APPLICATION OF FINITE ELEMENT METHOD TO THE
QUASI-STATIC THERMOELASTIC ANALYSIS OF
PRESTRESS IN MULTILAYER PRESSURE VESSELS

by

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CONTENTS

ACKNOWLEDGEMENTS	ii
ABSTRACT	iv
LIST OF FIGURES	v
NOMENCLATURE	vi
I. BACKGROUND	1
1.1 Introduction	1
1.2 Advantages of Multilayer Pressure Vessels	4
1.3 Limitations	7
1.4 Previous Work	9
II. ANALYSIS PROCEDURES	14
2.1 Introduction	14
2.2 The Finite Element Method	15
2.3 Problem Statement	18
2.4 Problem Analysis	20
2.5 The Finite Element Model	24
III. RESULTS AND DISCUSSION	36
3.1 Displacement Distribution	36
3.2 Stress Distributions	37
3.3 Effects of Friction	39
IV. CONCLUSION AND RECOMMENDATIONS	49
4.1 Conclusion	49
4.2 Recommendations	49
REFERENCES	51
APPENDIX A	57
APPENDIX B	60
APPENDIX C	69

ABSTRACT

Multilayered Wrapped vessel technology utilizes the compressive prestress induced during the construction process to gain a considerable advantage over the monoblock vessels. The compressive prestress allows for more efficient use of construction material and more uniform distribution of stress throughout the vessel's cross section. Analysis of the magnitude of prestress throughout the vessel's thickness has been previously reported Rasty [1988]. However, one major idealization in such analysis has been the assumption that the magnitude of induced prestress is constant around the circumference of the vessel. In this research, thermoelastic finite element method was utilized to simulate the construction process of one layer of the vessel. ABAQUS, a general purpose finite element code, was used for this purpose. It was concluded that the compressive residual stress induced by the weld shrinkage, varies through the circumference of the vessel by as much as 13.5%. Circumferential distribution of the prestress are presented and compared to the closed form solutions (constant prestress assumption) in earlier works.

LIST OF FIGURES

1.1	Different Types of Cylindrical Multilayer Pressure Vessels	12
1.2	Longitudinal Weld in Multilayer Pressure Vessel (not to scale) . .	13
2.1	A Continuous Function $\phi = \phi(x, y)$ and Typical Elements that might be used to Approximate it	30
2.2	The Finite Element Model (Inset: weld region)	31
2.3	Different Stages of Mesh Refinement	32
2.4	Boundary Conditions on Heat Transfer Model (not to scale)	33
2.5	Boundary Conditions on Static Model (not to scale)	34
2.6	Bounding Planes and the Nodes used for describing the Generalized Plane Strain Elements	35
3.1	Results of the Analysis showing the Layer Shrinkage due to Cooling of Weld (Inset: displacement magnified 10 times)	41
3.2	Circumferential Distribution of the Tangential Displacement of the Outer Surface	42
3.3	Radial Distribution of the Tangential Prestress due to Cooling of Weld (FEM results averaged over the circumference of the vessel) .	43
3.4	Circumferential Distribution of the Tangential Prestress at the Outer Surface	44
3.5	Circumferential Distribution of the Tangential Prestress at the Inner Surface	45
3.6	Circumferential Distribution of the Tangential Prestress Obtained from Tangential Displacements at the Outer Surface of the Finite Element Model	46
3.7	Radial Distribution of the Tangential Prestress for Different Friction Coefficients (results averaged over the circumference of the vessel)	47
3.8	Circumferential Distribution of the Tangential Prestress at the Inner Surface for Different Friction Coefficients	48

NOMENCLATURE

c	Specific heat, J/kg $^{\circ}$ K
E	Modulus of Elasticity, GPa
F	Force, MPa
h	Convective heat transfer coefficient, W/m 2 $^{\circ}$ K
K	Conductivity, W/m $^{\circ}$ K
P	Interference pressure between welded layer and its preceding layer, MPa
r	radius, m
R	Interface resistance, W/m 2 $^{\circ}$ K
T	Temperature, $^{\circ}$ C
T_o	Initial Temperature, $^{\circ}$ C
t	Time, sec
ΔT	Temperature difference, $^{\circ}$ C
u	displacement, mm
u_1	Radial deformation of welded layer, mm
u_2	Radial deflection of layers beneath welded layer, mm
v	velocity of dilatational waves in an elastic medium, m/s

W	width of longitudinal weld, mm
α	Coefficient of thermal expansion, mm/mm°C
Δ	Total interference due to weld shrinkage, mm
ϵ	Strain, mm/mm
λ	Lame's constant, GPa
μ	Shear Modulus, GPa
ν	Poissons Ratio
ρ	Density, kg/m ³
σ	Stress, MPa

Subscripts

i	inner
j	=1, 2, 3, ... , n=number of layers
k	=1, 2, 3, ... , n=number of layers after j th layer
n	layer number n (last layer welded)
r	radial
z	axial
θ	tangential

CHAPTER I

BACKGROUND

1.1 Introduction

The main difference between high-pressure vessels and ordinary vessels is the wall thickness. With the advances in welding technology in recent years, metal plates of any thickness can be joined, but it is not as easy to bend the metal plates if the thickness exceeds the capacity of the equipment available to do the bending and rolling. Bending a steel plate into a circular arc, which is 6 in. or more thick is quite a formidable task, even for the modern fabrication facilities. Yet for petrochemical work, pressure vessels with wall thickness of 7 in. or more are common.

Construction of high-pressure vessels made by forging is limited by the capacity of the forging mill. At present, the largest mills in the world can produce forgings of up to about 150 tons. However, some recent pressure vessels weigh as much as 600 tons. The limitations involved in solid wall construction compel the designer of pressure vessels to turn to alternate solutions.

Monoblock vessel, whether of welded or forged construction, are uneconomical for high-pressures because the outer portion of the vessel's wall is not fully utilized as far as the distribution of stress is concerned. The remedy is to use prestress in the vessel wall. For monoblock vessels, there is the well-known method of autofrettage, whereby a sufficiently high internal pressure is applied to the full length of the vessel, so that the inner portion of the solid wall undergoes plastic deformation. This plastic deformation causes microstress that must be removed

by careful annealing. Since the annealing must be performed on the full length of the vessel, including the ends and closures, the annealing furnace must be big enough to hold the complete vessel. With the increase in the size of pressure vessels in recent years, the lack of sufficiently large annealing furnaces has become a problem.

Limitations associated with confining large volumes of high-internal pressure in a single wall vessel have been recognized by pressure vessel designers for many years. With increasing demands of industrial processes for higher operating pressures, the multilayer vessel construction has emerged to handle the present day specialized requirements. Layered vessels were developed in the United States and Germany at about the same time during World War II. In Germany, they were used in ammonia plants as well as for producing gasoline from coal. In the United States, they were used for ammonia synthesis processes for the ultimate production of nitrates. Since World War II, the technology of building layered vessels has improved substantially. Today, layered vessels are used in a wide range of high-pressure applications in the petrochemical industry, such as, heat exchangers, urea reactors, ammonia converters, autoclaves, and coal gasification reactors.

Layered vessels consist of a multitude of layers wrapped tightly around an inner shell to form a pressure-retaining envelope. Layered vessels are constructed by various methods. The difference between these methods is in the thickness of individual layers, wrapping procedure, and welding technique. In general, layered vessel construction can be divided into three categories. The first is the concentric or spiral wrapped method where the layers consist of segments welded

together in a spiral or concentric fashion to form the required thickness as shown in Figure 1.1 (a) and Figure 1.1 (d). The second method, as shown in Figure 1.1 (c), is the shrink-fit method whereby layers are individually formed into cylinders and shrunk on each other to form the required total thickness. The third is the coil-wrapped method whereby a continuous sheet or strip is wound in spiral or helical fashion to form a cylinder, as shown in Figure 1.1 (b).

The earliest reference to layered vessels was made in the 1951 API-ASME Code. In later years when the API Standard and the ASME Code were separated, the layered vessel criteria were deleted from both. It was not until January 1979 that layered vessels were included in the ASME Code. In establishing the new layered vessel rules, consideration was given to the state of the art as well as the experience and research accumulated by the industry in the past 50 years. An effort was also made to provide rules to accommodate all types of known layered vessel construction.

Today, layered vessels are constructed in accordance with the ASME Code, Section VIII, Division 2. The majority of design equations given in the code for solid-wall vessels are applicable to layered vessels. For fabrication, the ASME Code, gives additional rules for layered vessel construction. One criteria for controlling wrapping tightness of layered shells is that the area of any gap between two adjacent layers, as measured from the end of a shell section, must not exceed the thickness of a layer expressed in square inches.

The most widely used layer vessel design is the Multilayer Vessel design, pioneered in the 1930's by A. O. Smith Corporation. Built up of many layers of relatively thin gauge steel, multilayer wrapped vessels are commonly employed

in high-pressure applications of up to 550 MPa. The innermost shell, or core, is constructed in the same manner as any solid-wall vessel. Subsequent layers, usually 6.4 mm (0.25 in.) thick, are progressively wrapped around the inner shell by mechanical means and butt welded along a longitudinal seam as shown in Figure 1.2. The welds are distributed around the circumference to minimize localization of any excessive stresses at or near the welded joints and are ground flush prior to adding of subsequent layers. Cylindrical rings are inserted at both ends of the inner shell during these operations to maintain a true cylindrical shape. After the vessel shell has been built up to the desired thickness with successive layers, the ends of the built-up shell are machined for welding groove for the attachment of the formed head. In the final vessel, 12 or more successive layers are often used.

1.2 Advantages of Multilayer Pressure Vessels

Multilayer pressure vessel technology offers distinct and significant advantages over monoblock vessels in the construction of high pressure vessels. Some of the many advantages are listed as follows:

1. Only the inner shell layer needs to be of environment resistant materials.

For example, in applications where the material choice is dictated by hydrogen temperature and partial pressure, hydrogen resistant Cr-Mo steel can be used for inner shell, and low alloy or plain carbon steel can be used for outer shells, thus effecting a tremendous savings on cost. Hydrogen can be vented out from the filler layer via mechanical venting system.

McDowell and Milligan [1966] have shown the effectiveness of this

construction technique in dealing with hydrogen attack for hydrocracker reactor construction.

2. The thin plates (usually 0.25 in. thick) used as layer material exhibit better mechanical properties than thick plates made to the same specification. The mechanical properties are known and recorded for every 1/4 in. of the total wall thickness, assuring uniform and known properties throughout the vessel wall.
3. The advantages in the welding of multilayer vessels and the metallurgical benefits inherent in the use of thin plates have been explained by Mantle et al. [1959].
4. Another feature of layered vessels is the safety mechanism provided by the vent holes. These vent holes are drilled through the vessel wall down to the outside surface of the inner shell. Any erosion, corrosion, or fatigue cracking that occurs in the inner shell due to operating temperature and pressure will result in leakage through these vent holes which is monitored by plant personnel. The vessel can then be shut down safely without any damage to its structural integrity.
5. Multilayer vessels have an inherent notch toughness. Experiments conducted by Norris et al. [1967] have shown improvements in fracture characteristics on the basis of impact energy, fracture appearance and lateral expansion measurements. In the same experiment, it was shown that 0.25 in. thick plates have superior properties over 2 in. thick plates subjected to similar heat treating operations.
6. Multilayer vessels have been observed to retard and/or arrest the propagation of cracks. Birbeck et al. [1969] have shown that the ductile to brittle

transition temperature is reduced with layers less than 1/4 in. thick. This arises because, the stress ahead of a crack in a thin laminate layer approaches biaxiality, which favors ductile propagation and suppresses the brittle fracture mode.

7. The inner layers of the shell are precompressed by the combination of wrapping load and weld shrinkage in the longitudinal seams, which reduces the magnitude of the tensile stress at the inside surface of the shell. Jasper et al. [1941], have shown that, the inner strains increase progressively but at reduced rates as subsequent layers are added.
8. The thin plate metallurgy and choice of layer materials precludes the necessity for postweld heat-treatment of layer sections. No stress relieving is necessary. H. C. Boardman, in his discussion to the paper by Jasper and Scudder [1941], considers heat treatment as inadvisable, because it would remove helpful compression and tension stresses induced by the wrapping and welding operations.

Here is a case where, locked up construction stresses are cleverly utilized to obtain under maximum service conditions a stress distribution nearly uniform over the shell thickness, thereby largely offsetting the Lamé's stress variations.

9. Complete design flexibility: There is no specific limit for the thickness and dimensions of a multilayer vessel that can be produced. The size of the vessel is restricted only to the physical capacity of the plant and transportation limitations. Also, no heating furnace or heavy rolling presses are required [Strelzoff et al. 1968].
10. Natural safe design against risk of brittle fracture. Yamauchi [1974], on discussing the applicability of ASME code, claims that since the steel plate

is relatively thin, the distribution of the three components of stress is not so complicated as in the case of the ordinary thick wall, where three-dimensional stress is prevailing, with two-dimensional being the maximum. Accordingly, even in the worst case of bursting, the layer plates are broken one by one by the shearing mode, and thus an outstanding feature that no fragments are shattered out.

11. The multilayer vessel sections are comparatively long, reducing the number of deep-groove girth seams required in the finished vessel. Davis [1970] has discussed the effects of the girth seams.
12. Since the layer plates are applied parallel to the axis of the vessel and since longitudinal seams are staggered, mill camber in the center of the layer plates is not incremental.

1.3 Limitations

Inspite of all the advantages of the multilayer pressure vessels described in the previous section, there are certain limitations to their applications. These are summarized below:

1. Extremely high magnitude of residual shear stress when the vessel is unpressurized. This limitation is due to the fact that the vessel must be designed for zero pressure as well as design pressure. Thus if the vessel is designed for very high design pressure, the stresses at zero pressure may be unacceptable.
2. In order to minimize the deviation of stress intensity from the ideal value, machining to extremely accurate tolerances is required. In designing very

long cylinders with thin walls, the tolerance problem is difficult to solve and can cause some increase in stresses.

3. Another limitation to shrink fit design is the longitudinal stress problem. Considerable axial stresses can be produced when two component cylinders are shrunk together. The magnitude of these stresses is unpredictable. These effects have been discussed by Davis [1970] and Sparanger [1950].
4. Stresses due to shrinkage are not uniformly distributed because the contact surfaces are not mechanically adjusted to each other [Noel 1970].
5. In his paper on the new fabrication method, McFarland [1973] in elucidating certain advantages of his method says that in multilayer vessels there are a multitude of longitudinal welds which cannot be fully tested by non-destructive methods.
6. Reduced through-wall overall thermal conductivity which is caused by layer to layer interface thermal barriers [Maruyama 1977].

If multilayered structure is used for high temperature conditions, it seems to cause large thermal stresses, especially on a transient operation such as starting-up or shutting-down, because of the large temperature difference between an inner surface and outer surface, due to thermal resistance between each layer, peculiar to a multilayered structure. These thermal stresses may cause fatigue failure during a cyclic transient.

These thermal effects have been reported by Mancuso [1979], Armstrong [1981], and Ueda [1986] and also reported in Struthers Wells Lab. Reports Nos. 959 and 1043 for developing the Standard 17.03.06.

1.4 Previous Work

To achieve high-pressures with a minimum compromise on factor of safety, it is important that the designer utilize accurate means for predicting both stresses and elastic breakdown in pressure vessels with combined loadings and optimize the various vessel design parameters.

Autofrettage as a means of pressurization was studied by Berman and Pai [1967], using two different methods to achieve the purpose, i.e., (1) Creep Autofrettage, in which pressurization takes place at a sufficiently elevated temperature for an adequate period of time to obtain second-stage creep stress distribution and (2) short-time pressurization at an elevated temperature, at which the yield stress is reduced from that at ambient temperature.

The multilayer method of construction has been investigated by a number of authors, with one of the earliest being, Cox [1936], who developed the general formulae for direct application to design of built up cylinders. Cox's analysis was later improved by Manning [1947, 50, 57], in which he developed the formulae for compound cylinders for high-pressure service and calculated the contact stresses introduced as a result. The analysis of multilayer shrink-fit vessels based on elastic stress limit has been discussed by Faupel [1954], Dalton [1987], Brownell [1959], Jawad [1972, 79, 83], Jasper [1941], McCabe [1971], Noel [1970], Rasty [1988], Rauchenplat [1971], Sabbaghian [1969], Strub [1953], and Yang [1970]. Design optimization of this type of vessel has been reported by Brock [1967, 71], Huddleston [1972], who also introduced thermal effects, Pimshtein [1972], and Sabbaghian [1982], who considered dissimilar materials for different layers in the optimization.

To overcome the limitations of multilayer design Berman [1966], came up with the idea of floating-head controlled fluid-fill layer vessel design. This particular design utilizes a series of vessels that are separated by fluid in which the pressure is externally controlled so that at each pressure, each vessel sustains the same maximum stress intensity based on all three principal stresses. Birchall [1947] designed an alternative form of pressure vessel of novel laminar construction. McFarland [1973] devised a new fabrication method for large high-pressure vessels. Wedellsborg [1989] designed multilayer pressure vessels, with interlayer metallic cushions. This particular method is still in the development stage, but has a very great advantage for Nuclear applications, i.e., implosive ductile failure mode. Brink [1964] considered sandwich cylinder construction for underwater pressure vessels. Mantle [1959] investigated experimentally into the stress distribution in a band-reinforced pressure vessel, in which half the tangential stress is carried by the barrel and half by the band reinforcement.

Solutions by Huddleston [1972] and Strub [1953] are based on Mises yield criteria; while those of Manning [1947, 50], Becker, and Mollick [1960], and Whalley, and Morris [1960] are based on maximum shear stress (Tresca) criteria.

Imperfect contact in multilayer pressure vessels has been a well-studied topic. Blanford and Tauchert [1985] developed a finite element formulation utilizing quadratic layer elements and linear interface elements to perform thermoelastic analysis of layered structures with imperfect layer contact. Chang and Kutlu [1989] developed an analytical model based on structural analysis for calculating the deformations and a fracture mechanics analysis for calculating the delamination growth in the structures. Rasty and Sabbaghain [1988] considered the

effect of imperfect contact between adjacent layers on the integrity of wrapped vessels. Fentzlaff and Van Gulick [1989] performed an elasto-plastic analysis of multilayer pressure vessels with initial interlayer gaps, using ABAQUS.

Clark and Terni [1972] and Weiler [1954] have given some general guidelines for the cost consideration in the design of pressure vessels. Fratcher [1954] and Lanchester [1964] discussed the suitability of various alloys for the manufacture of multilayer wrapped pressure vessels. A comprehensive review on the analysis of the laminated shells is given by Kapania [1989].

All the authors previously mentioned, assumed an axisymmetric solution in the analysis of multilayer pressure vessels. Boudjelida [1983] considered the effects of temperature distribution during the welding process on prestress of multilayer wrapped cylinders. In his solution, he neglected the temperature gradient in the radial direction, assuming negligible thickness of the added layer.

Figure 1.1: Different Types of Cylindrical Multilayer Pressure Vessels

Figure 1.2: Longitudinal Weld in Multilayer Pressure Vessel (not to scale)

CHAPTER II

ANALYSIS PROCEDURES

2.1 Introduction

Almost all physical phenomenon in nature can be described in terms of differential equations. In cases where both the given equation and the given domain are simple, the solution can be obtained in closed form, often in the form of infinite series. Efforts to develop new methods of exact solution to general partial differential equations are foiled by the irregular and geometrically complicated domains and the geometric and material nonlinearities of most practical problems. Consequently, various methods of finding suitable approximate solutions have been under continuous development since the last century. Among the methods of approximation, the finite difference method and variational methods have dominated the applications to problems in engineering. In recent decades, the finite element method, an off-shoot of classical variational method, has gained considerable popularity among practicing engineers as well as applied mathematicians.

The finite difference method is endowed with the simplicity of principle, and ease of formulation and implementation on a digital computer. The disadvantages of the method include the difficulty in implementing the boundary conditions on irregular boundaries, and the difficulty in accurately representing irregular domains.

The classical variational methods are based either on the minimization of a quadratic functional associated with the given problem or on the minimization of

the error in the approximation. Despite the simplicity and accuracy, both in the solution and its derivatives, the classical variational methods were not regarded as competitive with finite difference method. This was mainly due to the difficulty in constructing approximation functions for irregular domains. The finite element method overcomes the shortcoming of the classical variational methods by seeking the approximate solution on a collection of simple subdomains (elements) on which the approximation functions can be generated systematically.

2.2 The Finite Element Method

The finite element method is a variational procedure in which the approximating functions are systematically derived by representing the given domain as a collection of simple subdomains. The method differs in two ways from traditional variational methods in generating the algebraic equations of the problem. First, the approximating functions are often algebraic polynomials that are developed using ideas from the interpolation theory; second, the approximating functions are developed for subdomains into which a given domain is divided. The subdomains, or finite elements, are geometrically simple shapes that permit a systematic construction of the approximating functions. This representation is necessitated by two reasons: First, the derivation of the approximation functions using algebraic interpolation is possible only for certain well-defined (or simple) geometries, such as lines, triangles, rectangles, cubes, etc.; second, since the approximating functions are defined element-wise, the accuracy of the approximation can be improved by increasing the number of elements, i.e., refining the mesh.

In the finite element method, the approximating functions are conventionally algebraic polynomials, which facilitate accurate numerical evaluation of the coefficient matrices on a digital computer. The minimum degree of the polynomial used for approximation functions depends on the order of the differential equation being solved, and the degree of polynomial in turn dictates the number of interpolation points (nodes) to be identified in the element.

The approximation functions are also called interpolation functions because they are derived by interpolating the function and possibly its derivatives at the nodes of the elements. It should be realized that there are two entities to be approximated. The first is physical (the state variable) and the second is geometrical (the shape of the element). If the element is actually made of straight lines or planes, the coordinates of the primary nodes (those at the extremes of the elements) will define the element shape accurately. In this case, the geometric approximation does not enter into the picture. Because of this, discretization of the entire domain is most often made by straight line (linear) elements. For some problems, however, linear problems may introduce unacceptable errors, and discretization must be done by using isoparametric elements. A similar argument is of course, valid for the approximation of the state variable. It can be approximated in the form of a linear function or a higher function (i.e., quadratic, cubic, etc.).

Distinguishing features of isoparametric elements include their ability to have curved sides and their use of an intrinsic coordinate system for formulating the element stiffness matrix. The term "isoparametric," meaning same parameters, follows from the use of the same interpolation scheme to define both the geometry

and the displacement field of an element. Formulation of the problem using isoparametric elements is easier compared to non-isoparametric elements.

All elements contain boundary nodes which define their geometry. These boundary nodes connects the element to its neighbouring elements. The connectivity of the elements is accomplished by requiring that at a node common to adjacent elements, the nodal values from the elements are the same.

In a sense, finite elements are pieces of actual structure. The quadrilateral and triangular regions shown in Figure 2.1 are finite elements. Black dots are nodes where elements are connected to one another.

Figure 2.1 shows the essence of finite element method: piecewise approximation of a function ϕ , by means of polynomials, each defined over a small region (element) and expressed in nodal values of the function. Versatility is an outstanding feature of finite elements. The method can be applied to various problems. The region under analysis can have arbitrary shape, loads, and boundary conditions. The mesh can mix elements of different types, shapes and physical properties, exploiting each one's advantages. This great versatility often is contained within a single computer program: user-prepared input data control the selection of problem type, geometry, boundary conditions, element selection, and so on. Another feature of finite elements is the close resemblance between the mesh and the actual structure.

The finite element method also has disadvantages. A specific numerical result is obtained for a specific problem: there is no closed-form expression that permits analytical study of the effects of changing various parameters. A computer and a reliable program are essential. Experience and good engineering sense are needed

to construct a good mesh. Many input data are required, and voluminous output data must be sorted and understood.

If a stress or displacement field satisfies the equilibrium, compatibility and boundary conditions, then a solution has been found. If elements are based on displacement fields, so that the nodal displacements are the primary unknowns, then the compatibility condition is satisfied within elements. The principle of virtual work is the “weak form” of the equilibrium equations and is used as the basic equilibrium statement for the finite element formulation. Its advantage in this regard is that it is a statement of equilibrium cast in the form of an integral over the volume of the body, thus introducing approximations that are not entirely arbitrary, but whose variation is restricted to a finite number of nodal values. This approach provides a strong mathematical basis for studying the approximation, than the alternative of the direct discretization of the derivative in the differential equation of equilibrium at a point, which is the typical starting point for the finite difference approach to the same problem. Equilibrium equations and boundary conditions on stress are satisfied only approximately. The approximation improves as more degrees of freedom are used, barring computational difficulties, the exact solution is achieved in the limit of an infinitely refined mesh.

2.3 Problem Statement

The purpose of this research was to predict the residual stresses induced in the core of multilayer vessels due to the circumferential shrinkage of layers upon cooling of longitudinal welds. In order to compare the finite element results

with the analytical results, the following finite element model, similar to the case study by Rasty [1988] was utilized. The core cylinder, with an inside radius of 0.762 m (30.0 in.) and a thickness of 12.7 mm (0.5 in.), was assumed to be stress-free before the addition of first layer. A 6.4 mm (0.25 in.) thick layer of sheet steel was assumed to have been wrapped around the core and then welded along a longitudinal seam. The width of the weld was taken as 7.6 mm (0.3 in.). Initially, the weld was assumed to be at a temperature of 1132 °C(2070 °F) and the rest of the layer plate to be at 21 °C(70 °F). The added layer plate was also assumed to be stress-free prior to cooling of the weld. The weld was then allowed to cool by convective cooling, which was allowed to continue until the core, the welded layer and the weld were at room temperature. Due to symmetry of the problem, only one-half of the cross-section of the cylinder was modelled. All the related data were taken from the case study presented by Rasty [1988]. The material used in these simulations was AISI-1010 carbon steel ($Mn \leq 1\%$, $Si \leq 0.1\%$), with the following thermal and mechanical properties:

Density (ρ)	7832 kg/m ³ (0.28294 lb/in ³)
Specific heat (c)	434 J/kg °K (0.1036 Btu/lb °F)
Thermal Conductivity (K)	63.9 W/m °K (8.54E-4 Btu/in.sec.°F)
Coef. of Thermal expansion (α)	12.51mm/mm°C (6.95E-6 in/in°F)
Modulus of Elasticity (E)	207 GPa (30E6 psi)
Poissons Ratio (ν)	0.3
Tensile Strength	462 MPa (67E3 psi)
Yield Strength	379 MPa (55E3 psi)

2.4 Problem Analysis

The boundary value problem under consideration combines the theories of elasticity and of heat conduction under transient conditions. For a body with density of ρ , specific heat of c , conductivity of K , subjected to a temperature of T and a cooling rate of $\dot{T}$, the linear coupled thermoelastic energy equation can be written as Boley and Weiner [1960],

$$KT_{,ii} = \rho c \dot{T} + m T_o \dot{\epsilon}_{kk} \quad (2.1)$$

where

$$m = (3\lambda + 2\mu)\alpha. \quad (2.2)$$

Equations of motion in an elastic body are given by

$$\sigma_{ij,j} = \rho \ddot{u}_i. \quad (2.3)$$

while the thermoelastic stress-strain relationships are given by

$$\sigma_{ij,j} = \delta_{ij} \lambda \epsilon_{kk} + 2\mu \epsilon_{ij} - \delta_{ij} m (T - T_o) \quad (2.4)$$

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad (2.5)$$

where

$$\lambda = \frac{\nu E}{(1 + \nu)(1 - 2\nu)} \quad (2.6)$$

and

$$\mu = \frac{E}{2(1 + \nu)}. \quad (2.7)$$

The external load produces variations of strain within a body. The linear-coupled thermoelastic equation (2.1) shows that these variations of strain are in

general accompanied by variations in temperature and consequently by a flow of heat; the whole process thus gives rise to an increase of entropy and therefore to an increase in the energy stored in mechanically irrecoverable manner. This phenomenon, known as thermoelastic dissipation requires for its study, the use of coupled heat equation (2.1); thus rendering the mechanical term in the heat equation essential to the description of the dissipative process. However, the deformations due to external loads are accompanied only by small changes in temperature, and it would therefore appear reasonable to calculate these deformations without taking into account of the thermal expansion. Similarly, if strains are produced in a body by a non-uniform temperature distribution, it would seem intuitively clear that the influence of these strains on the temperature itself should not be too large. One may, therefore, disregard the coupling term appearing in the coupled thermoelastic equation (2.1)

This matter may be made plausible by the following reasoning. Neglecting inertia terms in the equations of motion (2.3), the coupled thermoelastic equation can be written as

$$KT_{,ii} = \rho c \dot{T} \left[1 + \delta \left(\frac{\lambda + 2\mu}{3\lambda + 2\mu} \right) \left(\frac{\dot{\epsilon}_{kk}}{\alpha \dot{T}} \right) \right] \quad (2.8)$$

where the non-dimensional parameter δ is defined by

$$\delta = \frac{(3\lambda + 2\mu)^2 \alpha^2 T_0}{\rho^2 c v^2}. \quad (2.9)$$

with the velocity of propagation of dilatational waves in an elastic medium being denoted by

$$v = \sqrt{\frac{(\lambda + 2\mu)}{\rho}}. \quad (2.10)$$

The term proportional to δ in equation (2.8) is the coupling term, and it is negligible compared to unity if

$$\frac{\dot{\epsilon}_{kk}}{3\alpha\dot{T}} \ll \left(\frac{\lambda + 2\mu/3}{\lambda + 2\mu} \right) \frac{1}{\delta}. \quad (2.11)$$

Here

$$\lambda = 119.4 \text{ GPa (17.3E6 psi);}$$

$$\mu = 79.6 \text{ GPa (11.5E6 psi);}$$

$$c = 434 \text{ J/kg}^\circ\text{K (0.1036 Btu/lb}^\circ\text{F);}$$

$$\rho = 7832 \text{ kg/m}^3 \text{ (0.2829 lb/in}^3\text{);}$$

$$\alpha = 12.51\text{E-6 mm/mm}^\circ\text{C (6.95E-6 in/in}^\circ\text{F) .}$$

From equation (2.10), v can be calculated as

$$v = 5964 \text{ m/s (19562 ft/sec).}$$

From equation (2.9), at $T_o = 1093^\circ\text{C (2000}^\circ\text{F)}$, δ can be calculated as

$$\delta = \frac{(3 \times 119.4\text{E9} + 2 \times 79.6\text{E9})^2 \times (12.51\text{E-6} - 6)^2 \times 1093}{7832^2 \times 434 \times 5964^2} = 0.0483.$$

Thus the coupling term will be small if,

$$\frac{\dot{\epsilon}_{kk}}{3\alpha\dot{T}} \ll \frac{1}{\delta}. \quad (2.12)$$

The preceding discussion makes it clear that the possibility of omitting the coupling term depends not only on the fact that the inequality

$$\delta \ll 1 \quad (2.13)$$

must hold (as it does for most metals), but also on the fact that strain rates must be of the same order of magnitude as temperature rates. The latter condition implies that the time history of the displacement closely follows that of the temperature. For temperature distributions with no sharp variations or discontinuities in their time histories, it is intuitively expected that the time rate of change of dilatation is of the same order of magnitude as that of the temperature; thus the disregard of coupling term, as described previously, appears to be reasonable.

With the omission of coupling, the general thermal stress problem separates into two distinct problems to be solved consecutively. The first is the boundary value heat conduction problem and the second is the determination of the resulting stress distribution.

In the ensuing analysis, the following assumptions were made:

1. Plain strain condition.
2. Both the core and the layer around it are free of stress prior to the cooling of the weld.
3. At time $t=0$, only the welded segment of the layer is assumed to be at 1132°C (2070°F) and the rest of the layer, as well as the core are at 21°C (70°F).
4. Welding is completed in one pass only.
5. The heat conduction due to welding does not go ahead of the weld arc. In other words, welding is fast enough. This is to ensure only two-dimensional heat flow.
6. Elements used to define the model are fully isoparametric, i.e., the coordinate interpolation is the same as displacement interpolation.

7. Material was assumed to be homogeneous, isotropic and linear elastic.
8. Material properties such as conductivity, coefficient of thermal expansion, Young's modulus and specific heat are constant and do not vary with temperature.
9. Heat transfer coefficient is constant.
10. Calculations are representative of the middle portion of the vessel and do not reflect closure or other discontinuity effects.

2.5 The Finite Element Model

The present problem involves finding an approximate solution for the temperature, displacements, deformations and stresses in a solid body that is subjected to some history of loading. The displacement finite element method is based on approximating the previously stated equilibrium requirement by replacing it with a weaker requirement, that equilibrium must be maintained in an average sense over a finite number of divisions of the volume of the body.

ABAQUS, a general purpose finite element code, was employed for the modelling purposes. Due to the symmetry of the problem, only one-half of the layer's cross-section was modelled, utilizing 2035 nodes and 623 elements (Figure 2.2). Such large number of elements were needed to maintain the correct aspect ratio of the quadratic, eight noded elements, required for modelling the core and the layer. The thickness of the core was modelled to be twice thicker than the layer above it. The finite element mesh in the weld region was refined to accommodate for the large gradients. The different stages of mesh refinement are shown in Figure 2.3. First stage shows the elements of equal length generated over the circumference. For the elements generated in this way the length was 0.6 in.,

which is three times the length of one-half of the weld element. Thus in the second stage, elements in the region of 5° near the weld were generated with decreasing length towards the weld. Having achieved the exact length of the weld element, in the third stage the weld element was divided into three layers using trapezoidal shape of the quadrilateral elements. One-dimensional interface elements were used between the core and the layer.

The analysis consisted of transient heat transfer analysis followed by a thermal stress analysis, in which the temperatures predicted by the heat transfer analysis were used as the loading for the problem. ABAQUS provides an output option in the heat transfer analysis to write the nodal temperatures to a file. Then in the stress analysis, another option is used to read this file of nodal temperatures back into the thermal stress analysis model. This mode of transferring the temperatures is based on node numbers -the temperature at node N on the output from the heat transfer analysis is applied at node N in the stress analysis. Thus, the node numbers must remain the same from the heat transfer model to the stress model. Total degrees of freedom was 2035 for the heat transfer analysis and 4378 for the thermal stress analysis.

The following boundary conditions were applied to the heat transfer analysis as shown in Figure 2.4.

(a) Because of symmetry, the heat flux in circumferential direction at $\theta = 0$ and $\theta = \pi$ vanishes; i.e.,

$$\frac{\partial T}{\partial \theta} = 0, \text{ at } \theta = 0, \theta = \pi, t > 0 \quad (2.14)$$

where $\frac{\partial}{\partial \theta}$ denotes differentiation in the direction of the outward normal to the surface.

(b) There is heat loss from the surface of the cylinder to the outside by convection; i.e.,

$$K \frac{\partial T}{\partial r} - h(\Delta T) = 0, \text{ at } r = a, r = b, t > 0. \quad (2.15)$$

The value of h , the heat transfer coefficient, was calculated by the simplified formula used for natural convection, Benham and Hoyle [1965]

$$h = 0.27(\Delta T)^{(0.25)} \text{ Btu/ft}^2\text{hrF} \quad (2.16)$$

where $\Delta T(^{\circ}\text{F})$ is the temperature difference between the surface of the body and the bulk of the gas surrounding it. With $\Delta T = 1093^{\circ}\text{C}$ (2000°F), for this model, h for this problem, from the above formula comes out to be, $10.52 \text{ W/m}^2\text{K}$ ($3.483\text{E-}06 \text{ Btu/in}^2\text{.sec.}^{\circ}\text{F}$).

(c) The layer coming in contact with the core cylinder introduces thermal resistance (R). At the interface of the layer and the core cylinder, the thermal resistance introduces the following condition for the heat flux being equal to the temperature difference across the surface; i.e.,

$$K \frac{\partial T}{\partial r} = \frac{1}{R}(\Delta T), \quad t > 0. \quad (2.17)$$

The value of this resistance (or interface heat conductance) for a tightly wrapped vessel was taken from Armstrong and Jawad [1981], to be $850 \text{ W/m}^2\text{K}$ ($2.8935\text{E-}04 \text{ Btu/in}^2 \text{ sec.}^{\circ}\text{F}$).

Boundary conditions for the thermal stress analysis are shown in Figure 2.5. Appendix B shows a typical input file for the heat transfer analysis. DELTMX parameter on the *HEAT TRANSFER option limits the maximum temperature

change that may occur in an increment, and thus determines the accuracy with which the transient temperature solution is integrated in time. It also implies the use of automatic time incrementation, which is very desirable, as we wish to carry the analysis through to steady state conditions, so that large increments are needed towards the end of the solution. Since the solution is to reach steady state, the time period specification has to be long enough to let it reach steady state. The parameter END=SS is used for this purpose. This indicates that the analysis should terminate when steady state conditions are reached. Steady state conditions are defined for the purpose of this parameter by the time rate of change of temperature at all nodes falling below the value given on the data card (0.1). The step terminates either when the steady state conditions have been reached or when the time period specified for the step has been completed, whichever comes first, so that a very large time period is generally used. As per the recommendations of the ABAQUS User's Manual, a minimum time increment was specified, to cover the possibility that a data error or unforeseen event in the solution causes the automatic time increment scheme to choose very small time increments, and eventually lead to infinite computer time usage.

The temperature-time history of the thermal analysis was the loading for the thermal stress model. Coefficient of friction at the interface was varied for different cases from 0.01 to 0.1 to study the effects of friction. The stress analysis model consisted of plane strain elements of the type CGPE10R, ten noded biquadratic quadrilateral with reduced integration. Generalized plane strain theory provides for the modeling of cases where the structure has constant curvature (and hence no gradients of solution variables) with respect to one material

direction - the “axial” direction of the model. The theory thus involves a model that lies between two planes, each orthogonal to the axial material direction of the model (Figure 2.6). These planes may move as rigid bodies with respect to each other and so cause strain in axial direction of the model which varies linearly with respect to position in the planes, the variation being due to the change in curvature in the axial direction. In the initial configuration, the bounding planes may be parallel or at an angle to each other. Each generalized plane strain element has eight conventional nodes. At each of these nodes, x and y coordinates, displacements, etc. are stored. These nodes determine the position and motion of the element in two bounding planes. Each element also has two extra nodes, which are usually the same nodes in all the generalized plane strain elements in the model. The first of these extra nodes has one degree of freedom, which is the change in length of the axial material fiber connecting this node and its image in the other bounding plane. This displacement is positive as the planes move apart, so that there is a tensile axial strain in the fiber. The second of these extra nodes has two degrees of freedom, which are the components of the relative rotation of one bounding plane with respect to the other. A typical input file for thermal stress analysis problem is shown in Appendix C.

ABAQUS provides three basic types of elements for modeling contact between deformable objects:

1. Gap Elements,
2. Interface Elements, and
3. Slideline Elements.

Gap Elements are used to describe situations in which discrete points on opposing surfaces likely to come into contact can be readily identified. Gap Elements, placed between pair of nodes on the two surfaces, allow them to contact or separate, as determined by their relative displacements in a specified direction, usually normal to the surfaces.

Interface Elements are intended for analyses describing contact over parts of surfaces where relative tangential or sliding motion may occur. They contain faces on each of the surfaces and use normal relative displacements of opposing faces to measure surface separation.

Slideline Elements may be used to describe contact problems in which large sliding motions, and/or arbitrary rotations, of contacting surfaces may occur. Sets of nodes on opposing surfaces are identified and their displacements tracked to determine when contact occurs at any point on the surfaces.

Gap elements were not found suitable for this problem, as a continuous contact was anticipated, throughout the loading history. Moreover, the nodes used to define these elements were coincident nodes. Slideline elements were ruled out as per ABAQUS User's Manual, as they are used for large relative displacements.

Interface elements were used for modelling the interface conditions at the core and the layer. One-dimensional interface elements, DINTER3, for use with two-dimensional heat transfer elements were used for heat transfer analysis and One-dimensional interface elements, INTER3, for use with two-dimensional plane strain elements were used for stress analysis. Figure 2.2 shows the finite element model, with an enlarged view of the weld region. The shaded elements represent the weld.

Figure 2.1: A Continuous Function $\phi = \phi(x, y)$ and Typical Elements that might be used to Approximate it

Figure 2.2: The Finite Element Model (Inset: weld region)

Figure 2.3: Different Stages of Mesh Refinement

Figure 2.4: Boundary Conditions on Heat Transfer Model (not to scale)

Figure 2.5: Boundary Conditions on Static Model (not to scale)

Figure 2.6: Bounding Planes and the Nodes used for describing the Generalized Plane Strain Elements

CHAPTER III

RESULTS AND DISCUSSION

Weld shrinkage is affected by such variables as speed of welding, type of welding process, and the amount of heat input. Due to localized longitudinal welding, there exists a temperature gradient in the circumferential direction of a layer. The analytical results of Appendix A do not take into account the effect of temperature gradient in the cylinder. A comparison is made of the radial and circumferential stresses and displacements, obtained by the finite element method and those obtained analytically. The analytical results are obtained from the equations presented in Appendix A. For the analytical results $u_r = u_r(r)$. But the finite element method results take into consideration, the fact that, $u_r = u_r(r, \theta)$.

3.1 Displacement Distribution

Figure 3.1 shows the displaced plot of the thermal stress analysis. Since the order of magnitude of the displacements is small, an inset is shown in Figure 3.1 in which the displacements are magnified 10 times for the purpose of clarity. It can be seen that the layer elements moved towards the weld element as it shrank.

Figure 3.2 shows the distribution of the tangential displacement around the circumference of the layer. The tangential displacement obtained from analytical solution is also plotted for comparison with the finite element results. Figure 3.2 indicates that the maximum displacement occurs at the weld location ($\theta = 0$) and exponentially decreases up to $\theta = 45^\circ$. Beyond $\theta = 45^\circ$, the tangential

displacement decreases almost linearly to a zero value at $\theta = 180^\circ$. This explains the higher magnitude of tangential stress near the weld area. It is interesting to note that the maximum value of the tangential displacement obtained from the finite element method is the same as that obtained from the analytical solution, but sharply deviates from the analytical solution for $\theta > 0$.

3.2 Stress Distributions

Figure 3.3, shows the radial distribution of the tangential prestress due to cooling of the weld, averaged over the circumference of the model. In this figure, 0% wall thickness refers to the vessel's inner surface and 100% wall thickness refers to its outer surface. Since the thickness of the core was twice that of the layer, 0-67% of wall thickness shows the stress in the core and 67-100% of wall thickness shows the stress in the welded layer. As shown in Figure 3.3, the tangential prestress in the core is purely compressive, balanced by tensile prestress in the welded layer. In order for the equilibrium of forces in the tangential direction to be satisfied, the areas under the compressive and tensile portions of Figure 3.3 must be equal ($\sum F_\theta = 0$). For the finite element analysis, these areas are -151 MPa (calculated per unit length and unit thickness of the vessel) and 151 MPa, respectively. The analytical equations of Appendix A also show the core to be in pure compression and the welded layer to be in pure tension. The areas under the compressive part of the analytical stress distribution is -100 MPa, while that of tensile region is 100 MPa. Figure (3.3) shows that the magnitude of the tangential stress distribution, obtained from finite element method, is consistently 50% larger than the magnitude of the stresses obtained from analytical solutions.

This could be partly attributed to the simplification of the problem in analytical solution, which assumes that the tangential and radial displacements are uniformly distributed around the circumference of the layer. Boudjelida [1983] in his analytical treatment of the effect of temperature distribution during the welding process on the prestress, showed an improvement of 38% of the total shrinkage over the conventional formula used in Appendix A. However, it should be noted that he neglected temperature gradient in the radial direction.

Figure 3.4, shows the circumferential distribution of the tangential stress at the outer surface of the vessel. Analytical results show a constant value of 3.07 MPa (433 psi), while finite element method results decrease from a maximum of 5.1 MPa (735 psi) at the weld ($\theta = 0$) to 4.46 MPa (648 psi) at $\theta = 180^\circ$. This corresponds to a 12.5% variation in the circumferential distribution of the tangential prestress at the outer surface.

The circumferential distribution of the tangential stress at the inner surface of the vessel is shown in Figure 3.5. Analytical results show a constant value of -1.51 MPa (-219.3 psi), while finite element method results varied from -2.52 MPa (-366 psi) at the weld ($\theta = 0$) to -2.18 MPa (-316 psi) at $\theta = 180^\circ$. This corresponds to a 13.5% variation in the circumferential distribution of tangential prestress at the inner surface. Upon cooling, the welded layer begins to shrink in the tangential direction. The analytical solutions assume that the tangential displacement of the layer is constant around the circumference.

Assuming that the finite element thermal analysis resulting in the tangential displacement profile of Figure 3.2 are correct, these displacements were substituted for the $W\alpha T$ term in the analytical equations of Appendix A, to yield an

analytical stress distribution based on the tangential displacement distribution obtained via the finite element method. This is shown by the dashed line in Figure 3.6 for comparison with the finite element results. All the above finite element results are for a friction factor of 0.01 between the layer and the core.

3.3 Effects of Friction

The results of different friction coefficients between the layer and the core cylinder are shown in Figures 3.7 and 3.8. Figure 3.7 shows the radial distribution of the tangential prestress, averaged over the circumference of the model, for friction coefficient of 0.0 and 0.1. Similar results were obtained for the two friction conditions.

The circumferential distribution of the tangential prestress at the inner surface of the vessel, with different coefficients of friction, is shown in Figure 3.8. The frictionless stress distribution shows a constant value of -2.39 MPa for the tangential prestress. As the friction increases, the stress distribution deviates from the constant value and for friction coefficient = 0.1, it varies from -2.00 MPa at $\theta = 0$ to -2.78 MPa at $\theta = 180^\circ$.

The discrepancy between the analytical and the finite element results can be attributed to parameters which were not accounted for in the analytical solution, such as friction between the core and the layer, contact resistance, convective cooling of the layer and also on the fact that temperature distribution is not symmetric.

In general, complex phenomena require mathematical models which are also complex. Therefore, all but the simplest and most trivial cases must be dealt

with by computational methods. The advent of extremely large and high speed computational devices is both an asset and a liability; an asset, since we can now consider more sophisticated and realistic models of the phenomena we wish to study, but also a liability since we are simultaneously faced with the responsibility of determining how well the computed solutions depict the behavior our model was designed to simulate. Too frequently this very real responsibility is swept aside and one mistakingly interprets the computed results as a simulation of the physical event, and not as what they are- only arrays of integers which may or may not approximate the behavior simulated by our model.

Models, on the other hand, are after all only schemes for simulating physics, so when we are confident that the simulations are mathematically sound, we must ultimately compare them with actual physical observations. If they prove to be inadequate to depict the observed phenomena, they must be abandoned, or expanded, improved and modified. Thus to evaluate the model, we must have confidence in its numerical approximation; we must have some guarantee of the reliability of the computed approximations of the model before we can adequately assess its adequacy as a simulation of actual events.

We cannot presume that a structure is obliged to behave as a computer says it should, no matter how expensive the program, how elegant the graphic display, or how many digits are printed out. Computation assists engineering judgment but must not replace it.

Figure 3.1: Results of the Analysis showing the Layer Shrinkage due to Cooling of Weld (Inset: displacement magnified 10 times)

Figure 3.2: Circumferential Distribution of the Tangential Displacement of the Outer Surface

Figure 3.3: Radial Distribution of the Tangential Prestress due to Cooling of Weld (FEM results averaged over the circumference of the vessel)

Figure 3.4: Circumferential Distribution of the Tangential Prestress at the Outer Surface

Figure 3.5: Circumferential Distribution of the Tangential Prestress at the Inner Surface

Figure 3.6: Circumferential Distribution of the Tangential Prestress Obtained from Tangential Displacements at the Outer Surface of the Finite Element Model

Figure 3.7: Radial Distribution of the Tangential Prestress for Different Friction Coefficients (results averaged over the circumference of the vessel)

Figure 3.8: Circumferential Distribution of the Tangential Prestress at the Inner Surface for Different Friction Coefficients

CHAPTER IV

CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

During the cooling of a welded layer, the core experiences compressive tangential stress while the layer itself is subjected to tensile stress. Unlike the analytical solutions, finite element results indicate that the tangential prestress is maximum at the weld ($\theta = 0$) and decreases by as much as 13.5% at $\theta = 180^\circ$. It should be noted that this paper presents the prestress distribution due to wrapping and welding of one layer only. Since the actual number of layers for a multilayer vessel could range between 10-30 layers, higher prestress magnitudes and higher circumferential variations are expected in multilayer vessels. Accurate prediction of stresses in a multilayer vessel requires a knowledge of such variations. Analytical solutions should be developed to account for the effect of friction, contact resistance between the layers and convective cooling of the layer.

The dependence of the prestress on the angular position of the weld necessitates the staggering of the welds in the successive layers of the fully built up vessels. In practice, the angle between the two successive welds is usually 30° .

4.2 Recommendations

The analysis was performed, assuming that boundary conditions and interface conditions were not a function of time. Thus, in the uncoupled quasi-static thermoelastic analysis with the boundary conditions not involving any derivative with respect to time, time simply plays the role of a parameter in the solution of

thermoelastic problem. But it is realized that boundary conditions are a function of time and hence certain recommendations for the improvement of the model are listed below.

1. The heat transfer process could be made more realistic, if we can prescribe a local heat transfer coefficient near the weld on the outer surface which varies with the temperature difference of the weld and the environment.
2. As the weld cools down, the tightness of the outer layer on the core cylinder increases. Thus the model could be made more realistic if the gap conductance is made to vary over time, with a low value initially and gradually increasing to the maximum towards the end of cooling.
3. Variation of physical and thermal properties of the core and layer material with temperature will simulate real conditions.

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APPENDIX A

CALCULATION OF WRAPPING PRESTRESS IN MULTILAYER CYLINDRICAL PRESSURE VESSEL ASSUMING AXISYMMETRIC CASE.

The total reduction in the radius due to weld shrinkage is

$$\Delta = \frac{W\alpha T}{2\pi}. \quad (4.1)$$

At the interface of the welded layer and the preceding layer,

$$u_1 + u_2 = \Delta = \frac{W\alpha T}{2\pi}. \quad (4.2)$$

The stresses in the last welded layer, (n th) can be calculated from the following equations, Boresi and Sidebottom [1985],

$$\sigma_\theta, \sigma_r = \frac{P_n r_n^2}{r_{n+1}^2 - r_n^2} \left[1 \pm \left(\frac{r_{n+1}}{r} \right)^2 \right]. \quad (4.3)$$

Similarly, the stresses in the layers inside the n th layer are

$$\sigma_\theta, \sigma_r = -\frac{P_n r_n^2}{r_n^2 - r_i^2} \left[1 \pm \left(\frac{r_i}{r} \right)^2 \right]. \quad (4.4)$$

Knowing that, for axisymmetric stress distribution

$$u_r = \frac{r}{E}(\sigma_\theta - \nu\sigma_r). \quad (4.5)$$

Evaluating u_1 and u_2 from from equations (4.3), (4.4), and (4.5), and then substituting back into equation (4.2) yields

$$\frac{r_n P_n}{E} \left[\frac{r_n^2 + r_i^2}{r_n^2 - r_i^2} - \nu \right] + \frac{r_n P_n}{E} \left[\frac{r_{n+1}^2 + r_n^2}{r_{n+1}^2 - r_n^2} + \nu \right] = \frac{W\alpha T}{2\pi}. \quad (4.6)$$

Solving for P_n , one obtains

$$P_n = \frac{W\alpha T E}{2\pi r_n \left(\frac{r_n^2 + r_i^2}{r_n^2 - r_i^2} + \frac{r_{n+1}^2 + r_n^2}{r_{n+1}^2 - r_n^2} \right)}. \quad (4.7)$$

Solving for P_n , one obtains

$$P_n = \frac{W\alpha TE}{2\pi r_n \left(\frac{r_n^2 + r_i^2}{r_n^2 - r_i^2} + \frac{r_{n+1}^2 + r_n^2}{r_{n+1}^2 - r_n^2} \right)}. \quad (4.7)$$

The stresses in any layer, j th, before welding of layers $(j+1)$ through n , can be obtained by direct substitution of P_n from (4.7) into (4.3)

$$\sigma_{\theta_j}, \sigma_{r_j} = \frac{W_j \alpha T E r_j \left[1 \pm \left(\frac{r_{j+1}}{r} \right)^2 \right]}{2\pi \left(\frac{r_j^2 + r_i^2}{r_j^2 - r_i^2} + \frac{r_{j+1}^2 + r_j^2}{r_{j+1}^2 - r_j^2} \right) (r_{j+1}^2 - r_j^2)}. \quad (4.8)$$

From equation (4.4), the stresses in the j th layer, due to wrapping of layers $(j+1)$ through n are

$$\sigma_{\theta_j}, \sigma_{r_j} = - \left[1 \pm \left(\frac{r_i}{r} \right)^2 \right] \sum_{k=j+1}^n \frac{P_k r_k^2}{r_k^2 - r_i^2}. \quad (4.9)$$

Substitution for R_k from equation (4.7) into equation (4.9) yields

$$\sigma_{\theta}, \sigma_r = - \left[1 \pm \left(\frac{r_i}{r} \right)^2 \right] \frac{\alpha T E}{2\pi} \times \sum_{k=j+1}^n \frac{W_k r_k}{\left(\frac{r_k^2 + r_i^2}{r_k^2 - r_i^2} + \frac{r_{k+1}^2 + r_k^2}{r_{k+1}^2 - r_k^2} \right) (r_k^2 - r_i^2)}. \quad (4.10)$$

Thus the total stress in the j th layer, are

$$\begin{aligned} \sigma_{\theta}, \sigma_r = & - \left[1 \pm \left(\frac{r_i}{r} \right)^2 \right] \frac{\alpha T E}{2\pi} \times \sum_{k=j+1}^n \frac{W_k r_k}{\left(\frac{r_k^2 + r_i^2}{r_k^2 - r_i^2} + \frac{r_{k+1}^2 + r_k^2}{r_{k+1}^2 - r_k^2} \right) (r_k^2 - r_i^2)} \\ & + \frac{W_j \alpha T E r_j \left[1 \pm \left(\frac{r_{j+1}}{r} \right)^2 \right]}{2\pi \left(\frac{r_j^2 + r_i^2}{r_j^2 - r_i^2} + \frac{r_{j+1}^2 + r_j^2}{r_{j+1}^2 - r_j^2} \right) (r_{j+1}^2 - r_j^2)}. \end{aligned} \quad (4.11)$$

The largest compressive prestress would be in the core (innermost layer) and can be calculated from

$$\sigma_{\theta}, \sigma_r = - \left[1 \pm \left(\frac{r_i}{r} \right)^2 \right] \frac{\alpha T E}{2\pi} \times \sum_{k=j+1}^n \frac{W_k r_k}{\left(\frac{r_k^2 + r_i^2}{r_k^2 - r_i^2} + \frac{r_{k+1}^2 + r_k^2}{r_{k+1}^2 - r_k^2} \right) (r_k^2 - r_i^2)}. \quad (4.12)$$

The largest tensile prestress is at the outer layer and is given by

$$\sigma_{\theta}, \sigma_r = \frac{W_j \alpha T E r_j \left[1 \pm \left(\frac{r_{j+1}}{r} \right)^2 \right]}{2\pi \left(\frac{r_j^2 + r_i^2}{r_j^2 - r_i^2} + \frac{r_{j+1}^2 + r_j^2}{r_{j+1}^2 - r_j^2} \right) (r_{j+1}^2 - r_j^2)}. \quad (4.13)$$

APPENDIX B
INPUT FILE FOR HEAT TRANSFER ANALYSIS

***HEADING**

HEAT TRANSFER MODEL WITH APPROPRIATE BOUNDARY CONDITIONS

**** NODE DEFINITIONS AND NODE GENERATION ****

***NODE**

1,0.,-30.0

4001,0.,-30.5

40001,0.,-30.5

6001,0.,-30.75

150,30.0,0.

4150,30.5,0.

40150,30.5,0.

6150,30.75,0.

305,0.,30.0

4305,0.,30.5

40305,0.,30.5

6305,0.,30.75

***NODE,SYSTEM=C**

291,30.0,85.16778525

1291,30.125,85.16778525

2291,30.25,85.16778525

3291,30.375,85.16778525

4291,30.5,85.16778525

40291,30.5,85.16778525

5291,30.625,85.16778525

6291,30.75,85.16778525

292,30.0,85.76527325

1292,30.125,85.76527325

2292,30.25,85.76527325

3292,30.375,85.76527325

4292,30.5,85.76527325

40292,30.5,85.76527325

5292,30.625,85.76527325

6292,30.75,85.76527325

293,30.0,86.30804237

1293,30.125,86.30804237

2293,30.25,86.30804237

3293,30.375,86.30804237

4293,30.5,86.30804237

3293,30.375,86.30804237
4293,30.5,86.30804237
40293,30.5,86.30804237
5293,30.625,86.30804237
6293,30.75,86.30804237

294,30.0,86.80117982
1294,30.125,86.80117982
2294,30.25,86.80117982
3294,30.375,86.80117982
4294,30.5,86.80117982
40294,30.5,86.80117982
5294,30.625,86.80117982
6294,30.75,86.80117982

295,30.0,87.24927986
1295,30.125,87.24927986
2295,30.25,87.24927986
3295,30.375,87.24927986
4295,30.5,87.24927986
40295,30.5,87.24927986
5295,30.625,87.24927986
6295,30.75,87.24927986

296,30.0,87.65649676
1296,30.125,87.65649676
2296,30.25,87.65649676
3296,30.375,87.65649676
4296,30.5,87.65649676
40296,30.5,87.65649676
5296,30.625,87.65649676
6296,30.75,87.65649676

297,30.0,88.02659063
1297,30.125,88.02659063
2297,30.25,88.02659063
3297,30.375,88.02659063
4297,30.5,88.02659063
40297,30.5,88.02659063
5297,30.625,88.02659063
6297,30.75,88.02659063

298,30.0,88.36296774
1298,30.125,88.36296774
2298,30.25,88.36296774
3298,30.375,88.36296774
4298,30.5,88.36296774
40298,30.5,88.36296774

5298,30.625,88.36296774
6298,30.75,88.36296774

299,30.0,88.66871584
1299,30.125,88.66871584
2299,30.25,88.66871584
3299,30.375,88.66871584
4299,30.5,88.66871584
40299,30.5,88.66871584
5299,30.625,88.66871584
6299,30.75,88.66871584

300,30.0,88.94663553
1300,30.125,88.94663553
2300,30.25,88.94663553
3300,30.375,88.94663553
4300,30.5,88.94663553
40300,30.5,88.94663553
5300,30.625,88.94663553
6300,30.75,88.94663553

301,30.0,89.199268
1301,30.125,89.199268
2301,30.25,89.199268
3301,30.375,89.199268
4301,30.5,89.199268
40301,30.5,89.199268
5301,30.625,89.199268
6301,30.75,89.199268

302,30.0,89.459888
1302,30.125,89.459888
2302,30.25,89.459888
3302,30.375,89.459888
4302,30.5,89.459888
40302,30.5,89.459888
5302,30.625,89.459888
6302,30.75,89.459888
303,30.0,89.72050728

1303,30.125,89.72050728
2303,30.25,89.72050728
3303,30.375,89.72050728
4303,30.5,89.72050728
40303,30.5,89.72050728
5303,30.625,89.72050728
6303,30.75,89.72050728

304,30.0,89.86025406
1304,30.125,89.86025406
2304,30.25,89.86025406
3304,30.375,89.86025406
4304,30.5,89.86025406
40304,30.5,89.86025406
5304,30.625,89.86025406
6304,30.75,89.86025406

501,30.0416667,89.329578
511,30.2083334,89.329578
521,30.2916667,89.329578
531,30.4583334,89.329578
541,30.5416667,89.329578
551,30.7083334,89.329578

502,30.0833334,89.459888
512,30.1666667,89.459888
522,30.3333334,89.459888
532,30.4166667,89.459888
542,30.5833334,89.459888
552,30.6666667,89.459888

503,30.0833334,89.65063307
513,30.1666667,89.65063307
523,30.3333334,89.65063307
533,30.4166667,89.65063307
543,30.5833334,89.65063307
553,30.6666667,89.65063307

504,30.0416667,89.72050728
514,30.0833334,89.72050728
524,30.1666667,89.72050728
534,30.2083334,89.72050728
544,30.2916667,89.72050728
554,30.3333334,89.72050728
564,30.4166667,89.72050728
574,30.4583334,89.72050728
584,30.5416667,89.72050728
594,30.5833334,89.72050728
604,30.6666667,89.72050728
614,30.7083334,89.72050728

505,30.0833334,89.86025406
515,30.1666667,89.86025406
525,30.3333334,89.86025406
535,30.4166667,89.86025406
545,30.5833334,89.86025406

```

555,30.6666667,89.86025406
*****
506,30.0416667,90.0
516,30.0833334,90.0
526,30.1666667,90.0
536,30.2083334,90.0
546,30.2916667,90.0
556,30.3333334,90.0
566,30.4166667,90.0
576,30.4583334,90.0
586,30.5416667,90.0
596,30.5833334,90.0
606,30.6666667,90.0
616,30.7083334,90.0
*****
** NODE GENERATIONS **
*****
*NGEN,NSET=SIDE1
1,6001,1000
*NGEN,NSET=SIDE2
150,6150,1000
*NGEN,NSET=SIDE3
291,6291,1000
*NGEN,NSET=SIDE4
305,6305,1000
*****
*NGEN,NSET=BORE1,LINE=C
1,150,1
*****
*NGEN,NSET=BORE2,LINE=C
150,291,1
*****
*NSET,NSET=BORE3
291,292,293,294,295,296,297,298,299,300,301,302,303,304,305
*****
*NGEN,NSET=OUT1,LINE=C
6001,6150,1
*****
*NGEN,NSET=OUT2,LINE=C
6150,6291,1
*****
*NSET,NSET=OUT3
6291,6292,6293,6294,6295,6296,6297,6298,6299,6300,6301,6302
,6303,6304,6305
*****
*NGEN,NSET=INTER1,LINE=C
40001,40150,1
*****

```

```

*NGEN,NSET=INTER2,LINE=C
40150,40291,1
*****
*NSET,NSET=INTER31
40291,40292,40293,40294,40295,40296,40297,40298,40299,40300
,40301,40302,40303
*****
*NSET,NSET=INTER32
40304,40305
*****
*NSET,NSET=INTER3
INTER31,INTER32
*****
*NFILL
BORE1,OUT1,6,1000
*NFILL
BORE2,OUT2,6,1000
*****
** WELD ELEMENT DEFINITION **
*****
*NSET,NSET=WELD
40303,40304,40305,5303,5305,6303,6304,6305
,586,596,606,616,545,555,584,594,604,614
*****
*NSET,NSET=REST,GENERATE
1,305,1
1001,1305,1
2001,2305,1
3001,3305,1
4001,4305,1
40001,40302,1
5001,5302,1
6001,6302,1
501,551,10
502,552,10
503,553,10
504,574,10
505,535,10
506,576,10
*****
** ELEMENT DEFINITION AND ELEMENT GENERATION **
*****
*ELEMENT,TYPE=DC2D8
1,1,2001,2003,3,1001,2002,1003,2
*ELGEN,ELSET=ALL1
1,2,2000,1,150,2,10
*****
*ELEMENT,TYPE=DC2D8

```



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4,40001,6001,6003,40003,5001,6002,5003,40002
*ELGEN,ELSET=ALL2
4,1,2000,1,150,2,10
*****
*ELEMENT,TYPE=DINTER3
3,4001,4002,4003,40001,40002,40003
*ELGEN,ELSET=INTERELE
3,152,2,10
*****
*ELEMENT,TYPE=DC2D8
2001,301,2301,512,502,1301,511,1302,501
*ELEMENT,TYPE=DC2D8
2002,2301,4301,532,522,3301,531,3302,521
*ELEMENT,TYPE=DC2D8
2004,40301,6301,552,542,5301,551,5302,541
*ELEMENT,TYPE=DC2D8
2011,301,502,514,303,501,503,504,302
*ELEMENT,TYPE=DC2D8
2012,502,512,524,514,1302,513,1303,503
*ELEMENT,TYPE=DC2D8
2013,512,2301,2303,524,511,2302,534,513
*ELEMENT,TYPE=DC2D8
2014,2301,522,554,2303,521,523,544,2302
*ELEMENT,TYPE=DC2D8
2015,522,532,564,554,3302,533,3303,523
*ELEMENT,TYPE=DC2D8
2016,532,4301,4303,564,531,4302,574,533
*ELEMENT,TYPE=DC2D8
2017,40301,542,594,40303,541,543,584,40302
*ELEMENT,TYPE=DC2D8
2018,542,552,604,594,5302,553,5303,543
*ELEMENT,TYPE=DC2D8
2019,552,6301,6303,604,551,6302,614,553
*ELEMENT,TYPE=DC2D8
2111,303,514,516,305,504,505,506,304
*ELEMENT,TYPE=DC2D8
2112,514,524,526,516,1303,515,1305,505
*ELEMENT,TYPE=DC2D8
2113,524,2303,2305,526,534,2304,536,515
*ELEMENT,TYPE=DC2D8
2114,2303,554,556,2305,544,525,546,2304
*ELEMENT,TYPE=DC2D8
2115,554,564,566,556,3303,535,3305,525
*ELEMENT,TYPE=DC2D8
2116,564,4303,4305,566,574,4304,576,535
*ELEMENT,TYPE=DC2D8
2117,40303,594,596,40305,584,545,586,40304
*ELEMENT,TYPE=DC2D8

```

```

2118,594,604,606,596,5303,555,5305,545
*ELEMENT,TYPE=DC2D8
2119,604,6303,6305,606,614,6304,616,555
*****
*ELSET,ELSET=BOUND1
1,2,4
*ELSET,ELSET=BOUND2
2111,2112,2113,2114,2115,2116,2117,2118,2119
*ELSET,ELSET=WELDELE
2117,2118,2119
*ELSET,ELSET=OUT1,GENERATE
4,1494,10
*ELSET,ELSET=OUT2
2019,2119
*ELSET,ELSET=OUT
OUT1,OUT2
*ELSET,ELSET=INN1,GENERATE
1,1491,10
*ELSET,ELSET=INN2
2011,2111
*ELSET,ELSET=INN
INN1,INN2
*****
** MATERIAL PROPERTY DEFINITION ***
*****
*SOLID SECTION,ELSET=ALL,MATERIAL=A1
*MATERIAL,NAME=A1
*CONDUCTIVITY
8.54692E-4
*DENSITY
0.283
*SPECIFIC HEAT
0.103
*****
** INTERFACE HEAT CONDUCTANCE ***
*****
*INTERFACE,ELSET=INTERELE
*GAP CONDUCTANCE
2.8935E-4,0.0
*****
** PRESCRIBING INITIAL TEMPERATURES ***
*****
*INITIAL CONDITIONS,TYPE=TEMPERATURE
WELD,2070.
REST,70.
*RESTART,WRITE,FREQ=5
*****
** START ANALYSIS WITH PRESCRIBED TOLERANCES AND TIME STEPS ***

```

```

*****
*STEP,INC=100
*HEAT TRANSFER,DELTMAX=800.,TEMTOL=1.,END=SS
.1,4.OE6,0.005,,1.E-6
*****
** SPECIFYING BOUNDARY CONDITIONS ***
*****
*FILM
WELDELE,F2,70.,3.4521E-6
INN,F4,70.,3.4521E-6
OUT,F2,70.,3.4521E-6
*DFLUX
BOUND1,S1,0.0
BOUND2,S3,0.0
*****
** WRITING OUT TEMPERATURE DATA TO A FILE ***
*****
*NODE FILE
NT
*PLOT,FREQ=10,COLORS=23
TEMPERATURE CONTOURS
*CONTOUR
NT11,21
*END STEP

```

APPENDIX C
INPUT FILE FOR THERMAL STRESS ANALYSIS

```
*HEADING
THE THERMAL STRESS MODEL, WITH TEMPERATURE-TIME HISTORY OF HEAT
TRANSFER MODEL
*****
**  NODE DEFINITIONS AND NODE GENERATION  **
*****
*NODE
999,
9999,
1,0.,-30.0
4001,0.,-30.5
40001,0.,-30.5
6001,0.,-30.75
150,30.0,0.
4150,30.5,0.
40150,30.5,0.
6150,30.75,0.
305,0.,30.0
4305,0.,30.5
40305,0.,30.5
6305,0.,30.75
*****
*NODE,SYSTEM=C
291,30.0,85.16778525
1291,30.125,85.16778525
2291,30.25,85.16778525
3291,30.375,85.16778525
4291,30.5,85.16778525
40291,30.5,85.16778525
5291,30.625,85.16778525
6291,30.75,85.16778525
*****
292,30.0,85.76527325
1292,30.125,85.76527325
2292,30.25,85.76527325
3292,30.375,85.76527325
4292,30.5,85.76527325
40292,30.5,85.76527325
5292,30.625,85.76527325
6292,30.75,85.76527325
*****
293,30.0,86.30804237
1293,30.125,86.30804237
```

1293,30.125,86.30804237
2293,30.25,86.30804237
3293,30.375,86.30804237
4293,30.5,86.30804237
40293,30.5,86.30804237
5293,30.625,86.30804237
6293,30.75,86.30804237

294,30.0,86.80117982
1294,30.125,86.80117982
2294,30.25,86.80117982
3294,30.375,86.80117982
4294,30.5,86.80117982
40294,30.5,86.80117982
5294,30.625,86.80117982
6294,30.75,86.80117982

295,30.0,87.24927986
1295,30.125,87.24927986
2295,30.25,87.24927986
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*NGEN,NSET=SIDE2
150,6150,1000
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291,6291,1000
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*NGEN,NSET=SIDE41
305,6305,1000
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*NGEN,NSET=SIDE42
506,616,10
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*NSET,NSET=SIDE4
SIDE41,SIDE42
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*NGEN,NSET=BORE2,LINE=C
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6001,6150,1

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,6303,6304,6305
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40150,40291,1
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,40300,40301,40302
*NSET,NSET=INTER32
40303,40304,40305
*NSET,NSET=INTER3
INTER31,INTER32
*****
*NFILL
BORE1,OUT1,6,1000
*NFILL
BORE2,OUT2,6,1000
*****
*NSET,NSET=WELD
40303,40304,40305,5303,5305,6303,6304,6305
,586,596,606,616,545,555,584,594,604,614
*NSET,NSET=REST,GENERATE
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1001,1305,1
2001,2305,1
3001,3305,1
4001,4305,1
40001,40302,1
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6001,6302,1
501,551,10
502,552,10
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505,535,10
506,576,10
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*** NORMAL DEFINITION FOR INTERFACE ELEMENTS ***
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1493,4301,1.239E-02,9.999E-01
1503,4301,1.239E-02,9.999E-01
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1513,4305,0.000E00,1.000E+00
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** ELEMENT DEFINITION AND ELEMENT GENERATION **
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1,1,2001,2003,3,1001,2002,1003,2,999,9999
*ELGEN,ELSET=ALL1
1,2,2000,1,150,2,10
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*ELEMENT,TYPE=CGPE10R
4,40001,6001,6003,40003,5001,6002,5003,40002,999,9999
*ELGEN,ELSET=ALL2
4,1,2000,1,150,2,10
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*ELEMENT,TYPE=INTER3
3,4001,4002,4003,40001,40002,40003
*ELGEN,ELSET=INTERELE
3,152,2,10
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*ELEMENT,TYPE=CGPE10R
2001,301,2301,512,502,1301,511,1302,501,999,9999
*ELEMENT,TYPE=CGPE10R
2002,2301,4301,532,522,3301,531,3302,521,999,9999
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2004,40301,6301,552,542,5301,551,5302,541,999,9999

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2013,512,2301,2303,524,511,2302,534,513,999,9999
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*ELEMENT,TYPE=CGPE10R
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*ELEMENT,TYPE=CGPE10R
2119,604,6303,6305,606,614,6304,616,555,999,9999
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*ELSET,ELSET=BOUND1
1,2,4
*****
*ELSET,ELSET=BOUND2
2111,2112,2113,2114,2115,2116,2117,2118,2119
*****
*ELSET,ELSET=WELDELE
2117,2118,2119
*****
*ELSET,ELSET=OUT1,GENERATE
4,1494,10

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*ELSET,ELSET=OUT2
2019,2119
*****
*ELSET,ELSET=OUT
OUT1,OUT2
*****
*ELSET,ELSET=INN1,GENERATE
1,1491,10
*****
*ELSET,ELSET=INN2
2011,2111
*****
*ELSET,ELSET=INN
INN1,INN2
*****
*ORIENTATION,SYSTEM=C,NAME=RADIAL
0.,0.,0.,0.,0.,1.
3,0.
*****
** MATERIAL PROPERTY DEFINITION **
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*SOLID SECTION,ELSET=ALL,MATERIAL=STEEL,ORIENTATION=RADIAL
*MATERIAL,NAME=STEEL
*ELASTIC
3.E7,.3
*EXPANSION,ZERO=70.
6.95E-6
*****
** BOUNDARY CONDITIONS **
*****
*BOUNDARY
SIDE1,XSMM
SIDE4,XSMM
1,2
9999,1,2
999,1
*****
** SPECIFYING INTERFACE FRICTION **
*****
*INTERFACE,ELSET=INTERELE
*FRICTION
0.01,1.E7
*****
*INITIAL CONDITIONS,TYPE=TEMPERATURE
WELD,2070.
REST,70.
*RESTART,WRITE,FREQ=10

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*STEP,INC=50,CYCLE=50
*STATIC,PTOL=10
.1,1.
*****
**  READING TEMPERATURE-TIME HISTORY FROM HEAT      ***
**                TRANSFER ANALYSIS                  ***
*****
*TEMPERATURE,FILE=15,BSTEP=1(INC=1),ESTEP=1(INC=21)
*****
*EL PRINT,FREQ=20
S
*END STEP

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