3(KOUNT)=F2MAX(KOUNT)/SF2MAX(KOUNT) MAP06070
RATIO4(KOUNT)=F1MAX(KOUNT)/SF1MAX(KOUNT) MAP06080
C ***** MAP06090
C III. PERFORMANCE CRITERIA MAP06100
C (METHOD OF NUMERICAL INTEGRATION--TRAPEZOIDAL RULE--IS EMPLOYED) MAP06110
C ***** MAP06120
C 1. STRESS MAP06130
C CALCULATE NORMAL STRESS MAP06140
DO 85 I=1,N MAP06150
STRESS(I)=YMOM1(I)*R1I2+(XF1(I)*COS(ANG1(I))+Z1(I)*SIN(ANG1(I)))/A12 MAP06160
1/A12 MAP06170
85 CONTINUE MAP06180
N2=N-1 MAP06190
C CALCULATE TOTAL STRESS MAP06200
SUM=0.0 MAP06210
DO 86 I=2,N2 MAP06220
86 SUM=SUM+STRESS(I) MAP06230
TOTSTR=TIME/2.0*(STRESS(1)+2.0*SUM+STRESS(N)) MAP06240
C 2. WORK, POWER, STRESS RATE MAP06250
C ***** MAP06260
C INITIALIZE WORK, POWER, STRESS RATE MAP06270
WORK(1)=0.0 MAP06280
POWER(1)=0.0 MAP06290
SSRATE(1)=0.0 MAP06300
C CALCULATE TOTAL WORK, POWER, AND STRESS RATE MAP06310
DO 41 I=2,N MAP06320
SSRATE(I)=SSRATE(I-1)+0.5*(ANGV1(I)-ANGV1(I-1))*(STRESS(I) MAP06330
1+STRESS(I-1)) MAP06340
WORK(I)=WORK(I-1)+0.5*(ANG1(I)-ANG1(I-1))*(YMOM1(I)+YMOM1(I-1)) MAP06350
POWER(I)=POWER(I-1)+0.5*(ANGV1(I)-ANGV1(I-1))*(YMOM1(I)+YMOM1(I-1)) MAP06360
1)) MAP06370
41 CONTINUE MAP06380
C ***** MAP06390
C 3. LINEAR AND ANGULAR IMPULSES MAP06400
C ***** MAP06410
ANGIMP=0.0 MAP06420
FIMP=0.0 MAP06430
DO 42 I=2,N2 MAP06440
ANGIMP=ANGIMP+YMOM1(I) MAP06450
FIMP=FIMP+F1(I) MAP06460
42 CONTINUE MAP06470
TANGIM=TIME/2.0*(YMOM1(1)+2.0*ANGIMP+YMOM1(N)) MAP06480
TFIMP=TIME/2.0*(F1(1)+2.0*FIMP+F1(N)) MAP06490
C ***** MAP06500
C STORE PERFORMANCE CRITERIA VALUES COMPUTED FOR ITERATION MAP06510
C ***** MAP06520
FCT1(KOUNT)=WORK(N) MAP06530
FCT2(KOUNT)=POWER(N) MAP06540
FCT3(KOUNT)=TANGIM MAP06550
FCT4(KOUNT)=TFIMP MAP06560
FCT5(KOUNT)=TOTSTR MAP06570
FCT6(KOUNT)=SSRATE(N) MAP06580
HHH(KOUNT)=H MAP06590
POINT1(KOUNT)=POINT MAP06600

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      P1(I(KOUNT))=P1
      P2(I(KOUNT))=P2
      P3(I(KOUNT))=P3
      P4(I(KOUNT))=P4
      P5(I(KOUNT))=P5
C *****
C      PRINT AND PLOT MOTION CHARACTERISTICS
C *****
      IF(NOW1.EQ.1) GO TO 813
C*****
C      PLOTTING STICK DIAGRAM FOR THE MOTION
C*****
      CALL STICK (ANG1,ANG2,N,L12,L23)
C PRINT AND PLOT HAND DATA
      WRITE(6,83)
      83 FORMAT(1H1,50X,9HHAND DATA)
      WRITE(6,951)
      951 FORMAT(1H0,15H X-ACCELERATION,3X,15H Z-ACCELERATION/1X,
        115H METER/SEC/SEC,3X,15H METER/SEC/SEC)
      DO 57 I=1,N
      57 WRITE(6,70) XA3(I),ZA3(I)
      WRITE(6,577)
      577 FORMAT(1H0,20X,'PLOTS OF LINEAR ACCELERATIONS'//1X,'X-ACCELERATION
        1 SYMBOL IS *,20X,'Z-ACCELERATION SYMBOL IS X')
      CALL PLOT(XA3,ZA3,N,0)
C PRINT AND PLOT ELBOW DATA FOR BOTH STATIC
C AND DYNAMIC CONDITIONS
      WRITE(6,80)
      80 FORMAT(1H1,50X,10HELLOW DATA)
      WRITE(6,901)
      WRITE(6,952)
      952 FORMAT(1H0,15H X-FORCE ,15H Z-FORCE ,15H RESULTANT-FORCE
        1E,15H MOMENT /1X,15H NEWTON ,15H NEWTON ,
        215H NEWTON ,15H NEWTON-METER )
      DO 92 I=1,N
      92 WRITE(6,70) SXF2(I),SZF2(I),SF2(I),TOR2(I)
      WRITE(6,902)
      WRITE(6,953)
      953 FORMAT(1H0,15H ANGULAR DISPL.,15H ANGULAR VEL. ,15H ANGULAR ACCEL
        1.,15H X-ACCEL. ,15H Z-ACCEL. /1X,
        215H RADIANS ,15H RAD/SEC ,15H RAD/SEC/SEC ,
        315H METER/SEC/SEC ,15H METER/SEC/SEC )
      DO 50 I=1,N
      50 WRITE(6,70) ANG2(I),ANGV2(I),ANGA2(I),XA2(I),ZA2(I)
      70 FORMAT(1H0,8F15.5)
      WRITE(6,952)
      DO 501 I=1,N
      501 WRITE(6,70) XF2(I),ZF2(I),F2(I),YMOM2(I)
      IF(NOW.EQ.0) GO TO 811
C*****
C      PLOTTING RESULTANT FORCE AND MOMENT AT THE ELBOW
C*****
      WRITE(6,701)
      701 FORMAT(1H1,20X,35HPLOTS OF RESULTANT FORCE AND MOMENT)
      WRITE(6,711)
      711 FORMAT(1H0,18HMOMENT SYMBOL IS *,20X,27H RESULTANT FORCE SYMBOL IS
        1X)
      CALL PLOT(YMOM2,F2,N,0)
C PRINT AND PLOT SHOULDER DATA
      811 WRITE(6,811)

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81 FORMAT(1H1,50X,13HSHOULDER DATA)
WRITE(6,901)
WRITE(6,952)
DO 93 I=1,N
93 WRITE(6,70) SXF1(I),SZF1(I),SF1(I),TOR1(I)
WRITE(6,902)
WRITE(6,954)
954 FORMAT(1H0,15H ANGULAR DISPL.,15H ANGULAR VEL. ,
115H ANGULAR ACCEL./1X,15H RADIANS ,15H RAD/SEC
215H RAD/SEC/SEC )
DO 55 I=1,N
55 WRITE(6,70) ANG1(I),ANGV1(I),ANGA1(I)
WRITE(6,952)
DO 551 I=1,N
551 WRITE(6,70) XF1(I),ZF1(I),F1(I),YMOM1(I)
IF(NOW.EQ.0) GO TO 812
C*****
C PLOTTING RESULTANT FORCE AND MOMENT AT THE SHOULDER
C*****
WRITE(6,701)
WRITE(6,711)
CALL PLOT(YMOM1,F1,N,0)
812 WRITE(6,82)
82 FORMAT(1H1,50X,30HWORK AND POWER AT THE SHOULDER)
WRITE(6,955)
955 FORMAT(1H0,15H WORK ,15H POWER /1X,
115H NEWTON ,15H NEWTON/SEC.)
DO 56 I=1,N
56 WRITE(6,70) WORK(I),POWER(I)
WRITE(6,84)
84 FORMAT(1H0,15HANGULAR-IMPULSE,15H LINEAR-IMPULSE,15H STRESS
1 )
WRITE(6,70) TANGIM,TFIMP,TOTSTR
813 KOUNT=KOUNT+1
IF(ITYPE.EQ.0.OR.ITYPE.EQ.1.OR.ITYPE.EQ.2.OR.ITYPE.EQ.3)GO TO 3003
IF(P5.LT.P5M) GO TO 2005
IF(P4.LT.P4M) GO TO 2006
IF(P3.LT.P3M) GO TO 2007
IF(P2.LT.P2M) GO TO 2008
IF(P1.LT.P1M) GO TO 2009
GO TO 3002
3003 PERCNT=PERCNT+5.0
IF(PERCNT.LE.PERMAX) GO TO 1003
IF(POINT.LE.80.0) GO TO 2003
3002 IEND=KOUNT-1
IF(IEND.LE.1) GO TO 1007
C IN CASE OF EXPERIMENTAL VALUES SKIP THE NEXT SEGMENT
C PRINT ITERATIONS SUMMARY IF SIMULATION ANALYSIS IS USED.
WRITE(6,87)
87 FORMAT(1H1,20X,39H S U M M A R Y O F I T E R A T I O N S/21X,
139H-----)
WRITE(6,68) ICODE,MOTDIS,MOTANG,WEIGHT,MOTIME
WRITE(6,681)
WRITE(6,682) XINT,YINT
WRITE(6,90)
90 FORMAT(1H0,48HMAXIMUM VALUES OF VEL.,ACCEL.,FORCES,AND MOMENTS)
WRITE(6,901)
901 FORMAT(1H0,20X,15HSTATIC ANALYSIS)
WRITE(6,956)
956 FORMAT(1H0,15H ELBOW FORCE ,15H SHOULDER-FORCE,15H ELBOW-MOMENT

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MAP07210
MAP07220
MAP07230
MAP07240
MAP07250
MAP07260
MAP07270
MAP07280
MAP07290
MAP07300
MAP07310
MAP07320
MAP07330
MAP07340
MAP07350
MAP07360
MAP07370
MAP07380
MAP07390
MAP07400
MAP07410
MAP07420
MAP07430
MAP07440
MAP07450
MAP07460
MAP07470
MAP07480
MAP07490
MAP07500
MAP07510
MAP07520
MAP07530
MAP07540
MAP07550
MAP07560
MAP07570
MAP07580
MAP07590
MAP07600
MAP07610
MAP07620
MAP07630
MAP07640
MAP07650
MAP07660
MAP07670
MAP07680
MAP07690
MAP07700
MAP07710
MAP07720
MAP07730
MAP07740
MAP07750
MAP07760
MAP07770
MAP07780
MAP07790
MAP07800

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1 ,15HSHOULDER-MOMENT)
DO 911 I=1,IEND
911 WRITE(6,921)SF2MAX(I),SF1MAX(I),TOR2M(I),TOR1M(I)
921 FORMAT(1H0,8F15.6)
WRITE(6,902)
902 FORMAT(1H0,20X,16HDYNAMIC ANALYSIS)
WRITE(6,957)
957 FORMAT(1H0,15H FOREARM-VEL. ,15H UPPER ARM-VEL.,15H FOREARM-ACCEL
1.,15HUPPER ARM-ACCEL)
DO 912 I=1,IEND
912 WRITE(6,921)AV2MAX(I),AV1MAX(I),AA2MAX(I),AA1MAX(I)
WRITE(6,956)
DO 915 I=1,IEND
915 WRITE(6,921) F2MAX(I),F1MAX(I),YMOHM2(I),YMOHM1(I)
WRITE(6,903)
903 FORMAT(1H0,20X,23H RATIO OF DYNAMIC/STATIC)
WRITE(6,958)
958 FORMAT(1H0,15H ELBOW-FORCE ,15H SHOULDER-FORCE,15H ELBOW-MOMENT
1 ,15HSHOULDER-MOMENT)
DO 913 I=1,IEND
913 WRITE(6,921)RATIO3(I),RATIO4(I),RATIO1(I),RATIO2(I)
WRITE(6,904)
904 FORMAT(1H0,20X,19HCRITERION FUNCTIONS)
WRITE(6,959)
959 FORMAT(1H0,9HITERATION,15H WORK ,15H POWER ,
115HANGULAR-IMPULSE,15H LINEAR-IMPULSE,15H STRESS ,15H STR
2ESS RATE )
DO 89 I=1,IEND
WRITE(6,88) I,FCT1(I),FCT2(I),FCT3(I),FCT4(I),FCT5(I),FCT6(I)
88 FORMAT(1H0,16,6F15.5)
89 CONTINUE
C PLOT TOTAL POWER AND ANGULAR IMPULSE FOR ALL ITERATIONS
WRITE(6,914)
914 FORMAT(1H0,34HPLOTS OF POWER AND ANGULAR IMPULSE//1X,17HPOWER SYMB
10L IS *,20X,27HANGULAR IMPULSE SYMBOL IS X)
CALL PLOT(FCT2,FCT3,IEND,1)
C DETERMINE THE ITERATION WHICH GIVES THE MINIMUM POWER.
CALL MINTIR(FCT2,IEND,SMALL,KK)
C DEFINE PARAMETERS FOR THE OPTIMUM MOTION
POINT=POINT1(KK)
P1=P11(KK)
P2=P21(KK)
P3=P31(KK)
P4=P41(KK)
P5=P51(KK)
HH=HHH(KK)
C PRINT THE OPTIMAL MOTION
WRITE(6,91)
91 FORMAT(1H1,20X,14HOPTIMAL MOTION/21X,14HXXXXXXXXXXXXXXXXXX)
WRITE(6,68) ICODE,MOTDIS,MOTANG,WEIGHT,MOTIME
WRITE(6,681)
WRITE(6,682) XINT,YINT
NOW=1
NOW1=0
C CALL SOLVER TO OBTAIN ANGULAR DISPLACEMENTS FOR THE OPTIMUM MOTION
CALL SOLVER(I12,L23,MOTDIS,MOTANG,ITIME,HH,N,ANG1,ANG2,ITYPE,
1XINT,YINT,POINT,NOW1,P1,P2,P3,P4,P5,M12,M23,I12,I23,R12,R23,
2WEIGHT)
C GO AND PERFORM DYNAMIC ANALYSIS FOR THE OPTIMUM MOTION
IF(NUM.EQ.0) GO TO 1006

```

MAP07810
MAP07820
MAP07830
MAP07840
MAP07850
MAP07860
MAP07870
MAP07880
MAP07890
MAP07900
MAP07910
MAP07920
MAP07930
MAP07940
MAP07950
MAP07960
MAP07970
MAP07980
MAP07990
MAP08000
MAP08010
MAP08020
MAP08030
MAP08040
MAP08050
MAP08060
MAP08070
MAP08080
MAP08090
MAP08100
MAP08110
MAP08120
MAP08130
MAP08140
MAP08150
MAP08160
MAP08170
MAP08180
MAP08190
MAP08200
MAP08210
MAP08220
MAP08230
MAP08240
MAP08250
MAP08260
MAP08270
MAP08280
MAP08290
MAP08300
MAP08310
MAP08320
MAP08330
MAP08340
MAP08350
MAP08360
MAP08370
MAP08380
MAP08390
MAP08400

```

1007 NUM=0
GO TO 1001
1002 CALL EXIT
END
C *****
C SUBROUTINE SOLVER
C *****
C PURPOSE:
C TO COMPUTE EULER'S ANGLES FOR THE UPPER ARM AND FOREARM
C DURING THE MOTION.
C DESCRIPTIONS OF PARAMETERS:
C XL12, XL23--ARM DIMENSIONS
C DIS, TANG1, ITIME--MOTION PARAMETERS (SEE MAIN PROGRAM)
C XINT, YINT--INITIAL ARM ANGLES.
C ITYPE--TYPE OF THE HAND PATH OF MOTION FUNCTION (SEE MAIN PROGRAM)
C EPS--THE ALLOWABLE ERROR IN DETERMINING EULER'S ANGLES
C IEND--MAXIMUM NUMBER OF ITERATIONS FOR SOLVING THE
C NONLINEAR EQUATIONS
C C1, C2--X AND Z COORDINATES OF THE HAND
C C11, C22--X AND Z DISPLACEMENTS OF THE HAND
C XMAX--MAXIMUM DISPLACEMENT IN THE X DIRECTION
C B-- VECTOR OF REGRESSION COEFFICIENTS
C KONT--CONTROL VARIABLE FOR DETERMINING THE TYPE OF INPUT VALUES
C KONT=0 GIVEN HAND PATH OF MOTION
C KONT=1 GIVEN HAND PATH OF MOTION OBTAINED
C BY DYNAMIC PROGRAMMING
C N--NUMBER OF DATA POINTS
C X,Y--OUTPUT VECTOR FOR EULER'S ANGLES
C ERRX, ERRY--ERRORS RESULT FROM SOLVING THE
C NONLINEAR EQUATIONS
C XPLOT, YPLOT--ANGULAR DISPLACEMENTS RECTORS FOR THE ARM
C TIM=TOTAL MOTION TIME
C REMARKS:
C THE FOLLOWING SUBROUTINES ARE REQUIRED
C -PARAB
C -SIMUL
C -PLOT
C *****
C SUBROUTINE SOLVER(XL12,XL23,DIS,TANG1,ITIME,H,N,X,Y,ITYPE,XINT
1,YINT,POINT,NOW1,P1,P2,P3,P4,P5,XM12,XM23,XI12,XI23,R12,R23,
2WEIGHT)
REAL L12,L23
DATA EPS,IEND/0.0001,50/
DIMENSION C1(20),C2(20),C11(20),C22(20)
DIMENSION X(20),Y(20)
DIMENSION XPLOT(20),YPLOT(20)
DIMENSION ERRX(20),ERRY(20)
DIMENSION B(20,20)
L12=XL12
L23=XL23
X(1)=XINT
Y(1)=YINT
TANG=TANG1*0.01745
X(1)=X(1)*0.01745
Y(1)=Y(1)*0.01745
C IF ITYPE EQUAL ZERO, GO AND READ DATA FOR THE HAND PATH OF MOTION

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MAP08410
MAP08420
MAP08430
MAP08440
MAP08450
MAP08460
MAP08470
MAP08480
MAP08490
MAP08500
MAP08510
MAP08520
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MAP08540
MAP08550
MAP08560
MAP08570
MAP08580
MAP08590
MAP08600
MAP08610
MAP08620
MAP08630
MAP08640
MAP08650
MAP08660
MAP08670
MAP08680
MAP08690
MAP08700
MAP08710
MAP08720
MAP08730
MAP08740
MAP08750
MAP08760
MAP08770
MAP08780
MAP08790
MAP08800
MAP08810
MAP08820
MAP08830
MAP08840
MAP08850
MAP08860
MAP08870
MAP08880
MAP08890
MAP08900
MAP08910
MAP08920
MAP08930
MAP08940
MAP08950
MAP08960
MAP08970
MAP08980
MAP08990
MAP09000

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```

      IF(IITYPE.EQ.0) GO TO 35
      XMAX=DIS*COS(TANG)
      XMAX1=0.5*XMAX
C   COMPUTE THE INITIAL X,Z OF THE HAND
      C1(1)=L12*COS(X(1))+L23*COS(Y(1))
      C2(1)=L12*SIN(X(1))+L23*SIN(Y(1))
      IF(IITYPE.EQ.3) CALL PARAB(XMAX,H,POINT,TANG,B)
      IF(IITYPE.EQ.4) CALL SIMUL(XMAX,TANG,P1,P2,P3,P4,P5,B)
C   USE THE DISPLACEMENT FUNCTION TO GENERATE X-COORDINATE
      I=1
      TIM3=ITIME
      TIM=TIM3/100.0
      ITIME1=ITIME-1
      DO 10 K=5,ITIME1,5
      I=I+1
      TIM1=K
      TIM2=TIM1/100.0
      TERM1=6.28*TIM2/TIM
      TERM2=TERM1-SIN(TERM1)
      C1(I)=C1(1)+XMAX*TERM2/6.28
      C11(I)=C1(I)-C1(1)
      TERM3=C11(I)*3.14/XMAX
C   ACCORDING TO THE FUNCTION REQUIRED COMPUTE Z-COORDINATE
      IF(IITYPE.EQ.1) C22(I)=H*SIN(TERM3)+C11(I)*TAN(TANG)
      IF(IITYPE.EQ.2) C22(I)=H*SQRT(1.0-((ABS(C11(I))-XMAX1)**2)/XMAX1**2)
      C2(I)=C2(1)+C22(I)*TAN(TANG)
      IF(IITYPE.EQ.3) C22(I)=B(1,1)+B(2,1)*C11(I)+B(3,1)*C11(I)**2
      IF(IITYPE.EQ.4) C22(I)=B(1,1)+B(2,1)*C11(I)+B(3,1)*C11(I)**2+
      1B(4,1)*C11(I)**3+B(5,1)*C11(I)**4+B(6,1)*C11(I)**5
      C2(I)=C2(1)+C22(I)
10  CONTINUE
      C1(N)=C1(1)+XMAX
      C2(N)=C2(1)+XMAX*TAN(TANG)
      GO TO 36
35  READ(5,2) KONT
      2  FORMAT(I6)
C   READ X AND Y COORDINATES OF THE HAND
      IF(KONT.EQ.0) READ(5,28) (C1(I),C2(I),I=1,N)
      IF(KONT.EQ.1) CALL DYNMC(L12,L23,XM12,XM23,XI12,XI23,R12,R23,XMAX,
      1ITIME,XINT,YINT,WEIGHT,C1,C2,N,TANG1)
      28  FORMAT(2F10.5)
C   *****
C   SOLVE TWO NONLINEAR EQUATIONS BY NEWTON'S METHOD
C   *****
36  ERRX(1)=0.0
      ERRY(1)=0.0
C   START NEWTON ITERATION SCHEME
      DO 30 I=2,N
      XIT=X(I-1)
      YIT=Y(I-1)
C   FX, FY, GX, GY--DERIVATIVES OF THE TWO EQUATIONS
      DO 20 J=1,IEND
      FX=-L12*SIN(XIT)
      FY=-L23*SIN(YIT)
      GX=L12*COS(XIT)
      GY=L23*COS(YIT)
      F=L12*COS(XIT)+L23*COS(YIT)-C1(I)
      G=L12*SIN(XIT)+L23*SIN(YIT)-C2(I)
      DIV=FX*GY-GX*FY+0.000001
      XIT1=XIT+(G*FY-F*GY)/DIV

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MAP09010
MAP09020
MAP09030
MAP09040
MAP09050
MAP09060
MAP09070
MAP09080
MAP09090
MAP09100
MAP09110
MAP09120
MAP09130
MAP09140
MAP09150
MAP09160
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MAP09180
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MAP09200
MAP09210
MAP09220
MAP09230
MAP09240
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MAP09290
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MAP09310
MAP09320
MAP09330
MAP09340
MAP09350
MAP09360
MAP09370
MAP09380
MAP09390
MAP09400
MAP09410
MAP09420
MAP09430
MAP09440
MAP09450
MAP09460
MAP09470
MAP09480
MAP09490
MAP09500
MAP09510
MAP09520
MAP09530
MAP09540
MAP09550
MAP09560
MAP09570
MAP09580
MAP09590
MAP09600

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      YIT1=YIT+(F*GX-G*FX)/DIV
      Z1=ABS(F)
      Z2=ABS(G)
C IF THE ERRORS ARE SMALL TERMINATE THE ITERATION PROCESS
      IF(Z1.LE.EPS.AND.Z2.LE.EPS) GO TO 25
      XIT=XIT1
      YIT=YIT1
20 CONTINUE
25 X(I)=XIT1
      Y(I)=YIT1
      ERRX(I)=Z1
      ERRY(I)=Z2
30 CONTINUE
      IF(NOW1.EQ.1) GO TO 100
C *****
C PRINT DISPLACEMENT DATA AND THE BASIS FOR GENERATIONG IT
C *****
      WRITE(6,11)
11 FORMAT(1H0,25X,17HDISPLACEMENT DATA)
      IF(ITYPE.EQ.0.AND.KONT.EQ.0) WRITE(6,4)
      IF(ITYPE.EQ.0.AND.KONT.EQ.1) WRITE(6,287)
      IF(ITYPE.EQ.1) WRITE(6,5)
      IF(ITYPE.EQ.2) WRITE(6,6)
      IF(ITYPE.EQ.3) WRITE(6,12)
      IF(ITYPE.EQ.4) WRITE(6,14)
4 FORMAT(1H0,7X,44HDATA FOR THE HAND PATH OF MOTION IS GIVEN )
5 FORMAT(1H0,7X,88H SINE CURVE ASSUMPTION IS USED AS A BASIS FOR
      GENERATING A PATH OF MOTION FOR THE HAND)
6 FORMAT(1H0,7X,88HELLIPSE CURVE ASSUMPTION IS USED AS A BASIS FOR
      GENERATING A PATH OF MOTION FOR THE HAND)
12 FORMAT(1H0,7X,'SECOND ORDER POLYNOMIAL IS USED AS A BASIS FOR GENE
      RATING A PATH OF MOTION FOR THE HAND')
14 FORMAT(1H0,7X,' SIMULATION ANALYSIS ')
287 FORMAT(1H0,7X,' DYNAMIC PROGRAMMING ')
      WRITE(6,7)
7 FORMAT(1H0,15H X-COOR. HAND,16H Z-COOR. HAND ,
      115HUPPER ARM ANGLE,15H ERROR ,15H FOREARM ANGLE,
      215H ERROR )
C PRINT TABLE OF ANGULAR DISPLACEMENTS
      DO 40 I=1,N
40 WRITE(6,3) C1(I),C2(I),X(I),ERRX(I),Y(I),ERRY(I)
3 FORMAT(1H0,6F15.7)
      DO 50 I=1,N
      XPLT(I)=X(I)-X(1)
50 YPLT(I)=Y(I)-Y(1)
C PLOT DISPLACEMENT DATA
      WRITE(6,8)
8 FORMAT(1H0,10X,29HLOTS OF ANGULAR DISPLACEMENT)
      WRITE(6,9)
9 FORMAT(1H0,28H UPPER ARM ANGLE SYMBOL IS *,20X,
      126HFOREARM ANGLE SYMBOL IS X )
      CALL PLOT(XPLT,YPLT,N,0)
100 RETURN
      END
C *****
C SUBROUTINE PARAB
C *****
C PURPOSE:
      GENERATE A PARABOLA--SECOND ORDER POLYNOMIAL--BY USING
      THREE POINTS. FIRST AND THIRD POINT ARE TAKEN TO BE THE

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MAP09610
 MAP09620
 MAP09630
 MAP09640
 MAP09650
 MAP09660
 MAP09670
 MAP09680
 MAP09690
 MAP09700
 MAP09710
 MAP09720
 MAP09730
 MAP09740
 MAP09750
 MAP09760
 MAP09770
 MAP09780
 MAP09790
 MAP09800
 MAP09810
 MAP09820
 MAP09830
 MAP09840
 MAP09850
 MAP09860
 MAP09870
 MAP09880
 MAP09890
 MAP09900
 MAP09910
 MAP09920
 MAP09930
 MAP09940
 MAP09950
 MAP09960
 MAP09970
 MAP09980
 MAP09990
 MAP10000
 MAP10010
 MAP10020
 MAP10030
 MAP10040
 MAP10050
 MAP10060
 MAP10070
 MAP10080
 MAP10090
 MAP10100
 MAP10110
 MAP10120
 MAP10130
 MAP10140
 MAP10150
 MAP10160
 MAP10170
 MAP10180
 MAP10190
 MAP10200

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C      INITIAL AND THE TERMINAL POINTS OF THE MOTION. MAP10210
C      MAP10220
C DESCRIPTION OF PARAMETERS MAP10230
C      MAP10240
C      XMAX--MOTION DISTANCE MAP10250
C      H--HEIGHT OF THE SECOND POINT, EXPRESSED AS A PERCENTAGE MAP10260
C      OF THE MOTION DISTANCE MAP10270
C      POINT--POSITION OF THE SECOND POINT, ON THE X-AXIS; MAP10280
C      EXPRESSED AS A PERCENTAGE OF THE MOTION DISTANCE MAP10290
C      TANG--MOTION ANGLE MAP10300
C      B--VECTOR OF REGRESSION COEFFICIENTS MAP10310
C      X,Y--INPUT VECTORS FOR REGRESSION SUBROUTINE MAP10320
C REMARKS MAP10330
C      SUBROUTINE REGRSS IS REQUIRED MAP10340
C ***** MAP10350
C      SUBROUTINE PARAB(XMAX,H,POINT,TANG,B) MAP10360
C      DIMENSION X(20),Y(20,4),B(20,20) MAP10370
C      X(1)=0.0 MAP10380
C      Y(1,1)=0.0 MAP10390
C      X(2)=POINT*XMAX/100.0 MAP10400
C      Y(2,1)=H+X(2)*TAN(TANG) MAP10410
C      X(3)=XMAX MAP10420
C      Y(3,1)=XMAX*TAN(TANG) MAP10430
C      CALL REGRSS(X,Y,3,2,1,B) MAP10440
C      RETURN MAP10450
C      END MAP10460
C ***** MAP10470
C      SUBROUTINE SIMUL MAP10480
C ***** MAP10490
C      MAP10500
C PURPOSE: MAP10510
C      MAP10520
C      TO FIT A FIFTH ORDER POLYNOMIAL REGRESSION FUNCTION TO SEVEN MAP10530
C      GRID POINTS. THE FIRST AND THE LAST POINTS ARE THE MAP10540
C      INITIAL AND TERMINAL POINTS OF THE MOTION. MAP10550
C      MAP10560
C DESCRIPTION OF PARAMETERS MAP10570
C      MAP10580
C      XMAX--MOTION DISTANCE MAP10590
C      TANG--ANGLE OF MOTION MAP10600
C      P1,P2,P3,P4,P5--HEIGHT OF GRID POINTS EXPRESSED AS PERCENTAGES MAP10610
C      OF THE MOTION DISTANCE MAP10620
C      X,Y--INPUT VECTORS FOR REGRESSION SUBROUTINE MAP10630
C      B--VECTOR OF REGRESSION COEFFICIENTS MAP10640
C REMARKS MAP10650
C      1. HORIZONTAL COORDINATES OF GRID POINTS ARE OBTAINED BY MAP10660
C      USING 0,20,40,50,60,80,100 OF THE MOTION DISTANCE MAP10670
C      2. SUBROUTINE REGRSS IS REQUIRED MAP10680
C ***** MAP10690
C      SUBROUTINE SIMUL(XMAX,TANG,P1,P2,P3,P4,P5,B) MAP10700
C      DIMENSION X(20),Y(20,4),B(20,20) MAP10710
C      X(1)=0.0 MAP10720
C      Y(1,1)=0.0 MAP10730
C      X(2)=20.0*XMAX/100.0 MAP10740
C      Y(2,1)=P1*XMAX/100.0+X(2)*TAN(TANG) MAP10750
C      X(3)=40.0*XMAX/100.0 MAP10760
C      Y(3,1)=P2*XMAX/100.0+X(3)*TAN(TANG) MAP10770
C      X(4)=50.0*XMAX/100.0 MAP10780
C      Y(4,1)=P3*XMAX/100.0+X(4)*TAN(TANG) MAP10790
C      X(5)=60.0*XMAX/100.0 MAP10800

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Y(5,1)=P4*XMAX/100.0+X(5)*TAN(TANG)
X(6)=80.0*XMAX/100.0
Y(6,1)=P5*XMAX/100.0+X(6)*TAN(TANG)
X(7)=XMAX
Y(7,1)=0.0+X(7)*TAN(TANG)
CALL REGRSS(X,Y,7,5,1,8)
RETURN
END
C *****
C SUBROUTINE SMOOTH
C *****
C PURPOSE:
C THE SUBROUTINE HAS TWO OBJECTIVES
C 1. FITTING POLYNOMIAL REGRESSION FUNCTIONS TO TWO SETS
C OF EXPERIMENTAL DATA.
C 2. USING THE RESULTING POLYNOMIAL FUNCTIONS, TWO NEW SETS
C OF DATA (SMOOTHED) ARE OBTAINED.
C DESCRIPTION OF PARAMETERS
C XX--INPUT VECTOR OF THE FIRST VARIABLE (SET)
C Y--INPUT VECTOR OF THE SECOND VARIABLE (SET)
C N--NUMBER OF ELEMENTS IN EACH INPUT VECTOR; XX,Y.
C Y1,XX1--TWO ARRAYS TO STORE THE INPUT VECTORS
C Y,XX--PREDICTED VECTORS WHICH REPLACES THE INPUT VECTORS
C XX0,Y0--DISPLACEMENT VECTORS OBTAINED FROM EXPERIMENTAL DATA
C XX10,Y10--DISPLACEMENT VECTORS OBTAINED BY REGRESSION
C TIME--TIME INCREMENT
C T--TIME ARRAY
C B1,B2--REGRESSION COEFFICIENTS
C ERROR1,ERROR2--VECTORS OF DIFFERENCES BETWEEN PREDICTED
C VALUES AND THE EXPERIMENTAL ONES
C REMARKS:
C SUBROUTINES REGRSS AND PLOT ARE REQUIRED
C *****
C SUBROUTINE SMOOTH(XX,Y,N,TIME)
C DIMENSION B1(20,20),B2(20,20)
C DIMENSION T(20),XX(20),Y(20)
C DIMENSION XX1(20),XX10(20),XX0(20)
C DIMENSION Y1(20),Y10(20),Y0(20)
C DIMENSION ERROR1(20),ERROR2(20)
C DO 5 I=1,N
C XX1(I)=XX(I)
C Y1(I)=Y(I)
C 5 CONTINUE
C GENERATE A TIME TABLE
C T(1)=0.0
C DO 10 I=2,N
C 10 T(I)=T(I-1)+TIME
C FIT A POLYNOMIAL REGRESSION FUNCTION (FIFTH ORDER) TO EACH VECTOR,
C THAT IS XX AND Y
C CALL REGRSS(T,XX,N,5,1,B1)
C CALL REGRSS(T,Y,N,5,1,B2)
C EVALUATE THE RESULTING REGRESSION FUNCTIONS AT THE PREVIOUSLY
C CALCULATED TIME POINTS
C DO 20 I=1,N
C XX(I)=B1(1,1)+B1(2,1)*T(I)+B1(3,1)*T(I)**2+B1(4,1)*T(I)**3+
C B1(5,1)*T(I)**4+B1(6,1)*T(I)**5
C Y(I)=B2(1,1)+B2(2,1)*T(I)+B2(3,1)*T(I)**2+B2(4,1)*T(I)**3+

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      2B2(5,1)*T(I)**4+B2(6,1)*T(I)**5
20 CONTINUE
C CALCULATE DISPLACEMENT DATA (TWO ANGLES) AS WELL AS THE ASSOCIATED
C ERRORS
      DO 30 I=1,N
        XX0(I)=XX(I)-XX(1)
        XX10(I)=XX1(I)-XX1(1)
        Y0(I)=Y(I)-Y(1)
        Y10(I)=Y1(I)-Y1(1)
        ERROR1(I)=XX(I)-XX1(I)
        30 ERROR2(I)=Y(I)-Y1(I)
C PRINT A TABLE FOR DISPLACEMENTS
      WRITE(6,1)
      1 FORMAT(1H1,20X,'ANGULAR DISPLACEMENTS---- EXPERIMENTAL DATA ')
      WRITE(6,2)
      2 FORMAT(1H0,'MEASURED ANGLE U.ARM   PREDICTED ANGLE U.ARM   ERROR
      1   MEASURED ANGLE F.ARM   PREDICTED ANGLE F.ARM   ERROR
      2')
      DO 40 I=1,N
40 WRITE(6,3) XX1(I),XX(I),ERROR1(I),Y1(I),Y(I),ERROR2(I)
      3 FORMAT(1H0,6F20.7)
C PLOT DISPLACEMENTS
      WRITE(6,4)
      4 FORMAT(1H0,'PLOT OF UPPER ARM ANGLE')
      WRITE(6,8)
      8 FORMAT(1H0,'MEASURED ANGLE SYMBOL IS *           PREDICTED ANGLE
      1SYMBOL IS X')
      CALL PLOT(XX10,XX0,N,0)
      WRITE(6,6)
      6 FORMAT(1H0,'PLOT OF FOREARM ANGLE')
      WRITE(6,8)
      CALL PLOT(Y10,Y0,N,0)
      RETURN
      END
C *****
C                               SUBROUTINE REGRSS
C *****
C                               MAP11770
C                               MAP11780
C PURPOSE:
C   TO FIT POLYNOMIALS OF SEVERAL DIFFERENT DEGREES TO A GIVEN SET
C   OF N DATA POINTS
C   DESCRIPTION OF PARAMETERS
C     N--NUMBER OF DATA POINTS
C     M--DEGREES OF THE POLYNOMIAL TO BE FITTED
C     X,Y--INPUT VECTORS
C     L--NUMBER OF Y-VALUES WHICH CORRESPOND TO EACH X(I), IN MOST
C     CASES L=1.
C     B--VECTOR OF REGRESSION COEFFICIENTS
C     A--COEFFICIENTS MATRIX
C REMARKS
C SUBROUTINE MATEQS IS REQUIRED
C *****
C SUBROUTINE REGRSS(X,Y,N,M,L,B)
      DIMENSION X(20),Y(20,4)
      DIMENSION A(20,20),B(20,20),C(20,20)
      DO 30 I=1,N
30 C(I,1)=1.0
      NPL=N+1

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DO 35 J=2,MP1
DO 35 I=1,N
35 C(I,J)=C(I,J-1)*X(I)
DO 40 I=1,MP1
DO 40 J=1,MP1
A(I,J)=0.0
DO 40 K=1,N
40 A(I,J)=A(I,J)+C(K,I)*C(K,J)
DO 45 J=1,L
DO 45 I=1,MP1
B(I,J)=0.0
DO 45 K=1,N
45 B(I,J)=B(I,J)+C(K,I)*Y(K,J)
CALL MATEQS(A,MP1,B,L,DET)
RETURN
END
*****
C *****
C SUBROUTINE MATEQS
C *****
C
C PURPOSE:
C TO PERFORM MATRIX INVERSION AND SIMULT LINEAR EQUATIONS.
C
C DESCRIPTION OF PARAMETERS:
C
C A--THE GIVEN COEFFICIENT MATRIX
C N--ORDER OF A;N>,1
C B--MATRIX OF CONSTANT VECTOR
C M--THE NUMBER OF COLUMN VECTOR IN THE METRIX OF CONSTANT
C VECTORS.
C DET--VALUE OF THE DETERMINANT |A|
C
C REMARKS:
C THIS SUBROUTINE IS OBTAINED FROM NUMERICAL METHODS AND
C COMPUTERS, SHAN S KUO, P.168.
C *****
C *****
C SUBROUTINE MATEQS(A,N,B,M,DET)
C DIMENSION A(20,20),B(20,20),IPVOT(20),INDEX(20,2),PIVOT(20)
C EQUIVALENCE (IR3W,JROW),(ICOL,JCOL)
57 DET=1.0
DO 17 J=1,N
17 IPVOT(J)=0.0
DO 135 I=1,N
T=0.0
DO 9 J=1,N
IF(IPVOT(J)-1) 13,9,13
13 DO 23 K=1,N
IF(IPVOT(K)-1) 43,23,81
43 IF(ABS(T)-ABS(A(J,K))) 83,23,23
83 IROW=J
ICOL=K
T=A(J,K)
23 CONTINUE
9 CONTINUE
IPVOT(ICOL)=IPVOT(ICOL)+1
IF(IROW-ICOL) 73,109,73
73 DET=-DET
DO 12 L=1,N
T=A(IROW,L)
A(IROW,L)=A(ICOL,L)

```

MAP12010
MAP12020
MAP12030
MAP12040
MAP12050
MAP12060
MAP12070
MAP12080
MAP12090
MAP12100
MAP12110
MAP12120
MAP12130
MAP12140
MAP12150
MAP12160
MAP12170
MAP12180
MAP12190
MAP12200
MAP12210
MAP12220
MAP12230
MAP12240
MAP12250
MAP12260
MAP12270
MAP12280
MAP12290
MAP12300
MAP12310
MAP12320
MAP12330
MAP12340
MAP12350
MAP12360
MAP12370
MAP12380
MAP12390
MAP12400
MAP12410
MAP12420
MAP12430
MAP12440
MAP12450
MAP12460
MAP12470
MAP12480
MAP12490
MAP12500
MAP12510
MAP12520
MAP12530
MAP12540
MAP12550
MAP12560
MAP12570
MAP12580
MAP12590
MAP12600

```

12 A(ICOL,L)=T
   IF(M) 109,109,33
33 DO 2 L=1,M
   T=B(IROW,L)
   B(IROW,L)=B(ICOL,L)
   2 B(ICOL,L)=T
109 INDEX(I,1)=IROW
   INDEX(I,2)=ICOL
   PIVOT(I)=A(ICOL,ICOL)
   DET=DET*PIVOT(I)
   A(ICOL,ICOL)=1.0
   DO 205 L=1,N
205 A(ICOL,L)=A(ICOL,L)/PIVOT(I)
   IF(M) 347,347,66
   66 DO 52 L=1,M
   52 B(ICOL,L)=B(ICOL,L)/PIVOT(I)
347 DO 135 LI=1,N
   IF(LI-ICOL) 21, 135, 21
   21 T=A(LI,ICOL)
   A(LI,ICOL)=0.0
   DO 89 L=1,N
   89 A(LI,L)=A(LI,L)-A(ICOL,L)*T
   IF(M) 135,135,18
   18 DO 68 L=1,M
   68 B(LI,L)=B(LI,L)-B(ICOL,L)*T
135 CONTINUE
222 DO 3 I=1,N
   L=N-I+1
   IF (INDEX(L,1)-INDEX(L,2)) 19,3,19
   19 JROW=INDEX(L,1)
   JCOL=INDEX(L,2)
   DO 549 K=1,N
   T=A(K,JROW)
   A(K,JROW)=A(K,JCOL)
   A(K,JCOL)=T
549 CONTINUE
   3 CONTINUE
   81 RETURN
   END
C *****
C SUBROUTINE STICK
C *****
C PURPOSE:
C TO PLOT A STICK DIAGRAM FOR THE ARM DURING THE MOTION
C DESCRIPTION OF PARAMETERS
C A--INPUT VECTOR OF THE UPPER ARM ANGLES
C B--INPUT VECTOR OF THE FOREARM ANGLES
C N--NUMBER OF DATA POINTS FOR EACH INPUT VECTOR
C X--LENGTH OF THE UPPER ARM
C Z--LENGTH OF THE FOREARM
C XE, ZE, XW, ZW--X AND Z COORDINATES OF THE HAND AND ELBOW
C WITH RESPECT TO SHOULDER JOINT
C XMAX1--VECTOR OF THE LARGEST ELEMENTS OF THE ELBOW AND
C HAND COORDINATES
C XMAX--LARGEST ELEMENT IN VECTOR XMAX1
C BLANK, DOT, XX, S SYMBOL--PLOTING SYMBOLS

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```

MAP12610
MAP12620
MAP12630
MAP12640
MAP12650
MAP12660
MAP12670
MAP12680
MAP12690
MAP12700
MAP12710
MAP12720
MAP12730
MAP12740
MAP12750
MAP12760
MAP12770
MAP12780
MAP12790
MAP12800
MAP12810
MAP12820
MAP12830
MAP12840
MAP12850
MAP12860
MAP12870
MAP12880
MAP12890
MAP12900
MAP12910
MAP12920
MAP12930
MAP12940
MAP12950
MAP12960
MAP12970
MAP12980
MAP12990
MAP13000
MAP13010
MAP13020
MAP13030
MAP13040
MAP13050
MAP13060
MAP13070
MAP13080
MAP13090
MAP13100
MAP13110
MAP13120
MAP13130
MAP13140
MAP13150
MAP13160
MAP13170
MAP13180
MAP13190
MAP13200

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C      GRAPH--A TWO DIMENSIONAL ARRAY FOR PLOTTING PURPOSES                                MAP13210
C  REMARKS                                                                              MAP13220
C      FUNTION BIG IS REQUIRED                                                            MAP13230
C  *****MAP13240
      SUBROUTINE STICK(A,B,N,X,Z)
      DIMENSION SYMBOL(20)
      MAP13250
C  DEFINE PLOTTING SYMBOLS
      DATA BLANK,DOT,XX,S,E,W/1H ,1H.,1H1,1H5,1HE,1HW/
      DATA SYMBOL/1H0,1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,
      11H0,1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9/
      MAP13260
      DIMENSION A(20),B(20)
      MAP13270
      DIMENSION XE(20),ZE(20),XW(20),ZW(20)
      MAP13280
      DIMENSION XMAX1(20)
      MAP13290
      DIMENSION GRAPH(61,61)
      MAP13300
C  CALCULATE THE ELBOW AND HAND X AND Z-COORDINATES
      MAP13310
      DO 10 I=1,N
      MAP13320
      XE(I)=X*COS(A(I))
      MAP13330
      ZE(I)=X*SIN(A(I))
      MAP13340
      XW(I)=XE(I)+Z*COS(B(I))
      MAP13350
      ZW(I)=ZE(I)+Z*SIN(B(I))
      MAP13360
      10 CONTINUE
      MAP13370
C  FIND THE LARGEST VALUE AMONG THE HAND AND ELBOW COORDINATES
      MAP13380
      XMAX1(2)=BIG(ZE,N)
      MAP13390
      XMAX1(1)=BIG(XE,N)
      MAP13400
      XMAX1(3)=BIG(XW,N)
      MAP13410
      XMAX1(4)=BIG(ZW,N)
      MAP13420
      XMAX=BIG(XMAX1,4)
      MAP13430
C  CLEAR ARRAY FOR GRAPH
      MAP13440
      DO 15 I=1,61
      MAP13450
      DO 15 J=1,61
      MAP13460
      15 GRAPH(I,J)=BLANK
      MAP13470
C  GENERATE GRAPH AXIS AND FRAME
      MAP13480
      DO 16 J=1,61
      MAP13490
      GRAPH(1,J)=DOT
      MAP13500
      GRAPH(31,J)=XX
      MAP13510
      GRAPH(61,J)=DOT
      MAP13520
      16 CONTINUE
      MAP13530
      DO 17 I=1,61
      MAP13540
      GRAPH(I,1)=DOT
      MAP13550
      GRAPH(I,31)=XX
      MAP13560
      GRAPH(I,61)=DOT
      MAP13570
      17 CONTINUE
      MAP13580
      GRAPH(31,31)=S
      MAP13590
C  CONVERT THE COORDINATES TO THE EQUIVALENT PLOTTING POSITIONS
      MAP13600
      DO 20 I=1,N
      MAP13610
      J=(XE(I)/XMAX+1.0)*30.0+1.5
      MAP13620
      K=(ABS(ZE(I)/XMAX-1.0))*30.0+1.5
      MAP13630
      L=(XW(I)/XMAX+1.0)*30.0+1.5
      MAP13640
      M=(ABS(ZW(I)/XMAX-1.0))*30.0+1.5
      MAP13650
      GRAPH(K,J)=SYMBOL(I)
      MAP13660
      GRAPH(M,L)=SYMBOL(I)
      MAP13670
      20 CONTINUE
      MAP13680
C  PLOT GRAPH TITLE AND THE MOTION STICK DIAGRAM
      MAP13690
      WRITE(6,21)
      MAP13700
      21 FORMAT(1H1,20X,27HSTICK DIAGRAM OF THE MOTION)
      MAP13710
      DO 30 I=1,61
      MAP13720
      WRITE(6,22) (GRAPH(I,J),J=1,61)
      MAP13730
      22 FORMAT(1H ,3X,61(A1,1H ))
      MAP13740
      30 CONTINUE
      MAP13750
      RETURN
      MAP13760
      MAP13770
      MAP13780
      MAP13790
      MAP13800

```

```

      END
C *****
C SUBROUTINE PLOT
C *****
C PURPOSE:
C
C TO PLOT TWO DEPENDENT VARIABLES VERSUS A COMMON BASE VARIABLE.
C THE DEPENDENT VARIABLES ARE PRESENTED AT EQUAL
C INTERVALS OF THE BASE VARIABLE.
C
C DESCRIPTION OF PARAMETERS:
C
C     A--INPUT VECTOR FOR THE FIRST DEPENDENT VARIABLE
C     B--INPUT VECTOR FOR THE SECOND DEPENDENT VARIABLE
C     TIME--A POSSIBLE BASE VARIABLE
C     ITER--A POSSIBLE BASE VARIABLE
C     GRAPH--A ONE DIMENSIONAL ARRAY FOR THE PLOT
C     BLANK, DOT, STAR, XX,--PLOTING SYMBOLS
C     AMAX, AMAX1, BMAX, BMAX1--LARGEST NUMBERS FOR ARRAYS A AND B
C
C REMARKS:
C FUNCTION BIG IS REQUIRED
C *****
C     SUBROUTINE PLOT(A,B,N,NN)
C     DIMENSION A(N),B(N)
C     DIMENSION GRAPH(100)
C     DIMENSION TIME(16)
C     DIMENSION ITER(16)
C
C DEFINE PLOTING SYMBOLS
C     DATA BLANK,DOT,STAR,XX/1H,1H,1H*,1HX/
C     DATA TIME/1HT,1H,1HI,1H,1HM,1H,1HE,1H,1H,1H,1H,1H,1H,1H,1H,1H,1H,1H/
C     DATA ITER/1HI,1HT,1HE,1HR,1HA,1HT,1HI,1H3,1HN,1H,1HN,1HU,1HM,1HB,1HE,1HR/
C
C FIND THE LARGEST NUMBER FOR ARRAYS A AND B
C     AMAX=BIG(A,N)
C     BMAX=BIG(B,N)
C     AMAX1=-AMAX
C     BMAX1=-BMAX
C
C PRINT GRAPH SCALE
C     WRITE(6,2) AMAX1,AMAX,STAR
C     WRITE(6,2) BMAX1,BMAX,XX
C     2 FORMAT(1H0,20X,F10.5,26X,1H0,26X,F10.5,2X,A1)
C     DO 10 I=1,63
C 10 GRAPH(I)=DOT
C     WRITE(6,1) (GRAPH(I),I=1,63)
C     1 FORMAT(1H0,25X,63A1)
C     DO 20 I=2,62
C 20 GRAPH(I)=BLANK
C     GRAPH(32)=DOT
C
C COMPUTE PLOTING POSITIONS
C     DO 40 I=1,N
C     J=(A(I)/(AMAX+0.000001)+1.0)*30.0+2.5
C     K=(B(I)/(BMAX+0.000001)+1.0)*30.0+2.5
C     GRAPH(J)=STAR
C     GRAPH(K)=XX
C
C PRINT GRAPH
C     IF(NN.EQ.0) WRITE(6,3) TIME(I),(GRAPH(L),L=1,63)
C     IF(NN.EQ.1) WRITE(6,3) ITER(I),(GRAPH(L),L=1,63)
C     3 FORMAT(1H0,22X,A1,2X,63A1)
C     DO 30 L=2,62

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30 GRAPH(L)=BLANK                                MAP14410
   GRAPH(32)=DOT                                  MAP14420
40 CONTINUE                                       MAP14430
   DO 50 I=1,63                                   MAP14440
50 GRAPH(I)=DOT                                  MAP14450
   WRITE(6,1) (GRAPH(I),I=1,63)                 MAP14460
   RETURN                                         MAP14470
   END                                           MAP14480
C *****MAP14490
C                                     FUNCTION BIG    MAP14500
C *****MAP14510
C PURPOSE—FIND THE LARGEST NUMBER FOR A ONE DIMENSIONAL ARRAY MAP14520
C                                     MAP14530
C DESCRIPTION OF PARAMETERS           MAP14540
C                                     MAP14550
C       X—ARRAY NAME                 MAP14560
C       M—NUMBER OF ELEMENTS IN THE ARRAY MAP14570
C REMARKS:                             MAP14580
C       NONE                         MAP14590
C *****MAP14600
C       FUNCTION BIG(X,M)             MAP14610
C       DIMENSION X(M)               MAP14620
C       BIG=ABS(X(1))                 MAP14630
C       DO 10 I=1,M                  MAP14640
C       IF(ABS(X(I)).LT.BIG) GO TO 10 MAP14650
C       BIG=ABS(X(I))                 MAP14660
C 10 CONTINUE                         MAP14670
C   RETURN                           MAP14680
C   END                             MAP14690
C *****MAP14700
C                                     SUBROUTINE MINTIR MAP14710
C *****MAP14720
C                                     MAP14730
C PURPOSE—FIND THE SMALLEST NUMBER AS WELL AS ITS POSITION MAP14740
C       OF ONE DIMENSIONAL ARRAY     MAP14750
C                                     MAP14760
C DESCRIPTION OF PARAMETERS           MAP14770
C                                     MAP14780
C       A—ARRAY NAME                 MAP14790
C       N—NUMBER OF ELEMENTS IN THE ARRAY MAP14800
C       SMALL—SMALLEST NUMBER        MAP14810
C       J—POSITION OF THE SMALLEST NUMBER MAP14820
C REMARKS:                             MAP14830
C       NONE                         MAP14840
C *****MAP14850
C       SUBROUTINE MINTIR(A,N,SMALL,J) MAP14860
C       DIMENSION A(N)               MAP14870
C       SMALL=ABS(A(1))               MAP14880
C       J=1                           MAP14890
C       DO 10 I=2,N                  MAP14900
C       IF(ABS(A(I)).GT.SMALL) GO TO 10 MAP14910
C       SMALL=ABS(A(I))               MAP14920
C       J=I                           MAP14930
C 10 CONTINUE                         MAP14940
C   RETURN                           MAP14950
C   END                             MAP14960
C *****MAP14970
C                                     SUBROUTINE DYNAMC    MAP14980
C *****MAP14990
C                                     MAP15000

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C	PURPOSE	MAP15010
C		MAP15020
C	TO FIND THE OPTIMAL PATH OF MOTION BY USING DYNAMIC	MAP15030
C	PROGRAMMING	MAP15040
C		MAP15050
C	DESCRIPTION OF PARAMETERS	MAP15060
C		MAP15070
C	L12,L23,M12,M23,R12,R23,I12,I23--ARM ANTHROPOMETRIC DATA	MAP15080
C	X0,Y0--INITIAL HAND COORDINATES	MAP15090
C	TOR2--INITIAL ELBOW MOMENT	MAP15100
C	SZF2--INITIAL ELBOW FORCE	MAP15110
C	ITIME--MOTION TIME X100	MAP15120
C	VEL--ANGULAR VELOCITY VECTOR FOR THE SHOULDER	MAP15130
C	VEL1--ANGULAR VELOCITY VECTOR FOR THE ARM	MAP15140
C	TORINT--INITIAL SHOULDER MOMENT	MAP15150
C	TOR--MOMENT VECTOR FOR THE SHOULDER	MAP15160
C	FCT--COST VECTOR (POWER)	MAP15170
C	KT--TIME INDEX	MAP15180
C	K--STAGE INDEX	MAP15190
C	X1,X2--HAND X-COORDINATES AT STAGES K AND K+1	MAP15200
C	J--Z--COORDINATE INDEX AT STAGE K+1	MAP15210
C	Y21--RATIO OF HAND Z-COORDINATE TO TOTAL MOTION DISTANCE	MAP15220
C	AT STAGE K+1	MAP15230
C	I--INDEX OF Z-COORDINATE AT STAGE K	MAP15240
C	Y2--HAND Z-COORDINATE AT STAGE K+1	MAP15250
C	Y11--RATIO OF HAND Z-COORDINATE TO TOTAL MOTION DISTANCE	MAP15260
C	AT STAGE K	MAP15270
C	Y1--HAND Z--COORDINATE AT STAGE K	MAP15280
C	A,B--ARM EULER'S ANGLES AT STAGE K	MAP15290
C	C,D--ARM EULER'S ANGLES AT STAGE K+1	MAP15300
C	ANGV--ANGULAR VELOCITY VECTOR FOR THE SHOULDER	MAP15310
C	ANGV3--ANGULAR VELOCITY VECTOR FOR THE ELBOW	MAP15320
C	XA2,ZA2--COMPONENTS OF ELBOW LINEAR ACCELERATION	MAP15330
C	XGA3,ZGA3--COMPONENTS OF FOREARM C.G. LINEAR ACCELERATION	MAP15340
C	YMON2--ELBOW MOMENT	MAP15350
C	XF2,ZF2--COMPONENTS OF ELBOW REACTIVE FORCE	MAP15360
C	POWER--TOTAL POWER COMPUTED AT THE SHOULDER	MAP15370
C	NODE--VECTOR OF GRID POSITIONS ALONG THE Z-AXIS	MAP15380
C	XOPT,YOPT--HAND COORDINATES FOR THE OPTIMAL PATH OF MOTION	MAP15390
C	FCTOPT--COST VECTOR (POWER) FOR THE OPTIMAL PATH OF MOTION	MAP15400
C	VELOPT--SHOULDER ANGULAR VELOCITY VECTOR FOR THE OPTIMAL PATH	MAP15410
C	OF MOTION	MAP15420
C	TOROPT--SHOULDER MOMENT VECTOR FOR THE OPTIMAL PATH OF MOTION	MAP15430
C	XC,ZC--X AND Z COORDINATES OF THE SMOOTHED OPTIMAL PATH OF	MAP15440
C	MOTION	MAP15450
C	REMARKS	MAP15460
C	THE FOLLOWING SUBROUTINES ARE REQUIRED	MAP15470
C	-NEWTON	MAP15480
C	-MINIM	MAP15490
C	-REGRSS	MAP15500
C	*****	MAP15510
C	SUBROUTINE DYNAMC(XL,YL,XM,YM,XI,YI,R12,R23,XMAX,ITIME,ANG1,ANG2,	MAP15520
C	IWEIGHT,XC,ZC,NM,TANG)	MAP15530
C	DIMENSION XC(20),ZC(20),T(20),W(20,20)	MAP15540
C	DIMENSION FCT(20,20),TOR(20,20),NODE(20,20)	MAP15550
C	DIMENSION POWER(20,20,20),VEL1(20,20),VEL(20,20)	MAP15560
C	DIMENSION FCTOPT(20),POINT(20),XOPT(20),YOPT(20),TIME(20)	MAP15570
C	DIMENSION ANGV(20),ANGV3(20),YMON(20),VELOPT(20),TOROPT(20)	MAP15580
C	DIMENSION ZCRG(20,10)	MAP15590
C	DIMENSION YMOME(20),TORE(20,20)	MAP15600

REAL L12,L23,M12,M23,I12,I23	MAP15610
REAL KT	MAP15620
C READ GRID PARAMETERS	MAP15630
READ(5,1) DELTA,Y21INT,Y21INC,Y11INT,Y11INC	MAP15640
1 FORMAT(5F5.2)	MAP15650
DEL=DELTA/100.0	MAP15660
L12=XL	MAP15670
L23=YL	MAP15680
M12=XM	MAP15690
M23=YM	MAP15700
I12=XI	MAP15710
I23=YI	MAP15720
A=ANG1*0.0175	MAP15730
B=ANG2*0.0175	MAP15740
C COMPUTE INITIAL COORDINATES OF THE HAND	MAP15750
XO=L12*COS(A)+L23*COS(B)	MAP15760
YO=L12*SIN(A)+L23*SIN(B)	MAP15770
C COMPUTE MOMENT AND REACTIVE FORCE AT THE INITIAL POINT	MAP15780
TOR2=-M23*9.8*R23*COS(B)-WEIGHT*L23*COS(B)	MAP15790
SFZ2=-WEIGHT-M23*9.8	MAP15800
TORINT=TOR2-M12*9.8*R12*COS(A)+SFZ2*L12*COS(A)	MAP15810
TIM1=ITIME	MAP15820
TIM=TIM1/100.0	MAP15830
C INITIALIZE VALUES FOR STAGE 1	MAP15840
DO 5 I=1,20	MAP15850
VEL(I,I)=0.0	MAP15860
VEL1(I,I)=0.0	MAP15870
TURE(I,I)=TOR2	MAP15880
TOR(I,I)=TORINT	MAP15890
5 FCT(I,I)=999999.9	MAP15900
FCT(I,I)=0.0	MAP15910
KT=0	MAP15920
C SET TIME INDEX EQUAL ZERO	MAP15930
K=0	MAP15940
WRITE(6,631)	MAP15950
631 FORMAT(1H1,' ***** STAGES SUMMARY *****')	MAP15960
10 TIM1=KT	MAP15970
K=K+1	MAP15980
TIM2=TIM1/100.0	MAP15990
TIM3=TIM2*DEL	MAP16000
TERM1=6.28*TIM2/TIM	MAP16010
TERM2=TERM1-SIN(TERM1)	MAP16020
TERM3=6.28*TIM3/TIM	MAP16030
TERM4=TERM3-SIN(TERM3)	MAP16040
C COMPUTE X-COORDINATE OF THE HAND AT STAGE K AND K+1	MAP16050
X1=XMAX*TERM2/6.28+XO	MAP16060
X2=XMAX*TERM4/6.28+XO	MAP16070
XOPT(K)=X1	MAP16080
TIME(K)=TIM2	MAP16090
J=0	MAP16100
C INITIALIZE Z-COORDINATE OF THE HAND AT STAGE K+1	MAP16110
Y21=Y21INT	MAP16120
20 J=J+1	MAP16130
Y21=Y21+Y21INC	MAP16140
Y2=Y21*XMAX/100.0+YO+X2*TAN(TANG)	MAP16150
I=0	MAP16160
C INITIALIZE Z-COORDINATE OF THE HAND AT STAGE K	MAP16170
Y11=Y11INT	MAP16180
30 I=I+1	MAP16190
Y11=Y11+Y11INC	MAP16200


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      Y1=Y11*XMAX/100.0+Y0+X1*TAN(TANG)
C  CALL NEWTON TO COMPUTE EULER'S ANGLES FOR THE ARM AT STAGE K
      CALL NEWTON(L12,L23,X1,Y1,A,B)
      C=A
      D=B
C  CALL NEWTON TO COMPUTE EULER'S ANGLES FOR THE ARM AT STAGE K+1
      CALL NEWTON(L12,L23,X2,Y2,C,D)
      AC=C
      BD=D
C  COMPUTE ANGULAR VELOCITIES AND ACCELERATIONS
      ANGV1=(C-A)/DEL
      ANGV(I)=ANGV1
      ANGV2=(D-B)/DEL
      ANGV3(I)=ANGV2
      ANGA1=(ANGV1-VEL(K,I))/DEL
      ANGA2=(ANGV2-VEL1(K,I))/DEL
C  COMPUTE LINEAR ACCELERATIONS
      XA2=-L12*SIN(AC)*ANGA1-L12*COS(AC)*(ANGV1**2)
      ZA2=L12*COS(AC)*ANGA1-L12*SIN(AC)*(ANGV1**2)
      XGA2=R12*XA2/L12
      ZGA2=R12*ZA2/L12
      XGA3=XA2-R23*SIN(BD)*ANGA2-R23*COS(BD)*(ANGV2**2)
      ZGA3=ZA2+R23*COS(BD)*ANGA2-R23*SIN(BD)*(ANGV2**2)
C  COMPUTE ELBOW MOMENT AND REACTIVE FORCE COMPONENTS
      YMOM2=I23*ANGA2-M23*R23*SIN(BD)*XGA3+M23*R23*COS(BD)*ZGA3-M23*9.8
      1R23*COS(BD)-WEIGHT*L23*COS(BD)
      YMOME(I)=YMOM2
      XF2=M23*XGA3
      ZF2=-WEIGHT-M23*9.8+M23*ZGA3
C  COMPUTE SHOULDER MOMENT
      YMOM1=YMOM2+I12*ANGA1-M12*R12*SIN(AC)*XGA2+M12*R12*COS(AC)*ZGA2
      2-M12*9.8*R12*COS(AC)-XF2*L12*SIN(AC)+ZF2*L12*COS(AC)
      YMOM(I)=YMOM1
C  COMPUTE TOTAL POWER EXPENDED FOR MOVING FROM THE INITIAL POINT TO
C  STAGE K+1 AND FOLLOWING LINK IJ BETWEEN STAGE K AND K+1
      POWER(K+1,J,I)=FCT(K,I)+0.5*(ABS(TOR(K,I))+ABS(YMOM1))*ABS(ANGV1
      1-VEL(K,I))
      IF(I.LT.20) GO TO 30
      KK=K+1
C  FIND LINK IJ WHICH GIVES THE MINIMUM TOTAL POWER
      CALL MINIM(POWER,KK,J,I,SMALL,II)
      FCT(K+1,J)=ABS(SMALL)
      NODE(K+1,J)=II
      VEL(K+1,J)=ANGV(II)
      VEL1(K+1,J)=ANGV3(II)
      TORE(K+1,J)=YMOME(II)
      TOR(K+1,J)=YMOM(II)
      WRITE(6,2) KK,J,SMALL,VEL(KK,J),TOR(KK,J),K,II
2  FORMAT(1H0,2I6,3F20.5,2I6)
      IF(J.LT.20) GO TO 20
      KT=KT+DELTA
      XXTIME=ITIME
      IF(KT.LT.XXTIME) GO TO 10
      TIME(K+1)=TIM
      XOPT(K+1)=XMAX+X0
      YOPT(K+1)=Y0
      TOROPT(K+1)=TOR(K+1,1)
      FCTOPT(K+1)=FCT(K+1,1)
      POINT(K+1)=0.0
      VELOPT(K+1)=0.0

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MAP16210
MAP16220
MAP16230
MAP16240
MAP16250
MAP16260
MAP16270
MAP16280
MAP16290
MAP16300
MAP16310
MAP16320
MAP16330
MAP16340
MAP16350
MAP16360
MAP16370
MAP16380
MAP16390
MAP16400
MAP16410
MAP16420
MAP16430
MAP16440
MAP16450
MAP16460
MAP16470
MAP16480
MAP16490
MAP16500
MAP16510
MAP16520
MAP16530
MAP16540
MAP16550
MAP16560
MAP16570
MAP16580
MAP16590
MAP16600
MAP16610
MAP16620
MAP16630
MAP16640
MAP16650
MAP16660
MAP16670
MAP16680
MAP16690
MAP16700
MAP16710
MAP16720
MAP16730
MAP16740
MAP16750
MAP16760
MAP16770
MAP16780
MAP16790
MAP16800

```

```

C TRACE THE OPTIMAL PATH OF MOTION
  K1=K+1
  J1=1
100 J1=NODE(K1,J1)
  FCTOPT(K1-1)=FCT(K1-1,J1)
  XJ1=J1
  POINT(K1-1)=XJ1*1.0-1.0
  VELOPT(K1-1)=VEL(K1-1,J1)
  TOROPT(K1-1)=TOR(K1-1,J1)
  YOPT(K1-1)=XMAX*POINT(K1-1)/100.0+YO
  K1=K1-1
  IF(K1.GT.1) GO TO 100
  YOPT(1)=YO
  FCTOPT(1)=0.0
  POINT(1)=0.0
  TOROPT(1)=TORINT
  KM=K+1
  DO 90 I=1,KM
90 WRITE(6,3) TIME(I),XOPT(I),YOPT(I),POINT(I),VELOPT(I),TOROPT(I),
  3FCTOPT(I)
  3 FORMAT(1H0,7F15.5)
C SMOOTH THE HAND PATH OF MOTION BY USING REGRESSION ANALYSIS
  KMM1=KM-1
  CALL REGRSS(XOPT,YOPT,KM,KMM1,1,W)
  NM=ITIME/10+1
  T(1)=0.0
  DO 50 I=2,NM
50 T(I)=T(I-1)+0.10
  DO 70 I=1,NM
  TER1=6.28*T(I)/TIM
  TER2=TER1-SIN(TER1)
  XC(I)=XMAX*TER2/6.28+XO
  ZCRG(I,1)=W(1,1)
  DO 77 L=2,KM
77 ZCRG(I,L)=ZCRG(I,L-1)+(W(L,1)*XC(I))*L-1))
  ZC(I)=ZCRG(I,KM)
70 CONTINUE
  ZC(NM)=YOPT(KM)
  ZC(1)=YOPT(1)
  RETURN
  END
C *****
C SUBROUTINE MINIM
C *****
C PURPOSE
C TO FIND THE SMALLEST NUMBER FOR THREE DIMENSIONAL ARRAY
C DESCRIPTION OF PARAMETERS
C APOW---ARRAY NAME
C K,J,I,---ARRAY INDICES
C SMALL---SMALLEST NUMBER
C II---POSITION OF THE SMALLEST NUMBER
C REMARKS
C NONE
C *****
C SUBROUTINE MINIM(APOW,K,J,I,SMALL,II)
  DIMENSION APOW(20,20,20)

```

```

MAP16810
MAP16820
MAP16830
MAP16840
MAP16850
MAP16860
MAP16870
MAP16880
MAP16890
MAP16900
MAP16910
MAP16920
MAP16930
MAP16940
MAP16950
MAP16960
MAP16970
MAP16980
MAP16990
MAP17000
MAP17010
MAP17020
MAP17030
MAP17040
MAP17050
MAP17060
MAP17070
MAP17080
MAP17090
MAP17100
MAP17110
MAP17120
MAP17130
MAP17140
MAP17150
MAP17160
MAP17170
MAP17180
MAP17190
MAP17200
MAP17210
MAP17220
MAP17230
MAP17240
MAP17250
MAP17260
MAP17270
MAP17280
MAP17290
MAP17300
MAP17310
MAP17320
MAP17330
MAP17340
MAP17350
MAP17360
MAP17370
MAP17380
MAP17390
MAP17400

```

```

        SMALL=APOW(K,J,1)
        I1=1
        DO 10 I1=1,I
        IF(APOW(K,J,I1).GT.SMALL) GO TO 10
        SMALL=APOW(K,J,I1)
        I1=I1
10 CONTINUE
    RETURN
    END
C *****MAP17410
C                                     MAP17420
C                                     MAP17430
C                                     MAP17440
C                                     MAP17450
C                                     MAP17460
C                                     MAP17470
C                                     MAP17480
C                                     MAP17490
C *****MAP17500
C                                     MAP17510
C                                     MAP17520
C                                     MAP17530
C                                     MAP17540
C                                     MAP17550
C                                     MAP17560
C                                     MAP17570
C                                     MAP17580
C                                     MAP17590
C                                     MAP17600
C                                     MAP17610
C *****MAP17620
C                                     MAP17630
C                                     MAP17640
C                                     MAP17650
C                                     MAP17660
C                                     MAP17670
C                                     MAP17680
C                                     MAP17690
C                                     MAP17700
C                                     MAP17710
C                                     MAP17720
C                                     MAP17730
C                                     MAP17740
C                                     MAP17750
C                                     MAP17760
C                                     MAP17770
C                                     MAP17780
C                                     MAP17790
C                                     MAP17800
C                                     MAP17810
C                                     MAP17820
C                                     MAP17830
C                                     MAP17840
C                                     MAP17850
C                                     MAP17860
C                                     MAP17870
C                                     MAP17880
C                                     MAP17890
C                                     MAP17900
C                                     MAP17910

        SUBROUTINE NEWTON
        PURPOSE
        TO SOLVE TWO NONLINEAR EQUATIONS
        REMARKS
        ALL THE PARAMETERS HAVE BEEN EXPLAINED BEFORE FOR SUBROUTINE
        SOLVER
        SUBROUTINE NEWTON(XL12,XL23,C1,C2,THETA1,THETA2)
        REAL L12,L23
        EPS=0.0001
        L12=XL12
        L23=XL23
        XIT=THETA1
        YIT=THETA2
        DO 20 J=1,50
        FX=-L12*SIN(XIT)
        FY=-L23*SIN(YIT)
        GX=L12*COS(XIT)
        GY=L23*COS(YIT)
        F=L12*COS(XIT)+L23*COS(YIT)-C1
        G=L12*SIN(XIT)+L23*SIN(YIT)-C2
        DIV=FX*GY-GX*FY+0.0000001
        XIT1=XIT+(G*FY-F*GY)/DIV
        YIT1=YIT+(F*GX-G*FX)/DIV
        Z1=ABS(F)
        Z2=ABS(G)
        IF(Z1.LE.EPS.AND.Z2.LE.EPS) GO TO 25
        XIT=XIT1
        YIT=YIT1
20 CONTINUE
25 THETA1=XIT1
    THETA2=YIT1
    RETURN
    END

```

NUMBER OF CARDS IS 1791

PROGRAM FLOW CHARTS:FLOW CHART--MAIN PROGRAM

FLOW CHART SOLVER

FLOW CHART PARAB

FLOW CHART SIMUL

FLOW CHART SMOOTH

FLOW CHART STICK

FLOW CHART PLOT

FLOW CHART BIG

FLOW CHART DYNMC

FLOW CHART MINTIR

SAMPLE OUTPUT

*

* DYNAMICAL ANALYSIS *
*

MOTION NUMBER 13415

MOTION CHARACTERISTICS#

DISTANCE...# 0.30480METER

ANGLE.....# 0.0 DEG.

WEIGHT.....# 0.0 KGS.

TIME.....# 0.60000SEC.

INITIAL ARM CCNFIGURATION

UPPER ARM ANGLE# 294.00000 DEG.

FOREARM ANGLE # 73.50000 DEG.

DISPLACEMENT DATA

DYNAMIC PROGRAMMING

X-COOR. HAND	Z-COOR. HAND	UPPER ARM ANGLE	ERROR	FOREARM ANGLE	ERROR
0.2080800	0.1117100	5.1302986	0.0	1.2825747	0.0
0.2092200	0.1125200	5.1361904	0.0000035	1.2770309	0.0000014
0.2168600	0.1175800	5.1773405	0.0000001	1.2823420	0.0000001
0.2357399	0.1277900	5.2606363	0.0000015	1.2814360	0.0000009
0.2676200	0.1383600	5.3589258	0.0000017	1.2489491	0.0000021
0.3107600	0.1419300	5.4429960	0.0000007	1.1700535	0.0000005
0.3604000	0.1355100	5.5106134	0.0000080	1.0549021	0.0000074
0.4100699	0.1238000	5.5795803	0.0000116	0.9294137	0.0000098
0.4532599	0.1148000	5.6612120	0.0000081	0.8194639	0.0000045
0.4852099	0.1122200	5.7451038	0.0000042	0.7395920	0.0000004
0.5041800	0.1135100	5.8083553	0.0000011	0.6924658	0.0000002
0.5118700	0.1148199	5.8376303	0.0000001	0.6732289	0.0000001
0.5130300	0.1150600	5.8422480	0.0000037	0.6703087	0.0000001

PLCTS CF ANGULAR DISPLACEMENT

PPER ARM ANGLE SYMBOL IS *	FOREARM ANGLE SYMBOL IS X	
-0.71195	C	C.71195 *
-C.61227	C	C.61227 X

STICK DIAGRAM OF THE MOTION

HAND DATA

X-ACCELERATION
METER/SEC/SEC

Z-ACCELERATION
METER/SEC/SEC

0.0	0.0
8.13558	4.11831
11.46517	-3.86398
8.16156	-1.90679
0.05610	-1.51620
-8.00587	-4.88528
-11.23330	3.75065
-7.97308	2.69784
0.0	0.0

PLOTS OF LINEAR ACCELERATIONS

X-ACCELERATION SYMBOL IS *

Z-ACCELERATION SYMBOL IS X

-11.46517	0	11.46517 *
-4.88528	0	4.88528 X

```

.....
T . X
.
. * X
I . X
. X
M . X
.X *
E . *
. *
.
. X
.....

```

ELBOW DATA

STATIC ANALYSIS

X-FORCE NEWTON	Z-FORCE NEWTON	RESULTANT-FORCE NEWTON	MOMENT NEWTON-METER
0.0	-10.94660	10.94660	-0.49754
0.0	-10.94660	10.94660	-0.47692
0.0	-10.94660	10.94660	-0.46414
0.0	-10.94660	10.94660	-0.61496
0.0	-10.94660	10.94660	-0.84767
0.0	-10.94660	10.94660	-1.08361
0.0	-10.94660	10.94660	-1.27440
0.0	-10.94660	10.94660	-1.35723
0.0	-10.94660	10.94660	-1.36844

DYNAMIC ANALYSIS

ANGULAR DISPL. RADIAN	ANGULAR VEL. RAD/SEC	ANGULAR ACCEL. RAD/SEC/SEC	X-ACCEL. METER/SEC/SEC	Z-ACCEL. METER/SEC/SEC
1.28257	0.0	0.0	0.0	0.0
1.29484	0.19841	-1.87340	7.48982	4.31592
1.30242	-0.83035	-39.27727	-2.11771	0.12924
1.21180	-2.37163	-22.37358	1.32435	2.82404
1.06525	-3.08577	-6.19240	-0.23456	2.56705
0.90322	-3.09975	5.63278	-4.26782	-3.42255
0.75528	-2.19774	30.44739	-2.44384	-3.04548
0.68345	-0.82042	24.64561	-2.17607	-4.03627
0.67324	0.0	0.0	0.0	0.0

X-FORCE NEWTON	Z-FORCE NEWTON	RESULTANT-FORCE NEWTON	MOMENT NEWTON-METER
0.0	-10.94660	10.94660	-0.49754
8.68615	-6.22365	10.68565	-2.35493
4.36596	-12.78121	13.50633	-16.88528
4.86770	-10.13666	11.24484	-10.22445
-0.11795	-10.10279	10.10348	-3.23637
-6.61967	-15.49449	16.84930	1.54472
-7.08568	-10.98034	13.06808	11.64576
-5.30357	-12.11780	13.22758	8.86581
0.0	-10.94660	10.94660	-1.36844

PLOTS OF RESULTANT FORCE AND MOMENT

MOMENT SYMBOL IS #	RESULTANT FORCE SYMBOL IS X		
-16.88528	0	16.88528	*
-16.84930	0	16.84930	X
.....			
T .	*.	X	.
.	*	X	.
I .*	.	X	.
.	*	X	.
M .	*	X	.
.	*	X.	.
E .	.	* X	.
.	.	* X	.
.	*.	X	.
.....			

SHOULDER DATA

STATIC ANALYSIS

X-FORCE NEWTON	Z-FORCE NEWTON	RESULTANT-FORCE NEWTON	MOMENT NEWTON-METER
0.0	-23.12798	23.12798	-2.19253
0.0	-23.12798	23.12798	-2.30357
0.0	-23.12798	23.12798	-2.72484
0.0	-23.12798	23.12798	-3.22323
0.0	-23.12798	23.12798	-3.85668
0.0	-23.12798	23.12798	-4.48319
0.0	-23.12798	23.12798	-4.88853
0.0	-23.12798	23.12798	-5.08480
0.0	-23.12798	23.12798	-5.11855

DYNAMIC ANALYSIS

ANGULAR DISPL. RADIAN	ANGULAR VEL. RAD/SEC	ANGULAR ACCEL. RAD/SEC/SEC
5.13030	0.0	0.0
5.16507	1.54077	33.81615
5.28438	2.21846	-6.70853
5.38691	2.32506	10.97260
5.51688	2.76475	5.61507
5.66337	2.41395	-20.64705
5.75828	1.51933	-15.13787
5.81532	0.69148	-17.97600
5.82742	0.0	0.0

X-FORCE NEWTON	Z-FORCE NEWTON	RESULTANT-FORCE NEWTON	MOMENT NEWTON-METER
0.0	-23.12798	23.12798	-2.19253
12.83724	-16.01302	20.52344	6.80918
3.19226	-24.89096	25.09482	-20.14380
5.60170	-20.75287	21.49559	-8.98528
-0.24795	-20.86143	20.86290	-4.45391
-8.98502	-29.57274	30.90756	-8.95219
-8.44013	-24.84962	26.24384	3.32614
-6.50961	-26.53621	27.32298	-0.24187
0.0	-23.12798	23.12798	-5.11855

PLOTS OF RESULTANT FORCE AND MOMENT

MOMENT SYMBOL IS #	RESULTANT FORCE SYMBOL IS X	
-20.14380	0	20.14380 *
-30.90756	0	30.90756 X
.....		
I .	*	X .
.	.	X .
I .*	.	X .
.	*	X .
M .	*	X .
.	*	X.
E .	*	X .
.	*	X .
.	*	X .
.....		