Multiple Mode Couplings in Waveguide Array for Broadband Near-zero
Dispersion and Supercontinuum Generation

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Saleha Fatema, B.S.

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Approved

Dr. Sangsik Kim
Chair of Committee

Dr. Ayrton Bernussi

Dr. Mark Sheridan
Dean of the Graduate School

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ABSTRACT

Photonic chips can enhance the light-matter interaction by confining light within a submicron-scale waveguide cross-section and have advanced many nonlinear optical processes, such as supercontinuum and Kerr frequency comb generations. These nonlinear frequency conversion processes rely on the phase-matching of an optical system, which can be achieved by engineering the dispersion of the photonic waveguides. A typical dispersion engineering method, however, is bounded by the material dispersions, limiting the spectral bandwidth and efficiency of many nonlinear processes. Moreover, it causes certain fabrication challenges; for example, a thick silicon nitride film is required for anomalous dispersion, which is easy to crack due to high film-stress and incompatible with the complementary metal-oxide-semiconductor (CMOS) foundry. Thus, to develop fully CMOS compatible on-chip photonic devices, further advanced dispersion engineering is required. This work presents a coupled waveguide array scheme that introduces multiple mode couplings at different wavelengths and achieves a near-zero dispersion profile at near-infrared. A broadband near-zero dispersion profile, spanning over the spectral range of 1350 – 1900 nm within ± 140 ps/nm/km, is obtained on a 300 nm-thick silicon nitride film, which is compatible with the CMOS process but shows high normal dispersion unless otherwise engineered. Using this engineered dispersion profile, an octave-spanning supercontinuum is numerically generated by pumping near the spectral regions of the mode couplings. The concept of utilizing multiple mode couplings to achieve broadband near-zero dispersion can be applied to other spectral regimes and material platforms as well, by engineering the structural parameters to shift the positions of the zero-dispersion-wavelengths. This CMOS compatible waveguide array design would allow mass production in supercontinuum generating chips, which can then be used in molecular sensing and spectroscopic applications.

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CHAPTER 1

INTRODUCTION

1.1 Motivation

Photonic chips can confine light within a submicron waveguide cross-section and boost the nonlinear optical processes with increased optical intensity. The tiny chip platform ensures a portable, robust, and scalable system. Additionally, if the devices are CMOS compatible, the wafer-scale fabrication can decrease the production cost. The integration of different photonic components on a single chip has made an extensive system transportable and enabled their usage in many optical applications, including telecommunication [1–4], biomedical [5–7], spectroscopy [8,9], light detection and ranging (LIDAR) [10,11]. The developments of these photonic chip technologies are accelerating with the advancement of nanotechnologies. The device footprints are being further reduced using novel phenomena such as metamaterial [12,13] and plasmonic nanostructures [14]. Moreover, due to the process compatibility and the ability to share the same wafer with current silicon microelectronic devices, the combination of silicon photonics and microelectronics is believed to be the future path of integrated circuits [15–17].

In ultrafast photonics, controlling the dispersion of the optical components is crucial [18–23]. Dispersion is the spreading of a light pulse with time as it propagates through an optical fiber or a system. Due to the spreading of light, the pulse is broadened while it travels, resulting in a signal overlapping. To avoid this, dispersion compensation is widely practiced in optical communications. In addition to preventing pulse distortion, engineering dispersion to obtain a near-zero dispersion also plays a key role in nonlinear frequency conversion processes since it assists in satisfying the necessary phase-matching criteria. Therefore, for nonlinear parametric processes such as supercontinuum and Kerr frequency comb generations, engineering dispersion to achieve a broadband near-zero dispersion (NZD) is highly required.

Supercontinua (SCs) can produce a broadband optical spectrum, and they have been used in a wide range of applications, such as coherent spectroscopy [24,25],

frequency metrology [26, 27], pulse compression [28], exotic nonlinear dynamics [29, 30], and optical coherence tomography [31, 32]. Precisely engineered dispersion enables the broadening of the spectrum through the controllable generation of dispersive wave (DW), also known as Cherenkov radiation [33]. DWs are the result of a phase-matched process that depends on the higher-order dispersions (HOD). DWs can widen the SC bandwidth by shifting the pump-energy to other spectra across the zero-dispersion wavelength (ZDW) [34–49]. If the position of the ZDW is engineered to be far away from the pump wavelength, a bandwidth of multiples of octaves can be achieved [50–55]. Thus, to extend the bandwidth of an SC, a broadband NZD profile with multiple ZDWs is highly desired.

The material platform also plays a significant role in an efficient and coherent generation of SC. Among the leading integrated waveguide platforms, Si_3N_4 is often an ideal choice due to its high nonlinearity, refractive index, and large bandgap, which allows nonlinear interactions that are free from the two-photon absorption. However, in the Si_3N_4 platform, a thick film thickness (>800 nm) is typically required for near-zero or anomalous dispersions at near-infrared (NIR) [56–58]. Due to high film stress, such thick Si_3N_4 film is easy to crack [57, 59], and the current CMOS process limits the Si_3N_4 thickness to less than 300 nm, which shows a highly normal dispersion in the near-infrared. Hence, to develop a fully CMOS compatible on-chip supercontinuum generation (SCG) device, further advanced dispersion engineering is required.

In integrated photonics, the waveguide geometries can be engineered to control the dispersion of a guided mode [60]; in general, the waveguide dispersion induces anomalous dispersion near the high confinement (with a larger cross-section area) and normal dispersion near the cut-off (with a smaller cross-section area). Note that, however, the overall dispersion is a combination of both modal and material dispersions; thus, the intrinsic material dispersion determines the achievable dispersion limit. Several waveguide structures have been proposed to overcome the material dispersion limit utilizing the mode-coupling effect [20, 50, 61–68]. A mode-coupling appears when two distinct modes are close by and coupled together, resulting in symmetric and antisymmetric supermodes. The coupled modes force the symmetric and antisymmetric modes toward normal and

anomalous dispersions, respectively, and the degree of shift can be controlled by their coupling strength. Such mode-coupling can be formed between different mode orders [20, 69], polarizations [70], bending radiuses [21, 71, 72], and materials [64, 73].

In this thesis, we introduce multiple mode-couplings in a 300 nm-thick CMOS compatible silicon nitride waveguide array to achieve a broadband near-zero dispersion at NIR. Four mode-couplings are introduced on a thin silicon nitride, which shows highly normal dispersion unless otherwise engineered. The engineered dispersion profile shows eight ZDWs, achieving the near-zero dispersions (within ± 140 ps/nm/km) over the spectral range of 1350 – 1900 nm. Leveraging this dispersion profile and the high nonlinearity of the Si_3N_4 waveguide, we numerically generated supercontinuum. The closely spaced ZDWs help the phase-matchings and DW generations, enabling octave-spanning SCGs by pumping at both normal and anomalous dispersions.

1.2 Outline of the Thesis

There are six chapters in this thesis. Chapter 2 outlines the physical mechanisms behind the supercontinuum generation (SCG). Chapter 3 discusses the importance of dispersion engineering as well as the different approaches used in photonics to engineer dispersion. Then a novel technique of coupling multiple different-order modes to engineer dispersion is presented in Chapter 4. The details of the design, along with the results obtained, are discussed in this chapter. In order to show a practical application of the obtained dispersion profile, a supercontinuum is generated using the engineered dispersion profile in Chapter 5. The Nonlinear Schrödinger equation is numerically solved for generating the supercontinuum by pumping at both normal and anomalous dispersion regions. In the final chapter, a summary of the achieved results and the potential of this technique in different applications are presented to conclude the thesis.

CHAPTER 2

SUPERCONTINUUM GENERATION

The electromagnetic (EM) wave propagation depends on the medium of propagation. In vacuum, the pulse propagation remains unchanged while in any other medium, the EM field interacts with the atoms of the medium resulting in loss and dispersion of the field. If the intensity of the source becomes high, the nonlinear phenomena also come into play. When a narrow-band pulse is transmitted in a nonlinear medium, it can undergo a massive spectral broadening due to the different nonlinear interplay of the medium. This spectral broadening is widely known as the supercontinuum (SC).

In this chapter, a theoretical overview of SC is briefly discussed. In section 2.1, the equation for numerically modeling the supercontinuum is presented. This section begins with the theory of electromagnetic wave propagation using Maxwell's equation, followed by the wave equation in a nonlinear medium, the nonlinear Schrödinger equation (NLSE), and the generalized nonlinear envelope equation (GNLE). The physical mechanisms modeled in these equations are discussed in section 2.2. In this section, the effects of the linear (frequency dependency of refractive index, i.e., dispersion) and nonlinear (intensity dependency of refractive index, i.e., Kerr effect) properties in pulse propagation are discussed briefly. Lastly, section 2.3 reviews the waveguide material platforms used for SCG.

2.1 Numerical Modeling of Supercontinuum

2.1.1 Maxwell's Equation

The propagation of the electromagnetic field can be represented by Maxwell's equations in differential form as below [74]:

1. Gauss's law:

$$\nabla \cdot \mathbf{D} = \rho_f \tag{2.1}$$

2. Gauss's law for magnetism:

$$\nabla \cdot \mathbf{B} = 0 \quad (2.2)$$

3. Faraday's law of induction:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2.3)$$

4. Ampère's circuital law:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (2.4)$$

Here, $\mathbf{E}$ and $\mathbf{H}$ are electric and magnetic field vectors, respectively, $\mathbf{D}$ and $\mathbf{B}$ are corresponding electric and magnetic flux densities, respectively, $\mathbf{J}$ is the electric current density, and ρ_f is the electric charge density. The current density vector $\mathbf{J}$ and the charge density ρ_f represent the sources for an electromagnetic field. In the case of optical propagation, the free charge is absent, resulting in $\mathbf{J} = \rho_f = 0$. The flux densities $\mathbf{D}$ and $\mathbf{B}$ are related to the electric and magnetic field, respectively, through the constitutive relationships as follows [74],

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \quad (2.5)$$

$$\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M} \quad (2.6)$$

Here, ϵ_0 is the vacuum permittivity, μ_0 is the vacuum permeability, and $\mathbf{P}$ and $\mathbf{M}$ are the induced electric and magnetic polarizations, respectively. Since optical fibers are nonmagnetic medium, $\mathbf{M} = 0$.

2.1.2 Wave Equation in Nonlinear Medium

The wave equation models the propagation of light in a medium, and it can be derived by taking the curl of Eq. (2.3) and substituting Eqs. (2.1-2.2) and Eqs. (2.4-2.6) where necessary, as,

$$\nabla \times \nabla \times \mathbf{E} = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} \quad (2.7)$$

where $c = 1/\sqrt{\mu_0\epsilon_0}$, is the speed of light. The polarization field $\mathbf{P}$ can be divided into a linear and a nonlinear part, i.e., $\mathbf{P}(r, t) = \mathbf{P}_L(r, t) + \mathbf{P}_{NL}(r, t)$, where $\mathbf{P}_L(r, t) = \epsilon_0\chi^{(1)}\mathbf{E}$, and $\mathbf{P}_{NL}(r, t) = \epsilon_0\chi^{(2)}\mathbf{E}^2 + \epsilon_0\chi^{(3)}\mathbf{E}^3 + \epsilon_0\chi^{(4)}\mathbf{E}^4 + \dots$. Here, $\chi^{(i)}$ indicates the i -th order nonlinearity. This makes the wave equation as [74],

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = \mu_0\frac{\partial^2\mathbf{P}_L}{\partial t^2} + \mu_0\frac{\partial^2\mathbf{P}_{NL}}{\partial t^2} \quad (2.8)$$

Equation (2.8) is the wave equation in the nonlinear medium [74].

2.1.3 Nonlinear Schrödinger Equation (NLSE)

The nonlinear wave equation can be further modified to model the light-propagation in a nonlinear medium. The modified equation is known as the Nonlinear Schrödinger Equation (NLSE) and is widely used to model the light-propagation in nonlinear optical fibers and planar waveguides [75].

$$\frac{\partial\mathbf{E}}{\partial z} + j\frac{\beta_2}{2}\frac{\partial^2\mathbf{E}}{\partial t^2} - j\gamma|\mathbf{E}|^2\mathbf{E} = 0 \quad (2.9)$$

where β_2 is the group velocity dispersion (GVD), and γ is the nonlinear parameter.

2.1.4 Generalized Nonlinear Envelope Equation (GNLE)

When ultra-short optical pulses propagate through a guided mode, the interplay between nonlinear and linear effects generates new frequencies, eventually resulting in a spectral broadening. For high-intense pulses, the pulse spectra can experience a broadening exceeding 10 THz [74]; this pulse broadening is known as supercontinuum generation (SCG). While modeling SC, nonlinear effects such as self-phase modulation (SPM), cross-phase modulation (XPM), modulation instability (MI), four-wave mixing (FWM), stimulated Raman scattering (SRS) should be considered. Including these nonlinear effects and the contributions of

higher-order dispersions, the modified NLSE becomes as follows,

$$\begin{aligned} \frac{\partial \mathbf{E}}{\partial z} + \frac{1}{2} \left(\alpha + j\alpha_1 \frac{\partial}{\partial t} \right) \mathbf{E} + \sum_{m=2}^M j^{m-1} \frac{\beta_m}{m!} \frac{\partial^m \mathbf{E}}{\partial t^m} \\ = j \left(\gamma + j\gamma_1 \frac{\partial}{\partial t} \right) \mathbf{E}(z, t) \int_{-\infty}^{\infty} R(t') |\mathbf{E}(z, t - t')|^2 dt' \end{aligned} \quad (2.10)$$

Here, α represents the loss, and $\alpha_1 = \alpha/\omega$ models the higher-order loss. α_1 should be included since loss may not remain constant over the entire SC bandwidth. β_m indicates the m -th order dispersion coefficient, γ_1 represents the higher-order nonlinearity, and $R(t)$ models the Raman response. This equation is known as Nonlinear Envelope Equation (NLE) or Generalized Nonlinear Envelope Equation (GNLE) [75].

2.2 Physical Mechanisms of Supercontinuum Generation

The loss, dispersion, and nonlinearity each have separate roles in the formation of an SC. In this section, to clearly understand their contribution, each phenomenon is considered and discussed separately. Another parameter that plays a critical part in SC formation is the pump source; depending on the initial pulse width T_0 and the peak power P_0 of the source, either dispersion or nonlinearity dominates throughout the medium. Their effects are included in two distinct lengths named, dispersion length L_D and nonlinear length L_{NL} [74]:

$$L_D = \frac{T_0^2}{|\beta_2|} \text{ and } L_{NL} = \frac{1}{\gamma P_0} \quad (2.11)$$

Based on the relative extent of propagation length L , L_D , and L_{NL} , the propagation behavior in a medium can be categorized into the following four classes:

1. $L \ll L_{NL}$ and $L \ll L_D$: Neither dispersion nor nonlinearity plays a significant role during pulse propagation, and the optical pulse maintains its shape throughout the propagation length. By ignoring the dispersion and nonlinear

effects, the wave equation becomes,

$$\frac{\partial \mathbf{E}}{\partial z} = 0 \quad (2.12)$$

2. $L \ll L_{\text{NL}}$ and $L \approx L_D$: The pulse propagation is controlled by GVD with a negligible effect of nonlinearity.
3. $L \approx L_{\text{NL}}$ and $L \ll L_D$: The pulse propagation is controlled by SPM, leading to spectral broadening, with a negligible effect of dispersion.
4. $L \approx L_{\text{NL}}$ and $L \approx L_D$: Dispersion and nonlinearity act together, and their interplay affects the propagation behavior in an entirely different manner. In the anomalous dispersion regime, the interaction of nonlinearity generates soliton, while in the normal dispersion, it results in pulse compression.

The cases 2-4 are discussed in section (2.2.1-2.2.6).

2.2.1 Dispersion

In a guided mode, the dispersion arises from the frequency dependence of the effective index of a specific mode. The optical spectrum contains multiple wavelength components that travel at different velocities in a dispersive medium, and the pulse spectrum tends to broaden temporally [76–79]. Mathematically, dispersion D is the second-order derivative of the propagation constant $\beta(\omega)$ with respect to the wavelength λ . The second-order derivative of $\beta(\omega)$ with respect to the angular frequency ω is termed as group velocity dispersion (GVD). Mathematically, GVD and D are represented as follows,

$$\text{GVD} = \beta_2 = \frac{\partial^2 \beta}{\partial \omega^2} \quad (2.13)$$

$$D = -\frac{\lambda}{c} \frac{\partial^2 \beta}{\partial \lambda^2} = -\frac{2\pi c}{\lambda^2} \beta_2 \quad (2.14)$$

Dispersion can be categorized into anomalous and normal dispersions based on their signs. From the mathematical representation, it can be understood that although β_2 and D represent the same physical mechanism, the signs of β_2 and D

Figure 2.1. The effects of dispersion in a Gaussian pulse. The effects of (a) second-order dispersion (β_2), (b) third-order dispersion (β_3), (c) second and third-order dispersion, and (d) higher-order dispersion. A loss-less and purely linear medium is considered for every case. The z/L_D parameter indicates the ratio of propagation distance z to dispersion length L_D . For initial condition, i.e., $z = 0$, $z/L_D = 0$.

are opposite for the normal and anomalous dispersions. The normal dispersion is indicated by $\beta_2 > 0$ and $D < 0$ while the anomalous dispersion is indicated by $\beta_2 < 0$ and $D > 0$.

The role of dispersion in light-propagation can be understood more clearly in a purely linear medium. If the loss and nonlinearity are ignored, the NLSE becomes as follows:

$$\frac{\partial \mathbf{E}(z, t)}{\partial z} + j \frac{(\text{sgn})\beta_2}{2} \frac{\partial^2}{\partial t^2} \mathbf{E}(z, t) = 0 \quad (2.15)$$

The effects of dispersion can be observed by solving this equation, considering a

Gaussian pulse as input as follows [74],

$$\mathbf{E}(0, T) = \exp\left(-\frac{T^2}{2T_0^2}\right) \quad (2.16)$$

where T_0 is the input pulse-width. Figure 2.1(a) shows the propagation of a Gaussian pulse modeled by Eq. (2.15) at propagation length $z = 2L_D$ and $z = 4L_D$. It can be observed that, for $\beta_2 = 1 \text{ ps}^2/\text{m}$, the pulse is broadened significantly with distance.

2.2.1.1 Third-order dispersion

Although the contribution of β_2 is the most dominant, in many cases, the third-order dispersion (TOD) coefficient β_3 also becomes important [51, 74, 80]. If the pulses are pumped at (or near) the zero-dispersion wavelength, the GVD approaches to zero, and the TOD dominates. Additionally, while pumping in the femtosecond regime, it is compulsory to include the TOD, even if $\beta_2 \neq 0$, since the expansion parameter $\Delta\omega/\omega_0$ is no longer negligible [74]. When TOD is considered in the NLSE along with GVD, the equation becomes as follows,

$$\frac{\partial \mathbf{E}(z, t)}{\partial z} + j \frac{(\text{sgn})\beta_2}{2} \frac{\partial^2}{\partial t^2} \mathbf{E}(z, t) - \frac{(\text{sgn})\beta_3}{6} \frac{\partial^3}{\partial t^3} \mathbf{E}(z, t) = 0 \quad (2.17)$$

Dispersion length corresponding to the TOD term is $L'_D = T_0^3/|\beta_3|$, and the effects of TOD become significant only if $L'_D \leq L_D$ [74].

The effect of TOD considering $\beta_2 = 0$ is demonstrated in Fig. 2.1(b). The combined effects of GVD and TOD are plotted in Fig. 2.1(c), by plugging in both the Taylor coefficients β_2 and β_3 in the NLSE. Although in a purely linear regime, the sign of β_2 does not affect the pulse broadening, this, however, is not the case for TOD. The TOD results in the oscillatory structures near the trailing edge of the pulses observed in Figs. 2.1(b-c), and the sign of TOD determines the spectral position of these oscillatory structures. In these cases, the β_3 coefficient used to model the pulse is positive ($\beta_3 = 1 \text{ ps}^3/\text{m}$). If the β_3 parameter with a negative coefficient is used, the oscillatory spectrum is observed on the opposite edge of the pulse. This effect is shown more clearly using a chirped-Gaussian pulse as input

Figure 2.2. The effects of the sign of third-order dispersion in a chirped-Gaussian pulse, with chirp parameter $C = 3$. A loss-less, purely linear medium with only the third-order dispersion is considered; (a) β_3 is positive ($\beta_3 = 1 \text{ ps}^3/\text{m}$), and (b) β_3 is negative ($\beta_3 = -1 \text{ ps}^3/\text{m}$). The z/L_D parameter indicates the ratio of propagation distance z to dispersion length L_D . For initial condition, i.e., $z = 0$, $z/L_D = 0$.

as follows,

$$\mathbf{E}(0, T) = \exp\left(-\frac{1 + jC}{2} \frac{T^2}{T_0^2}\right) \quad (2.18)$$

where C is a chirp parameter [81, 82] ($C > 0$ indicates an increase in frequency, known as up-chirp, and $C < 0$ indicates a decrease in frequency, known as down-chirp). Figure 2.2 shows that changing the sign of TOD, while keeping the other parameters unchanged, changes the extent of the red and blue edges of the spectrum [74]. For a positive β_3 , the oscillations are developed in the trailing side of the pulse, whereas for a negative β_3 , they are observed in the leading side. In the case of the SCG as well, if $\omega_{\text{pump}} \approx \omega_{\text{ZDW}}$, the GVD becomes smaller, and the basic shape depends primarily on the sign and magnitude of the TOD [51, 79].

2.2.1.2 Higher-order dispersion

While expanding the propagation constant $\beta(\omega)$ in terms of the power of ω , if the higher-order terms are not ignored, the wave equation takes the form as below in

the absence of loss and nonlinearity,

$$\frac{\partial \mathbf{E}(z, t)}{\partial z} = j \sum_{n=2}^N j^n \frac{\beta_n}{n!} \frac{\partial^n \mathbf{E}(z, t)}{\partial t^n} \quad (2.19)$$

where β_n indicates the n -th order dispersion coefficient. Figure 2.1(d) shows the propagation of a Gaussian pulse modeled by Eq. (2.19) at propagation lengths, $z = 2L_D$ and $z = 4L_D$, with the dispersion coefficient, $\beta_n = 1 \text{ ps}^n/\text{m}$ up to $N = 20$.

In the case of the ultrafast pulse propagation, or more specifically, SCG and Kerr comb generation, higher-order dispersion is considered to be an important parameter since they contribute to generating dispersive waves (DWs) to assist the widening of the bandwidth. A DW generation, also commonly known as Cherenkov radiation [33], results from a phase-matching in the presence of higher-order dispersions, and it can transfer the pump-energy to a wavelength away from the pump across the ZDW [83, 84]. Therefore, it is often essential to include all the higher-order dispersion (HOD) coefficients, which can be achieved by taking the infinite series at the frequency domain as follows,

$$\mathcal{F} \left[\sum_{n \geq 2} j \frac{j^n \beta_n}{n!} \frac{\partial^n \mathbf{E}(z, t)}{\partial t^n} \right] = [\beta(\omega) - \beta(\omega_0) - \beta_1(\omega_0)(\omega - \omega_0)] \mathbf{E}(\omega) \quad (2.20)$$

where $\mathcal{F}$ is the Fourier transform operator, ω_0 is the angular pump frequency, and $\beta_1(\omega)$ is the first-order derivative of the propagation constant $\beta(\omega)$.

The spectral envelope of supercontinuum is primarily determined by the HOD of the medium [51, 80, 85–87]. The phase-matching criteria play a vital role in determining the nonlinear frequency conversion efficiency, and both linear and nonlinear contributions to the overall phase mismatch must be considered. Moreover, dispersion influences both the phase and group velocities; even under the conditions where a particular process is phase-matched, group velocity mismatch can play a significant role in limiting the interaction length [88]. On the contrary, the matching of group velocity at different wavelengths can result in mutual trapping between widely separated spectral components [88]. Additionally, the HOD alters the development of the Raman-soliton and decreases the extent of the soliton self-frequency shift [80]. In addition to the HOD, polarization mode

dispersion, higher-order inter-modal dispersion between transverse modes also modify the propagation, and their effects should be considered when necessary [88].

2.2.2 Self-phase Modulation (SPM)

The time-dependent intensity of a pulse causes a corresponding modulation to the local refractive index of the waveguide [75]. This, in turn, creates a time-dependent phase delay to the same pulse (SPM) or the co-propagating pulse (XPM). The effect of this modulation is similar to applying a corresponding nonlinear chirp, resulting in the generation of new frequency components.

In the presence of different dispersions (normal or anomalous), the effects of SPM are different. In order to understand them clearly, three cases are considered: first, SPM is considered in the absence of loss and dispersion, then in a lossy but dispersion-less medium, and lastly, in the presence of anomalous and normal dispersions.

- **Case 1: Neglecting dispersion and loss:** Considering only the time-dependent intensity and ignoring the dispersion and loss, the NLSE takes the form as follows:

$$\frac{\partial \mathbf{E}(z, t)}{\partial z} = -j\gamma |\mathbf{E}(z, t)|^2 \mathbf{E}(z, t) \quad (2.21)$$

Integrating the equation, we get,

$$\mathbf{E}(z, t) = \mathbf{E}(0, t) e^{-j\gamma |\mathbf{E}(0, t)|^2 z} = \mathbf{E}(0, t) e^{j\Delta\phi} \quad (2.22)$$

where $\mathbf{E}(0, t)$ is the input temporal field profile, and $\Delta\phi = -\gamma |\mathbf{E}(0, t)|^2 z$ is the time-dependent nonlinear phase shift. The temporal profile of $\Delta\phi(z, t)$ is directly proportional to that of the intensity, and a temporally varying phase indicates that the instantaneous optical frequency changes across the pulse from its central value ω_0 . The instantaneous frequency can be written as,

$$\omega(z, t) = \omega_0 + \Delta\omega(z, t) \quad (2.23)$$

Figure 2.3. (a) The effects of self-phase modulation in an un-chirped Gaussian pulse. A pulse with peak power $P_0 = 1$ W is used in a waveguide with nonlinear parameter $\gamma = 1 \text{ W}^{-1}\text{m}^{-1}$ for a propagation length $L = 4.5\pi$ m. This results in a chirped-output pulse with 5 peaks (phase shift is 4.5π). (b) The zoomed view of the output pulse, clearly showing the frequency chirping.

where $\Delta\omega(z, t)$ is the first-order derivative of $\Delta\phi(z, t)$ with respect to time. Thus, the instantaneous frequency is modulated according to the time derivative of the intensity. For a propagation length L and normalized $\mathbf{E}$ (e.g., $|\mathbf{E}(0, 0)| = 1$), the maximum phase shift is, $\phi_{\max} = \gamma P_0 L$, where P_0 is the peak pulse power. As mentioned before, the time-dependency of $\Delta\omega$ is identical to the frequency chirping phenomenon. Chirping in a signal results in increased (up-chirping) or decreased (down-chirping) oscillation. In the case of SPM introduced chirping, the chirp increases in magnitude since new frequency components are produced continuously with propagation, and these SPM induced frequency components widen the spectrum. The amount of chirping, i.e., the number of peaks increases linearly with $\phi_{\max}$, and the number of peaks observed in an un-chirped input pulse is equal to $\Delta\phi_{\max}/\pi + 1$ [75]. In the case of the chirped input pulses, however, the sign of the C parameter changes the peak numbers; for $C > 0$, the peak number is increased whereas for $C < 0$, this number is decreased.

The effects of SPM in the absence of dispersion and loss in an un-chirped Gaussian pulse is observed in Fig. 2.3. A pulse with a peak power of 1 W is applied in a nonlinear medium, with $\gamma = 1 \text{ W}^{-1}\text{m}^{-1}$ and $L = 4.5\pi$ m. Therefore,

the maximum phase shift $\phi_{\max} = 4.5\pi$, which results in a total of 5 peaks in the output, as observed in Fig. 2.3.

- **Case 2: Neglecting dispersion but considering loss:** The nonlinear propagation equation takes the form as,

$$\frac{\partial \mathbf{E}'(z, t)}{\partial z} = -j\gamma |\mathbf{E}'|^2 \mathbf{E}' e^{-\alpha z} \quad (2.24)$$

where $\mathbf{E} = \mathbf{E}' e^{-\alpha z/2}$. Since $|\mathbf{E}'|$ remains unchanged during propagation, integrating the equation, we get

$$\mathbf{E}'(L, t) = \mathbf{E}'(0, t) e^{-j\gamma |\mathbf{E}'(0, t)|^2 L_{\text{eff}}} \quad (2.25)$$

where $L_{\text{eff}} = [1 - e^{-\alpha L}]/\alpha$. For a low loss medium, when $\alpha L \ll 1$, the effective length remains almost unchanged, i.e., $L_{\text{eff}} \approx L$. But in a lossy medium, when $\alpha L \gg 1$, L_{eff} approaches to α^{-1} , that is, the accumulated nonlinear phase shift becomes loss-limited. The maximum phase shift, in this case, is $\phi_{\max} = \gamma P_0 L_{\text{eff}}$. Therefore, in a lossy medium, the extent of nonlinearity is reduced.

- **Case 3: Including dispersion and loss:** For ultra-short high power pulses, both the L_D and L_{NL} become comparable to the propagation length resulting in a combined effect. When considering SPM in isolation (no dispersion) for a temporally symmetric un-chirped input pulse, the spectral broadening is also symmetric, and the time-domain envelope of the pulse is unchanged. On the other hand, in a purely linear regime, an initially un-chirped input pulse will temporarily broaden in an identical way irrespective of the sign of β_2 . However, in the presence of nonlinearity, the sign of dispersion influences the propagation dynamics significantly.

SPM and Normal Dispersion: In the normal dispersion region, the red components travel faster than the blue ones, leading to a temporal broadening of the pulse. When SPM works along with the normal dispersion, the positive chirps due to normal dispersion are added to the positive chirps created from the SPM, resulting in a much more rapid pulse broadening [74, 89, 90]. Initially, the effect of SPM is more dominant than that of dispersion [75], and the spectrum

Figure 2.4. The effects of SPM in a Gaussian pulse in the presence of (a) normal and (b) anomalous dispersion for a propagation length $z = 5L_D$. The upper plots show the evolution of pulse shapes, and the lower plots show the optical spectra for both cases. The waveguide and pulse parameters are chosen in a way that the dispersion length and nonlinear length are equal for both cases ($T_0 = 1$ fs, $P_0 = 1$ W, $|\beta_2| = 1$ ps²/m, and $\gamma = 1$ W⁻¹m⁻¹), creating a soliton with order $N = 1$.

begins to broaden by generating new frequency components. Frequencies near the leading side are then red shifted, while those of the trailing side are blue shifted. As the pulse is broadened, its intensity is decreased, reducing the effects of SPM. At a sufficiently long propagation distance, dispersion starts to play an increasingly influential role, reducing the phase shift $\phi_{NL} (= \gamma P_0 L)$ produced by the SPM. Therefore, the extent of nonlinear spectral broadening is limited in the normal dispersion regime. Figure 2.4(a) shows the propagation

of a Gaussian pulse with an input power $P = 1$ W and nonlinear parameter $\gamma = 1 \text{ W}^{-1}\text{m}^{-1}$, for a distance of $z = 5L_D$. This results in a maximum phase shift $\phi_{\max} = \gamma PL = 5 \approx 1.6\pi$, and therefore, 2 peaks were expected at the output. However, Fig. 2.4(a) shows that $\phi_{\max}$ is below π since most of the deep modulations are washed out, and the power spectra become substantially smoother.

SPM and Anomalous Dispersion: In the anomalous GVD regime, the dispersion induced chirp is negative [74]. Since the SPM creates positive chirping, the interplay between the SPM and anomalous dispersion is similar to the SPM of a down-chirped pulse. For $L_D = L_{NL}$, the two contributions of these opposite chirps nearly nullify each other at the central portion of the Gaussian pulse. Therefore, the generated spectrum is narrower than the SPM induced broadening in the absence of GVD, as observed in Fig. 2.4(b). The rate of pulse broadening is also much lower [74]. During propagation, the pulse shape adjusts itself to achieve the cancellation of opposite chirps, and the dispersion and SPM maintain a chirp-free pulse. This combination results in a stable or periodic evaluation of solitons [91].

2.2.3 Cross-phase Modulation (XPM)

When the input field consists of two or more distinct input waves, in addition to modulating the same pulse, one wave can modulate the phase of the other waves as well [92]. For two input fields $\mathbf{E}_1(z,t)$ and $\mathbf{E}_2(z,t)$, the wave equation becomes as follows [75]:

$$\frac{\partial (\mathbf{E}_1 + \mathbf{E}_2)}{\partial z} - \frac{j\beta_2}{2} \frac{\partial^2 (\mathbf{E}_1 + \mathbf{E}_2)}{\partial t^2} = -j\gamma |\mathbf{E}_1 + \mathbf{E}_2|^2 (\mathbf{E}_1 + \mathbf{E}_2) \quad (2.26)$$

Multiplying the nonlinear terms, we get coefficients for ω_1 , ω_2 , $2\omega_1 - \omega_2$, and $2\omega_2 - \omega_1$. The equations for ω_1 and ω_2 are as follows [75]:

$$\frac{\partial \mathbf{E}_1}{\partial z} - \frac{j\beta_2}{2} \frac{\partial^2 \mathbf{E}_1}{\partial t^2} = -j\gamma (|\mathbf{E}_1|^2 + 2|\mathbf{E}_2|^2) \mathbf{E}_1 \quad (2.27a)$$

$$\frac{\partial \mathbf{E}_2}{\partial z} - \frac{j\beta_2}{2} \frac{\partial^2 \mathbf{E}_2}{\partial t^2} = -j\gamma (|\mathbf{E}_2|^2 + 2|\mathbf{E}_1|^2) \mathbf{E}_2 \quad (2.27b)$$

It can be observed that, in addition to the intensity $|\mathbf{E}_1|^2$, the equation for ω_1 (Eq. (2.27a)) depends on the intensity $|\mathbf{E}_2|^2$ as well, and vice-versa. This time-dependent phase delay to the co-propagating pulse is termed as the cross-phase modulation (XPM).

2.2.4 Higher-Order Nonlinear Effects

For optical frequencies well below the electronic transitions (which typically fall in the ultra-violet spectrum for transparent media), the electronic contribution to the nonlinear index should have a response time in the order of the inverse of the typical transition frequencies [75]. Since this is faster than one optical cycle, the electronic contribution to the nonlinear index is considered approximately instantaneous. For optical frequencies well below the electronic transitions, nonlinear indices are affected by the electronic transitions as well. Therefore, for ultra-short pulses, in addition to the self and cross-phase modulation, other nonlinear effects, including self-steepening and Raman scattering, play a significant contribution. By considering these higher-order nonlinear effects, the wave equation becomes as following [74]:

$$\frac{\partial \mathbf{E}}{\partial z} + \sum_{m=2}^M i^{m-1} \frac{\beta_m}{m!} \frac{\partial^m \mathbf{E}}{\partial t^m} = j\gamma \left(1 + js \frac{\partial}{\partial T} \right) |\mathbf{E}|^2 \mathbf{E} - \tau_R \frac{\partial |\mathbf{E}|^2}{\partial T} \quad (2.28)$$

where the variables s and τ_R model the impacts of self-steepening and intra-pulse Raman scattering, respectively.

2.2.4.1 Self-steepening

The time derivative operator on the right-hand side of Eq. (2.28) results in an intensity-dependent group velocity; this phenomenon is known as self-steepening [93]. Due to the intensity dependency of group velocity, the peak moves at a slower speed than the wings. This leads to the shifting of the peak towards the trailing edge, and the pulse becomes asymmetric. The shifting of the peak also makes the trailing edge steeper, and this steepening continues with propagation until the pulse develops a region where the temporal intensity becomes discontinuous [75,94]. This

phenomenon is known as an optical shock front. This optical shock is characterized by the time scale of $\tau_{\text{shock}} = 1/\omega_0$. A more accurate definition of τ_{shock} includes the frequency dependency of the effective area A_{eff} [80, 95, 96], and the equation is as follows,

$$\tau_{\text{shock}} = \tau_0 + \frac{d}{d\omega} \left[\ln \frac{1}{n_{\text{eff}}(\omega) A_{\text{eff}}(\omega)} \right]_{\omega_0} \quad (2.29)$$

where $\tau_0 = 1/\omega_0$, and ω_0 is the pumping frequency.

2.2.4.2 Raman scattering

The nonlinear response of the nuclei of a medium to the intensity results in a vibration of the medium [75]. The resonant frequency of the vibrational motions of the nuclei is much lower and, since the time scale for the nonlinear response is dependent on the vibrational period, ultra-short pulses experience time-delayed or non-instantaneous vibrational contribution. These vibrations can be due to optical phonons or acoustical phonons. The vibration due to the optical phonon is termed as Raman scattering (while the other one is Brillouin scattering) [74].

When two distinct pulses with different frequencies travel in a Raman-active medium, the shorter frequency (the Stokes wave) experiences an optical amplification at the expense of the longer frequency pulse. The Raman gain depends on the corresponding shifting of stokes waves and can be significant when the stoke shift is of several THz. The Raman response function is modeled as follows [74, 95]:

$$R(t) = (1 - f_R)\delta(t) + f_R h_R(t) \quad (2.30)$$

where f_R is the fractional Raman contribution to nonlinear polarization, and h_R is set by the vibrations of material molecules induced by the optical field. Mathematically,

$$h_R(t) = (\tau_1^{-2} + \tau_2^{-2}) \tau_1 \exp\left(-\frac{t}{\tau_2}\right) \sin\left(\frac{t}{\tau_1}\right) \quad (2.31)$$

The values of τ_1 and τ_2 are determined experimentally.

Figure 2.5. Different orders of solitons generated in a nonlinear medium in the presence of anomalous GVD; Soliton of (a) order 2, (b) order 3 by increasing the peak power for a fixed full width at half maximum (FWHM), and (c) order 3 by increasing the FWHM with reduced peak power. The waveguide and pulse parameters used to generate the solitons are summarized in Table 2.1.

2.2.5 Soliton Generation

While propagating in the anomalous dispersion regime, the negative chirp due to dispersion cancels the positive chirp created from SPM, resulting in a stable or periodic evaluation of solitons [91]. The order of soliton N is proportional to the ratio of dispersion length L_D , and nonlinear length L_{NL} ; thus, it depends on both the pulse and the waveguide parameters. Mathematically,

$$N^2 = \frac{L_D}{L_{NL}} = \frac{T_0^2 \gamma P_0}{|\beta_2|} \quad (2.32)$$

where T_0 is the initial pulse width, and P_0 is the peak power of the input pulse. When $L_D = L_{NL}$, the generated positive and the negative chirp nearly nullify each other, creating a soliton of order $N = 1$. In this fundamental soliton, the spectral and temporal profiles remain unchanged, and they show a periodic behavior for the higher-order solitons ($N > 1$).

Figure 2.5 shows two different orders of solitons in a Si_3N_4 waveguide for a hyperbolic secant pulse; Fig. 2.5(a) shows a second-order soliton and Figs. 2.5(b-c) show solitons with third-order. In order to increase the soliton order from $N = 2$ to $N = 3$, the power P_0 is increased in Fig. 2.5(b), keeping the pulse duration T_0 the same, whereas in Fig. 2.5(c), the pulse duration T_0 is increased with a reduced

Table 2.1. Parameters of solitons evaluated in Fig. 2.5

Figure	β_2 (ps/nm/km)	γ (W ⁻¹ m ⁻¹)	α (dB/m)	Power (W)	FWHM (fs)	N	z_{sol} (mm)
Fig. 2.5(a)	0.051×10^{-24}	1.5763	0.45	160	50	2	28.45
Fig. 2.5(b)	0.051×10^{-24}	1.5763	0.45	361	50	3	28.45
Fig. 2.5(c)	0.051×10^{-24}	1.5763	0.45	90	100	3	99.4

P_0 . The waveguide properties, i.e., GVD $|\beta_2|$ and nonlinear parameter γ with a loss of 0.45 dB/m [97], are kept constant for each case. Table 2.1 summarizes the waveguide and pulse parameters used to generate the solitons shown in Fig. 2.5, along with the achieved soliton order and period. The soliton period z_{sol} depends on the dispersive length ($z_{sol} = \pi/2L_D$), i.e., the pulse width or the corresponding intensity full width at half maximum (FWHM). This is observed in Fig. 2.5(c); when the soliton order N is increased by expanding the FWHM, the soliton period is also increased, whereas in Fig. 2.5(b), the soliton period stays the same as the fundamental soliton, since increasing order with high power does not affect z_{sol} .

2.2.6 Soliton Perturbation: Raman Effects and Dispersive Wave Generation

Although an ideal soliton propagation is observed only under extremely stringent conditions, the injection of high-power pulses changes their propagation dynamics. Higher-order solitons are intrinsically unstable and, in the presence of perturbation, readily break-up or decay into a series of lower amplitude sub-pulses. This process is known as soliton fission. In the femtosecond regime, HOD and Raman scattering are the two most significant effects that perturb higher-order soliton propagation and induce this pulse break-up. The dominance of their contribution depends on the pulse-width; for pulses with pulse-width less than 20 fs, the break-up is created by the HOD, and for those with pulse-width greater than 200 fs, this break-up is due to the Raman effects [80]. The pulses with a pulse-width between 20-200 fs are perturbed by both effects. The effects of these perturbations are as the following:

1. A soliton propagating near the ZDW can transfer a portion of its energy to the

normal GVD regime to generate dispersive wave (DW) [98, 99], also known as Cherenkov radiation [33]. The position of the dispersive waves can be found using the following equation:

$$\omega_{DW} = \omega_{pump} + (\beta(\omega_{DW}) - \beta(\omega_{pump})) \times \nu_{g,pump} \quad (2.33)$$

where ω_{pump} is the angular frequency of pump, $\beta(\omega_{DW})$ is the propagation constant at the angular frequency of DW ω_{DW} , $\beta(\omega_{pump})$ is the propagation constant at ω_{pump} , and $\nu_{g,pump}$ is the group velocity at ω_{pump} . A DW can transfer the pump-energy to a wavelength away from the pump across a zero-dispersion wavelength [83, 84].

2. Even if perturbed by DW generation, solitons can experience continuous displacement to longer wavelength due to soliton self-frequency shift since the individual soliton bandwidths overlap the Raman gain [80]. This soliton self-frequency shift contributes significantly as well. The dynamics of the frequency shift ν_R can be related as $\frac{d\nu_R}{dz} \propto \frac{|\beta_2|}{T_0^4}$ [100].

Both of these effects can occur at the same time, and their interaction can be subtle. The propagation length where the fission occurs corresponds to a point where the evolving soliton reaches its maximum bandwidth ($L_{fiss} \propto \frac{L_D}{N}$) [80].

In this thesis, the expansion of the continuum is mainly attributed to the DW generation, which was possible due to the engineered dispersion profile. The contribution of the DW generation is explained in Fig. 2.6 using a soliton with an order $N = 7$. The soliton is generated assuming an input hyperbolic secant pulse with 100-fs FWHM and 50 pJ pulse-power. The soliton period is 85 mm, and a 200 mm long waveguide is chosen to show two periods of solitons. If we consider only the GVD and ignore the HOD, the soliton period is not perturbed, as observed in Fig. 2.6(a). However, when HOD is included in the NLSE, a DW is observed after ≈ 8 mm of wave propagation, as shown in Fig. 2.6(b). This DW perturbs the periodic soliton evolution resulting in soliton fission. Moreover, the pump-energy is also transferred across the DWs, expanding the bandwidth. If we consider a two-period length with the same properties used in Fig. 2.6(b), the effects of fission, resulting in a wideband supercontinuum can be observed in

Figure 2.6. (a) Propagation of a soliton with an order $N = 7$ and a period $z_{sol} = 85$ mm, in the absence of HOD. (b) Dispersive wave generation in the presence of HOD with a propagation length near 8 mm. (c) Expansion of bandwidth for a long propagation length up to 200 mm. A Gaussian pulse with a pulse-energy of 50 pJ and an FWHM of 100-fs is used as input for every case. The waveguide parameters (β_2 , γ , and α) are kept the same as in Table 2.1.

Fig. 2.6(c).

However, increasing the length is often not the best choice in integrated photonics, as it might introduce unwanted loss and dispersion issues. To avoid this and achieve wideband supercontinuum at the same time, the conventional approach is to increase the soliton order. The effects of each nonlinear property can be enhanced significantly by increasing the soliton order. Since the waveguide parameters depend on the medium itself and are somewhat limited, the approach is generally to use pulses with long FWHM and high power. The effects of using a pulse with long FWHM pulse and high power, as shown in Fig. 2.7, can be summarized as follows:

1. Figure 2.7(a) shows the effects of increasing pump power to increase the soliton order: the pulse energy is increased to 200 pJ, keeping the FWHM the same as 100-fs, and a soliton with order $N = 13$ is generated, increasing the bandwidth significantly for a short length of 10 mm.
2. Figures 2.7(b-c) show the effects of increasing FWHM to increase the soliton order: if the FWHM is increased to 200 fs for the same pulse energy of 200 pJ, the power required to achieve the same soliton order is reduced from 2 kW to 1 kW.

Figure 2.7. Effects of increasing soliton order in a 10 mm Si_3N_4 waveguide: the soliton order is increased by increasing (a) power (200 pJ 100 fs pulse with $N = 13$), (b) pulse FWHM (200 pJ 200 fs pulse with $N = 18$), (c) pulse FWHM (200 pJ 200 fs pulse with $N = 18$) with a longer propagation length of 20 mm, and (d) both power and FWHM (400 pJ 200 fs pulse with $N = 26$). The waveguide parameters (β_2 , γ , and α) are kept the same as in Table 2.1.

Since increasing T_0 results in an extended soliton period, the DW generation, in this case, takes a longer propagation length, as shown in Fig. 2.7(b). However, since soliton order is quadratically related to T_0 , the order becomes 18 for doubling the FWHM. The significance of increasing FWHM can be understood

more clearly for a longer propagation length. Figure 2.7(c) shows that with half of the power used to generate supercontinuum in Fig. 2.7(a), SC with an equal bandwidth can be generated.

3. Figure 2.7(d) shows the effects of increasing both the pump power and FWHM to increase the soliton order: A supercontinuum is generated in a 10 mm length Si_3N_4 waveguide with 400 pJ pulse energy and 200 fs FWHM.

2.3 Material Platforms for Supercontinuum

The strength of the nonlinear interplay depends on the nonlinear phase shift, $\phi_{\text{NL}} = \gamma PL$, where P and L are the pulse energy and the interaction length, respectively. The γ is the nonlinear parameter defined by $\gamma = (2\pi n_2)/(A_{\text{eff}}\lambda)$, where n_2 is the nonlinear index of the material, λ is the vacuum wavelength, and A_{eff} is the effective modal area. Thus, to increase the nonlinearity, either L , P , or γ should be increased. For example, in the case of a silica-based optical fiber, the nonlinear index is relatively low ($n_2 = 3 \times 10^{-20} \text{ m}^2\text{W}^{-1}$), requiring a long propagation length (e.g., a couple of meters) for an SCG [51,80,101–110]. Although silica has a low linear loss, selecting a long L often results in a large optical loss and dispersion issues. In order to avoid these issues, a high-energy input pulse can also be used to increase the nonlinear phase shift, but this often causes damage to the fiber ends [52,111]. Thus, to minimize the issues with a long L and a high P , it is often highly desired to increase the nonlinear parameter γ of the system.

The obvious approach to increase γ is to select a material with a high n_2 . Silicon (Si) has a relatively high nonlinear index ($n_2 = 5 \times 10^{-18} \text{ m}^2\text{W}^{-1}$), and a 3/10 octave-spanning SCG was observed in a 4.7 mm long Si waveguide pumping at $\lambda = 1300 \text{ nm}$ [112]. Despite having a relatively high n_2 , the extent of SCG in Si is limited at the telecommunication band due to the intrinsic two-photon absorption (TPA), and consequently, the free carrier absorption [113,114]. This led to investigating Si chip-based SCG in the mid-infrared region where the photon energies are close to the half bandgap of Si. Researches have shown that, in this regime, the effects of TPA are low [115,116], resulting in a broader spectrum [117] and DW generation [80], eventually leading to an octave-spanning

Table 2.2. Optical properties of different material platforms

Material	Refractive index	Nonlinear index (m^2W^{-1})	Transparency (μm)
SiO ₂ [110]	1.45	3×10^{-20}	0.2-4
Si [50]	3.47	5×10^{-18}	1.2-8
Si ₃ N ₄ [83]	2	2.5×10^{-19}	0.4–4.6
AlN [49]	2.12	2.3×10^{-19}	0.2–13.6
LiNbO ₃ [132]	2.21	1.8×10^{-19}	0.35-5
As ₂ S ₃ [121]	2.43	3.8×10^{-18}	0.6-11
As ₂ Se ₃ [141]	2.81	2.4×10^{-17}	1-14

supercontinuum [34, 50, 118]. Other material platforms such as indium gallium phosphide [119], chalcogenide [120–126], lithium niobate (LN) [127, 128] have also been investigated for on-chip SCG, and efficient SCs have been generated even though these platforms are not compatible with the current CMOS process. The recent advances on a low loss thin-film LN-on-insulator technology [129] enabled to use its excellent $\chi^{(2)}$ nonlinearity ($r_{33} = 31 \times 10^{-12}$ m/V) [130–132], a large nonlinear index ($n_2 = 2.5 \times 10^{-19}$ m²W⁻¹) [132], and a broad transparent window from 400 nm to 5000 nm [130] in a monolithically integrated platform. A coherent SCG spanning up to octave [133] and two octaves [132] have already been demonstrated using this thin film LN nanophotonic waveguides. Additionally, a variety of other material platforms like titanium dioxide [134, 135], silicon germanium [136, 137], germanium [138, 139], aluminum nitride [49, 140] have also been explored for SCG at different spectral bands. The optical properties of some of these materials evaluated at 1.55 μm are summarized in Table 2.2.

Silicon nitride (Si₃N₄) is one of the most widely used nonlinear platforms due to its low linear and nonlinear losses, wide transparent window covering visible to mid-IR region (400 nm–4600 nm) [83], and a moderately large $\chi^{(3)}$ nonlinearity ($n_2 = 2.5 \times 10^{-19}$ m²W⁻¹) [36, 83]. The large bandgap (≈ 5 eV) of Si₃N₄ [57] allows for nonlinear interactions that are free from TPA at near-infrared, and the moderately high refractive index of this material helps tight light confinement for a lower effective area A_{eff} , i.e., a higher γ . Leveraging all these properties, supercontinuum spectra broader than an octave have been explored

from visible to mid-infrared [35–47, 142]. The absence of substantial Raman gain in Si_3N_4 waveguides also assists in generating coherent supercontinua with higher nonlinearity and lower energies [42, 143].

CHAPTER 3

DISPERSION ENGINEERING

The demand for high-speed data communication and signal transmission is ubiquitous in our everyday life. With the advancement of technology, this demand is increasing exponentially, and continuous research has been going on to improve the various components of the optical communication system. The phenomenon that significantly limits from using optical systems to its full potential is dispersion [144, 145]. Dispersion is the spreading of the light pulse with time while it propagates through an optical fiber or system. Due to the spreading of light, the pulse is broadened as it travels, resulting in a signal overlapping. This overlapping results in distortion of signals and inter-symbol interference [146, 147] degrading the quality of communication. A longer bit interval is required to avoid this overlapping, which significantly limits the capacity of data transmission; this necessitates the dispersion engineering in optical fiber or system.

3.1 Different Techniques of Dispersion Engineering

The continuous research on dispersion compensation has provided various photonic structures over the years [148–152]. The oldest and most used approach in an optical communication system is dispersion compensation fiber (DCF) [152–154]. DCF is designed to have a negative dispersion coefficient, which helps to mitigate the current dispersion of the standard single-mode fiber. However, since this negative dispersion coefficient is very small, it requires a relatively long length of DCF [62, 151], which results in a bulky optical system with additional loss, crosstalk, and unwanted nonlinear effects. New categories of fibers like large effective area fiber (LEAF) [149, 150], reverse dispersion fiber (RDF) [155] are also used; however, dispersion in these fibers are also not zero.

Fiber Bragg grating (FBG) is another widely used approach [61, 156–158] for dispersion engineering. Fiber grating induces a periodic variation of the refractive index in an optical fiber; the grating is usually half the effective wavelength of the signal enabling first-order Bragg reflection. If a chirp is introduced in FBG, the

Figure 3.1. Effects of dispersion: (a) the gradual degradation of a pulse with time. The broadening of the pulse due to dispersion leads to overlapping of pulse, eventually making them undetectable with time. (b) The input and output signals in a dispersive fiber.

reflected pulse of the non-monochromatic light experiences a delay, resulting in different frequency components to be reflected at different depths [61, 157, 158]. By utilizing this wavelength-dependent delay, dispersion is engineered by optimizing the geometrical parameters. The re-compression of pulses in a long 200 m highly dispersive fiber ($D = 120 \text{ ps}^2/\text{km}$) [159], and even compensation of higher-order dispersion [160] are achieved using this approach. The additional benefit of this system is that the bandwidth can also be tuned by controlling the sign and magnitude of chirp [157], facilitating the accommodation of ultra-short pulses with high spectral content. Coupling of CBGs also helps to simultaneously compensate for GVD and reroute the compensated data [161]. Another similar approach is to use a long period grating to couple between the core and cladding mode [151]. However, the necessity of fiber coupler and circulator to avail the reflection often results in a complicated and unstable system [151, 161].

In addition to compensating dispersion, engineering dispersion property to achieve either normal or anomalous dispersion also plays an essential role in different fields of photonics, including ultrafast pulse processing [75, 162], soliton generation [163–165], and dispersive wave formation [99, 166, 167]. In integrated waveguides, the high-index contrast and tight mode confinement result in strong waveguide dispersion, which can be used to negate the inherent material dispersion.

This waveguide induced dispersion can be engineered by structural modification of the waveguides [60, 168–170], or by adding a new structure [50, 65–67]. In addition to waveguides, dispersion engineering is also widely practiced in microresonators [21, 171–174]. Through cavity enhancement, high- Q microresonators allow many vital applications, including a cascaded modulator [171, 172], frequency comb generation [173, 174], and harmonic generation [175, 176]. These applications also employ dispersion engineering in the microresonators by manipulating structural parameters [174, 177], coating different materials [178], precisely microstructuring the resonator boundary [179], or by adding an additional wedge angle [180].

3.2 Dispersion Engineering Using Mode Coupling

Mode coupling is a widely used approach to engineer dispersion. Many approaches described in section 3.1 use mode couplings to engineer dispersion. A mode-coupling appears when two different modes are close by and coupled together, resulting in symmetric and antisymmetric supermodes. The coupled modes force the symmetric and antisymmetric modes toward normal and anomalous dispersions, respectively. The degree of shift can be controlled by their coupling strength, which depends on the separation distance between the two modes. Such mode coupling can be formed between different mode orders [20, 69, 181], polarizations [70], bending radiuses [21, 72], and materials [64, 73].

In cases of the integrated waveguide photonics, dual-core slot waveguides [20, 62, 63], strip/slot hybrid waveguides [50, 65], bilayer waveguides [64] and even combination of different material waveguides [20, 66–68] have been used, where the dispersion is engineered through the concept of mode coupling. Slot waveguides provide an extra degree of freedom in tailoring dispersion [182–184]. The effective index of a slot waveguide varies rapidly with wavelength; at a longer wavelength, the effective index of the slot mode becomes closer to that of the substrate, resulting in a mode coupling and negative dispersion [185]. Since dispersion can be tuned by changing the group velocity of light in the slot and slabs, it can be easily manipulated by changing the dimensions, and even the material of the slabs and slot [186, 187]. Moreover, the bandwidth of the engineered dispersion can be increased by coupling two slot-waveguides [187], and even a slot and a strip

Figure 3.2. (a) Simulated effective refractive indices (n_{eff}) and (b) the corresponding dispersion profiles of each mode: independent TE_{slot} (orange dashed) and TE_0 (blue dashed) modes without coupling, and the coupled symmetric (orange solid) and antisymmetric (blue solid) modes. The inset shows the wavelength position of the avoided mode-crossing. The heights of the waveguides are set to 300 nm, and the widths of the waveguide with TE_{slot} and TE_0 mode are optimized as 805 nm and 1080 nm, respectively, with a gap of 525 nm in between them.

waveguide [188].

In this thesis, we started the waveguide array by coupling a TE_{slot} mode to the fundamental TE_0 mode. Figures 3.2(a) and 3.2(b) show the simulated n_{eff} and the corresponding dispersions, respectively, when we consider the mode coupling between the TE_{slot} and TE_0 modes only: independent TE_{slot} (orange dashed) and TE_0 (blue dashed) modes without coupling, and coupled symmetric (orange solid) and antisymmetric (blue solid) modes. Note that, in Fig. 3.2(a), the n_{eff} of the TE_{slot} mode (orange dashed) matches with that of the TE_0 mode (blue dashed) at a particular wavelength ($\lambda \approx 1420$ nm); once we locate these two modes together (here, with $g_{12} = 525$ nm), they couple together resulting in symmetric (orange solid) and antisymmetric (blue solid) modes. The coupled modes force the symmetric and antisymmetric modes toward normal ($D_\lambda < 0$, $D_\omega > 0$) and anomalous dispersions ($D_\lambda > 0$, $D_\omega < 0$), respectively; this property is observed in Fig. 3.2(b). Notice that the dispersion of the antisymmetric mode (blue solid-line) exhibits a near-zero dispersion (zoomed-in), while the other dispersions are highly normal. Since our goal is to shift the normal dispersion of a thin silicon nitride

waveguide mode to a near-zero dispersion, we will use this antisymmetric mode for our dispersion.

3.3 Mathematical Analysis of Mode Coupling

The mode coupling can be analyzed numerically by solving the equation for propagation constant and dispersion in a coupled mode [73]. The propagation constant $\beta(\omega)$ can be expanded using Taylor expansion, with respect to angular frequency ω , as follows,

$$\beta(\omega) = \beta(\omega_0) + (\omega - \omega_0) \frac{\partial \beta}{\partial \omega} + \frac{1}{2!} (\omega - \omega_0)^2 \frac{\partial^2 \beta}{\partial \omega^2} + \frac{1}{3!} (\omega - \omega_0)^3 \frac{\partial^3 \beta}{\partial \omega^3} + \dots \quad (3.1)$$

where ω_0 is the angular frequency at the mode-coupling. The first-order derivative of $\beta(\omega)$ is related to the group velocity as,

$$\nu = \left(\frac{\partial \beta}{\partial \omega} \right)^{-1} \quad (3.2)$$

and, as mentioned before, the second-order derivative of the $\beta(\omega)$ is the group velocity dispersion (GVD) D_ω . Plugging-in these values, and ignoring the third- and higher-order dispersions, we get,

$$\beta(\omega) \approx \beta(\omega_0) + \frac{\omega - \omega_0}{\nu} + \frac{D_\omega}{2} (\omega - \omega_0)^2 \quad (3.3)$$

When two isolated modes β_1 and β_2 are coupled, they create symmetric (β_+) and antisymmetric (β_-) supermodes. The propagation constants of the newly formed modes are as follows [73, 189]:

$$\beta_{\pm} = \frac{1}{2} (\beta_1 + \beta_2) \pm \sqrt{\frac{1}{4} (\beta_1 - \beta_2)^2 + |\kappa|^2} \quad (3.4)$$

where $|\kappa|$ is the coupling coefficient that depends on the distance between these two modes, and can be calculated as [189, 190],

$$\kappa = \frac{\omega \epsilon_0}{4} \int \Delta \epsilon(x, y) \mathbf{E}_1(x, y) \cdot \mathbf{E}_2^*(x, y) dx dy \quad (3.5)$$

where $\mathbf{E}_1(x, y)$ and $\mathbf{E}_2(x, y)$ are the normalized electric fields of each waveguide modes when they are isolated, and $\Delta\varepsilon(x, y)$ is the dielectric perturbation between them. Plugging-in the approximated β_1 and β_2 as Eq. (3.3) into Eq. (3.4), the propagation constants of the coupled mode become as,

$$\beta_{\pm}(\omega) = \frac{\beta_1(\omega_0) + \beta_2(\omega_0)}{2} + \frac{(\omega - \omega_0)}{2} \left(\frac{1}{\nu_1} + \frac{1}{\nu_2} \right) + \frac{(\omega - \omega_0)^2}{4} (D_{\omega_1} + D_{\omega_2}) \pm \sqrt{\frac{1}{4} \left[\left(\frac{1}{\nu_1} - \frac{1}{\nu_2} \right) (\omega - \omega_0) + \frac{(\omega - \omega_0)^2}{2} (D_{\omega_1} - D_{\omega_2}) \right]^2 + |\kappa|^2} \quad (3.6)$$

The GVD of the coupled mode D_{ω} can be achieved by taking the second-order derivative of Eq. (3.6) with respect to the angular frequency ω as the following:

$$D_{\omega}^{\pm} = \frac{\partial^2 \beta_{\pm}}{\partial \omega^2} = \frac{1}{2} (D_{\omega_1} + D_{\omega_2}) \pm \frac{1}{4|\kappa|} \left[\left(\frac{1}{\nu_1} - \frac{1}{\nu_2} \right) + (D_{\omega_1} - D_{\omega_2}) (\omega - \omega_0) \right]^2 (\tilde{\omega}^2 + 1)^{-\frac{3}{2}} \pm \frac{1}{4|\kappa|} (D_{\omega_1} - D_{\omega_2}) \tilde{\omega} (\tilde{\omega}^2 + 1)^{-\frac{1}{2}} \quad (3.7)$$

where $\tilde{\omega} = (\omega - \omega_0) / \delta\omega$, with the characteristic bandwidth of,

$$\delta\omega = 2|\kappa| \left| \left(\frac{1}{\nu_1} - \frac{1}{\nu_2} \right) + \frac{(D_{\omega_1} - D_{\omega_2})}{2} (\omega - \omega_0) \right|^{-1} \quad (3.8)$$

By assuming $D_{\omega_1} \approx D_{\omega_2} = D_{\omega_0}$ [63], Eqs. (3.7) and (3.8) can be approximated as:

$$D_{\omega}^{\pm} = \frac{\partial^2 \beta_{\pm}}{\partial \omega^2} \approx D_{\omega_0} \pm \frac{1}{4|\kappa|} \left(\frac{1}{\nu_1} - \frac{1}{\nu_2} \right)^2 (\tilde{\omega}^2 + 1)^{-\frac{3}{2}} \quad (3.9)$$

$$\delta\omega = 2|\kappa| \left| \frac{1}{\nu_1} - \frac{1}{\nu_2} \right|^{-1} \quad (3.10)$$

For the mode-coupling angular frequency $\omega = \omega_0$, the dispersions reach their peaks, and therefore, the maximum dispersion can be calculated by plugging $\tilde{\omega} = 0$ in Eq. (3.9), as follows,

$$D_{\omega, \max}^{\pm} = D_{\omega_0} \pm \frac{1}{4|\kappa|} \left(\frac{1}{\nu_1} - \frac{1}{\nu_2} \right)^2 \quad (3.11)$$

where c is the speed of light in vacuum, and n_g is the group index. In terms of wavelength,

$$D_\lambda^\pm = -\frac{\lambda}{c} \frac{\partial^2 n_{\text{eff}}}{\partial \lambda^2} = -\frac{2\pi c}{\lambda^2} D_\omega^\pm \quad (3.12)$$

Since the sign for the normal and anomalous dispersions are opposite for D_ω and D_λ , $D_\omega > 0$ and $D_\lambda < 0$ denote normal dispersions, and $D_\omega < 0$ and $D_\lambda > 0$ denote anomalous dispersions.

CHAPTER 4

MULTIPLE MODE COUPLINGS FOR BROADBAND NEAR-ZERO DISPERSION

In this chapter, the structural details to achieve a broadband near-zero dispersion (NZD) profile using a thin CMOS compatible Si_3N_4 waveguide array are presented, along with a discussion of the results obtained. Four mode-couplings are introduced on a 300 nm-thick silicon nitride, which shows highly normal dispersion unless otherwise engineered. The engineered dispersion profile achieves a near-zero dispersion (within ± 140 ps/nm/km) over the spectral range of 1350 – 1900 nm with eight ZDWs. Utilizing this dispersion profile and the high nonlinearity of the silicon nitride waveguide, we numerically examined the SC generation in Chapter 5. The closely spaced ZDWs help the generations of multiple dispersive waves (DWs), enabling octave-spanning SCs by pumping at both normal and anomalous dispersion regions in the near-infrared.

4.1 Structure for Multiple Mode Couplings

The cross-section of the proposed structure, along with the geometric parameters, is presented in Fig. 4.1. An array of five silicon nitride waveguides (blue) are cladded in SiO_2 (gray). The height of the waveguides is set to $h = 300$ nm to leverage the CMOS compatibility. The waveguide array starts with a single slot waveguide, formed by the first two silicon nitride waveguides (with w_1 - g_{11} - w_1 at the leftmost side), followed by a series of waveguides with a gradual increment in their width. The slot waveguide mode TE_{slot} has a faster group velocity compared to the other transverse electric (TE) modes and helps to achieve a sharp rise in dispersion. The widths of the waveguides after the slot waveguide are increased one after the other so that a consecutive order of modes can be assigned. The fundamental TE (TE_0) mode is placed in the waveguide with the width w_2 , then the first-order TE (TE_1) mode in the waveguide with w_3 , followed by the second-order TE (TE_2) mode in the waveguide with w_4 , and lastly, the third-order TE (TE_3) mode in the waveguide with w_5 [181].

Figure 4.1. Schematic cross-section of the proposed structure (blue: Si_3N_4 waveguide array, gray: SiO_2 cladding). The thickness of the waveguides is set to $h = 300$ nm. The widths w_i are optimized to $w_1 = 805$ nm, $w_2 = 1080$ nm, $w_3 = 2560$ nm, $w_4 = 4000$ nm, and $w_5 = 5510$ nm, with gap sizes g_{ij} as $g_{11} = 120$ nm, $g_{12} = 525$ nm, $g_{23} = 750$ nm, $g_{34} = 1620$ nm, and $g_{45} = 2240$ nm.

4.2 Engineering the Geometric Parameters

The widths of the waveguides are optimized so that each mode of waveguide couples to the nearest waveguide modes at distinct wavelengths. The peak dispersions $D_{peak}^\pm$ and the bandwidth $\delta\omega$ of the coupled symmetric (D_ω^+) and antisymmetric (D_ω^-) modes, derived in Chapter 3, are as follows,

$$D_{peak}^\pm = D_0 \pm \frac{1}{4|\kappa|} \left(\frac{1}{\nu_1} - \frac{1}{\nu_2} \right)^2 \quad (4.1)$$

$$\delta\omega = 2|\kappa| \left| \frac{1}{\nu_1} - \frac{1}{\nu_2} \right|^{-1} \quad (4.2)$$

where ν_1 and ν_2 are the group velocity of the two separate modes without coupling, and κ is the coupling coefficient between them. The group velocity dispersion (GVD) of the two independent modes are assumed to be equal to D_0 . Notice that, in Eq. (4.1), the magnitudes of the peak dispersions are inversely proportional to the coupling coefficient κ . This κ can be tuned by adjusting the gap size between the two modes; i.e., a larger gap size reduces κ , increasing the magnitude of peak dispersions, and it is the opposite for a smaller gap size. Also, there exists a trade-off between the peak values in dispersion (Eq. (4.1)) and the bandwidth (Eq. (4.2)); i.e., a weaker coupling induces a higher dispersion peak but a narrower bandwidth, and it is the opposite for a stronger coupling. Since our goal is to achieve a broadband NZD to overcome the highly-normal dispersion of a thin silicon nitride waveguide at near-infrared, we used coupling coefficients that are small enough to show slight anomalous dispersion but as large as possible to achieve

a broader bandwidth. To extend the near-zero dispersion bands further, we also introduce mode couplings multiple times at different wavelengths by cascading waveguides in an array.

To engineer the specific waveguide widths, we first fixed the geometric parameter for the slot waveguide ($w_1 = 805$ nm and $g_{11} = 120$ nm). Then the n_{eff} of TE_{slot} mode is plotted as a function of wavelength, along with the n_{eff} of the TE_0 mode with different widths [181]. The n_{eff} profile of the TE_{slot} mode intersects (i.e., gets matched) with the n_{eff} profile of the TE_0 mode allotted in different waveguide widths at different wavelengths. Therefore, if one fixes the desired coupling wavelength, the width required for the coupling can be found. As our goal is to engineer dispersion at near-infrared, we selected the first coupling wavelength as $\lambda_1 \approx 1420$ nm. The width of the waveguide containing the TE_0 mode for which the n_{eff} is equal to that of the TE_{slot} mode is found to be $w_2 = 1080$ nm. If a coupling wavelength is required to be less than 1420 nm, one can simply reduce the waveguide width to match the n_{eff} of the TE_0 mode at a shorter wavelength. Similarly, to achieve the mode coupling at a wavelength longer than 1420 nm, one can increase the w_2 . Figure 4.2(a) shows the dispersion profile of the antisymmetric mode by coupling the TE_{slot} and TE_0 modes; it shows a near-zero dispersion near the coupling wavelength $\lambda_1 = 1420$ nm. The gap size g_{12} is optimized to introduce the slightly anomalous but close to near-zero dispersion. The same approach is followed to determine the widths and gaps of the next waveguides. In Fig. 4.2(a), the dispersion turns back to the normal region after the coupling wavelength. In order to leverage this regime to the near-zero dispersion, the next coupling is introduced at $\lambda_2 = 1605$ nm by coupling the TE_0 and TE_1 modes. At this wavelength, the n_{eff} of the TE_0 mode (at $w_2 = 1080$ nm) matches that of the TE_1 mode at $w_3 = 2560$ nm. This mode coupling results in the second peak of the dispersion profile, as observed in Fig. 4.2(b).

The steps are repeated to add two additional mode couplings at $\lambda_3 = 1770$ nm and $\lambda_4 = 1870$ nm, and the optimized waveguide widths for the TE_2 and TE_3 modes are found as $w_4 = 4000$ nm and $w_5 = 5510$ nm, respectively, with the gap sizes of $g_{34} = 1620$ nm and $g_{45} = 2240$ nm. The evolutionary dispersion profiles with the introduction of additional TE_2 and TE_3 modes are shown in Figs. 4.2(c)

Figure 4.2. The gradual evolution of near-zero dispersion profile by cascading waveguides in an array: mode couplings between (a) TE_{slot} and TE_0 , (b) $\text{TE}_{\text{slot}}-\text{TE}_0$ and TE_1 , (c) $\text{TE}_{\text{slot}}-\text{TE}_0-\text{TE}_1$ and TE_2 , and (d) $\text{TE}_{\text{slot}}-\text{TE}_0-\text{TE}_1-\text{TE}_2$ and TE_3 . Inset schematics represent the cascaded waveguide array. Detailed geometric parameters are the same as in Fig. 4.1.

and 4.2(d), respectively. Figure 4.3(a-d) show the mode profiles ($|\mathbf{E}|$) at each mode coupling wavelength, and clearly show the couplings between different orders of modes.

4.3 Dispersion Profile of the Waveguide Array

Figure 4.4 shows the engineered dispersion profile achieved from the multiple mode couplings along with the dispersion of each mode when they are isolated. The blue solid-line in Fig. 4.4(b) shows the final dispersion. The black circles denote

Figure 4.3. Normalized electric fields ($|\mathbf{E}|$) of the coupled modes at different coupling wavelengths: (a) $\lambda_1=1420$ nm, (b) $\lambda_2 = 1605$ nm, (c) $\lambda_3 = 1770$ nm, and (d) $\lambda_4 = 1870$ nm.

the positions of the ZDWs, and eight ZDWs have been obtained. The dispersion profiles of each mode, when they are isolated, are also plotted to compare: TE_0 (orange), TE_1 (purple), TE_2 (green), and TE_3 (cyan). The discontinuities in TE_0 and TE_1 are due to the modal cut-off at longer wavelengths. Notice that, when each waveguide is separated, every mode shows a highly normal dispersion (< -300 ps/nm/km) in the given spectral region. Due to this high normal dispersion, thin ($h < 300$ nm) silicon nitride waveguides are not suitable for many nonlinear applications such as SC and Kerr combs generation at NIR. Here the multiple mode couplings can result in multiples of the anomalous dispersion peaks that lead to a broadband NZD profile, as shown in Fig. 4.4. For the excitation of a specific mode, we can use an adiabatic mode transition to effectively couple to the desired pump mode [14, 21].

Figure 4.4. Overall dispersion of the optimized structure with four mode-couplings (blue solid). The dashed black line denotes the zero-dispersion, and the circles indicate the ZDWs. The dispersions of individual modes, when they are isolated, are also plotted: TE_0 (orange), TE_1 (purple), TE_2 (green), and TE_3 (cyan).

4.4 Effect of Coupling Coefficient

The coupling coefficient plays a vital role in determining the value of the dispersion. From Eq. (4.1), we can see that increasing the gap, i.e., reducing the coupling strength, increases the dispersion peak. This effect is observed in Fig. 4.5(a). Therefore, by engineering the gap size, this configuration can also be used for the applications requiring high anomalous dispersion [21]. The near-zero dispersion with multiple ZDWs is maintained even after changing the gap size up to ± 20 nm. However, changing the gap between the two modes affect the total dispersion profile. Figure 4.5(a-d) shows the effect of changing different gap sizes, g_{12} , g_{23} , g_{34} , and g_{45} up to ± 20 nm from the respective g_{ij} mentioned in Fig. 4.1. It is observed that although changing the gap between any two modes affect the coupling strength of the other mode coupling, near-zero broadband dispersion is maintained throughout all the cases.

Figure 4.5. Dispersion profiles showing the effect of the coupling strength by changing the gap sizes: (a) $g_{12} = 505 - 545$ nm, (b) $g_{23} = 730 - 770$ nm, (c) $g_{34} = 1600 - 1640$ nm, and (d) $g_{45} = 2220 - 2240$ nm. The other geometric parameters are the same as in Fig. 4.1.

CHAPTER 5

BROADBAND SUPERCONTINUUM GENERATION VIA MULTIPLE MODE COUPLINGS

5.1 Integrated Dispersions and Dispersive Waves

In order to extend the bandwidth of an SCG, the generation of dispersive waves (DWs) is highly desired [80, 83]. A DW generation, also commonly known as Cherenkov radiation [33], results from a phase-matching in the presence of higher-order dispersions, and it can transfer the energy to a wavelength away from the pump across the ZDW [83, 84]. The presence of more than one ZDW allows multiple DWs to appear and span further the SC bandwidth [51, 191]. In our case, eight ZDWs would generate up to four DWs.

The spectral position of a DW can be estimated by evaluating the integrated dispersion β_{int} ; the DWs appear near the wavelength where $\beta_{\text{int}} = 0$. The β_{int} is defined as [38],

$$\beta_{\text{int}} = \beta(\omega) - \beta(\omega_p) - (\omega - \omega_p)\beta'(\omega_p) \quad (5.1)$$

where $\beta(\omega_p)$ and $\beta'(\omega_p)$ are the propagation constant and the first-order derivative of the β , respectively, at the pumping frequency ω_p . Since for dispersive waves, $\beta_{\text{int}} = 0$, the angular frequency of a DW can be estimated by setting Eq. (5.1) equal to 0, and we get,

$$\omega_{\text{DW}} = \omega_p + (\beta(\omega_{\text{DW}}) - \beta(\omega_p)) \times \nu_{g,p} \quad (5.2)$$

where $\nu_{g,p}$ is the group velocity at the pump frequency, and $\beta(\omega_{\text{DW}})$ is the propagation constant of the DW. To estimate the spectral locations of DWs and the corresponding SC bandwidth of our coupled waveguide array, we swept the pump wavelength near the mode coupling wavelengths and plotted the map of β_{int} in Fig. 5.1(a); the y-axis indicates the pump wavelength λ_p , and the x-axis shows the corresponding spectrum of the β_{int} . The white line in the diagonal indicates the pump wavelength, where $\beta_{\text{int}} = 0$. The other white lines or the intersection between blue and orange colors represent the spectral locations of the phase-matched

Figure 5.1. (a) Map of the integrated dispersion β_{int} as functions of the spectrum λ (x -axis) and the pump wavelength λ_p (y -axis). The diagonal white line indicates the pump wavelength. The rest of the white lines (i.e., discontinuity of blue and orange colors) correspond to the wavelengths at which the $\beta_{\text{int}} = 0$ and dispersive waves are generated. (b) Dispersion curve of the waveguide array. The anomalous dispersion region is shaded with green color. The black solid-line denotes the zero-dispersion, and the dashed lines are the spectral positions of the zero-dispersion wavelengths (ZDWs).

DWs, i.e., $\beta_{\text{int}} = 0$. The dispersion profile of the waveguide array is also plotted in Fig. 5.1(b), guiding the dispersion regimes of the pump wavelength λ_p . The anomalous dispersion regime ($D_\lambda > 0$) is shaded with green color, and the other white background is the normal dispersion regime ($D_\lambda < 0$). Notice that, in Fig. 5.1(a), there are a couple of regimes where four DWs are observed. These regimes are located near the ZDWs in both normal and anomalous dispersions and are expected to generate SCs efficiently.

5.2 Supercontinuum Generation

To examine the effectiveness of the SCG with the dispersion engineered waveguide array, we numerically simulated the SCG using the generalized Nonlinear Schrödinger Equation (GNLE); this equation has been widely used to examine SC generations in different types of waveguides [35, 36, 45–47], including waveguide arrays [39, 48, 49], addressing experimental results fairly accurately. The Raman effect in Si_3N_4 waveguides is considered to be very weak [42, 143], and has not been included in the numerical model. Therefore, the GNLE we used to model the SC takes the form as, [47, 80]

$$\left(\frac{\partial}{\partial z} + \frac{\alpha}{2} - j \sum_{k>2} j^k \frac{\beta_k}{k!} \frac{\partial^k}{\partial t^k} \right) \mathbf{E}(z, t) = j\gamma \left(1 + j\tau_{\text{shock}} \frac{\partial}{\partial t} \right) |\mathbf{E}(z, t)|^2 \mathbf{E}(z, t) \quad (5.3)$$

where α is the propagation loss, β_k is the k -th order chromatic dispersion derived from the Taylor expansion of the wavenumber β , and γ is the modal nonlinear parameter. The time derivative term models the self-steepening and optical shock formation, which are characterized by τ_{shock} . Addressing the frequency-dependent A_{eff} in our waveguide array, a correction has been made to the τ_{shock} , as follows [80],

$$\tau_{\text{shock}} = \tau_p + \frac{d}{d\omega} \left[\ln \frac{1}{n_{\text{eff}}(\omega) A_{\text{eff}}(\omega)} \right]_{\omega_p} \quad (5.4)$$

where $\tau_p = 1/\omega_p$, with the angular pumping frequency ω_p . The $A_{\text{eff}}(\omega)$ was obtained through the modal simulation. The NLSE was numerically solved using the split-step algorithm, assuming an input hyperbolic secant pulse with a full width at half maximum (FWHM) of 100-fs. We pumped in both normal and anomalous regions, considering a 0.45 dB/m propagation loss [97]. The waveguide length is set to 10 mm to ensure the coherency.

5.2.1 Supercontinuum Generation by Pumping in the Anomalous Dispersion Region

The dispersion of our waveguide array has four anomalous regions, and we observed SCGs by pumping in each of them at $\lambda_p^{a_i}$; the superscript a_i indicates the i -th anomalous region in the dispersion curve. Figures 5.2(a-d) show the SC

Figure 5.2. Supercontinuum (SC) generation by pumping in the different anomalous dispersion regions. SC spectra pumping at (a) $\lambda_p^{a1} = 1450$ nm, (b) $\lambda_p^{a2} = 1560$ nm, (c) $\lambda_p^{a3} = 1750$ nm, and (d) $\lambda_p^{a4} = 1860$ nm. Each pump wavelength is marked with a vertical dashed line. The integrated dispersion β_{int} (blue) of each pump is plotted on the top of each panel of (a-d), together with the dispersion curve (gray). The waveguide lengths are set to 10 mm, and the pump energy is marked on each panel.

spectra by pumping in four different anomalous dispersion regions. The pump wavelength is marked by a dashed vertical line in every case. The β_{int} profiles pumping at each λ_p are plotted on the top panel of each Fig. 5.2(a-d) (with blue lines); they depict the positions of DWs, i.e., when $\beta_{\text{int}} = 0$. The gray line on the same panel shows the dispersion profile and guides the dispersion of the pump wavelength.

Figure 5.2(a) shows SCG with $\lambda_p^{a_1} = 1450$ nm; here, two DWs are observed simultaneously at around 190 pJ. The DW at a longer wavelength is away from the pump and required a higher power to transfer the energy across that DW. Ripples observed at a shorter wavelength are attributed to the soliton fission effect, by which the injected pulse is divided into distinct solitons leading to spectral interference [38]. With pump energy of 350 pJ, the SC spanned to a near-octave. In Fig. 5.2(b), we chose to pump near the ZDW at $\lambda_p^{a_2} = 1560$ nm, which would allow the extended interaction length for a broader SCG [51]. Three distinct DWs are observed; one of them is at a wavelength longer than λ_p . With the multiple DWs near the pump, an efficient SC covering >1000 nm is generated with a low pump-energy of 200 pJ.

In Fig. 5.2(c), the pump wavelength is also close to the ZDW at $\lambda_p^{a_3} = 1750$ nm. With pump energy as low as 60 pJ, a DW is generated at the shorter wavelength. However, even with a pump as high as 350 pJ, the DW in the longer wavelength is not seen. This is because of the poor nonlinearity at longer wavelengths. The nonlinear parameter γ is inversely proportional to the effective area A_{eff} , which increases with a higher-order mode. At a longer wavelength, the near-zero dispersion is induced with the TE_3 mode, which has a relatively high A_{eff} ; thus, in turn, γ decreases with a poor SCG efficiency. For the same reason, the conversion efficiency of the SC pumping at $\lambda_p^{a_4} = 1860$ nm is quite poor (Fig. 5.2(d)); even with the input pulse energy of 300 pJ, the energy is not transferred to the DW in the short wavelength region. With pulse energy of 500 pJ, a prominent DW peak begins to be observed; then, pump energy as high as 1000 pJ is required to see an octave-spanning SC spectrum.

5.2.2 Supercontinuum Generation by Pumping in the Normal Dispersion Region

Pumping in the normal dispersion region leads to a coherent SC since the spectral broadening does not rely on soliton dynamics [83]; this is highly desired in self-referenced comb offset stabilization [192, 193]. However, pumping in a normal regime results in a temporal broadening and rapid reduction of peak power, limiting the extent of nonlinear broadening [88]. In our case, the ZDWs are placed closely and quickly transfer the pump energy to the anomalous regime, which in turn

Figure 5.3. Supercontinuum (SC) generations by pumping in the different normal dispersion regions. SC spectra pumping at (a) $\lambda_p^{n_1} = 1350$ nm, (b) $\lambda_p^{n_2} = 1525$ nm, (c) $\lambda_p^{n_3} = 1710$ nm, and (d) $\lambda_p^{n_4} = 1840$ nm. Each pump wavelength is marked with a vertical dashed line. The integrated dispersion β_{int} (blue) of each pump is plotted on the top of each panel of (a-d), together with the dispersion curve (gray). The waveguide lengths are set to 10 mm, and the pump energy is marked on each panel.

creates a DW back in the normal dispersion regions away from the pump [194–196]. Figures 5.3(a-d) show the SC spectra pumping at wavelengths of different normal dispersion regimes: (a) $\lambda_p^{n_1} = 1350$ nm, (b) $\lambda_p^{n_2} = 1525$ nm, (c) $\lambda_p^{n_3} = 1710$ nm, and (d) $\lambda_p^{n_4} = 1840$ nm. The superscript n_i indicates the i -th normal region in the dispersion curve.

Figure 5.3(a) shows SCG by pumping in a highly normal dispersion regime (-250 ps/nm/km). Two DWs are expected according to the β_{int} curve; however, only one DW near the pump is observed, due to the low nonlinearity at a longer wavelength regime. Notice that, even with high input energy, no ripples are observed in the generated SC spectrum, which indicates the absence of soliton fission. For the second case, as in Fig. 5.3(b), there are four DWs, two on each side of the pump. Two of the DWs are close to the pump and are not distinguishable in the SC spectrum. With a 30 pJ pump-energy, a prominent peak is observed at ≈ 2000 nm, indicating the DW generation. With an increased pump, the spectrum becomes flattened with intermediate frequency components, and the SC spectrum extends to almost an octave (from 1100 nm to 2180 nm).

Figure 5.3(c) shows a similar trend; however, the DW at the longer wavelength was not prominent, due to a lower nonlinearity in this regime. For the last case, in Fig. 5.3(d), a DW away from the pump is observed with a 200 pJ pump, but higher pump energy (400 pJ) is required to fill the spectral gap between the pump and the DW; again, this is due to the lower effective nonlinearity at a longer wavelength. Increasing the number of mode couplings would expand the bandwidth of flattened near-zero dispersion regions and increase the SC bandwidth. However, one may need to be careful in choosing the pump wavelength, as this scheme has a lower effective nonlinearity at a longer wavelength regime. The NLSE simulations suggest that pumping in near the ZDW, which is located in the slightly shorter wavelength regime of the near-zero dispersion band, generates the SCs most efficiently (e.g., as in the cases of Figs. 5.2(b) and 5.3(b)).

CHAPTER 6

CONCLUSION

Silicon photonics is considered the building block of the integrated photonics due to its large bandwidth, low power consumption, and full compatibility with the current complementary metal-oxide-semiconductor (CMOS) foundry [197, 198]. The large refractive index contrast between the silicon core and low indexed claddings (e.g., silica or air) has enabled confinement of optical fields within the chip, resulting in a compact integration of photonic components [199]. However, this optical confinement often results in a rapidly changing waveguide dispersion, making it difficult to obtain a broadband near-zero dispersion profile [186]. In this thesis, broadband near-zero dispersion is achieved in a CMOS compatible 300 nm-thick Si_3N_4 film with a base dispersion profile that is highly normal and unsuitable for nonlinear frequency conversions unless otherwise engineered. The achieved accomplishments are as follows:

1. The concept of mode coupling is used to engineer the dispersion by coupling multiple different-order modes. The coupling coefficients are designed to be strong for a broader bandwidth but are weak enough to show slight anomalous dispersions. The coupling is repeated between different orders of modes at different wavelengths to extend the bandwidth, and a broadband near-zero dispersion profile within ± 140 ps/nm/km is achieved, spanning over the spectral range of 1350 – 1900 nm.
2. The presence of closely spaced zero dispersion wavelengths (ZDWs) in the designed dispersion profile assists in the phase-matching process and dispersive wave generation, enabling wideband nonlinear frequency conversions. Eight ZDWs are achieved from the four mode-couplings, and up to four DWs can be generated for a pump wavelength near the ZDWs.
3. Using the engineered dispersion profile, SCs are generated in the near-infrared spectral region, overcoming the strong normal material dispersion due to the proximity to the material bandgap [200]. SC covering in the near-infrared has

potential applications such as bio-imaging [201,202] and Raman spectroscopy [201,203] since tissue and cells possess low absorption and scattering coefficients in this spectral window [47,204].

6.1 Future Work

The concept of utilizing multiple mode couplings to achieve a broadband near-zero dispersion applies to other spectral regimes as well, by engineering the structural parameters to shift the position of the ZDWs. Easy optimization of the location of ZDWs is another bright side of this approach; the dispersion profile having a ZDW at a distant wavelength can create a long wave DW generation, resulting in a broader SC expanding up to the mid-IR region [34–39]. Since many molecules have strong vibrational transitions in the mid-IR region, this holds great potential in molecular spectroscopy, opening doors for pharmaceutical and environmental analysis [37–39,205]. However, generating SC expanding up to mid-IR region is still a challenge in the 300 nm-thick Si_3N_4 platforms as the waveguide dispersion approaches the waveguide cut-off region. But this problem can be overcome by using a thicker Si_3N_4 . A recent demonstration has shown that, with 900 nm-thick Si_3N_4 , coupling the fundamental TE_{00} and TE_{10} modes can help to increase the bandwidth of dispersion profile by 1000 nm, assisting a 3500 nm wide SC expanding from 500-4000 nm [39]. This approach can be extended to multiple mode couplings using our technique. If the confinement loss due to the cladding absorption in the mid-IR region can be overcome, this will assist in SCG beyond 4 μm .

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