A COLLEGE APPROACH TO FRACTALS

IN MIDDLE SCHOOL

by

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ABSTRACT

The algebra and geometry of complex numbers were presented to eighth and ninth grade mathematics classes. The purpose of the presentations was to determine if this college level mathematics would have an influence on the algebra and geometry skills of the K-12 students. Pre and post surveys were employed. Results showed an increase in both student mathematics skills and student interest in the ninth grade class. In the eighth grade class there was not a significant improvement. It may be conjectured that ninth grade students benefit from this kind of intervention, but that the average eighth grade student does not have the mathematical skills required to handle the college level material.

CHAPTER I
INTRODUCTION

The study was implemented with two goals in mind. The first goal was to improve students abilities in the areas of algebra and geometry. The second goal was to improve their attitudes and feelings toward mathematics. College level mathematics were introduced, specifically in the area of complex analysis. The improvement of student abilities was of utmost importance. Achievement of American middle school students is low. [6] Many students arrive at college poorly prepared for college algebra. They lack a solid understanding of fractions, and of operations involving fractions. This a very basic, but extremely important, building block for all of mathematics. Shortcomings exist in other areas as well, such as operations involving radicals and fractional exponents. This study hoped to alleviate some of those difficulties for the students involved.

The concept of outreach is explained in great detail in the literature review. It involves a collaboration between institutions of higher learning and K-12 schools. This collaboration takes place at the K-12 school. Professors and graduate students visit the classroom, either during or after school. Sometimes lessons are presented, other times there is just an interaction with perhaps a challenging math game involved. The benefits of these interactions have also been explored with greater detail in the literature review in this paper.

This study was aimed toward middle school students. There are eight middle schools in the City of Lubbock. The Lubbock Independent School District website gives figures for the demographics of the whole district. The school district population has the following apportionment: Anglo/Other - 40%, Hispanic - 45%, and African-American - 15%. The demographics of the junior high schools that were visited were noted also. The school where the ninth grade class was held was 87.8% Hispanic, 6.7% White/Other, and 6.5% African-American. The eighth grade class was form a school that was 49.7% Hispanic, 40.9% White/Other, and 9.3% African-American.

There were fourteen students in the ninth grade class, seven boys and seven girls. The eighth grade class had seventeen students, seven boys and ten girls.

Pre and post tests in the areas of algebra and geometry skills and mathematics attitudes were administered on the first and last days of the study. A series of seven lessons covering topics in complex analysis were presented to the students. These lessons were fifteen to twenty minutes in length and were presented daily. On the next to last day of this study the students were allowed to go the computer lab and play a challenging math game. In all, the students were visited for ten consecutive school days. The study was presented to a ninth grade class first, and then to an eighth grade class.

Again the purpose of this study was to increase their algebra and geometry skills and improve their attitudes toward mathematics. Our objectives were to not only measure quantitatively the change that took place over the two week period in skills and attitudes, but also to notice qualitative aspects such as student attitudes during the lesson, student participation, and the methods with which the student used to solve problems presented both verbally and on the written test.

There has been another similar study done, by Leah Chenault, a Texas Tech graduate student. A summary of her study, along with other related issues, appears in the literature review. The procedures chapter gives even more detail about the specifics of this study and how it was conducted.

A thorough description of the items in the problem set follows the procedures chapter. Each lesson is then described as does an explanation of the interest inventory. Statistical analyses of both the problem set and the interest inventory are listed in great detail. Analysis was done on both the overall results and individual items. Conclusions and suggestions for future studies conclude this study.

CHAPTER II
LITERATURE REVIEW

A search of the literature in the field of mathematics education did not provide any studies directly relating to the introduction of college level mathematics in K-12 schools to motivate or excite students. There were also no studies that aspired to increase students mathematical abilities by introducing more complex material. A study done by a Texas Tech graduate student in the spring of 2005 did not come up in the literature search. The study was done by Leah Chenault, titled "Improving the Algebra and Geometry Skills of High School Students Using Complex Variables and Complex Transformations." The study was performed at a Lubbock area low-income high school. Lessons were presented to twenty nine algebra and geometry students concerning transformations in the complex plane, material that is usually not covered until a student attends college. Pre and post skills tests were administered, and the increases in student ability were significant. Pre and post interest surveys were also administered, with the result showing a very modest increase in student attitude toward mathematics.

Since the goal of this study was to increase student ability and improve student attitudes toward mathematics, and this was attempted using college intervention, we will take a look at the roles that departments of mathematics at institutions of higher education play in K-12 education. Byrne states that engagement or outreach will be the defining characteristic of the university of the future. [2] On page 127 of their article, Cohen, Raudenbush, and Ball state that "Teachers and students are more likely to exert themselves if schools are linked to institutions of higher education." [3] Thus the presence of Texas Tech University graduate students and professors was intended to create a positive presence and to motivate the students to learn.

The University of Tennessee started an outreach program. Outreach "is loosely defined as any activity that enhances the teaching and learning of mathematics outside the department, in particular in K-12 education and community colleges." [6]

The program endeavored to interact with area schools, impact the K-12 mathematics content, and examine how future mathematics teachers should be taught. [5] Lower-level undergraduates were sent to assist in K-12 mathematics classrooms to increase the visibility of the mathematics department. [6] The classroom teacher, the mathematician, and the undergraduate student all should work together and strive to help ensure the success of the mathematics students. It is important to note that this collaboration is just that, a collaboration. It is a voluntary relationship. The graduate student or professor who enters the classroom is not judging the teacher, only trying to supplement, assist, and work in a partnership for the greater benefit of the students. Conway, Davis and Dwyer reported some exciting and promising results generated by this outreach program. [4] An invitation to serve on the textbook committee for the Knoxville area was one result. Sponsorship and increased participation in the Tennessee Math Contest was another. Another great result was an increase in enrollment in mathematics educations programs at the University of Tennessee. These successes have also sparked similar interests at other mathematics departments, including the University of Kentucky, Iowa State University, and Texas Tech University.

Voluntary summer programs, such as the Texas Pre-freshman Engineering Program (TexPREP), are another example of K-12 students working with colleges. It is available to high school and junior high school students. According to the web page sponsored by TexPREP-Lubbock, the students who participate in this program are high-achieving students with an interest in science, mathematics, and engineering. [15] Special attention must be made so that outreach programs are not restricted to students who meet certain criteria, but must be available to all students. The outreach programs at the University of Tennessee and at Texas Tech University are examples of programs that are available to all students.

Other attempts to improve mathematics education involve curriculum changes, improvements in teacher education, and improving or revising national testing standards. One of the more well known policies is the No Child Left Behind Act of

2001. In order to effectively enforce this act, accountability became a major focus. [12] States and school districts began curriculum and standards changes in order to comply with this federal legislation. This in turn prompted school districts to more closely look at their teacher training standards in order to also ensure compliance. One of the methods of measuring accountability are standardized tests. In Texas, the Texas Assessment of Knowledge and Skills (TAKS) test is an example of such a test. The corresponding curriculum, the Texas Essential Knowledge and Skills (TEKS) was developed and refined in order to better prepare students for both college level mathematics and life after high school for those students who do not attend college. The problems assigned to students in this study addressed specific TEKS standards.

In addition to adjusting curricula, states are also trying to improve their teacher training programs. Widespread agreement exists that mathematics education in the United States needs improvement. [8] Good content knowledge, though extremely important, is only part of being a successful teacher. There are teachers who are brilliant in their area but can't teach very well. There are also teachers who teach well, but lack in content knowledge. They may may not be able to "provide multiple interpretations of concepts - particularly representations that provide concrete explanations or tie-ins to the real world." [10] There are may articles and ideas about what makes a good teacher. Ball explains that teachers have to be good listeners, be flexible, and be able to communicate mathematical ideas in a variety of ways in order to reach all of their students. [1] Professional development programs and teacher workshops are becoming more prevalent in order to address the issue of improving the quality of mathematics teaching.

CHAPTER III
PROCEDURES

3.1 Procedure Summary

The students involved in this project were from Lubbock area junior high schools. Each of these junior high schools is considered low income by federal standards. [13] The school where the ninth grade class was held was 87.8% Hispanic, 6.7% White/Other, and 6.5% African-American. There were fourteen students in the class, seven boys and seven girls. The eighth grade class was form a school that was 49.7% Hispanic, 40.9% White/Other, and 9.3% African-American. The class consisted of seventeen students, seven boys and ten girls. The research was done in the spring of 2004. The original plan was to conduct this study with a group of ninth grade students. Once this work was completed, a second study was undertaken. The purpose was to have another class with which to compare results, not only within each class, but between the two classes. Unfortunately, the second class was not another ninth grade class but was actually an eighth grade class. This difference in grades provided an additional analysis between grade levels for the study. The work was done from April 15 through April 28 with the ninth grade class and from May 11 through May 24.

These particular classes were chosen because of a previous working relationship between Texas Tech and the teachers in these two classrooms. The teacher of the ninth grade class is a graduate of Texas Tech and worked regularly with a professor from Texas Tech, allowing him to come into her classroom and present fun and challenging mathematics problems. The teacher of the eighth grade class allowed graduate students from Texas Tech to come to school in the afternoons for a math club tutoring and problem solving session.

The study had two goals, increase student abilities and proficiency in mathematics and improve student interest in mathematics. Students were given a consent form and asked to take it home for their parent(s) or guardian(s) to sign. A problem set designed to test student proficiency was administered on the first day of this study. This problem set is found in the Appendix. The students also completed an interest inventory on the first day. This survey can also be found in the Appendix. The same problem set and interest inventory were also administered on the last day of the study. These forms were numbered in order to directly compare individual differences. The study also consisted of a series of seven lessons presented to the students over a period of two weeks, each lasting approximately fifteen to twenty minutes. The lessons were presented each day. A benefit of everyday lessons was that the measurement of differences in the problem set would hopefully be attributable to the project and not just due to normal learning within a longer time frame. The day before the post tests were administered was spent in the computer lab. The students were directed to a web site with interactive activities involving fractals and other challenging mathematics activities. The latter mentioned mathematics activities, though not related to this study, were a nice diversion and were enjoyed by the students very much.

This study used a mixed methods approach. The majority of work done was quantitative and of a statistical nature. Scores were recorded for both the problem set and the interest inventory. Analysis was performed not only on each individual set, but also on the difference between pre and post tests as well. There was also a qualitative component to this study. Attention was also paid to the work done by the students in areas other than scores. We wanted to look at the processes students used in solving problems, both good and bad.

3.2 Problem Set

3.2.1 Problem Set Summary

This study aimed to not only introduce the students to college-level mathematics, but to increase the students' proficiency in their algebra class as a result of this exposure. The students were given a set of four problems on the first day of this study, and also on the last day of the study. The problem sets were the same both times. The problems were selected in order to not only test skills that are outlined by Texas Essential Knowledge and Skills for Mathematics (TEKS), but also skills that were used in the lessons. These skills relate to both the work with complex variables and fractals for the study and also to the concepts presented in the students' everyday algebra lessons. Each problem was graded on a scale of five points, with partial credit for completion of concepts within the problem.

3.2.2 Problem 1

Problem 1 asked "What is the distance from the point (4,3) to the origin?" The solution is 5. This problem was designed to test a several concepts. First of all, the problem tested the student's knowledge of how to plot a point in the cartesian plane. This concept is referenced in the TEKS standards, Chapter 111.24(b)(7)(D), which states the student is expected to "locate and name points on a coordinate plane using ordered pairs of rational numbers." [TEKS] With the point plotted correctly, we next wanted to see if the student recognized that the distance from the point to the origin was equal to the length of the hypotenuse of a right triangle, and then use the Pythagorean Theorem to find this distance. Chapter 111.24(b)(7)(C)states the student is expected to: "use pictures or models to demonstrate the Pythagorean Theorem." The concept of distance from a point to the origin is equivalent to the modulus of a complex number. Later on, the students would need to find the modulus of a complex number to see if the modulus was increasing or decreasing after many iterations in a function.

3.2.3 Problem 2

Problem 2 asked "If $f(z) = z^2$, what is the value of $f(0)$?" The solution is 0. This problem was chosen to see if the student could evaluate a function for a particular value. TEKS, Chapter 111.32(b)(4)(A) states "The student finds specific function values." [14] The problem was relevant to this study because fractals involve many iterations of a complex number in a function. Unfortunately, this problem should have been changed to more accurately test the students' ability to evaluate the function. Even just adding a constant to the end, such as $f(z) = z^2 - 2$, would have been an improvement because the student would have to plug in the value and do more than just squaring the number to get the answer. In this study of fractals, the students were expected to evaluate the function $f(z) = z^2$ for complex numbers such as $2 + 3i$, so a different choice of functions would have better demonstrated the students abilities in this area.

3.2.4 Problem 3

Problem 3 asked "What is the value of $\sqrt{-16}$?" The solution is $\sqrt{-16} = 4i$. This problem was a test to see if the students had an idea how to find the value of an imaginary or complex number. The relevant TEKS standard is found in Chapter 111.33(b)(2)(B) and states "the student uses complex numbers to describe the solutions of quadratic equations." [14] The quadratic formula for finding roots of quadratic equations is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} . \tag{3.1}$$

Whenever the discriminant $b^2 - 4ac$ is negative, the solution is imaginary or complex. Because fractals are constructed using functions with complex numbers, an understanding of this concept was desired.

3.2.5 Problem 4

Problem 4 stated "The diagram below shows a circle with center at (0, 0) and radius of 1. Find the distance from the y-axis of the point indicated where x= .75". The diagram is shown in Figure 3.1.

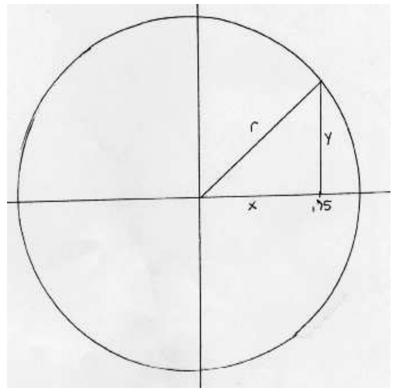

Figure 3.1: Problem 4

The solution is 0.6614 or $\frac{\sqrt{7}}{4}$. The problem was intended to supplement the use of the Pythagorean Theorem with the introduction of the unit circle. Unfortunately, the problem has several drawbacks. The problem asked for the distance from the y-axis when it should have asked for the distance from the x-axis. Next, the problem specifically asked for a distance from an indicated point, yet no point was actually indicated. Another shortcoming was the absence of units in the distance. When the problem set was administered, the students were advised to find the length of y. The students should have also been advised to make corrections in their problem set. Since the corrections were only verbal, if a student didn't understand or wasn't paying attention they weren't aware of the correction. Some students answered the problem as it was asked, or didn't know how to do the problem because of its ambiguity. The same standards were expected of the students as in Problem 1. This duplication of concepts with Problem 1 was another shortcoming of this particular problem.

3.3 Lessons

3.3.1 Lesson Summary

The lessons were designed to complement the skills already possessed by the students. Theory normally presented in undergraduate complex analysis classes was not covered in this study. A plan was mapped out in advance to progress the students through the steps necessary to produce basic fractal images. Though it was not covered in this study, the students could take what they learned from these lessons and construct fractals of a much more complex nature. Restricting our studies to a basic fractal, the *Julia set*, was also intended to keep the students from feeling overwhelmed by the complexity of the mathematics. After new concepts were presented the students worked several examples to solidify their understanding and skill with those concepts. References were made continually to known material in order to assist the students understanding. It should also be noted that the students each had scientific calculators at their disposal. This really helped in some of the more involved calculations. Unfortunately, some students use calculators for even the most basic calculations instead of using a pen and pencil to arrive at a solution.

3.3.2 Lesson 1: An Introduction to Complex Numbers

Lesson 1 was an introduction to complex numbers. We first defined $i = \sqrt{-1}$. The students were reminded that this same thing occurs when the discriminant is negative in the quadratic formula. A couple of more examples were given, such as $\sqrt{-4} = 2i$ to give the students some practice. The students were then introduced to one notation for complex numbers, $a+bi$. They were shown this form first because of the fact that any number can be written in this form, the difference being that real numbers have a complex component of 0. We then defined z to be a complex number in the form $z = x+iy$, where x is the real part of the number and y is the imaginary part. We wrote the complex number as an ordered pair, $z = (x, y)$. With this notation, a natural step was to introduce the complex plane. The students quickly became adept at plotting points such as $4 + 2i$ and $-3 + 2i$ in the complex plane.

These basic concepts were important because fractals are created using complex numbers and the complex plane. A solid basic understanding would assist later lessons and success of this study.

3.3.3 Lesson 2: Adding and Subtracting Complex Numbers

Lesson 2 instructed the students how to add, subtract and multiply complex numbers. The students were easily able to understand addition and subtraction of complex numbers. The process of adding and subtracting like terms is used over and over again in solving algebraic equations, and posed no problems for the students. They very quickly could do problems such as

$$(3 + 4i) + (2 - 3i) = 5 + i \; . \tag{3.2}$$

The students also noticed the result of adding or subtracting complex numbers when they plotted the resulting complex number. They realized that adding a number such as $2 - 3i$ was the same as taking the original point and moving two units to the right (the real component) and going down three units (the complex component). This was one of the shorter lessons, which allowed time for us to talk with the students and received feedback about the first couple of days in this study. The students were understanding the material and were very happy to continue. When students don't understand, their motivation and attitudes suffer. This was not apparent so far in this study.

Working with complex numbers and understanding operations involving them would also be beneficial to the students' advancement to the more difficult concepts presented later in the lessons.

3.3.4 Lesson 3: Multiplication of Complex Numbers

The process of multiplication of complex numbers was next introduced by first reminding the students about multiplying two binomials, such as

$$(x + 3y)(2x - y) \; . \tag{3.3}$$

The multiplication of complex numbers is very similar.

$$(x_1 + iy_i)(x_2 + iy_2) \tag{3.4}$$

When this process is carried out, the students didn't immediately know what to do with the i^2 component of the answer. When they were reminded that $i = \sqrt{-1}$, they knew to square both sides to come up with $i^2 = -1$. They could now simplify their previous answer and reduce it to the standard form of a complex number with a real and an imaginary component. This process was repeated to solidify the concept. Special emphasis was placed on squaring complex numbers. A common problem with many students arose when they were asked to square a complex number.

$$(3 + 4i)^2 = 9 + 16i^2 \rightarrow \; 9 - 16 \rightarrow \; -7 \qquad (3.5)$$

Many algebra students make this mistake by only squaring the first and last terms of a binomial, forgetting about the inner products. A solution for this mistake is to have the student write the term out twice, side by side, to remind them to fully use the distributive principle. A couple of the students in this class were guilty of this mistake, which was quickly corrected. The students also discovered another result of multiplication of complex numbers by noting the location of the resulting complex number. The students quickly noticed that when a complex number is squared, the angle which is formed by a line from the the original complex number to the origin and the x-axis is doubled. This was confirmed when they took a number such as $2 + 2i$ which lies at a 45 degree angle from the x-axis, and squared it. The resulting complex number, $8i$ lies directly on the y-axis, 90 degrees from the x-axis.

Multiplication of complex numbers is one of the most important concepts in the creation of fractals. A complex number is multiplied by itself, usually many times. Though computers would later be used when the calculations became too plentiful to be done by hand, a basic understanding of this process was absolutely essential for this study.

3.3.5 Lesson 4: Modulus of Complex Numbers

The next topic was the explanation of the modulus of a complex number. Modulus was defined for the students as the distance between the origin and the point

$z = (x, y)$. More than one student commented that was just like Problem 1 in the problem set, a good observation. The students were shown the formula to calculate the modulus and reminded of its relation to the Pythagorean Theorem.

$$|z| = \sqrt{x^2 + y^2} \tag{3.6}$$

When one student asked why absolute value was used, another student replied that distance can never be negative. This type of student interaction and cooperation is very encouraging. Sometimes an idea or suggestion will be more well received from another student than if it came from a teacher. The students were also asked about what happens to the modulus when a complex numbers are multiplied. To investigate, students were asked to find the modulus of $3 + 4i$, which they found to be 5, using the previous formula. They were also asked to find the modulus of $6 + 8i$, which they found to be 10. They were then asked to find the product of the two complex numbers, and then to find the modulus of that resulting complex number. They first found the product.

$$(3 + 4i)(6 + 8i) = -14 + 48i \tag{3.7}$$

The modulus was then calculated.

$$| - 14 + 48i| = \sqrt{(-14)^2 + 48^2} \rightarrow \sqrt{2500} \rightarrow 50 \tag{3.8}$$

The students were not immediately told about the relation of the modulus of the product in relation to the modulus of the two factors. They were asked to come up with the relation on their own. There was a bit of silence, so equation (3.7) was written on the board with the modulus of each component written directly below the corresponding component. They immediately saw that the modulus of the resulting product of the two complex numbers was the product of the modulus of each of the complex numbers. Another example was given, and the students quickly guessed the answer before the problem was completed. A visual interpretation was also given by plotting both of the original complex numbers and the resulting product all in the complex plane. One final example of modulus was presented. The modulus of $2 + 3i$

was assigned. The result, $\sqrt{13}$, reminded the students that answers are not always integers.

When a complex number is multiplied by itself over and over, it was very useful to see what happened to the resulting complex number. The modulus will be later used to determine convergence or divergence, concepts integral to the creation or construction of fractals.

3.3.6 Lesson 5: Functions

As mentioned previously in Problem 2 in the Problem Set Summary, the students have already been introduced to functions. They are taught that a function is a representation of one quantity that is dependent on another. They have also have been taught to find specific function values. The lesson began with a review of the very basic idea that in the expression $y = x + 3$ the value of y is dependent on the value of x. We next used the notation $f(x)$ to replace y. The students were given a function such as $f(x) = 2x+7$ and asked to evaluate it for some value of x, say 3. We wrote $f(3) = 2(3)+7$ and found the solution to be 13. They were instructed to finish the problem and find a specific value, rather than leaving the answer as $2(3) + 7$. We then progressed to evaluating functions with complex variables, starting with a basic function $f(z) = z^2$. The students were asked to evaluate this function with $z = 2-4i$. This also gave the students another opportunity to use the multiplication process from the previous lesson. The solution was shown to be $-12 - 16i$ without much of a problem. The students worked another problem for practice.

Functions of the form $f(z) = z^2 + c$ are used to create the *Julia set*, one of many fractals. The student must be able to evaluate functions of this type using complex numbers in order to create fractals.

3.3.7 Lesson 6: Julia Sets

The type of fractal we were planning to create was a *Julia set*. Fractals can be extremely complicated, but this particular fractal is not as difficult as many others are.

The *Julia set* is constructed using the principles taught in the previous lessons. We first began by re-introducing the function $f(z) = z^2$. Another form of this function, specifically $f(z) = z^2 + c$ was also shown. In this latter form, c can be any real number. This lesson was taught with the simplest form, with c = 0, with the understanding that the same concepts apply for other values of c also. It just happens that the results are very nice and clean when c = 0. One additional process with this function was explained. The students were instructed in the process of iteration. They were told that once they evaluated a function for a specific complex number, they would use that result and evaluate the function again for the resulting number. They would continue this process for two or three steps and then examine the result. A random complex number was selected by the students, $1 + 2i$, and the function $f(z) = z^2$ was evaluated.

$$f(1 + 2i) = (1 + 2i)^2 = -3 + 4i \tag{3.9}$$

$$f(-3 + 4i) = (-3 + 4i)^2 = -7 - 24i \tag{3.10}$$

$$f(-7 - 24i) = (-7 - 24i)^2 = -527 - 336i \tag{3.11}$$

The students were then asked to find the modulus of the original starting complex number, along with each of the resulting complex numbers. Table 3.1 summarizes these results.

Table 3.1: Modulus of Iterations of $(1 + 2i)$

Complex Number	Modulus
$(1 + 2i)$	$\sqrt{5}$
$(-3 + 4i)$	5
$(-7 - 24i)$	25
$(-527 - 336i)$	625

When asked about the modulus after repeated iterations, the students quickly responded that it was "getting really big." They were asked what the modulus would be of the next iteration, they responded 625^2, which is correct.

The students were next assigned to do the same operation, only with a number assigned to them. They were assigned the complex number $\frac{1}{2} + \frac{1}{2}i$. They were given extra assistance with this part of the lesson. The results follow.

$$f(\frac{1}{2} + \frac{1}{2}i) = (\frac{1}{2} + \frac{1}{2}i)^2 = \frac{1}{2}i \tag{3.12}$$

$$f(\frac{1}{2}i) = (\frac{1}{2}i)^2 = -\frac{1}{4} \tag{3.13}$$

$$f(-\frac{1}{4}) = (-\frac{1}{4})^2 = \frac{1}{16} \tag{3.14}$$

Table 3.2 lists the modulus of each result.

Table 3.2: Modulus of Iterations of $(\frac{1}{2} + \frac{1}{2}i)$

Complex Number	Modulus
$(\frac{1}{2} + \frac{1}{2}i)$	$\sqrt{.5}$
$(\frac{1}{2}i)$	.5
$(-\frac{1}{4})$	.25
$(\frac{1}{16})$	.0625

Again the students were asked to comment on the modulus of the complex number as it was iterated. Again they responded correctly, noting that it was decreasing. One more example was presented, using the complex number $\frac{\sqrt{3}}{2} + \frac{1}{2}i$. This point was chosen because it lies on the unit circle. The calculations were done before the lesson and presented on the board. The results for the iterations are shown below.

$$f(\frac{\sqrt{3}}{2} + \frac{1}{2}i) = (\frac{\sqrt{3}}{2} + \frac{1}{2}i)^2 = \frac{1}{2} + \frac{\sqrt{3}}{2}i \tag{3.15}$$

$$f(\frac{1}{2} + \frac{\sqrt{3}}{2}i) = (\frac{1}{2} + \frac{\sqrt{3}}{2}i)^2 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i \tag{3.16}$$

$$f(-\frac{1}{2} + \frac{\sqrt{3}}{2}i) = (-\frac{1}{2} + \frac{\sqrt{3}}{2}i)^2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i \tag{3.17}$$

$$f(-\frac{1}{2} - \frac{\sqrt{3}}{2}i) = (-\frac{1}{2} - \frac{\sqrt{3}}{2}i)^2 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i \tag{3.18}$$

Table 3.3: Modulus of Iterations of $(\frac{\sqrt{3}}{2} + \frac{1}{2}i)$

Complex Number	Modulus
$(\frac{\sqrt{3}}{2} + \frac{1}{2}i)$	1
$(\frac{1}{2} + \frac{\sqrt{3}}{2}i)$	1
$(-\frac{1}{2} + \frac{\sqrt{3}}{2}i)$	1
$(-\frac{1}{2} - \frac{\sqrt{3}}{2}i)$	1
$(-\frac{1}{2} + \frac{\sqrt{3}}{2}i)$	1

Table 3.3 lists the modulus of each result. The repetition or cycle of these results was pointed out to the students. The subjects of convergence and divergence were then introduced to the students. The terms were unfamiliar, so these terms were explained in a way so they could be understood. When the modulus of the iterations continued to grow until it got ”really big” as earlier noted was explained as divergence. When the modulus of the iterations continued to get really small and continually decrease, we called it convergence. We recalled what happened to the modulus when numbers are squared from Lesson 3. Form this, the students were instructed that when the initial number has a modulus of greater than 1, the modulus will grow with repeated iterations. If the modulus of the number is less than 1, the modulus will continue to shrink until it is infinitely small. If the modulus is exactly 1 to begin with, the students noticed that it always stayed at 1.

As mentioned earlier, The *Julia set* is created using the function $f(z) = z^2 + c$. As also mentioned, the we wanted to create the set with c = 0, because it is not as complicated as the sets with other values of c. The set of iterates given by our function for each value are called *orbits*. Let K_0 denote the set of all orbits that are bounded, that is they don't increase without bound. In our case, this is all of the points inside of and including the unit circle. The boundary of K_0 is known as the *Julia set* for the function. [9] In our case, the boundary of the bounded orbits would be the unit circle itself. This is the *Julia set* we were looking for.

Because this lesson was lengthy many of the calculations were done in advance to better utilize the time with the students. The actual construction of the *Julia set* was the culmination of the previous lessons.

3.3.8 Lesson 7: Computer Programming

It can be seen from the previous lesson that the actual calculations for the iterations of complex numbers can be quite difficult. There are also an infinite number of complex numbers in the plane with which to work, thereby creating another obstacle to creating a *Julia set* by hand. A computer would be an invaluable tool to assist in this process. The lesson for this day was an interactive trouble shooting session to find out how to use a computer to assist in our creation of the *Julia set*. This particular idea of student interaction and using computers was well received by the students. There were a couple of students who had not contributed a great deal before who now were very eager to contribute. We first wanted a way to evaluate the function with all of the complex numbers. It was decided that we would use a small part of the plane, from -2 to 2 on the both real x-axis and the imaginary y-axis. This would give us good coverage in the center, and could be adjusted later if necessary. The next part was to figure out how to use all of the numbers in this area. The students suggested going row by row or column by column across our region, so we just had to figure where to start. One student suggested that we start in the upper left corner, at the point $-2+2i$, and go down all of the way to the point $-2-2i$. We could then go across and do each column until we got to the point $2-2i$. Everyone decided that this would be a good plan. The problem arose when the students were advised that there were still infinite points in this region, so it was suggested to them that they use small increments when going from point to point in the plane. This idea satisfied them that the plane would be aptly covered.

The students were then asked what would happen at each individual point. Suppose you start at $-2+2i$, what are you supposed to do next? The students responded by saying that you had to evaluate the function at that point, and then evaluate it again at the previous result, and continue to do so. They weren't sure of how this could be accomplished, but they did know what needed to be done.

The next issue to be addressed was what was happening to the results of these repeated evaluations. The students said that they needed to know if they were growing larger, becoming smaller, or staying the same. They did not know exactly how to do this, but they again knew what needed to be done.

The students were then shown a copy of a *Maple* program that had been written to create the *Julia set*. Methods for coverage of the complex numbers were explained, along with methods for iterating each of these numbers and evaluating the results. The students were shown a finished picture of what we had been working for throughout the lessons. This program is included in the Appendix.

3.3.9 Lesson 8: Computer Time

This next to last day of the study there was no lesson taught. we had already reached the final goal of creating a fractal, the *Julia set*. We had already set up a day for the students to go to the computer lab, kind of as a reward for participating in the study and for working hard to try to understand the complex mathematics that had been presented to them. They were first instructed to go to a particular website http://math.bu.edu/DYSYS/applets/chaos-game.html. They played a game called Chaos, where they would try to manipulate a point to a certain region by a series of moves. A best possible score is shown, which the students try to match. There are many levels, each of increasing difficulty. The students were very eager to be challenged. They would call out to the instructor and show how well they had done. It was a nice break from the classroom. They really only had about ten minutes or so in the lab because of going to and from the classroom to the computer lab, but they really enjoyed their time in the lab.

3.3.10 Lesson Summary

The ninth grade class followed along with every lesson, and the comments expressed above in the different lessons apply to them. The eight grade class had far more difficulties, in part because they have not yet been exposed to some of the concepts. They understood plotting in the complex plane, probably because of its similarity to the cartesian plane. When asked to add and subtract complex numbers, they did very well. They had trouble when asked to multiply complex numbers. The FOIL method (first, outside, inside, last) was presented, but they did not follow. A different approach was taken, using the distributive property. Some of the students were able to catch on, but not all of the them. Several more examples were done, but some students never did catch on. The students also were completely unfamiliar with the concept of a function. After the explanation, they could somewhat follow, but once complex numbers were used in conjunction with functions the level of understanding dropped. Unfortunately, these concepts are integral in the creation of fractals such as the *Julia set*.

3.4 Interest Inventory

In order to observe if the lessons had an impact on student attitudes toward mathematics, the students were asked to fill out an interest inventory on the first day of the study. This inventory was developed by Tara Stevens and Arturo Olivarez Jr. of Texas Tech University. [11] Their interest inventory consisted of sixty items. We thought that the students would lose interest with such a lengthy survey, so we shortened it to twenty items. The survey also asked the students their ethnicity, age, gender and expected grade in the class. It also had a space for student comments asking if any items were difficult. All of the responses on this last particular item were positive, stating that it was easy. They were also given the same inventory on the last day of the study. The interest inventory consisted of twenty items. This survey is included in the Appendix. The student were asked to rate their response from one to seven, with one indicating "sounds like me not at all" and seven indicating "sounds like me." This type of response scale, known as a Likert scale, has been shown to provide more accurate and reliable responses than a single item scale. "By comparing the reliability of a summated, multi-item scale versus a single-item question, the authors show how unreliable a single item is; and therefore it is not appropriate to make inferences based upon the analysis of single-item questions which are used in measuring a construct." [7] The items explored different attitudes about mathematics. These could be broken down intro three sub categories: attitudes toward mathematics, students' mathematics work ethic, and importance of learning mathematics. Multiple items with in the survey tested each of these sub categories. Some items on the survey utilized a reverse scale, such as Item 17, "I am wasting my time on math." Usually a response of seven indicates a positive response, but in this case a response of seven would indicate the opposite. One benefit of having items with reverse scales is that it keeps the student from just arbitrarily marking high responses such as six or seven or even on all the way down the list. It was intended to induce the student to carefully read and respond to each question. In the analyses of the interest inventory responses, the responses on those items with inverted response scales were adjusted to facilitate the analysis and put all of the responses on the same scale.

CHAPTER IV
GENERAL RESULTS

4.1 Problem Set Results

We now look at the results of the problem set tests. As described earlier, there were four problems that tested particular concepts relating to our study. Each problem was graded on a five-point scale. The possible total score ranged from zero to twenty. The students were awarded points for sub-concepts within each problem. The particular appropriations of points and concepts are detailed within each problem analysis. We will first analyze the overall totals and then each individual problem. Summary statistics for the problem set, including the mean score for each grade level and the maximum and minimum recorded scores are shown in Table 4.1.

Table 4.1: Problem Set Statistics

	N	Mean	Min	Max
Pre-problem set - Grade 9	14	8.21	0	20
Post-problem set - Grade 9	14	15.43	10	20
Pre-problem set - Grade 8	17	1.71	0	7
Post-problem set - Grade 8	17	3.06	0	12

Clearly both grades had increased scores in the post-test than in the pre-test. Each class almost doubled their average. The item that stands out the most is the very large discrepancy between the scores for the eighth grade and ninth grade classes. Even after the lessons, the mean for the eighth grade students was less than half of the mean of the ninth grade students before the lessons. The mathematics behind some of the lessons and problems had not yet been introduced to the eighth grade students. They were not familiar with functions and multiplication of binomials, which was necessary for the iterations of complex numbers.

Histograms showing the distribution of the scores in the pre and post tests for the ninth grade students are shown in Figures 4.1 and 4.2, respectively. There is a clear shift to the right of the distribution. In the pre test, five students scored between 0 and 5, and in the post test no student scored below a 6. In the pre test three students scored between 15 and 20, while in the post test at least six students scored in that range. Histograms showing the distribution of the scores in the pre and post tests for the eighth grade students are shown in Figures 4.3 and 4.4, respectively. In this case, the shift to the right is not as dramatic as was the case with the ninth grade students. The maximum score rose seven to twelve, otherwise the scores flattened out without a large change.

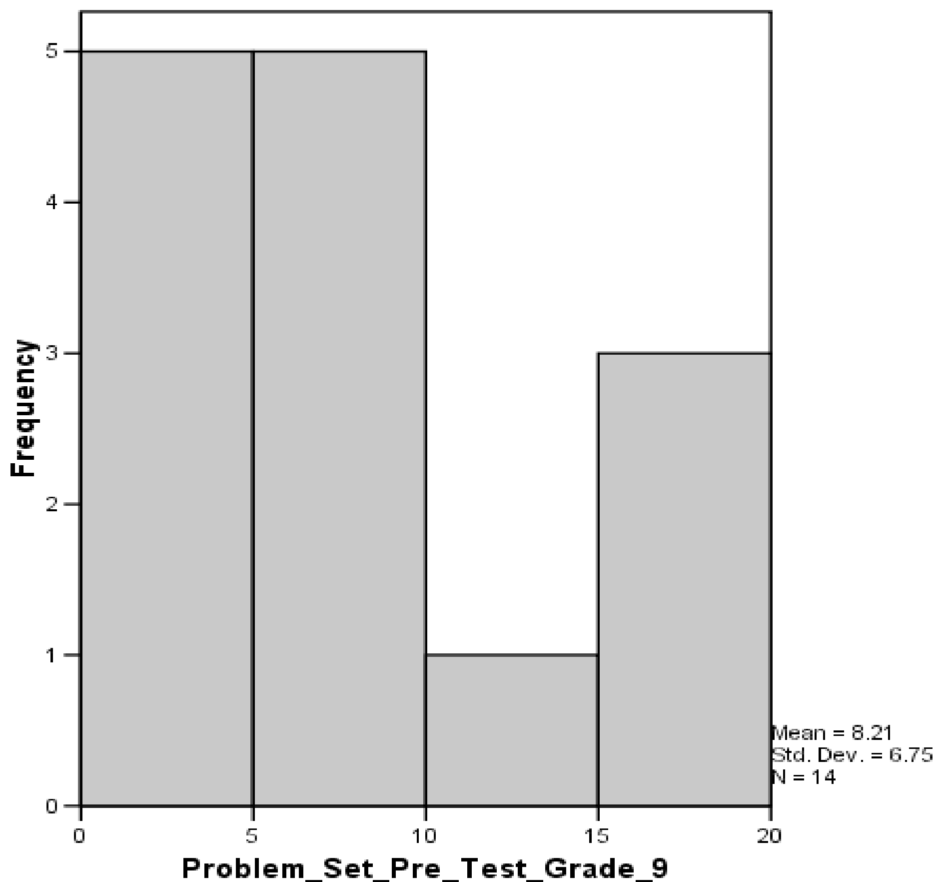

Figure 4.1: Pre Test Problem Set Scores - Grade 9

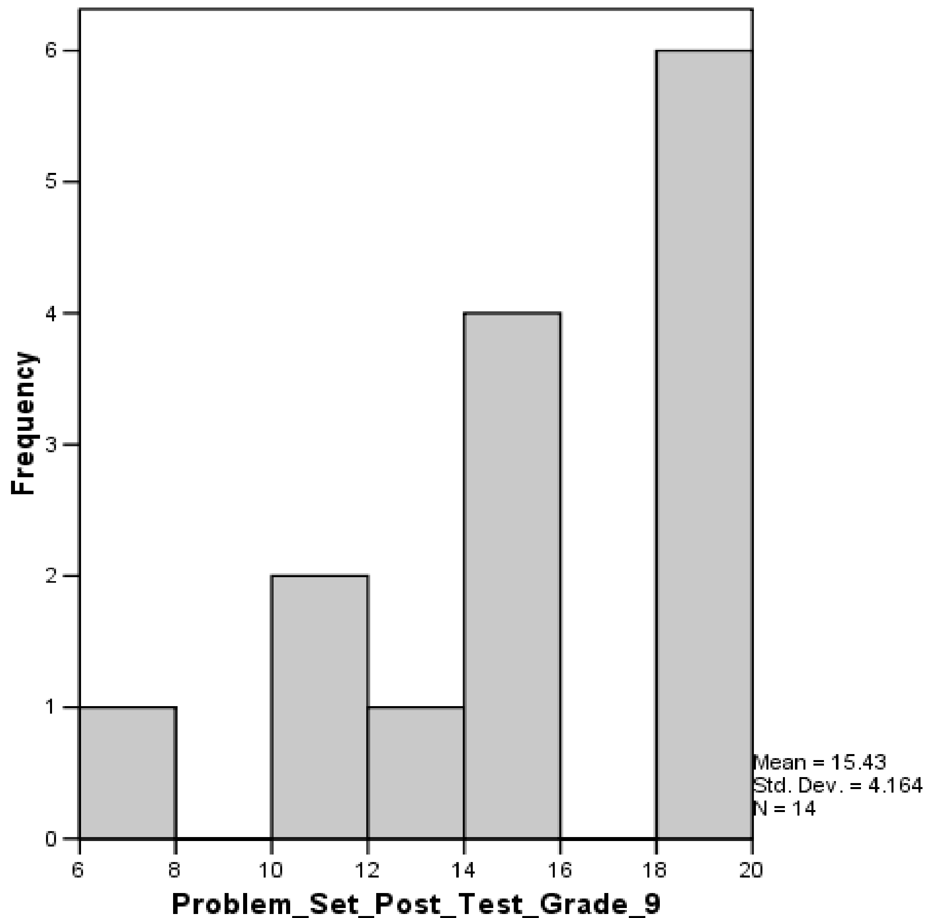

Figure 4.2: Post Test Problem Set Scores - Grade 9

A paired sample t-test was also performed to determine if the increases were statistically significant. This test has the same hypotheses and concepts as the paired sample t-tests as those performed on the interest inventory items in the previous section. The degrees of freedom (d.f.), t test statistic, and level of significance for these comparisons are shown in Table 4.2.

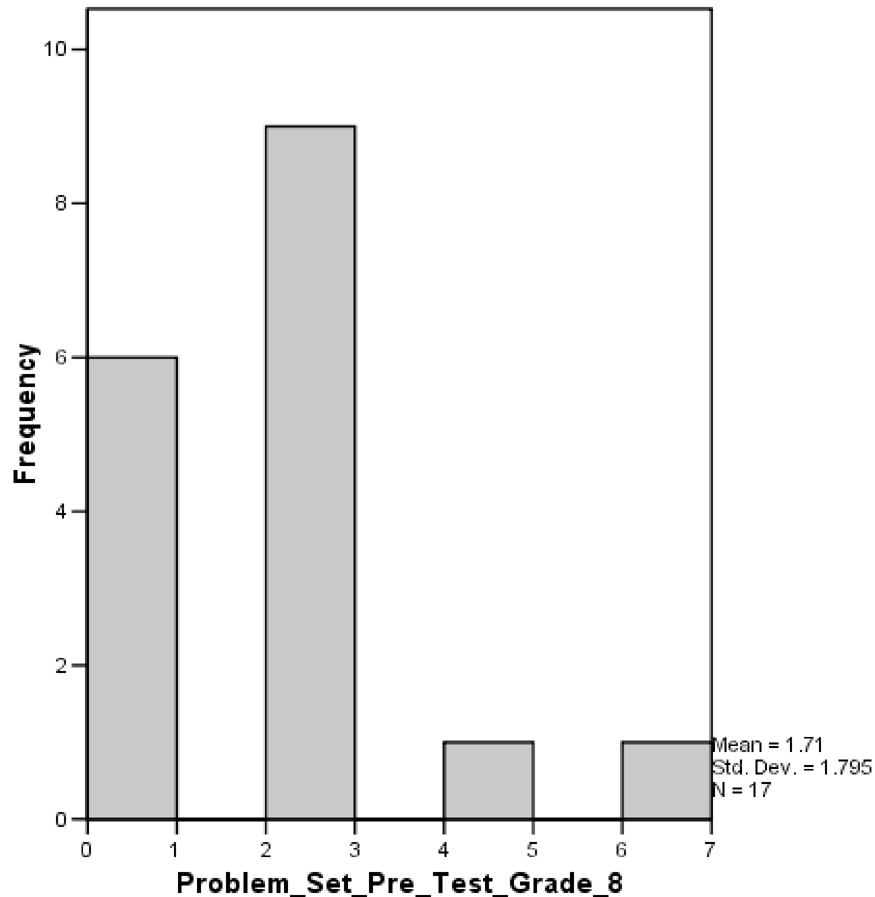

Figure 4.3: Pre Test Problem Set Scores - Grade 8

The significance level of 0.001 for the ninth grade class suggests that the difference was highly statistically significant. This indicates a difference between the pre and post problem set scores. The 95% confidence interval for the true mean difference, (-11.059, -3.370), confirms this. We do not believe the true overall difference to be zero, since zero is not in our interval. The significance level of 0.138 for the eighth grade class suggests that even though we reported a difference, it was not statistically significant. The 95% confidence interval for the true mean difference, (-3.188, .482), shows that zero is a possible value for the true mean difference. If zero was the true mean, this would indicate no difference in pre and post problem set scores.

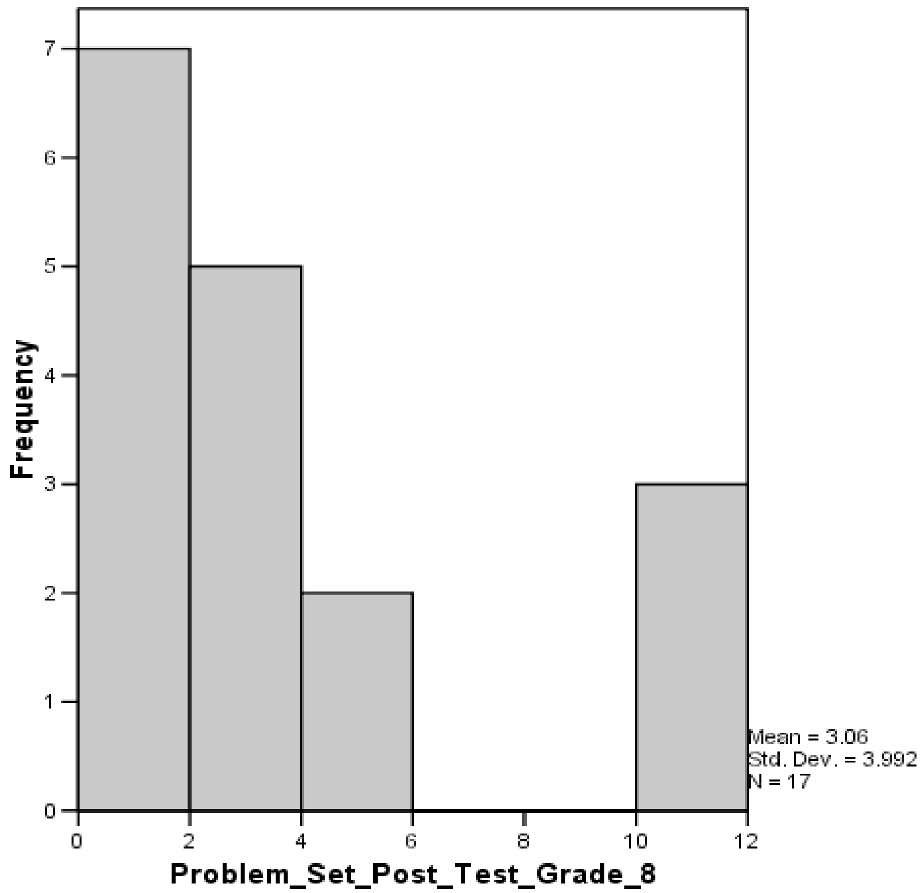

Figure 4.4: Post Test Problem Set Scores - Grade 8

4.2 Interest Inventory Results

The interest inventory was administered to assess the students overall attitude towards mathematics. We hoped that through this study these attitudes would improve. A Likert scale with values ranging from one to seven was utilized, with one indicating "Sounds like me not at all" and seven indicating "Sounds a lot like me." The interest survey is attached as Appendix B. Cronbach's Alpha Reliability Coefficient for Likert-Type Scales was calculated to establish overall scale internal consistency reliability. As an extra measure, the coefficient was calculated for both the pre and post interest inventories. Table 4.3 lists these results.

Table 4.2: Paired t-Test Statistics

	d.f.	t	Significance
Pre - Post - Grade 9	13	-4.054	.001
Pre - Post - Grade 8	16	-1.563	.138

Table 4.3: Cronbach's Alpha Reliability Coefficient - Overall

	N	Alpha
Pre-interest - Grade 9	14	.859
Post-interest - Grade 9	14	.855
Pre-interest - Grade 8	17	.769
Post-interest - Grade 8	17	.885

When Cronbach's Alpha Reliability Coefficient gets closer to 1, the greater the internal consistency of the items in the scale [1]. A good rule of thumb for the Cronbach's values follows: Alpha $>$ .9 - Excellent, Alpha $>$ .8 - Good, Alpha $>$ .7 - Acceptable, Alpha $>$ .6 - Questionable, Alpha $>$ .5 - Poor, Alpha $<$ .5 - Unacceptable [2]. According to these standards, both the pre-interest and post-interest results for both ninth grade and eighth grade are at least at the Acceptable level. With coefficients each above 0.850, the ninth grade rates in between Good and Excellent.

As mentioned before in Chapter 3, the interest inventory set was also broken down into three smaller sub-categories: 1) attitudes toward mathematics, 2) students' mathematics work ethic, and 3) importance of learning mathematics. The Alpha Reliability Coefficient was also calculated for these sub-categories, with the results shown in Table 4.4.

The lower values of the Reliability Coefficients in the sub-categories may be attributable to the small number of responses. The Attitudes Towards Mathematics coefficients are reasonably high, 0.778 for the post-interest coefficient and 0.768 for the pre-interest coefficient. The eighth grade results have the pre-interest numbers a bit lower, 0.679, but with a large improvement in the post-interest coefficient, 0.868.

Table 4.4: Cronbach's Alpha Reliability Coefficient

	Number of Responses	Alpha - Pre	Alpha - Post
Attitudes - Grade 9	9	.768	.778
Work Ethic - Grade 9	6	.720	.619
Importance - Grade 9	5	.427	.706
Attitudes - Grade 8	9	.679	.868
Work Ethic - Grade 8	6	.147	.378
Importance - Grade 8	5	.573	.798

The numbers were again reasonable for Students' Mathematics Work Ethic for the ninth grade, but were very low for the 8th grade, 0.147 and 0.378. Surprisingly the eighth grade values were higher than the ninth grade in the category Importance of Learning Mathematics in both the pre and post surveys, with the numbers becoming more reasonable in the post-interest surveys.

Now that the reliability of the interest inventory has been established, we must address the issue of validity. As mentioned earlier, this interest inventory was modelled after one developed by Tara Stevens and Arturo Olivarez Jr. of Texas Tech University. In their paper on this survey, the validity of the survey was established. However, we cannot make the same claim about the interest inventory we used due to the fact that it had been abbreviated.

A closer look can be taken at the individual surveys. Since there were twenty questions, each answer ranging in scale from one to seven, the minimum score was 20 and the maximum score was 140. Since some of the questions had an inverted scale (where seven would be the lowest score instead of the highest score), the numbers have been inverted on those questions so that all questions and analysis can be examined on an even basis. A summary of the statistics on the pre-inventory and post-inventory scores is shown in Table 4.5.

Table 4.5: Summary Statistics for Inventory Scores

	N	Mean	Item Mean	Std. Dev.	Median	Min	Max
Pre-interest - Grade 9	14	86.79	6.20	18.96	88.5	57	130
Post-interest - Grade 9	14	91.00	6.50	17.91	90	60	135
Pre-interest - Grade 8	17	74.35	4.37	17.49	68	50	102
Post-interest - Grade 8	17	67.29	3.96	21.10	65	38	104

As shown in Table 4.5, all of the numerical indicators for the ninth grade students showed a slight increase except standard deviation. This shows that while all of the scores went up, the amount of variation within the scores actually decreased by a small amount. The differences that immediately stand out are the fact that the ninth grade overall scores increased while the eighth grade scores decreased. The mean response per item went from 6.20 to 6.50 for the ninth grade students. This number is rather high, since the highest possible average would be 7.00. The eighth grade mean responses were much lower. The eighth grade post survey average of 3.96 is more than 2.5 points lower than the average for the ninth grade students. The fact that the minimum total survey score actually went down from 50 to 38 for the eighth grade students was surprising. This result is counter-intuitive to what we expected to happen with this procedure. We expected an improvement in the student interest, not a reduction. This result might have occurred because the students were frustrated at the complexity of the lessons. As mentioned before, some of the skills necessary for this study have not been introduced to them yet. Another reason for the drop may have been that this study with the eighth grade class took place right at the very end of the school year. Perhaps the students were more interested in their upcoming summer activities than in learning complex mathematics.

The histograms in Figures 4.5 and 4.6 show the pre and post interest inventory distributions, respectively, for the ninth grade students. There is a slight shift to the right, with both the minimum and maximum scores increasing, as also shown in Table 4.5 The histograms in Figures 4.7 and 4.8 show the pre and post interest inventory distributions, respectively, for the eighth grade students. In this case, there is a definite shift to the left indicating lower interest after the study.

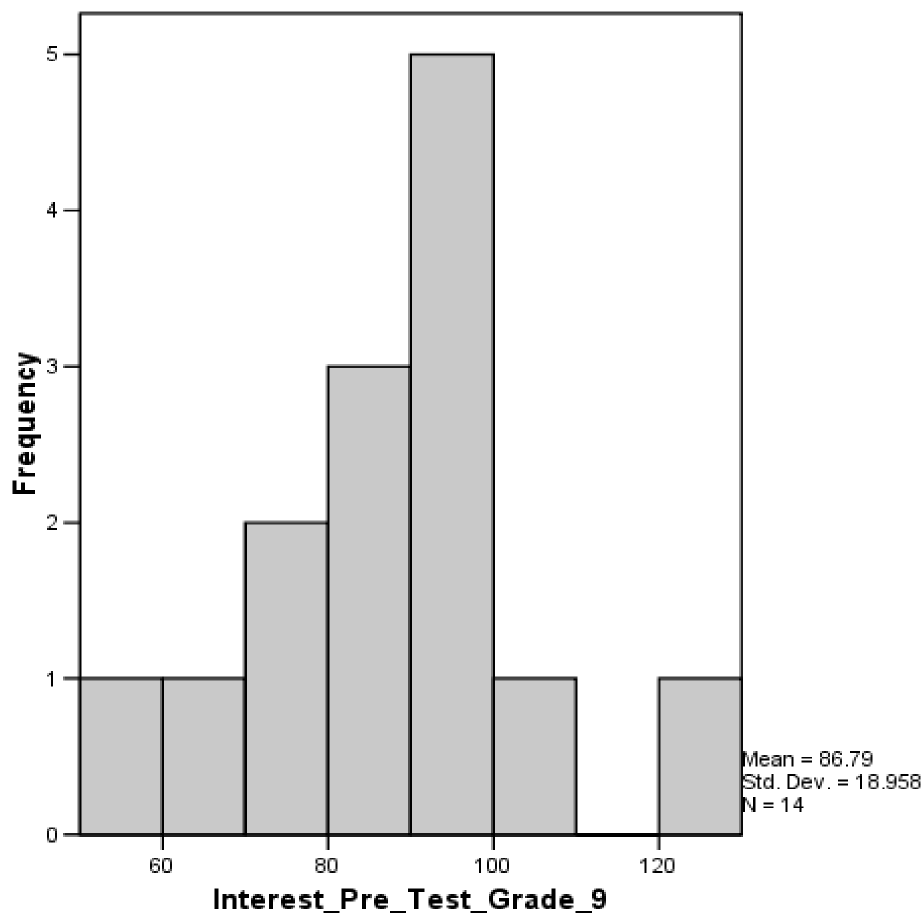

Figure 4.5: Pre Test Interest Inventory Scores - Grade 9

After reviewing the overall mean scores, we wanted to determine if the change between pre-interest and post-interest mean scores was statistically significant. The procedure to compare these means is a paired sample t-test. The t-test is used because of the small sample size. This is a hypothesis test procedure. A hypothesis is formulated about the significance of the difference, data is analyzed, and a conclusion is drawn from the data. We are testing the hypotheses listed below.

$$H_0 : D = 0 \tag{4.1}$$

$$H_1 : D \neq 0 \tag{4.2}$$

In these hypotheses the letter D stands for difference. The null hypothesis, $H_0 : D = 0$, claims the difference between the two means is zero, or there is no difference in the pre-test and post-test means. The alternate hypothesis, $H_1 : D = 0$, claims the difference between the two means is not zero, therefore there is a difference in the pre-test and post-test means of the responses. Though there was obviously a difference in our sample means as shown in the previous tables, we are testing whether or not the difference is statistically significant. Our sample size was relatively small and we must test to see if this difference could be projected to be accurate or true for the entire population. The formula for the test statistic is shown in Equation 4.3.

$$t = \frac{\overline{D}}{\frac{s_D}{\sqrt{n}}} \tag{4.3}$$

$\overline{D}$ represents the mean or average of all the individual differences, s_D represents the standard deviation of these differences, and n is the sample size. The test statistic for the ninth grade class is -1.339, which implies a significance value of 0.203. This value suggests that we do not reject $H_0 : D = 0$, and we therefore believe that the difference between pre-interest and post-interest scores is zero, or not statistically significant. The 95% confidence interval for the true mean difference is (-11.013, 2.584), which also indicates that there is no difference, because zero is included in the interval. If D=0, there is no difference. Even though all of the statistics showed an increase, it must be noted that because of our small sample size we cannot conclude that the difference was statistically significant.

The same t-test was performed for the eighth grade class, with a resulting test statistic of 2.846, and a corresponding significance value of 0.012. With this result, we would reject $H_0 : D = 0$ and conclude that the difference is indeed not zero and is statistically significant. We did affect a change in the eighth grade overall mathematics attitudes. Unfortunately, this change was in the negative direction. The possible reasons for this decrease were discussed earlier in this section. The 95% confidence interval for the true mean difference is (1.801, 12.316), which also shows that there is a difference because zero is not included in the interval.

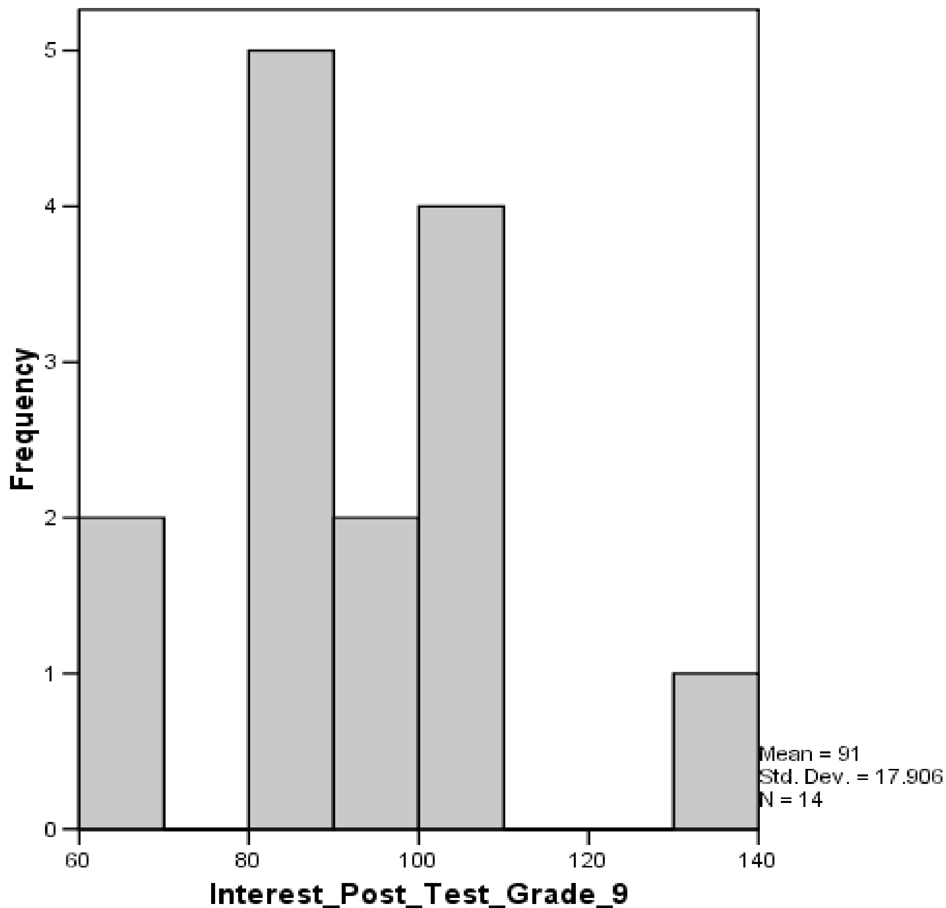

Figure 4.6: Post Test Interest Inventory Scores - Grade 9

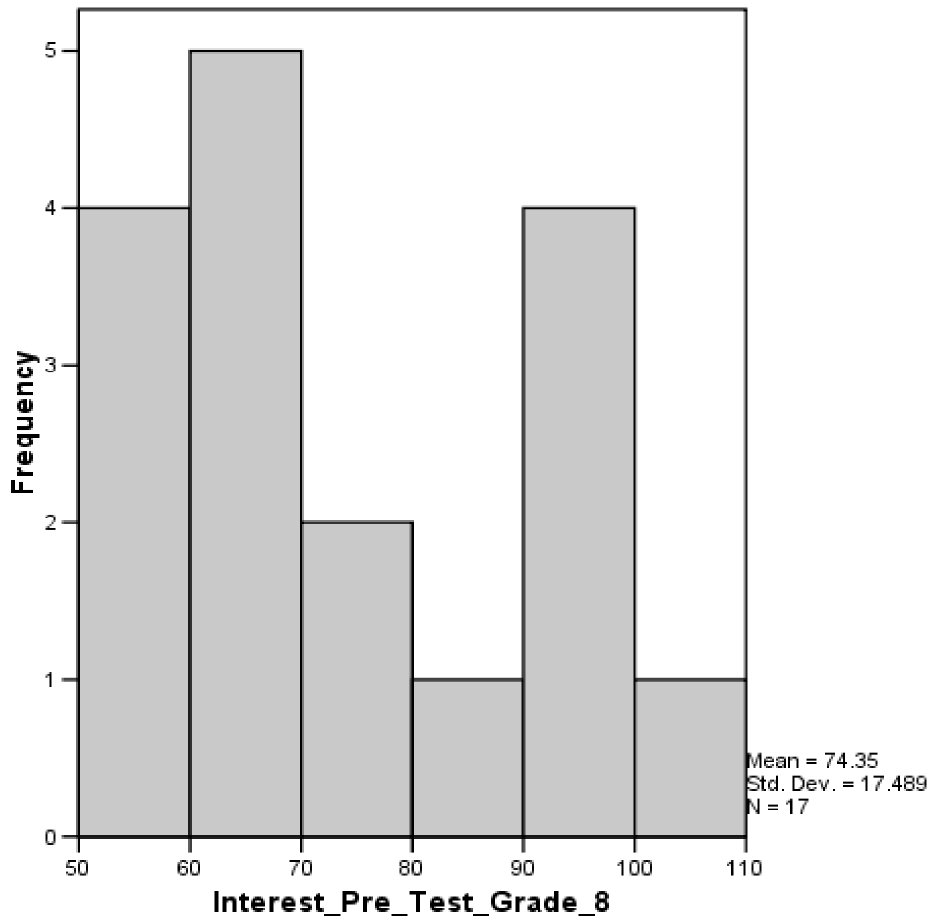

Figure 4.7: Pre Test Interest Inventory Scores - Grade 8

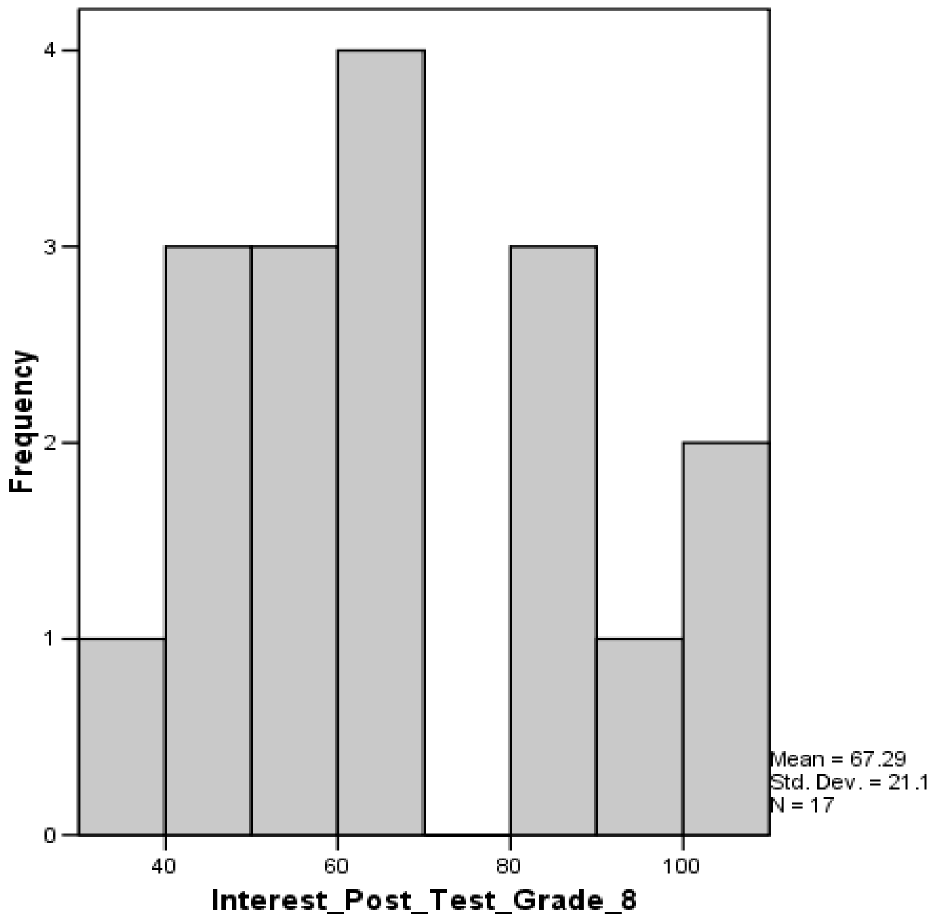

Figure 4.8: Post Test Interest Inventory Scores - Grade 8

CHAPTER V
DETAILED RESULTS

5.1 Problem Set Summary

In addition to the analyses of the overall results, we also can examine the results for each problem and student individually. Perhaps more important than whether or not the results were different or if the mean score increased is the manner in which they differed. We wanted to examine the students' results more closely to see which concepts were understood and which were not. We also want to find out the different types of mistakes that are being made. These results are of great interest in the assessment not only of our study but also of the students' understanding of the lessons and complex mathematical material. For each individual problem we show not only the results of the t-test for statistical significance of a difference, but also some common students responses and solutions. The ninth grade students will be identified C1, C2, C3... and the eighth grade students as S1, S2, S3, etc.

5.2 Problem 1 Analysis

Problem 1 asked "What is the distance from the point (4, 3) to the origin?" Points were given for various concepts within this problem, with the following breakdown. Two points were awarded if the student recognized the need for the Pythagorean Theorem. One point was awarded if values were correctly assigned in the Pythagorean Theorem or if a correct picture was drawn. One point was awarded if the student arrived at the equation $c^2 = 25$. Finally, one point was awarded for the correct answer. Table 5.1 shows the summary statistics for analysis of Problem 1. The table shows the statistics for each of the individual groups and also shows the results of the paired samples t-test in the second and fourth rows for the comparison between the pre-test and the post-test for each grade.

Table 5.1: Problem 1 Summary Statistics

	N	Mean	95% Conf. Int.	Significance
Pre - Grade 9	14	1.57		
Post - Grade 9	14	3.29	(-3.024, -.405)	.014
Pre - Grade 8	17	0.00		
Post - Grade 8	17	1.29	(-2.411, -.177)	.026

From the table we can see that both grades increased their scores on this problem, with the ninth grade increasing from 1.57 to 3.29 and the eighth grade increasing from 0.00 to 1.29. Since the t-test produced a significance value for each grade that is less than 0.05, with the ninth grade value of 0.014 and the eighth grade value of 0.026, the difference is statistically significant for both grades. This result is also confirmed by the 95% confidence intervals.

On the ninth grade pre-test, eight students out of fourteen got a zero. They didn't draw a picture to plot the point. They either left the problem blank or put down a number that wasn't correct or didn't make sense. Four students got the correct answer, with only one of these actually using the pythagorean theorem. The other three either made a good guess or were familiar with a 3-4-5 pythagorean triple. According to the TEKS standards, the "student identifies and applies patterns from right triangles to solve problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples" [14] when they take a high school geometry course. The other two students plotted the point in a cartesian plane, but didn't know where to go from there. On the post-test, the number of students receiving a zero was cut in half, while the number of students receiving five points for the correct answer more than doubled. Most of these students used a cartesian plane to plot the point. They plotted it correctly, and drew a line from that point to the origin. This shows that even if they couldn't solve the problem, they at least knew that the distance from the point (4, 3) to the origin was the length of that line segment. Student A14 was the one student who wrote out the following steps for the solution in the pre-test:

$$3^2 + 4^2 \tag{5.1}$$

$$9 + 16 = \sqrt{25} = 5\ . \tag{5.2}$$

This same student only wrote the answer, 5, on the post-test. All of the other students who got this problem correct just drew pictures of the plotted point, labelled the sides, and wrote the correct answer. The fact that most of the students understood the concepts of this problem may be due to the fact that they have already been exposed to some coordinate geometry and applications of the Pythagorean Theorem. As shown in the next paragraph, these concepts are taught in the eighth grade.

On the eighth grade pre-test, no student even got a single point on this problem. Many student wrote "Don't understand." Only two students even ventured a guess, and no student knew to draw a cartesian plane and plot the point. On the post-test, four students wrote the correct answer, but did not use the pythagorean theorem to get their answer. They remembered the 3-4-5 pythagorean triple. The closest any student got to being correct while showing work was student S17, who wrote the lengths of the two legs, 4 and 3 on a line. Right below, the student wrote the equation:

$$16 + 9 = 23\ . \tag{5.3}$$

He or she forgot to take the square root after incorrectly adding the squares of the two sides of the right triangle. In the end, most students still could not solve this problem. According to the TEKS standards, eighth grade students: "use pictures or models to demonstrate the Pythagorean Theorem", "locate and name points on a coordinate plane using ordered pairs of rational numbers", and "use the Pythagorean Theorem to solve real-life problems." [14] Perhaps this lack of knowledge is due to the fact that they are just now learning the major concepts of this problem.

Complete lists of all of the responses from the ninth grade and eighth grade students to Problem 1 are shown in Tables 5.2 and 5.3.

Table 5.2: Problem 1 Responses - Grade 9

Student	Pre-Test Response	Post-Test Response
C1	4,3	partial picture
C2	2	5, picture, values
C3	5, picture, values	5
C4	plotted point	5, partial picture
C5		5, picture, values
C6	3.5	$3.5i$
C7	4	4 down, 3 to the left
C8		
C9	5	5, picture, values
C10	3.5	5, picture, values
C11	5	5
C12		
C13	partial picture	5, picture, values
C14	$3^2 + 4^2$ $9 + 16 = \sqrt{25} = 5$	5

Table 5.3: Problem 1 Responses - Grade 8

Student	Pre-Test Response	Post-Test Response
S1	?	?
S2	don't know	don't know
S3		5
S4	I don't know	5
S5	I don't know	
S6	I don't know	I forgot how to do it
S7	I don't know	
S8	don't know	?
S9	I don't know	
S10		
S11	don't know	I don't know
S12	170 - guessed	90
S13	4	5
S14	I don't know	don't remember
S15		1
S16	I don't understand	5
S17		4 3 16+9=23

5.3 Problem 2 Analysis

Problem 2 asked "If $f(z) = z^2$, what is the value of $f(0)$?" It was a little more difficult deciding how to award points for sub-concepts within this problem. Since the answer is 0, and $0^2 = 0$, it was kind of hard to miss this problem. It must be noted that if a student did answer 0^2, they were given four points, but not full credit. An extra point was awarded only for an answer of zero. The purpose of evaluating a function is to come with a value. Even though it is true that $0^2 = 0$, if the function had been given as $f(z) = z^2 - 2$, an answer of $0^2 - 2$ would not be acceptable either. A better problem next time would be to add something to the end, such as $f(z) = z^2 - 3$ and probably have the student evaluate it at some point other than 0. Table 5.4 shows the summary statistics for analysis of Problem 2. The table shows the statistics for each of the individual groups and also shows the results of the paired samples t-test in the second and fourth rows for the comparison between the pre-test and the post-test for each grade.

Table 5.4: Problem 2 Summary Statistics

	N	Mean	95% Conf. Int.	Significance
Pre - Grade 9	14	2.79		
Post - Grade 9	14	4.93	(-3.643, -.643)	.009
Pre - Grade 8	17	0.53		
Post - Grade 8	17	0.88	(-1.380, .674)	.477

From the table we can see again that both grades increased their scores on this problem, with the ninth grade increasing from 2.79 to 4.93 and the eighth grade increasing from 0.53 to 0.88. Since the t-test produced a significance value for the ninth grade of 0.009, which is less than 0.05, the difference is statistically significant for that grade. The increase for the eighth grade was very small, and therefore not statistically significant, since the significance value was 0.477. These results are also confirmed by the 95% confidence intervals.

On the ninth grade pre-test, seven students out of fourteen got the correct answer, while six students got a zero. This problem was kind of hit-or-miss, without a lot of in-between. On the post-test, thirteen out of the fourteen students got the answer correct. A couple of good examples of students who did learn the concept are students C10, and C13. Student C10 answered 2 on the pre-test and student C13 left the question blank on the pre-test. They both had the same correct work on the post-test.

Their work is shown below.

$$f(0) = 0^2 \tag{5.4}$$

$$f(0) = 0\ . \tag{5.5}$$

One student on the pre-test and one student on the post-test left their answer as 0^2, but everyone else got the problem correct. These students did pretty well on this problem, perhaps because they have already been introduced to function concepts. For Algebra I, the TEKS standards state "the student finds specific function values" [14]

On the eighth grade pre-test, fifteen out of seventeen students didn't get any points on this problem, with most of them leaving the space blank or answering with a question mark. On the post-test, fourteen students still could not get the correct answer. Student S12 answered correctly on the pre-test, but responded with a 70 on the post-test. This would have been a good time for an interview to see where that answer came from. Student S13 answered 0^2 on the pre-test and i on the post-test. These were the only answers other than "0" or "don't know." There was not much of a change between the pre-test and the post-test. This is reinforced by the fact that the differences were not statistically significant as a result of the t-test. As stated earlier, the problem should have been changed to more accurately assess the students' knowledge of this concept.

Complete lists of all of the responses from the ninth grade and eighth grade students to Problem 2 are shown in Tables 5.5 and 5.6.

Table 5.5: Problem 2 Responses - Grade 9

Student	Pre-Test Response	Post-Test Response
C1	0	$f(0) = 0^2$
C2	0	0
C3	0	0
C4	0	0
C5		0
C6	2	0
C7	0^2	0
C8	$f(0) = z^2$ $f = z^2$	0
C9	0	0
C10	2	$f(0) = 0^2$ $f(0) = 0$
C11	0	0
C12	0	0
C13		$f(0) = 0^2$ $f = 0$
C14	0	0

5.4 Problem 3 Analysis

Problem 3 asked "What is the value of $\sqrt{-16}$?" Table 5.7 shows the summary statistics for analysis of Problem 3. Two points were given for the knowledge that the square root of 16 = 4, and two more points were given for the concept that the square root of -1 is i. One more point was given for the correct final answer. Table 5.7 shows the statistics for each of the individual groups and also shows the results of the paired samples t-test in the second and fourth rows for the comparison between the pre-test and the post-test for each grade.

Table 5.6: Problem 2 Responses - Grade 8

Student	Pre-Test Response	Post-Test Response
S1	?	?
S2	0	don't know
S3	0	0
S4	I don't know	0
S5	I don't know	
S6	I don't know	I forgot how to do it
S7	I don't know	
S8	don't know	?
S9	?	
S10		
S11	don't know	don't know
S12	0	70
S13	0^2	i
S14	I don't know	don't remember
S15		
S16	I don't understand	0
S17		

Table 5.7: Problem 3 Summary Statistics

	N	Mean	95% Conf. Int.	Significance
Pre - Grade 9	14	2.36		
Post - Grade 9	14	4.29	(-3.158, -.699)	.005
Pre - Grade 8	17	1.18		
Post - Grade 8	17	0.82	(-.191, .896)	.188

From the table we see that the ninth grade exhibited a large increase in the mean score from 2.36 to 4.29. However, the eighth grade mean actually decreased from 1.18 to 0.82. Since the t-test produced a significance value of 0.005 for the ninth grade, and this value is less than 0.05, the difference is statistically significant for that grade. The result was not statistically significant for the eighth grade, since the significance value was 0.188. The ninth grade showed a marked improvement while the eighth grade class did not. These results are also confirmed by the 95% confidence intervals.

On the ninth grade pre-test, four students out of fourteen got the correct answer. On the post-test, the number of students who got the answer correct increased to ten. Of the students who missed the problem, some responses were interesting. Students C2, C6, and C10 all responded "an imaginary number" on the pre-test. Students are taught to classify numbers as whole, natural, integers, rational, irrational, and imaginary. The concept of imaginary numbers is not taught until Algebra II, which is the next level for the ninth grade students in the survey. Students C1 and C3 both answered with -4, a reasonable incorrect answer. Student C1 answered the same way on the post-test, while student C3 responded with 4 on the post-test. Student C5 answered $\sqrt{-8}$ on the pre-test. Some students tend to confuse square roots with multiplication by one-half, but this student forgot to remove the radical. C5 did answer correctly on the post-test. Student C7 answered $1i$ on both the pre-test and the post-test. He or she already knew the concept of i but forgot about the $\sqrt{16} = 4$. Student C8 wrote -4·4=-16. He or she was intent on getting the correct answer of -16. This solution is not uncommon, students just forget that square roots mean that both numbers are the same. Student C13 went from leaving the answer blank to getting the correct answer. Student C4 wrote nothing on the pre-test, and $16i$ on the post-test, showing that he or she understood the concept of i, but forgetting to take the square root of 16.

On the eighth grade pre-test, none of the seventeen students answered correctly. No student even got the part about i. Most students answered either 4 or -4. Student C13 answered -8, incorrectly thinking that the square root of 16 is 8, and then applying the negative sign. On the post-test, again no student answered correctly. Only one student, C4, answered with i. Unfortunately, his or her answer was only i, without the 4 that should have gone with it. The post-test also produced an unusual result. Five students actually went backwards. They all had some kind of answer on the pre-test, but did not answer at all on the post-test. It has been suggested that maybe when a student answered with -4 on the pre test and ”I don't know” on the post test that perhaps this is not backwards at all. Perhaps the student learned enough throughout the course of this study to know that -4 is not the correct answer. They didn't know what the correct answer was, but they knew it couldn't be -4. This is another instance where an interview would have been extremely helpful in knowing what the student was thinking and why they responded the way they did. The other answers were again either 4 or -4. As mentioned earlier, imaginary numbers are not taught until Algebra II, and these students are even further away from Algebra II than the ninth grade students.

Complete lists of all of the responses from the ninth grade and eighth grade students to Problem 3 are shown in Tables 5.8 and 5.9.

Table 5.8: Problem 3 Responses - Grade 9

Student	Pre-Test Response	Post-Test Response
C1	-4	-4
C2	imaginary number	$4i$
C3	-4	4
C4		$16i$
C5	$\sqrt{-8}$	$4i$
C6	imaginary number	$4i$
C7	$1i$	$1i$
C8	$-4 \cdot \cdot 4 = -16$	$4i$
C9	$4i$	$4i$
C10	it's imaginary	$4i$
C11	$4i$	$4i$
C12	$4i$	$4i$
C13		$4i$
C14	$4i$	$4i$

5.5 Problem 4 Analysis

On Problem 4 the students were given a diagram of a right triangle inscribed inside a circle (see Appendix C). The problem stated "The diagram below shows a circle with center at (0, 0) and radius of 1. Find the distance from the y-axis of the point indicated where x= .75". This problem was really not a very good problem for a couple of reasons. First of all, the point was not clearly marked. There should have been a clearly labelled point right on the unit circle at the end of the hypotenuse. Second, the problem should have asked for the distance from the x-axis instead of the distance from the y-axis. When the problem set was given to the students, they were notified of these shortcomings and directions were given as to what the problem meant to ask. Still another shortcoming was the omission of units in the problem.

Table 5.9: Problem 3 Responses - Grade 8

Student	Pre-Test Response	Post-Test Response
S1	?	?
S2	4	don't know
S3	-4	nothing
S4	I don't know	i
S5	4	4
S6	I don't know	I forgot how to do it
S7	I don't know	
S8	don't know	?
S9	-4	
S10	-4	-4
S11	-4	-4
S12	4	-4
S13	-8	no answer
S14	I don't know	don't remember
S15	-4	4
S16	-4	
S17	-4	-4

Much emphasis is given on units these days, right down to elementary school students. The number, 0.75, would have much more meaning if there had been units given with the problem. This is another Pythagorean Theorem problem, involving the unit circle and fractions or decimals. Two points again were given as in Problem 1 for recognizing the need for the Pythagorean Theorem, one point was awarded for plugging values into the correct variables, and one point was awarded for arriving at the equation $.75^2 + y^2 = 1$. One final point was awarded for the correct answer. Some students answered the question exactly as it was asked, finding the distance to the y-axis as the problem stated instead of to the x-axis as it was intended. Since this answer is correct according to how the problem was stated, they received full credit. Table 5.10 shows the summary statistics for analysis of Problem 4. The table shows the statistics for each of the individual groups and also shows the results of the paired samples t-test in the second and fourth rows for the comparison between the pre-test and the post-test for each grade.

Table 5.10: Problem 4 Summary Statistics

	N	Mean	95% Conf. Int.	Significance
Pre - Grade 9	14	1.50		
Post - Grade 9	14	2.93	(-2.763, -.094)	.038
Pre - Grade 8	17	0.00		
Post - Grade 8	17	0.06	(-.184, .066)	.332

From the table we see that the ninth grade exhibited a large increase in the mean score from 1.50 to 2.93. The eighth grade mean had a very small increase from 0.00 to 0.06. Since the t-test produced a significance value of 0.038 for the ninth grade, and this value is less than 0.05, the difference is statistically significant for that grade. The result was not statistically significant for the eighth grade, since the significance value was 0.332. The ninth grade showed a large improvement on this concept while the eighth grade experienced a very slight improvement. These results are also confirmed by the 95% confidence intervals.

On the ninth grade pre-test, two students out of fourteen got the correct answer, while five students got no points at all. On the post-test, seven students got the correct answer, while four students missed the problem completely. One thing to look for on this problem is recognition of the application of the Pythagorean Theorem. The problem stated that the circle had a radius of 1, so the students needed to assign 1 to the hypotenuse. Many students did so, but did not know what to do after that.

Student C4 started out very well on the pre-test. He or she wrote:

$$.75^2 + b^2 = 1 \ . \tag{5.6}$$

His or her work stopped there. On the post-test, the same student showed the correct solution with the following work:

$$.75^2 + y^2 = 1 \tag{5.7}$$

$$.5625 + y^2 = 1 \tag{5.8}$$

$$y^2 = 1 - .5625 \tag{5.9}$$

$$\sqrt{y^2} = \sqrt{.4375} \tag{5.10}$$

$$y = .6614 \ . \tag{5.11}$$

Student C5 got really close, transposing the hypotenuse and one of the legs. The student responded:

$$.75^2 + 1^2 = y^2 \tag{5.12}$$

$$.5625 + 1 = y^2 \tag{5.13}$$

$$1.5625 = y^2 \tag{5.14}$$

$$1.25 = y \ . \tag{5.15}$$

The same student responded the same way in the post-test. This was good work and was very close, but not quite correct. The student probably didn't remember a particular feature of a right triangle, the fact that neither leg can be longer than the hypotenuse. A couple of more students must have made the same mistake, because they also answered 1.25. They didn't show their work, but it must have been similar to student C5. A few other students got the correct answer but didn't show their work. Student C8 did not write anything on either problem set. There were a couple of other random answers, 0.15 and 0.25. Without an interview or work shown it is difficult to determine the process which produced those answers. Surprisingly, no student converted the 0.75 length of a triangle leg to a fraction. Solving this equation with fractions would have led to a very simple answer, $\frac{\sqrt{7}}{4}$, which does equal 0.6614. Some of the lessons, as mentioned earlier, did reference the unit circle and particular values of the coordinate pairs along the circle. As in Problem 1, these students were already somewhat familiar with the Pythagorean Theorem and its applications.

On the eighth grade pre-test, none of the seventeen students answered correctly or even received partial credit. Many students responded "I don't know." On the post-test, again no student answered correctly. Sixteen out of the seventeen students again received no credit at all for the problem. The students did not know to mark the hypotenuse length of 1 and apply the Pythagorean Theorem. There was almost no improvement on this problem. This may again be due to the fact that these students are just now learning the Pythagorean Theorem and its applications.

Complete lists of all of the responses from the ninth grade and eighth grade students to Problem 4 are shown in Tables 5.11 and 5.12.

5.6 Interest Inventory Analysis

Analysis of the individual questions in the interest inventory yielded some interesting items. For the ninth grade students, in the categories Importance of Learning Mathematics and Students' Mathematics Work Ethic, about half of the questions showed an increase and half showed a decrease. Of the nine items referring to Attitudes Towards Mathematics, eight items showed an increase with only one item having a lower mean in the post-survey than in the pre-survey, Item 19, "I want to figure out new ways to solve math problems." The largest single change was Item 1, "I work carefully when doing math." The mean increased from 4.14 to 5.50, a 33% increase. The eighth grade students' mean responses were generally down across the board, with only four items out of twenty having higher mean responses in the post-survey than in the pre-survey.

Table 5.11: Problem 4 Responses - Grade 9

Student	Pre-Test Response	Post-Test Response
C1	.25	.25
C2	.15	.75
C3	$\sqrt{.5625}$	.5625
C4	$.75^2 + b^2 = 1^2$	$.75^2 + y^2 = 1^2$ $y^2 = 1 - .5625$ $\sqrt{y^2} = \sqrt{.4375}$ $y = .6614$
C5	$a^2 + b^2 = c^2$ $.75^2 + 1^2 = c^2$ $.5625 + 1 = c^2$ $\sqrt{1.5625} = c^2$ $1.25 = y$	$.75^2 + 1^2 = y^2$ $.5625 + 1 = y^2$ $1.5625 = y^2$ $1.25 = y$
C6	.15	.75
C7	.25	$y = .6614$
C8		
C9	.66	.66
C10		$.75 + y^2 = 1^2$ $.56 + y^2 = 1 - .56$ $\sqrt{y^2} = \sqrt{.44}$ $y = .66$
C11	$y = .6614$	.66
C12		.25
C13		
C14	$y = \sqrt{1.5625}$ $y = 1.25$	1.06

Table 5.12: Problem 4 Responses - Grade 8

Student	Pre-Test Response	Post-Test Response
S1	?	?
S2	don't know	don't know
S3		
S4		1
S5	I don't know	
S6	I don't know	I forgot how to do it
S7	I don't know	
S8	don't know	?
S9	?	
S10		
S11	don't know	don't know
S12	don't know	don't know
S13	.50	.45
S14		
S15		
S16	I don't understand	
S17		$y = 75$

Importance of Learning Mathematics was split about 50-50 as far as increases and decreases, but both Students' Mathematics Work Ethic and Attitudes Towards Mathematics each only had one item with an increase in mean response. The biggest changes were Item 5, "I want to learn more about math," and Item 20, "I become excited when close to solving a math problem." Each of the items showed about a 30% drop in mean response scores. The twenty items with their pre-interest and post-interest means are shown below in Table 5.13.

Paired samples t-tests were also performed on each individual pair of items (pre-interest and post-interest) to observe if any of the differences in the means were statistically significant. The procedure is the same as before, with the same hypotheses and the same formula for the test statistic. A significance level of 0.05 is a widely used value for hypothesis testing. Just as before, if the significance level is greater than 0.05, we do not reject $H_0: D = 0$ and conclude the difference is not statistically significant. If the significance level is less than 0.05, we then reject $H_0: D = 0$ and conclude that the results are statistically significant and there is indeed a difference in the mean responses. Table 5.14 below shows the significance values of the two-tailed hypothesis tests for each item.

The table confirms that two items for the ninth grade students were statistically different, Items 1 and 15. Item 1 was already referred to in the last paragraph because of its large change. Item 15 stated "I enjoy challenging math problems." It is good to see that the students want to learn more and be challenged. The only significant items for the eighth grade students were Items 5 and 20, both mentioned in the previous paragraph because of the large differences in means.

In addition to the twenty interest items the students were also asked their age, ethnicity, current grade, and gender. Each of these factors can be analyzed separately against each of the responses. A Chi-Square test is used to determine if any of these factors has an influence on the responses of the twenty items, or if they are independent. A plain words way of looking at the appropriate hypotheses for the is test is: H_0: Responses and factor are independent, and H_1: Responses and factor are not independent. The data are entered into a frequency table with separate columns for gender and separate rows for the value of each response. An expected frequency is calculated by multiplying the row total by the column total and dividing that result by the sample size. If there is no difference between gender, then the values should be distributed about evenly.

To test this, we employ the following formula for the Chi-Square test statistic:

$$\chi^2 \ = \ \frac{(O-E)^2}{E} \ . \tag{5.16}$$

In the preceding formula, O represents the observed frequency in each cell of the table and E represents the expected value, as mentioned before. This value of the test statistic is compared against the Chi-Square distribution, and an appropriate significance value is calculated. Again the significance level determines whether we reject H_0 or we do not reject H_0. When the significance level is below 0.05, we reject H_0.

When analyzing the age factor, it should be noted that if a fifteen year old student just had a birthday and a fourteen year old student has a birthday next week, they could be only days apart in age. The analysis of the responses against age were insignificant, showing no relation between the two for either the ninth or the eighth grade students. There was also no significant relation when both ethnicity and current grade were tested against interest. In the ninth grade class twelve out of the fourteen students were Hispanic, as were fourteen out of the seventeen eighth grade students. The overwhelming majority of one ethnicity may have been a factor in the lack of significance. There was a better distribution of expected grades, but still no significant relation between the responses and the current grade. The ninth grade class was split evenly in gender, seven boys and seven girls. The eighth grade class had seven boys and ten girls. With this even gender distribution, a closer look at the effects of gender against the interest responses might prove interesting. Table 5.15 shows the significance levels for all of the pre-interest and post-interest responses against gender.

The only item with a significance value less than 0.05 was Item 12 in the pre-interest survey, "I am able to concentrate easily when working on math." The same item was not significant in the post-interest survey. The frequency table for the pre-interest survey is shown in Table 5.16 and for the post-interest survey is shown in Table 5.17 Gender 1 is for male and Gender 2 is for female.

Table 5.16 shows the males' responses are somewhat higher, with four students answering with a response of five or higher, compared to zero females with responses in that range. Table 5.17 shows the responses for both genders were lower. The significance for the pre-interest survey was 0.042, just barely under the 0.05 cutoff value. Both genders seemed to think less of their abilities to concentrate easily on math after the experiment, with the difference in genders lessening afterwards.

Table 5.13: Individual Item Comparisons of Means

Grade	Q1 pre	Q1 post	Q2 pre	Q2 post	Q3 pre	Q3 post
9	4.14	5.50	3.71	3.79	2.71	2.93
8	4.12	3.71	3.00	2.53	2.18	1.71
Grade	Q4 pre	Q4 post	Q5 pre	Q5 post	Q6 pre	Q6 post
9	4.29	4.86	5.07	5.43	4.71	5.57
8	3.18	2.59	4.24	3.00	3.47	4.35
Grade	Q7 pre	Q7 post	Q8 pre	Q8 post	Q9 pre	Q9 post
9	2.57	3.21	3.36	3.71	6.07	5.71
8	3.59	3.18	3.12	3.24	4.76	4.00
Grade	Q10 pre	Q10 post	Q11 pre	Q11 post	Q12 pre	Q12 post
9	3.93	3.43	4.36	4.57	3.93	3.86
8	4.47	3.88	3.18	2.94	3.29	3.06
Grade	Q13 pre	Q13 post	Q14 pre	Q14 post	Q15 pre	Q15 post
9	4.93	4.71	3.64	3.86	3.00	3.57
8	3.88	3.53	2.65	2.24	1.88	1.88
Grade	Q16 pre	Q16 post	Q17 pre	Q17 post	Q18 pre	Q18 post
9	5.79	5.57	5.29	5.07	5.64	5.86
8	5.76	6.29	5.12	4.12	4.76	5.24
Grade	Q19 pre	Q19 post	Q20 pre	Q20 post		
9	5.36	5.29	4.29	4.50		
8	4.24	3.41	3.47	2.41		

Table 5.14: Paired Samples t-Tests on Individual Items

	Significance - Grade 9	Significance - Grade 8
Q1 pre - Q1 post	.003	.394
Q2 pre - Q2 post	.861	.216
Q3 pre - Q3 post	.720	.466
Q4 pre - Q4 post	.252	.347
Q5 pre - Q5 post	.239	.001
Q6 pre - Q6 post	.075	.140
Q7 pre - Q7 post	.168	.069
Q8 pre - Q8 post	.418	.829
Q9 pre - Q9 post	.239	.137
Q10 pre - Q10 post	.451	.400
Q11 pre - Q11 post	.459	.466
Q12 pre - Q12 post	.893	.602
Q13 pre - Q13 post	.671	.455
Q14 pre - Q14 post	.640	.464
Q15 pre - Q15 post	.026	1.000
Q16 pre - Q16 post	.711	.144
Q17 pre - Q17 post	.678	.084
Q18 pre - Q18 post	.630	.426
Q19 pre - Q19 post	.869	.140
Q20 pre - Q20 post	.487	.024

Table 5.15: Pearson Chi-Square Test - Interest vs. Gender

Grade	Q1 pre	Q1 post	Q2 pre	Q2 post	Q3 pre	Q3 post
9	.736	.881	.353	.258	.306	.247
8	.153	.424	.560	.121	.285	.367
Grade	Q4 pre	Q4 post	Q5 pre	Q5 post	Q6 pre	Q6 post
9	.247	.340	.475	.228	.279	.493
8	.409	.369	.705	.392	.567	.820
Grade	Q7pre	Q7post	Q8pre	Q8post	Q9 pre	Q9 post
9	.615	.193	.458	.462	.387	.132
8	.403	.589	.210	.210	.173	.165
Grade	Q10 pre	Q10 post	Q11 pre	Q11 post	Q12 pre	Q12 post
9	.291	.649	.649	.475	.970	.458
8	.519	.263	.328	.832	.042	.234
Grade	Q13 pre	Q13 post	Q14 pre	Q14 post	Q15 pre	Q15 post
9	.306	.392	.632	.639	.247	.291
8	.549	.580	.120	.247	.517	.155
Grade	Q16 pre	Q16 post	Q17 pre	Q17 post	Q18 pre	Q18 post
9	.856	.639	.258	.760	.102	.753
8	.416	.556	.611	.235	.549	.694
Grade	Q19 pre	Q19 post	Q20 pre	Q20 post		
9	.363	.424	.504	.856		
8	.319	.716	.465	.506		

Table 5.16: Item 12 - Pre

	Gender		
Response	1	2	Total
1	0	3	3
2	2	1	3
3	1	2	3
4	0	4	4
5	3	0	3
6	0	0	0
7	1	0	1

Table 5.17: Item 12 - Post

	Gender		
Response	1	2	Total
1	0	5	5
2	1	1	2
3	2	1	3
4	2	2	4
5	0	0	1
6	1	0	1
7	1	0	1

CHAPTER VI
CONCLUSIONS

Complex variables, functions and other operations involving complex variables were introduced to eighth and ninth grade algebra students in two different junior high schools. There are no previous studies or projects of this nature with which to compare results. Some of these concepts will be introduced to the students in high school, while many of the concepts that were introduced would only be seen in college.

Pre and post tests were administered to each student on the first and last days of the study. First, a mathematics interest inventory was given to allow the student to rate their interest in and their attitude towards mathematics. A second test containing problems designed to measure the students' mathematical abilities was also administered. These problems followed TEKS standards and also were pertinent to the overall mission of the study.

The sample sizes were small with fourteen ninth grade and seventeen eighth grade students. All statistical analyses were performed with the proper methods for the small sample. As with any small sample, we must assume that the samples were drawn from a population that was normally distributed. Though this assumption is not suspect, we still must not place too much emphasis on exact results. However, most of the results in this review are well beyond questionable borders. The conclusions drawn are quite reasonable and well supported. The pre and post interest inventory results were not different enough to be considered statistically significant for the ninth grade class. However, the difference was found to be statistically significant for the eighth grade class. These results were reversed for the problem sets. The ninth grade difference in pre and post test problem set scores was highly statistically significant, but not so for the eighth grade students. The ninth grade class had a quicker grasp and better understanding of the material. The eighth grade students had many difficulties understanding the material.

This study did produce some excellent results for the ninth grade students. It was shown that the material presented was challenging and yet good results were attainable for ninth grade students. It would also be helpful to interview the students before and after the study. This would definitely give more insight into the thinking and solving process of the individual student. Also much more care must be taken in the formulation of the problem set to ensure that it evaluates the concepts pertinent to the study. This study had two problems that basically addressed the same concept, which was unnecessary. Another problem could have been used to test another useful concept. The students should also be required or at least encouraged to show their work so that their answers can be analyzed more completely.

The students did show an eagerness to work with a student from Texas Tech University. Studies and projects of this nature enhance the relationship between Texas Tech University and the schools in the Lubbock Independent School District and surrounding districts also. Other studies are taking place in different subject areas by the Texas Tech University Department of Mathematics and Statistics. An analysis could be done combining these individual studies to evaluate the success or failure of these interactions. With this increased visibility and the expected success with students in grade nine or above, an outreach program may be formed to hopefully raise mathematic performance and interest in mathematics.

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APPENDIX A
PARENT/STUDENT CONSENT FORM

Your son/daughter is invited to participate in the research study being conducted at your LISD school. Our names are Dr. Jerry Dwyer, Dr. Jennifer Wilhelm, and Billy Duke. Dr. Dwyer and Dr. Wilhelm are professors at Texas Tech University, Dr. Dwyer is in the Department of Mathematics and Statistics, and Dr. Wilhelm is in the College of Education. Billy Duke is a Graduate Part-Time Instructor with the Department of Mathematics and Statistics. The goal of this research study is to increase students' interests and ability levels in mathematics. We are asking for permission to include your son/daughter in this study. The data obtained from this study will be collected during the 2004 spring semester.

All students will continue to participate in all classroom activities. Researchers from Texas Tech will attend class and introduce exciting new mathematical concepts with the purpose of enhancing the students' algebra and geometry skills. They will also record observations about the classroom activities, and review students' responses to project materials. If you allow your child to be a part of this study, his/her normal classroom participation will be analyzed as part of our research data. Additionally, he/she may be invited to participate in one-on-one interviews with project researchers. These interviews will be audio and/or video taped during the normal school day. Students do not have to answer every question during these interviews. Cassettes will be coded so that no personally identifying information is visible on them, they will be kept in a secure place, and they will be heard or viewed only for research purposes by the investigators and their associates. At the conclusion of the study the tapes will be retained by the investigators for future analysis.

Any information that is obtained in connection with this study (and that can be identified with your son/daughter) will remain confidential and will only be disclosed with your permission. His or her responses will not be linked to his or her name or your name in any written or verbal report of this research project.

Your decision whether or not to allow your son/daughter to participate will not affect your own or his/her present or future relationship with Texas Tech University. Also, students' grades will not be affected by your decision to allow him/her to participate in this study. The participating students in this study will benefit from the opportunity to engage in interdisciplinary projects, which are designed to increase their understandings of content aligned with state and national standards. There are no known risks as a result of participation in this study.

If you have any questions about the study, please contact Dr. Jerry Dwyer or Billy Duke. Dr. Dwyer can be reached at 806-742-2580 and Billy Duke can be reached at 806-742-2566. You will be given a copy of this consent form for your records.

You are making a decision about allowing your son/daughter to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. If you later decide that you wish to withdraw your permission for your son/daughter to participate in the study, simply tell us by phoning the researchers: Dr. Jerry Dwyer or Billy Duke at 806-742-2566 or by contacting your son/daughter's classroom teacher. You may discontinue his or her participation at any time.

____

Name of son/daughter/or ward

____

Signature of Parent(s) or Legal Guardian Date

____

Signatures of Investigators Date

I agree to participate in this interdisciplinary study. This study was explained to my (mother/father/parents/guardian) and (she/he/they) said that I could take part. The only people who will know about what I say and do in the study will be the people in charge of the study.

In this study I will be asked questions about how I write, solve problems, and perform tasks. If I decide to quit the study, all I have to do is tell the person in charge.

____

Signature of Student Date

APPENDIX B

INTEREST INVENTORY

How well does each statement that is listed below describe your feelings about math?

If you believe the statement sounds a lot like you, pick a number from the far right side of the scale and write it in the space beside the item. If you believe the statement sounds like you not at all, pick a number from the far left, and if you believe the statement sounds like you somewhere between these two extremes, then pick a number from someplace in the middle of the scale.

Sounds like me not at all	1	2	3	4	5	6	7	Sounds a lot like me

1. I work carefully when doing math. ______
2. I feel good when it comes to working on math. ______
3. I prefer easy math over math that is hard. ______
4. Time goes by quickly when I am working on math. ______
5. I want to learn more about math. ______
6. I see how I can use math in everyday life. ______
7. When working on math, I want to stop and begin working on something else. ______
8. I tend to know more about math than other kids. ______
9. I think learning math is important. ______
10. I would rather be working on something else besides math. ______
11. I am interested in math. ______

Sounds like me not at all	1	2	3	4	5	6	7	Sounds exactly like me

12. I am able to concentrate easily when working on math. ______

13. I think about how I can use math in a future career. ______

14. I like to answer questions in math class. ______

15. I enjoy challenging math problems. ______

16. I feel let down when I have to stop working on math. ______

17. I am wasting my time on math. ______

18. Knowing a lot about math is helpful. ______

19. I want to figure out new ways to solve math problems. ______

20. I become excited when close to solving a math problem. ______

Please respond to the following:

21. How old are you? ______

22. Circle the word that best describes you.

Hispanic Black White Native American Asian Other

23. What is your current math grade.

A B C D F

24. Boy Girl

Were any of the above items confusing, difficult to understand, or odd in any way? If so, please describe below. Your comments are much appreciated.

THANK YOU!

APPENDIX C

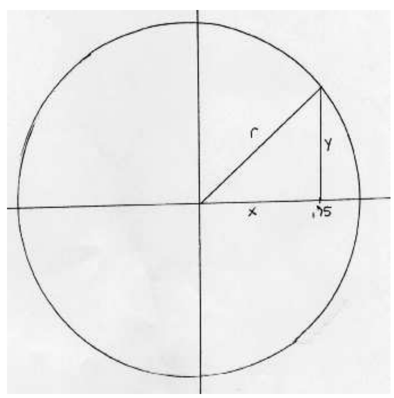
PROBLEM SET

1. What is the distance from the point (4,3) to the origin?

2. If $f(z) = z^2$ what is the value of $f(0)$?

3. What is the value of $\sqrt{-16}$?

4. The diagram below shows a circle with center at (0,0) and radius of 1. Find the distance from the y-axis of the point indicated where x=.75.



APPENDIX D
MAPLE PROGRAM

```
with(plots);
a:=-2.0;
b:=2.0;
c:=-2.0;
d:=2.0;
k1:=0;
k2:=0;
eps:=0.001;
bound:=100.0;
LIM:=300;
dx:=(b-a)/LIM;
dy:=(d-c)/LIM;
A:= array(1..LIM,1..LIM);
for ik from 1 to LIM do
for jk from 1 to LIM do
A[ik,jk]:= a + ik*dx + I*(c + jk*dy)
od
od:
for k from 1 to LIM do
for kk from 1 to LIM do
z1:=A[k,kk]:
dd:=1:
for j from 1 to 8 while ((dd>eps)and(dd>bound)) do
z2:=z1^2;
dd:=abs(z2-z1);
z1:=z2
od;
if (j>8) then
k1:= k1+1;
xp1[k1]:=Re(A[k,kk]);
yp1[k1]:=Im(A[k,kk]);
```

```
else
k2:= k2+1;
xp2[k2]:=Re(A[k,kk]);
yp2[k2]:=Im(A[k,kk]);
fi
od
od;
L1:=seq([xp1[i],yp1[i]],i = 1..k1):
L2:=seq([xp2[i],yp2[i]],i = 1..k2):
K1:=pointplot(L1,color=red,symbol=POINT):
K2:=pointplot(L2,color=blue,symbol=POINT):
display(K1,K2);
```

PERMISSION TO COPY

In presenting this thesis in partial fulfillment of the requirements for a master's degree at Texas Tech University or Texas Tech University Health Sciences Center, I agree that the Library and my major department shall make it freely available for research purposes. Permission to copy this thesis for scholarly purposes may be granted by the Director of the Library or my major professor. It is understood that any copying or publication of this thesis for financial gain shall not be allowed without my further written permission and that any user may be liable for copyright infringement.

Agree (Permission is granted.)

______Billy J. Duke___________________________________07-10-05_______

Student Signature   Date

Disagree (Permission is not granted.)

____   _________________

Student Signature   Date