ADAPTIVE FUZZY NONLINEAR INTERNAL MODEL CONTROL STRATEGY

by

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A DISSERTATION

IN

ELECTRICAL ENGINEERING

Submitted to the Graduate Faculty
of Texas Tech University in
Partial Fulfillment of
the Requirements for
the Degree of

DOCTOR OF PHILOSOPHY

Approved

December, 1996

ACKNOWLEDGMENTS

To accomplish this dissertation, I am deeply indebted to Dr. Donald L. Gustafson, the chairman of my committee. His expertise, encouragement, guidance and patience have supported me throughout the process and will never be forgotten.

I would also like to extend my gratitude to the members of my committee: Dr. Sunanda Mitra, Dr. Michael G. Giesselmann and Dr. William J.B. Oldham for their guidance, help and support throughout my education. Finally, I would like to thank all my friends for their support.

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ABSTRACT

Proportional-Integratal Derivative like Fuzzy Logic Controllers (PID-FLCs). have been used for a variety of nonlinear control problems. Basically, a PID-FLC contains a control algorithm in the form of linguistic fuzzy rules. The problem with PID-FLCs is that there is no systematic design for developing fuzzy rules. It is also difficult to develop the controllers to meet specific requirements on control performances.

In this dissertation, a nonlinear internal model control (NIMC) structure and an adaptive fuzzy NIMC strategy have been proposed to overcome the problems of PID-FLCs. One of the attractive features of the NIMC structure is that the relations between some designed parameters and the performance of the control system can be found explicitly. Thus, this control structure allows designers to systematically construct the fuzzy control. An adaptive fuzzy NIMC strategy has been proposed. The proposed strategy has two attractive features. First, the strategy provides an on-line adaptation to improve control performance and to keep the closed-loop system stable. Second, a fuzzy basis function (FBF) expansion is used to implement the controller. The use of the FBF expansion enhances the ability of the strategy to control practical nonlinear systems whose exact mathematical models are difficult to obtain.

Finally, Simulation studies of controlling four nonlinear systems (e.g., a pendulum, an inverted pendulum, a forced Van der Pol equation, and a two-link cylindrical robot manipulator) have been conducted. The simulation results show that the proposed strategy has successfully controlled the four nonlinear systems.

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CHAPTER 1

INTRODUCTION

1.1 Background

A control system is a mechanism which makes physical variables of a system behave in a prescribed manner [52]. According to the architecture of control systems, control systems may be classified into two groups: open-loop control systems and closed-loop control systems. So far, most control theories and research have focused on closed-loop control systems.

A closed-loop system is one in which the plant input is determined, at least in part, by the output of the system [93]. By using output information to affect the control input of the system, feedback is being applied to that system. There are many ways to utilize the feedback signal. It is often the case that the signal feedback from the system output is compared with a reference signal, r. Then, the result of this comparison is used to obtain the actuating system input. Such a closed-loop system is shown in Figure 1.1.

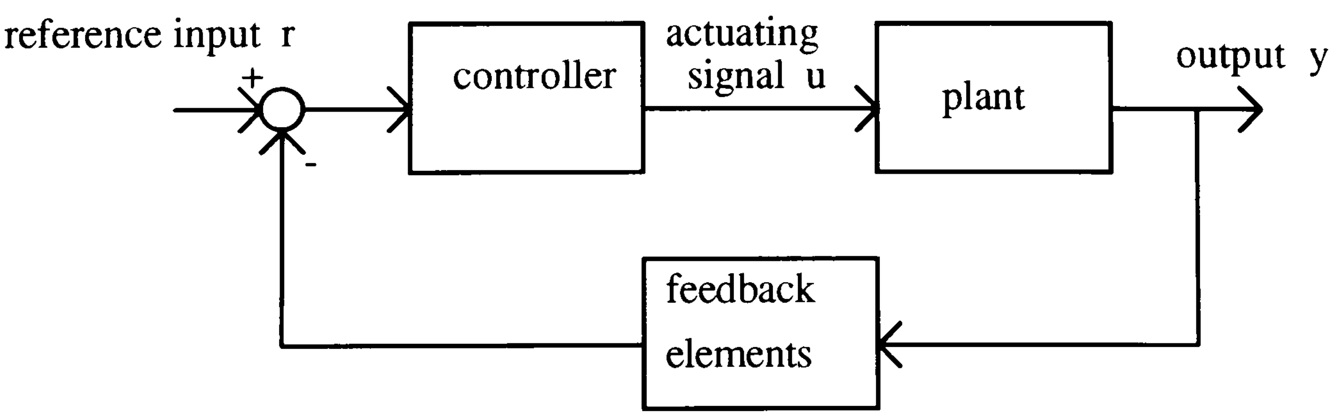


Figure 1.1: A closed-loop system

For closed-loop control systems, the general control problem is to construct a controller, given a plant and a feedback configuration so that requirements are achieved. For linear feedback control systems there exist many mathematical techniques for design and analysis purposes. Unlike linear feedback control systems, nonlinear feedback control systems are very difficult to design and analyze the control systems mathematically. In addition, since there are many different forms of nonlinear systems, there are no general design techniques that can be used for every class of nonlinear systems.

Unfortunately, linear systems do not exist in practice. All physical systems are nonlinear to some extent. Linear feedback control systems are idealized models that are used for the simplicity of analysis and design. If the operating range of a control system is small, then the control system may be approximated by a linearized model. Typically, the linearized model can be obtained by the Jacobian linearization at a nominal operating point [93]. However, when the magnitudes of the signals are extended beyond the range of the linear operation, the system should no longer be considered linear. If this is the case, design techniques for nonlinear systems are necessary.

Conventionally, there are two commonly used nonlinear control design methods: the feedback linearization method [36, 68, 106] and the nonlinear model predictive control method [25, 88, 108]. Here, only these methods will be presented.

1.1.1 Feedback Linearization

A feedback linearization method is based on the idea of transforming nonlinear dynamics into a linear form by using state feedback. Unlike the Jacobian linearization, which linearizes a nonlinear model only at the operating point, the feedback linearization transforms a nonlinear model into a linear model in a neighborhood of the operating point. Thus, the control system that uses the feedback linearization method can be operated over a wider range, when compared with the system that uses the Jacobian linearization.

The theoretical developments of this method can be found in Slotine [106], and Marino [68]. In addition, the recent developments of this method can be found in the literature [36]. For introduction purposes, the presentation will avoid mathematical difficulty and focus on the basic concept.

Here, it is assumed that a nonlinear system is described by

$$\dot{x} = f(x, u). \tag{1.1}$$

The linearization problem is to find a state transformation,

$$z = \phi(x), \tag{1.2}$$

and an input transformation,

$$u = \psi(x, v), \tag{1.3}$$

such that the map between the new input, v, and the output, y, is linear in the form

$$\dot{z} = Az + Bv, \tag{1.4}$$

for all values of the state x and the input u. Typically, these transformations exist only in a local sense. However, in this region, the input-output map is exactly linear. Once a linear relationship between v and y is available, linear controller design techniques can be employed. The control system under this method is represented in the block diagram in Figure 1.2 .

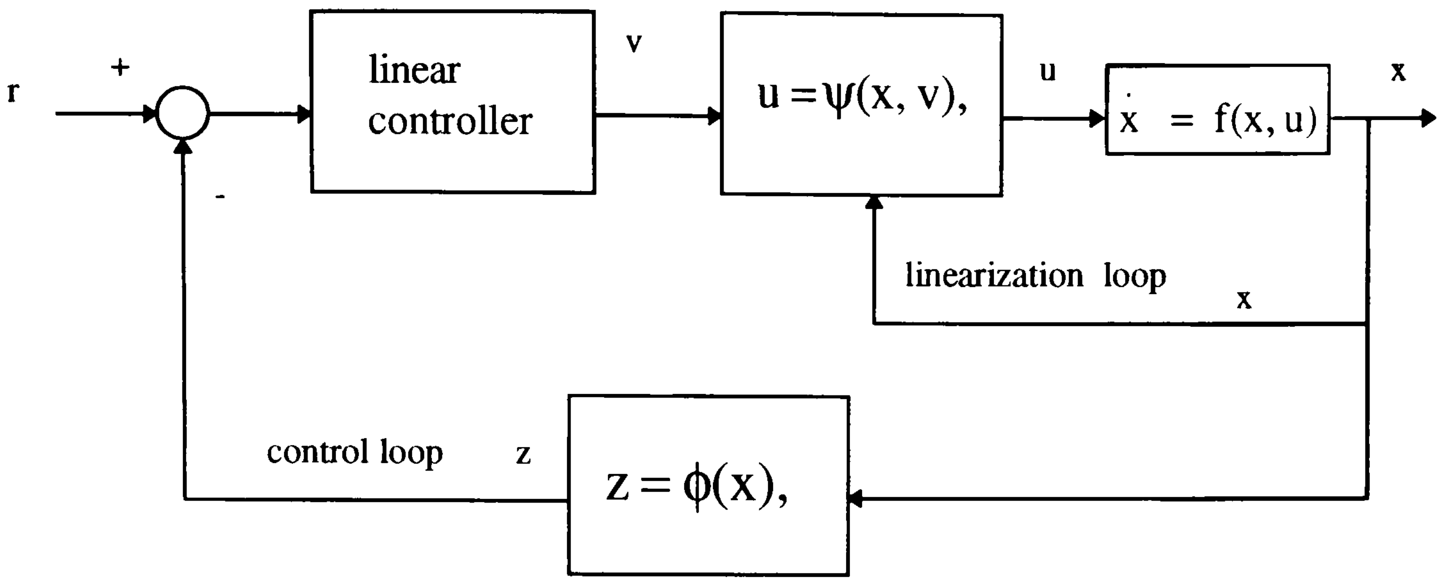


Figure 1.2: Control system under feedback linearization method [106]

This method has two major limitations. First, determining state transformations and input-output transformations is generally not systematic and require extensive mathematical techniques. Second, the method is based on the assumption that the exact mathematical model of nonlinear systems is available in performing feedback linearization. In practice, the exact mathematical model can not be obtained in general.

1.1.2 Model Predictive Control

The concept of a model predictive control system is that the controller uses knowledge of the process dynamics (an explicit model) to find the input of the process that optimize certain performance objectives [108]. The theoretical developments can be found in the literature [25, 88, 108]. So far, the model predictive control method is developed for discrete-time models. However, it has been known that a continuous-time model can be accurately discretized to a discrete-time model if a high enough sampling rate is used [74]. Thus, this method can also be used to control a continuous model.

In this section, the model predictive control will be presented in discrete time. The control system under this method is shown in Figure 1.3.

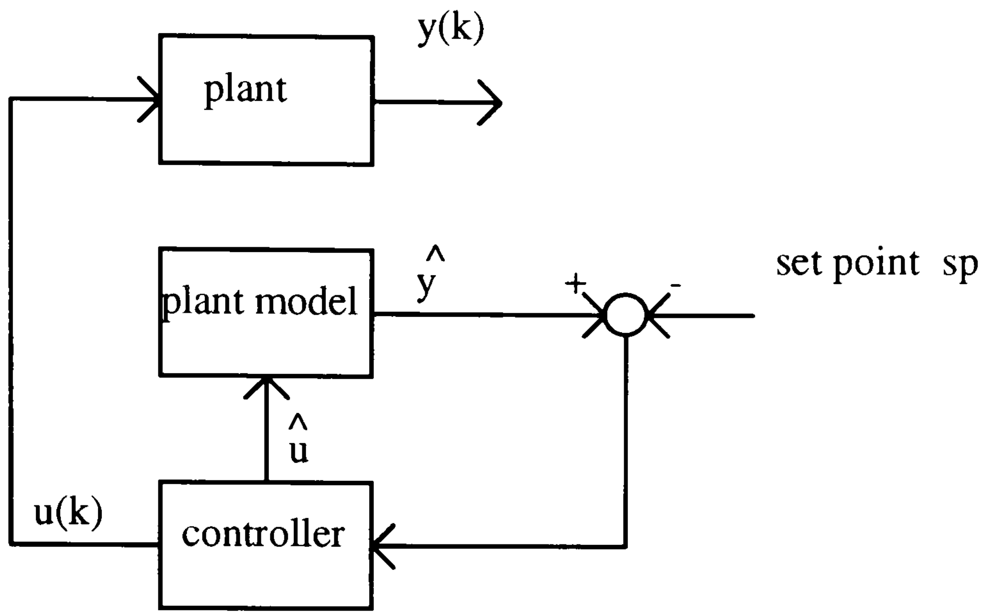


Figure 1.3: Model Predictive control system

From the block diagram, $\hat{u}, \hat{y}, sp$ are defined as follows:

$$
\begin{aligned}
\hat{u} &= [\, u(k), \dots\dots\dots, u(k + H_p - 1)]^T, \\
\hat{y} &= [\, \hat{y}(k), \dots\dots\dots, \hat{y}(k + H_p)]^T, \\
sp &= [\, sp(k), \dots\dots\dots, sp(k + H_p)]^T.
\end{aligned}
\tag{1.5}
$$

H_p, called the prediction horizon, is the number of time step ahead prediction of $\hat{y}$. The predictive controller calculates a future controller output sequence, $\hat{u}$, so that the predicted output of the model, $\hat{y}$,is close to the desired output, sp.

In order to define how well the predicted process output, $\hat{y}$, tracks the desired output, sp, a criterion function, J, is used. Typically, a criterion function is a function of

$\hat{u}, \hat{y}$ and sp . As an example , a simple criterion function is

$$J = \sum_{i=1}^{H_p} [\, \hat{y}(k+i) - sp(k+i)]^2 \,. \tag{1.6}$$

The controller output sequence $\hat{u}$ over the prediction horizon is obtained by the optimization of J with respect to $\hat{u}$. Typically, nonlinear programming methods (NLP) [88] are applied to solve this problem.

Rather than using the controller output sequence determined in the above way to control the plant in the next H_p steps, only the first element of this controller output sequence, i.e., u(k), is used to control the plant. Then, at the next time step, the procedure is repeated by using the latest measured information of the plant. This is called the receding horizon principle. The reason for using the receding horizon approach is that this allows the controller to compensate for future disturbances or modeling errors.

Since a criterion function is the function of model predictive values, $\hat{y}$, modeling errors could cause the process to deviate from the optimal results and often produce poor performance. In addition, calculating an optimum controller output requires intensive computing effort. It may not be possible to implement the algorithm on a small computer.

A common feature of conventional control methods is that the control algorithms are analytically described by mathematical equations. Thus, the syntheses of the conventional control algorithms require the description of the plant by exact mathematical models. Unfortunately, exact mathematical models are too difficult or even infeasible to be obtained in practice. These limit the use of conventional control methods.

1.1.3 Fuzzy Control

An alternative to conventional control is fuzzy control. Fuzzy control is based on the fact that a human operator can sometimes effectively control a complex process without the exact knowledge of its dynamics [66]. Therefore, the idea of fuzzy control is to incorporate the knowledge of a human operator in control algorithms by the methods of fuzzy logic sets, without using knowledge about the plant in the form of mathematical models.

Fuzzy control has been successfully used for a variety of nonlinear control problems [90]. In most applications, fuzzy logic systems are used as nonlinear controllers that are called Proportional-Integral-Derivative-like-Fuzzy Logic controllers (PID-FLCs). The control structure of a PID-FLC is shown in Figure 1.4.

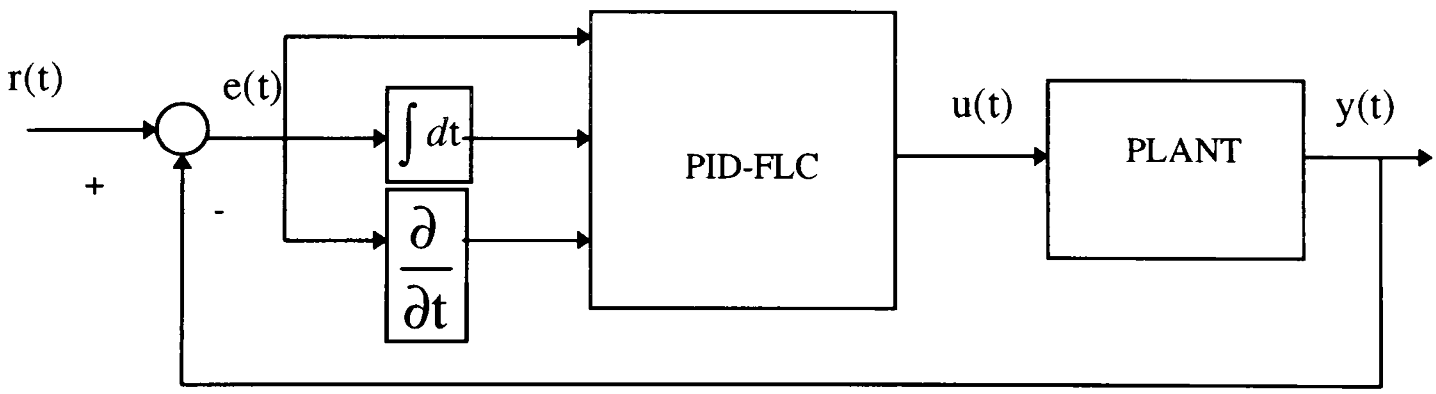


Figure 1.4: The control structure of PID-FLC.

The success of PID-FLCs in controlling complex systems, whose precise models are not possible to obtain, have been reported in several applications [47, 61, 64, 66, 67, 83].

From the Figure 1.4, the PID-FLC contains the control algorithm in the form of linguistic rules which describe the relationship between the inputs and the output of the controller. The set of linguistic rules is called a rule base. The rule base is the main part of the PID-FLC.

The construction of the rule base is the most difficult aspect of a PID-FLC design. In general, there are no systematic tools for deriving the rule base. Typically, the rule base is derived from the experience of operators. However, the expert description of the rules may not be complete and correct in general. In addition, the relationship between the rules and the performance of the control system cannot be found explicitly. Thus, it is difficult to design the rule base such that the system performance meets specific achievable requirements on the control performance, e.g., rise time, percentage of overshoot, etc., without repeatedly iterating on the design. This is a very time consuming process.

In this work, as an alternative method to alleviate the problems of the PID-FLC, an IMC structure will be introduced. This control structure allows designers to systematically construct a fuzzy controller to meet specific requirements on system performance [77]. The background of the IMC structure will be presented in the next section.

1.1.4 IMC Structure for Nonlinear System

The IMC system is a model based control system. The IMC structure basic structure is shown in Figure 1.5. Here, P denotes the plant, C denotes the controller and M denotes the plant model. Basically, the IMC scheme makes use of the process model to infer the effect of unmeasurable disturbances on the process output. Then, the controller generates an input to the system to counteract that effect. The detailed analysis of this control structure can be found in Chapter 3. Here, only results based on an ideal condition will be presented.

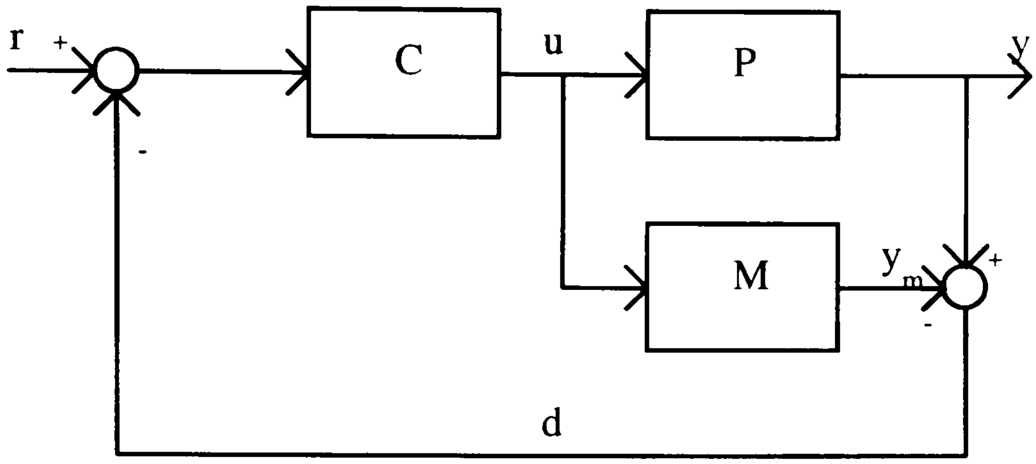


Figure 1.5: The Basic Structure of IMC

From the block diagram, the input-output relationship is

$$\begin{aligned} y &= PC(r - d) \\ d &= (P - M)C(d), \end{aligned}$$

where P, C, M are the nonlinear operators which represent the plant, the controller, and the plant model respectively. If the plant, P, is the same as the plant model, M, and the controller, C, is the model inverse of the plant (i.e., P=M and C= $M^{-1} = P^{-1}$) the relationship is reduced to

$$y=r.$$

From this equation, it means that the output response, y, can completely follow the reference signal, r. Based on this analysis, the design procedure for the IMC structure is to construct the process model and the inverse model for the plant model and the controller respectively. Thus, constructing the IMC structure is convenient and systematic because the function of each designed block, i.e., C and M, is well defined.

The idea of applying the IMC structure for nonlinear systems, which is called the nonlinear internal model control (NIMC) structure, was first introduced by Economou [22]. Then Henson and Serborg proposed an improved NIMC strategy for nonlinear systems [37]. However, both strategies can be used only when processes are well described by mathematical equations. Thus, the NIMC strategies become impossible to be implemented in most systems because most real processes cannot be exactly described by mathematical equations. Recently, several NIMC strategies for neural networks have been proposed [39, 40, 78]. These strategies have been successfully used for controlling complex systems or partially unknown systems.

However, the NIMC strategies for neural networks, the Economou approach and the Henson and Serborg approach share a number of drawbacks. First, the control schemes do not provide an on-line adaptation to deal with the plant-model mismatch due to the varying of plant parameters. Secondly, there lacks a systematic technique to keep the closed-loop system stable. This is the case because a stability criterion for the closed-loop system was not derived explicitly. Thus, to stabilize the closed-loop system, some parameters of the control system have to be adjusted by a trial and error method. Third, the implementation of their approach, which requires the construction of both the plant model and the model inverse of the plant model, introduces the steady state error problem caused by the imperfect model inverse of the plant model.

In this work, besides using the NIMC structure for fuzzy control to alleviate some of the problems of PID-FLC, it is the intention to overcome the drawbacks of the previous NIMC approach. The formal research objectives are addressed in the next section.

1.2 Research Objectives

As mentioned before, the problem with PID-FLCs is that there is no systematic design for developing fuzzy rules. It is also difficult to develop the controllers to meet specific requirements on the control performance.

Therefore, the first objective is to develop a fuzzy control structure that can be conveniently and systematically designed such that the specific requirements on the control performance are achieved. To achieve this objective, an internal model control (IMC) structure will be proposed. One of the attractive features of the IMC structure is that based on an operator approach, the relation between some designed parameters and the performance of the control system can be found explicitly. Thus, this control structure allows designers to systematically construct, with little trial and error effort, the fuzzy control.

The second objective is to develop the fuzzy NIMC scheme such that it can overcome the drawbacks of the previous IMC strategies. As mentioned before, the previous IMC strategies have three drawbacks. First, the control schemes do not provide an on-line adaptation to deal with the plant-model mismatch due to the varying of plant parameters. Secondly, there lacks a systematic technique to keep the closed-loop system stable. Third, the implementation of their approach, which requires the construction of both the plant model and the model inverse of the plant model, introduces the steady state error problem caused by the imperfect model inverse of the plant model.

To overcome the first drawback, an on-line adaptive algorithm (also called an on-line identification algorithm) will be used to adjust the parameters of the fuzzy system.

However, different kinds of fuzzy systems usually require different identification algorithms. Generally, the identification algorithms of fuzzy systems that are nonlinear in their adjustable parameters require more computational complexity than that of fuzzy systems that are linear in their adjustable parameters. Here, the fuzzy basis function (FBF) expansion, which is linear in its adjustable parameters, will be proposed as the controller in the NIMC structure. In addition, the least square identification method will be proposed to obtain the parameters of the FBF expansion.

To overcome the second drawback, a systematic method for stabilization of the NIMC will be proposed. In this work, based on the small gain theorem, a stability criterion, which is a function of some parameters of the NIMC structure, is proposed. Then, unlike the stabilization methods in previous work that rely on trial and error methods of adjusting some parameters of the NIMC structure, the proposed stabilization method keeps the closed-loop system stable by systematically adjusting the parameters of the NIMC structure, which relates to the stability criterion.

Finally, to overcome the third drawback, a modified NIMC structure will be proposed in this work. The implementation based on this modified structure does not require the designer to construct the plant model and directly construct the inverse model from the plant. The reason is to avoid the imperfect model inverse of the plant model, which caused the steady state error problem.

1.3 Outline of The Dissertation

The organization of this proposal is as follows. This chapter has provided research objectives. A review of control systems and some conventional design methods for nonlinear systems also appear in this chapter. Chapter 2 presents a review of fuzzy sets and fuzzy logic systems. Then, fuzzy logic systems as fuzzy basis function expansions and their properties are discussed. In addition, the linear least square identification algorithm is proposed to obtain the parameters of fuzzy basis function expansions. The derivations of the identification algorithm in both on-line and off-line versions are also presented

In chapter 3, the NIMC structure is first introduced. Then, an operator approach is used to explore some important properties of the NIMC structure. Then, the modified NIMC structure is introduced. In addition, an approach based on the small gain theorem is proposed to study the stability of the modified NIMC structure. Then, fuzzy logic techniques are proposed to implement the modified NIMC structure. To be more specific, a fuzzy basis function expansion is proposed as the controller in the modified NIMC structure. Then, an on-line identification architecture for the fuzzy controller is also proposed. As the final topic, an adaptive fuzzy NIMC strategy is proposed for controlling nonlinear systems that can be represented by a continuous function in the form of

$$\vec{f}(\vec{y}^{(n)}, \vec{y}^{(n-1)}, \ldots, \dot{\vec{y}}, \vec{y}, \vec{u}) = \vec{0},$$

where $\vec{u}$ is the input vector, $\vec{y}$ is the output vector, $\dot{\vec{y}}$ is the first derivative of the output vector and similarly for $\ddot{\vec{y}}, \ldots, \vec{y}^{(n)}$.

Simulation studies of controlling four nonlinear systems, e.g., an inverted pendulum, a pendulum, a forced Van der Pol equation and a two-link cylindrical robot manipulator, are presented in Chapter 4. Finally, the conclusion and future research are presented in Chapter 5.

CHAPTER 2

BASIC CONCEPTS OF FUZZY SETS AND FUZZY SYSTEMS

This chapter introduces some basic concepts of fuzzy sets. In addition, fuzzy logic systems, the systems having a direct relationship with fuzzy concepts, are discussed. Finally, fuzzy systems as fuzzy basis function expansions are introduced.

2.1 Fuzzy Sets

Fuzzy sets were proposed by L.A Zadeh in 1965. Fuzzy set theory is primarily designed to allow imprecise and qualitative information to be expressed in linguistic form. The theories try to express an abstract set by means of the concepts of sets.

An abstract representation of the fuzzy subject of set U can be shown in Figure 2.1.

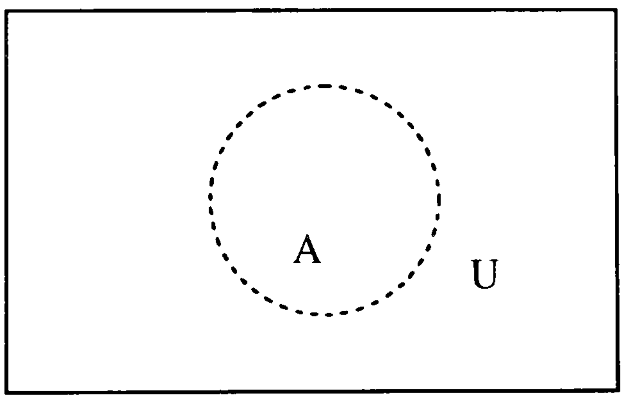


Figure 2.1: Fuzzy Set

The rectangular form represents set U; the dotted circle, a fuzzy subset of U, represents the ambiguous border of what is inside and outside. Fuzzy set theory defines the degree to which the element x of set U is included in this subset. The function that gives the degree to which it is included in A is called the membership function. For example, the degree of element x in area A is expressed by

$$\mu_A(x_1) = 1, \mu_A(x_2) = .8 \text{ etc.}$$

μ is the membership function. The subscript of μ, A,shows that μ_A is the membership function of A. A formal definition of a fuzzy set is shown as follows:

Definition : The membership function μ_A of a fuzzy set A is a function μ_A: $U \rightarrow [0 \;\; 1]$. So, every element x from U has a membership degree A is completely determined by the set of tuples $A = \{ (x, \;\; \mu_A(x)) | \; x \in U \}$.

A fuzzy subset A in U may be represented as a set of ordered pairs of an element, x, and its membership degree as

$$A = \int \mu_A(x)/x. \tag{2.1}$$

When U is a finite set, a fuzzy set A is represented as

$$A = \sum_{i=1}^{n} \mu_A(x_i) / x_i . \tag{2.2}$$

A fuzzy set may considered as a generalization of a classical set. A classical set is defined as a collection of elements that can be finite or countable. Each single element can either belong to or not belong to a set A. Therefore, the membership function of classical set A, called the characteristic function of subset A, is defined as

$$\chi = \begin{cases} 1 \text{ if } x \in A \\ 0 \text{ otherwise} \end{cases} . \tag{2.3}$$

2.2 Operations on Fuzzy Sets

The operations on a fuzzy set are defined in a similar manner to those of a classical set. The basic fuzzy set operations include union, intersection and complement. The operations are defined via their membership function. Many operations on fuzzy sets have been developed. Based on different interpretations which range from intuitive argumentation to empirical justifications, several operators have been suggested in the literature [131]. As examples, t-norm and t-conorm were proposed for the intersection and union operations, respectively. Here, only the basic operations proposed by Zadeh [128] are presented as follows:

Definition : The membership function $\mu_C(x)$ of the intersection $C = A \cap B$ is pointwise defined by $\mu_C(x) = \min\{ \mu_A(x), \mu_B(x) \} \; x \in U$.

Definition : The membership function $\mu_C(x)$ of the intersection $C = A \cup B$ is pointwise defined by $\mu_C(x) = \max\{ \mu_A(x), \mu_B(x) \} \; x \in U$.

Definition : The membership function of the complement of a fuzzy set A, is defined by $\mu_{CA}(x) = 1 - \mu_C(x) \; x \in U$.

2.3 Fuzzy Logic Systems

A fuzzy logic system is a knowledge-based system that uses fuzzy logic to represent knowledge and to make inference [117]. The kernel of a fuzzy logic system is a set of linguistic rules that relate inputs to outputs. These rules are the representations of the qualitative knowledge. The basic configuration of a fuzzy logic system is shown in Figure 2.2. Fuzzy logic systems perform a mapping from U to V. Fuzzy logic systems consist of four blocks: a fuzzifier, a fuzzy rule base, a fuzzy inference, and a defuzzifier. The description of each of the four blocks will be presented in detail.

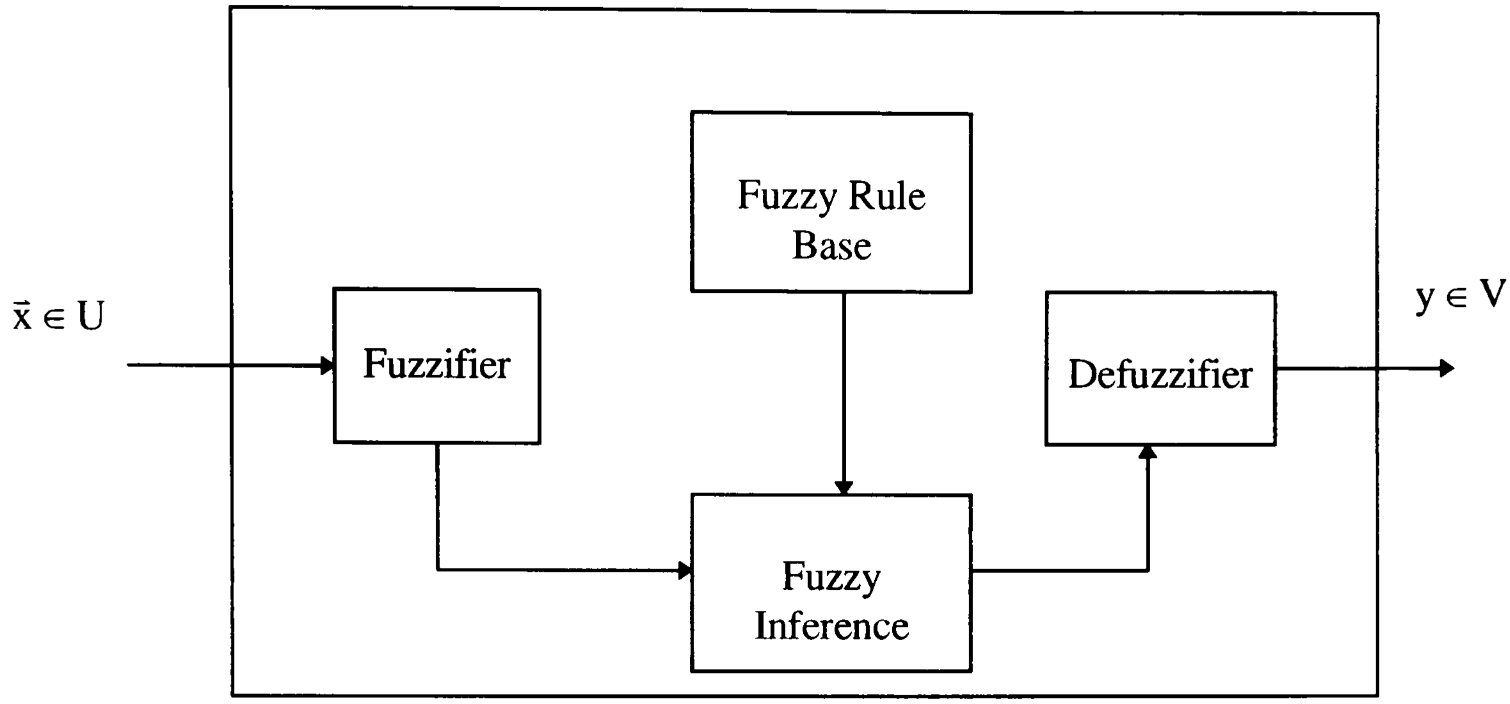


Figure 2.2: Fuzzy Logic Systems

2.3.1 Fuzzy Rule Base

The fuzzy rule base is a collection of the knowledge of the relationship between x and y. The fuzzy rule base is usually in the form of "if-then" statements. As an example, a fuzzy rule can be written as follows:

$$\text{Rule}^{(l)}: \text{if } x_1 \text{ is } F_1^l \text{ and ...and } x_n \text{ is } F_n^l \text{ then y is } G^l,$$

where $\vec{x} = (x_1, \ldots, x_n) \in U$ and $y \in V$ are the input and output of the fuzzy logic system respectively, $F_1^l, F_2^l \ldots, F_n^l$ and G^l are labels of fuzzy sets in U and V respectively.

Each fuzzy "if-then" rule defines a fuzzy implication $F_1^l \times \ldots \times F_n^l \rightarrow G^l$ in $U \times V$. Many fuzzy implication rules have been proposed in the fuzzy logic literature [55, 56]. As examples, some commonly used interpretations for the fuzzy "if-then" rules are shown as follows:

*Mini-operation rule of fuzzy implication

$$\mu_{A\to B}(x,y) = \min[\ \mu_A(x),\ \mu_B(y)] \tag{2.4}$$

*Product-operation rule of fuzzy implication

$$\mu_{A\to B}(x,y) = \mu_A(x)\cdot\mu_B(y) \tag{2.5}$$

*Maxmin rule of fuzzy implication

$$\mu_{A\to B}(x,y) = \max[\min[\ \mu_A(x),\ \mu_B(y)],\ 1-\mu_A(x)] \tag{2.6}$$

*Boolean rule of fuzzy implication

$$\mu_{A\to B}(x,y) = \max[\mu_B(y),\ 1-\mu_A(x)] \tag{2.7}$$

*Arithmetic rule of fuzzy implication

$$\mu_{A\to B}(x,y) = \min[1,\ 1-\mu_A(x)+\mu_B(y)]. \tag{2.8}$$

In the above rules, A is $F_1^l \times \ldots \times F_n^l$, B is G^l, and $\mu_A(x) = \mu_{F_1^l\times\ldots\times F_n^l}(x)$ is defined by

$$\mu_{F_1^l\times\ldots\times F_1^l}(x) = \min[\mu_{F_1^l}(x),\ \ldots,\mu_{F_n^l}(x)] \tag{2.9}$$

or

$$\mu_{F_1^l\times\ldots\times F_1^l}(x) = \mu_{F_1^l}(x)\cdot\ \ldots\ \cdot\mu_{F_n^l}(x). \tag{2.10}$$

2.3.2 Fuzzy Inference Engine

The fuzzy inference engine is used to combine the fuzzy "if-then" rules in the fuzzy rule base into a mapping from the input fuzzy set in U to the output fuzzy set in V. Let A be a fuzzy set in U and $Rule^{(l)}$ be an "if-then" rule. Then, the fuzzy set $B^{(l)}$ of V which is induced by A is given by the composition of $Rule^{(l)}$ and A. This can be expressed a mathematical equation

$$B^{(l)} = A \circ Rule^{(l)}, \qquad (2.11)$$

where $\circ$ denotes the composition operator. According to the sup-star composition rule of inference, $\mu_B(y)$ is defined as

$$\mu_{B^l}(y) = \sup_{x \in U}[\mu_A(x) * \mu_{F_1^l \times \ldots \times F_n^l}(x, y)], \qquad (2.12)$$

where $\mu_{F_1^l \times \ldots \times F_n^l}(x, y)$ is determined by a fuzzy implication rule and

$$\mu_A(x) * \mu_{F_1^l \times \ldots \times F_n^l}(x, y) = \begin{cases} \min[\mu_A(x),\ \mu_{F_1^l \times \ldots \times F_n^l}(x, y)] \text{ (fuzzy intersection) or} \\ \mu_A(x) \cdot \mu_{F_1^l \times \ldots \times F_n^l}(x) \text{ (algebraic product) or} \\ \max[0,\ \mu_A(x) + \mu_{F_1^l \times \ldots \times F_n^l}(x, y) - 1] \text{ (bounded product).} \end{cases} \qquad (2.13)$$

As a general form of inference, the fuzzy set B of V, determined by all L rules in the fuzzy rule base, is obtained by combining $\mu_{A \circ Rule^l}(y)$ for l=1, 2, 3, ..., L :

$$\mu_B(y) = \mu_{B^1}(y) \dot{+} \ldots \dot{+} \mu_{B^L}(y), \qquad (2,14)$$

where $\dot{+}$ denotes the t-conorm: the most commonly used operations for t-conorm are

$$u \dot{+} v = \begin{cases} \max(u, v) \text{ (fuzzy union) or} \\ u + v - uv \text{ (algebraic sum) or} \\ \min(1, u + v) \text{ (bounded sum).} \end{cases} \qquad (2.15)$$

<u>2.3.3 Fuzzifier</u>

The fuzzifier performs a mapping from a crisp point onto a fuzzy set. This can be done by assigning the degree of which a crisp point belongs to each fuzzy set. The degree is determined by the membership functions of each fuzzy set. There are several types of membership functions. Here, three common types of membership functions are introduced.

Let U be a universal set. Then, the membership function ,μ_A, is given by

$$\mu_A : U \rightarrow [0\ \ 1].$$

1. Gaussian membership function

$$\mu_A(x) = \exp\left[-\left(\frac{x - x_j}{\sigma}\right)^2\right] \tag{2.16}$$

2. Triangular membership function

$$\mu_A(x) = \begin{cases} 0 & x \le a \\ \dfrac{(x-a)}{(b-a)} & a < x \le b \\ \dfrac{(x-c)}{(b-c)} & b < x \le c \\ 0 & x > c \end{cases} \tag{2.17}$$

3. Trapezoidal membership function

$$\mu_A(x) = \begin{cases} 0 & x \le a \\ \dfrac{(x - a)}{(b - a)} & a < x \le b \\ 1 & b < x \le c \\ \dfrac{(x - d)}{(c - d)} & c < x \le d \\ 0 & x > d \end{cases} \tag{2.18}$$

Typically for a universe of discourse, there is more than one fuzzy set. As an example, the U domain can be described as in Figure 2.6. Here, the domain consists of five fuzzy sets: “Negative Big”(NB), “Negative Small” (NS) , “Zero”(ZE), “Positive Small”(PS), “Positive Big”(PB).

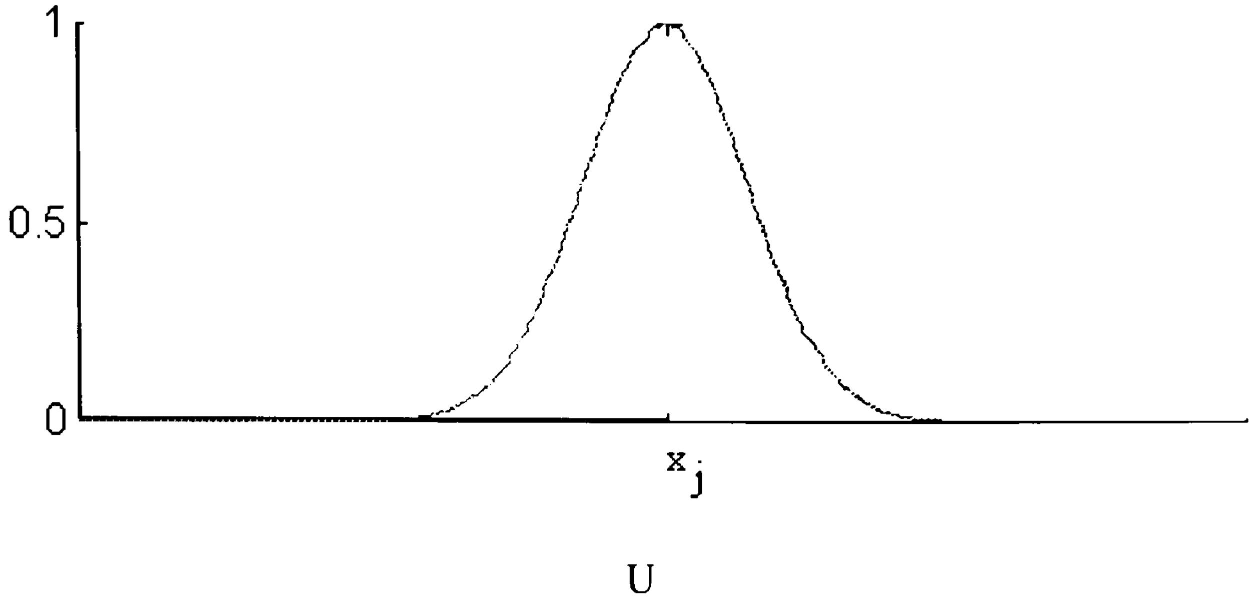


Figure 2.3: Gaussian membership function

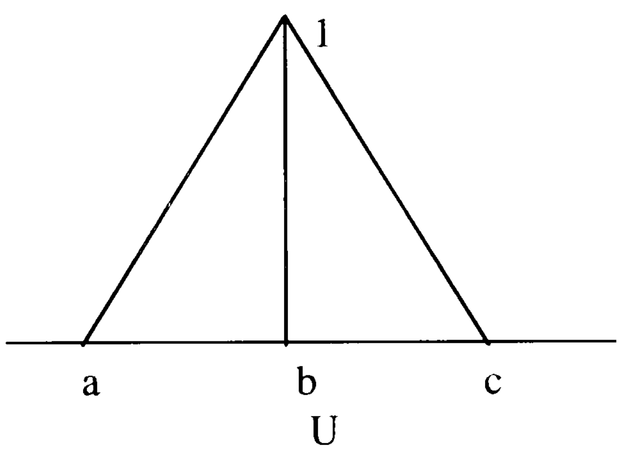


Figure 2.4: Triangular membership function

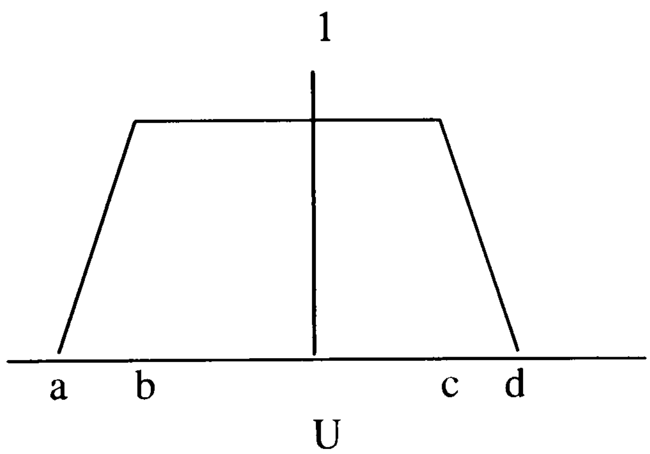


Figure 2.5: Trapezoidal membership function

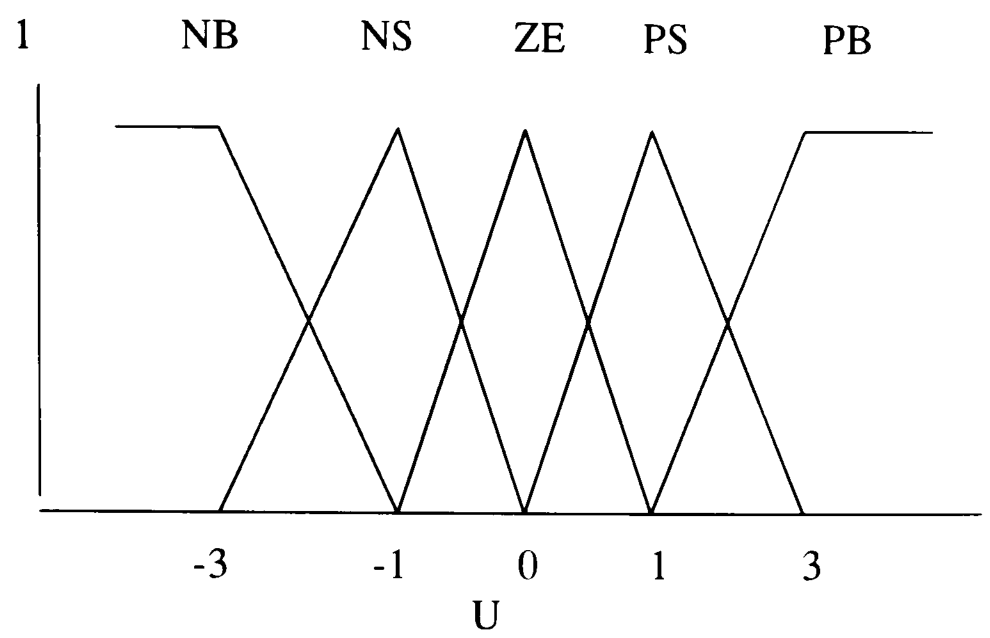


Figure 2.6: Fuzzy Sets for The U Domain

2.3.4 Defuzzifier

The defuzzifier performs a mapping from the fuzzy set in V to a crisp point $y \in V$. There are several strategies for defuzzification. Among them, the maximum defuzzifier and the center average defuzzifier are commonly used.

The maximum defuzzifier is the simplest method. The defuzzifier produces the control value with maximum membership function, that is

$$y = \max_{y \in V} \{ y \in V \mid \mu_V(y) = \max[\mu_{B^1}(y), \ldots, \mu_{B^L}(y)] \}. \qquad (2.19)$$

This is a simple method but not a very good method. Specifically, when more than one membership function reaches the same value, the value of defuzzification is not unique.

Another method is the center average defuzzifier. This defuzzifier is the most well-known method. In the discrete case, $Y = \{\bar{y}^1, \bar{y}^2, \ldots, \bar{y}^L\}$, the center averaging defuzzifier is defined as

$$y = \frac{\sum_{l=1}^{L} \bar{y}^l (\mu_{B^l}(\bar{y}^l))}{\sum_{l=1}^{L} \mu_{B^l}(\bar{y}^l)} \tag{2.20}$$

where $\bar{y}^l$ is the center of the fuzzy set G^l, that is the point in V at which $\mu_{G^l}(y)$ achieves it maximum value.

In the continuous case, the defuzzifier is

$$y = \frac{\int_y y\, \mu_B(y)dy}{\int_y \mu_B(y)dy}. \tag{2.21}$$

This method determines the center of the area below the combined membership function. Figure 2.7 shows this operation in a graphical way. It can be seen that this defuzzification method considers the area of V as a whole. Therefore, if the areas of two fuzzy sets overlap, then the overlapping area is not reflected in the above formula. This operation is computationally rather complex.

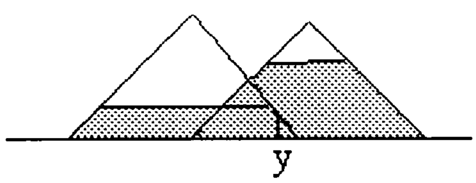


Figure 2.7: Center Averaging Defuzzification

2.4 Fuzzy Systems as Fuzzy Basis Function Expansion

As seen in sections 2.3.1 and 2.3.2, there are many different rules of fuzzy implication and also different kinds of the fuzzy inference rules. Also in section 2.3.3 and

2.3.4, there are different types of fuzzifiers and defuzzifiers. Thus, various kinds of fuzzy logic systems can be constructed from these inference engines, fuzzy implication rules, fuzzifiers and defuzzifiers.

As mentioned in Chapter 1, one objective of this research is to develop an on-line adaptive algorithm for the fuzzy NIMC structure. This requires adjusting the parameters of fuzzy system. However, different kinds of fuzzy systems usually require different identification algorithms. Generally, the identification algorithms of fuzzy systems that are nonlinear in their adjustable parameters require more computational complexity than that of fuzzy systems that are linear in their adjustable parameters. In this thesis, the fuzzy system as the fuzzy basis function (FBF) expansion, which is linear in its adjustable parameters, will be used. The fuzzy system as the FBF expansion will be introduced in this section.

Definition [115] : Fuzzy logic systems with a center average defuzzifier, algebraic product inference, and singleton fuzzifier consist of all functions of the form

$$y(\vec{x}) = \frac{\sum_{l=1}^{L} \bar{y}^{l} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i) \right)}{\sum_{l=1}^{L} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i) \right)} , \tag{2.22}$$

where $\bar{y}^{l}$ is the point at which μ_{G^l} achieves its maximum value that is assumed to be one.

The derivation of Eqs.(2.22) can be found in Wang [115]. From this definition, if $\mu_{F_i^l}(x_i)$'s are fixed and $\bar{y}^{l}$'s are considered adjustable parameters, $y(\vec{x})$ can be expressed as

$$y(\vec{x}) = \frac{\sum_{l=1}^{L} \theta_j (\prod_{i=1}^{n} \mu_{F_i^l}(x_i))}{\sum_{l=1}^{L} (\prod_{i=1}^{n} \mu_{F_i^l}(x_i))}, \tag{2.23}$$

where the θ_j's are adjustable parameters.

Then, the output of the fuzzy system, $y(\vec{x})$, can be viewed as a linear combination of fuzzy basis functions that are defined as follows.

Definition [115]: Fuzzy basis functions(FBF) are defined as

$$p_j(\vec{x}) = \frac{\prod_{i=1}^{n} \mu_{F_i^j}(x_i)}{\sum_{j=1}^{L} (\prod_{i=1}^{n} \mu_{F_i^j}(x_i))}, \qquad j = 1, 2, 3, \ldots, L.$$

Therefore, the fuzzy system in Eqs.(2.23) is equivalent to a fuzzy basis function (FBF) expansion:

$$y(\vec{x}) = \sum_{j=1}^{L} \theta_j p_j(\vec{x}). \tag{2.24}$$

From this definition, a FBF corresponds to a fuzzy "if-then" rule. A FBF for Rule$^{(j)}$ can be computed as follows. First, the product of all the membership functions for the linguistic terms in the if-part of Rule$^{(j)}$ is calculated. This product is called a pseudo-FBF of Rule$^{(j)}$. Then, the pseudo-FBFs for all the L fuzzy rules are calculated. The FBF for Rule$^{(j)}$ is determined by dividing the pseudo-FBF for Rule$^{(j)}$ by the sum of all pseudo-FBFs.

As an example, a one-input fuzzy system with four fuzzy rules can be written as

$$y(x) = \sum_{j=1}^{4} \theta_j p_j(x), \tag{2.25}$$

where

$$p_j(x) = \frac{\mu_{A^j}(x)}{\sum_{i=1}^{4}\mu_{A^i}(x)}. \tag{2.26}$$

It is assumed that

$$\mu_{A^j}(x) = \exp(-\frac{1}{2}(x - x_j)^2), \tag{2.27}$$

where $x_j = -3, -1, 1, 3$ for $j = 1, 2, 3, 4$, respectively. Thus, the fuzzy basis functions are

$$p_j(x) = \frac{\exp(-\frac{1}{2}(x - x_j)^2)}{\sum_{j=1}^{4}\exp(-\frac{1}{2}(x - x_j)^2)}, \; j = 1, 2, 3, 4, \tag{2.28}$$

where $x_j = -3, -1, 1, 3$ for $j = 1, 2, 3, 4$, respectively.

One important property of FBF expansions is that FBF expansions are capable of approximating any real continuous function. Wang showed that FBF expansions are universal function approximators [115]. The theory is shown below.

Theorem [115]: For any given real continuous function g on a compact set (closed and bounded in a finite dimensional space) and arbitrary $\varepsilon > 0$, there exists a fuzzy logic system f(.) in the form of the FBF expansion such that

$$\sup_{\bar{x}} |f(\bar{x}) - g(\bar{x})| < \varepsilon. \tag{2.29}$$

The proof of this theorem is given in Wang [115]. This theorem gives a justification for using FBF expansions to model any dynamic system which is usually described by continuous functions. In this work, the FBF expansions will be applied to modeling a controller for the proposed control structure.

This theorem only shows that there exists a FBF expansion that can approximate any given function to an arbitrary accuracy. However, it does not show how to construct the FBF expansions. An identification technique will be presented in the next section.

2.5 A Learning Algorithm for The FBF Expansions

There are two main approaches in constructing a fuzzy model [124]. The first approach in constructing a fuzzy model is based on the use of an expert's description of the actual system. In this approach, a fuzzy model is expressed by a set of linguistic rules that are derived from the experience of experts. However, this approach has difficulty in describing the behavior of a complex system because a large number of linguistic rules need to be specified. In addition, an expert description of the rule may not be complete and correct in general.

The second approach is based on the use of input-output data. This approach is also regarded as a system identification approach. In this approach, the parameters of a fuzzy model, such as the parameters of membership functions, are estimated from input-output data. The second approach is one way to train a fuzzy model in an on-line fashion. In this section, a learning algorithm for FBF expansions based on input-output data will be presented.

As mentioned in section 2.4, a fuzzy system can be expressed in the form of the fuzzy basic function expansion as

$$y(\vec{x}) = \sum_{j=1}^{L} \theta_j p_j(\vec{x}), \tag{2.30}$$

where y(.) is the output of the fuzzy system, the θ_j's are constants, the L consists of a number of fuzzy rules, $\bar{x}$ is the inputs of the fuzzy system and the $p_j(\bar{x})$'s are the fuzzy basic functions. The fuzzy basic function, $p_j(\bar{x})$, is defined as

$$p_j(\bar{x}) = \frac{\prod_{i=1}^{n} \mu_{A_i^j}(x_i)}{\sum_{j=1}^{L}\left(\prod_{i=1}^{n} \mu_{A_i^j}(x_i)\right)} \qquad j = 1, 2, \ldots, L. \tag{2.31}$$

From the equation of the fuzzy basis function expression, the θ_j's are considered as adjustable parameters and $p_j(\bar{x})$'s are consider as regressors. It can be seen that the FBF expansion is linear in the parameters. Thus, linear model identification techniques can be applied to search for the parameters. Here, the Least Square (LS) algorithm is proposed to identify the parameters.

To derive the LS algorithm, the FBF expansion is viewed as a linear regression model,

$$y = \sum_{j=1}^{L} \hat{\theta}_j p_j(\bar{x}) + \varepsilon, \tag{2.32}$$

where ε , called prediction error, is the equation error. Assuming that there are N observations, the problem is to estimate the parameters, $\hat{\theta}_j$'s, from the data. As a mathematical equation, the statement can be written as

$$\begin{aligned} y^1 &= \sum_{j=1}^{L} \hat{\theta}_j p_j^1 + \varepsilon^1 \\ &\;\;\vdots \\ y^N &= \sum_{j=1}^{L} \hat{\theta}_j p_j^N + \varepsilon^N. \end{aligned} \tag{2.33}$$

This can be written in matrix notation as

$$Y = P\Theta + E, \tag{2.34}$$

where

$$Y = \begin{bmatrix} y^1 \\ \cdot \\ \cdot \\ \cdot \\ y^N \end{bmatrix}, \tag{2.35}$$

$$P = \begin{bmatrix} p_1^1 & \cdot & \cdot & \cdot & p_L^1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ p_1^N & \cdot & \cdot & \cdot & p_L^N \end{bmatrix} = \begin{bmatrix} \mathbf{p}^1 \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{p}^N \end{bmatrix}, \tag{2.36}$$

$$E = \begin{bmatrix} \varepsilon^1 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon^N \end{bmatrix}. \tag{2.37}$$

The LS estimation of Θ is defined as the vector $\hat{\Theta}$ that minimizes the loss function

$$\begin{aligned} V(\hat{\Theta}) &= \frac{1}{2}E^T E \\ &= \frac{1}{2}[Y - P\hat{\Theta}]^T[Y - P\hat{\Theta}] \\ &= \frac{1}{2}[\hat{\Theta} - (P^T P)^{-1} P^T Y]^T [P^T P][\hat{\Theta} - (P^T P)^{-1} P^T Y] \\ &\quad + \frac{1}{2}[Y^T Y - Y^T P(P^T P)^{-1} P^T Y]. \end{aligned} \tag{2.38}$$

By taking the gradient of the loss function and setting the gradient to zero, this can be written as

$$\frac{\partial V(\hat{\Theta})}{\partial \hat{\Theta}} = 0$$
$$= \hat{\Theta}^T (P^T P) - Y^T P. \tag{2.39}$$

Therefore, $\hat{\Theta}$ must satisfy the following equation:

$$(P^T P)\hat{\Theta} = P^T Y. \tag{2.40}$$

When $P^T P$ is a nonsingular matrix, $\hat{\Theta}$ is simply obtained by

$$\hat{\Theta} = (P^T P)^{-1} P^T Y. \tag{2.41}$$

When $P^T P$ is a singular matrix, there are infinitely many solutions. $\hat{\Theta}$ is obtained by

$$\hat{\Theta} = A' P^T Y, \tag{2.42}$$

where A' is pseudo-inverse of $P^T P$. The pseudo-inverse of $P^T P$ is defined as

$$A' = [A^*(P^T P)]^{-1} A^*. \tag{2.43}$$

A^* must satisfy the following condition:

$$A^*(P^T P)\hat{\Theta} = A^* P^T Y. \tag{2.44}$$

There are a number of different algorithms to calculate the pseudo-inverse A', such as Gaussian Elimination, LU decomposition and singular value decomposition [44].

So far, all data are used simultaneously to find the parameters. This is called an off-line identification. The algorithm is not suitable for a real-time application. Thus, it is desirable to reformulate the algorithm in order that it is used for real-time identification. In real-time (also called on-line) identification methods, the estimated parameters are computed recursively in time. As an example, if $\hat{\Theta}_{t-1}$ is based on data up to time t-1, then $\hat{\Theta}_t$ is computed by some modification of $\hat{\Theta}_{t-1}$.

The LS algorithm can be rewritten in the recursive form:

$$\hat{\Theta}_t = [\sum_{i=1}^{t} p_i p_i^T]^{-1}[\sum_{i=1}^{t} p_i y_i]. \tag{2.45}$$

Then, Q_t is defined as

$$\begin{aligned} Q_t &= [\sum_{i=1}^{t} p_i p_i^T]^{-1} \\ &= [P^T P]^{-1}. \end{aligned} \tag{2.46}$$

It follows that

$$\begin{aligned} \hat{\Theta}_t &= Q_t[\sum_{i=1}^{t-1} p_i y_i + p_t y_t] \\ &= Q_t[Q_{t-1}^{-1}\hat{\Theta}_{t-1} + p_t y_t]. \end{aligned} \tag{2.47}$$

Since

$$Q_t^{-1} = Q_{t-1}^{-1} + p_t p_t^T,$$

$\hat{\Theta}_t$ is written as

$$\hat{\Theta}_t = \hat{\Theta}_{t-1} + Q_t p_t[y_t - p_t^T \hat{\Theta}_{t-1}]. \tag{2.48}$$

Thus, the recursive Least square algorithm is

$$\begin{aligned} \varepsilon_t &= y_t - p_t \hat{\Theta}_{t-1} \\ Q_t &= [\sum_{i=1}^{t} p_i p_i^T]^{-1} = Q_{t-1} - \frac{Q_{t-1} p_t p_t^T Q_{t-1}}{[1 + p_t^T Q_{t-1} p_t]}, \\ K_t &= Q_t p_t, \\ \hat{\Theta}_t &= \hat{\Theta}_{t-1} + K_t \varepsilon_t \end{aligned} \tag{2.49}$$

The RLS algorithm needs initial values $\hat{\Theta}_0$ and Q_0. Typically, both initial values are obtained from off-line identification.

The above algorithm is suitable for an unchanging process because it gives equal weighting to old data and new data. However, it is desirable to further modify the algorithm to be used for a changing system, i.e., the parameters of the plant change with time. This can be done by introducing a " forgetting factor" λ and modifying the loss function of the least square method as

$$V_t(\Theta) = \sum_{i=1}^{t} \lambda^{t-1} \varepsilon_t^2, \quad 0<\lambda<1. \tag{2.50}$$

This means that the old information will get less attention than the new information. Since the derivation of the modified algorithm [71] is similar to the RLS algorithm, only the results will be presented. The modified algorithm is

$$\begin{aligned}
\varepsilon_t &= y_t - p_t^T \hat{\Theta}_{t-1} , \\
Q_t &= \frac{\{Q_{t-1} - \dfrac{Q_{t-1} p_t p_t^T Q_{t-1}}{[\lambda + p_t^T Q_{t-1} p_t]}\}}{\lambda} \\
K_t &= Q_t p_t , \\
\hat{\Theta}_t &= \hat{\Theta}_{t-1} + K_t \varepsilon_t .
\end{aligned} \tag{2.51}$$

The modified algorithm in Eqs(2.51) can be used to track time varying parameters. In this thesis, it is assumed that the change of parameters is slower than the convergence time of the algorithm. In other words, parameters are constant for long period of time and then jump from one value to another.

It was mentioned by Astrom [3] that under the assumption that an input signal that is used to generate the data excites all the possible modes of the system, the parameter convergence of this algorithm can be guaranteed and the algorithm has exponentially convergence rate. The proofs of these statements are shown in Appendix A.

2.6 Summary

This chapter provides the basic concepts of fuzzy sets and fuzzy logic systems. Particularly, fuzzy logic systems as fuzzy basis function expansions and a learning algorithm for the FBF expansions are also discussed. A fuzzy logic system as a fuzzy basis function expansion will be used as the controller in the internal model control (IMC) structure. This control structure allows designers to systematically construct, with little trial and error effort, an adequate fuzzy controller to meet requirements on system performance. This control structure, which is considered as the alternative method to alleviate some of the problems of the PIC-FLC, will be introduced in the next chapter.

CHAPTER 3

ADAPTIVE FUZZY NONLINEAR INTERNALl MODEL CONTROL STRATEGY

In this chapter, an alternative control method will be proposed to avoid the problems of the PID-FLC. The chapter will begin briefly with a basic idea of the IMC structure. Then, theoretical analysis will be presented in detail to deal with a system in a general manner. Finally, the NIMC structure and strategy for fuzzy logic controllers will be discussed.

3.1 IMC Structure

The IMC system is a model based control system. It is a part of the standard approach of control system design. A review of the IMC schemes for linear systems was presented by Garsia [28].

The IMC basic structure is shown in Figure 3.1. Here, P denotes the plant, C denotes the controller and M denotes the plant model. The controller, C, determines the value of the input, u. The control objective is to keep the output, y, close to the reference signal, r. The IMC scheme makes use of the process model to infer the effect of unmeasurable disturbances on the process output. Then the controller generates an input to the system to counteract that effect.

So far, most theories of IMC strategies have been developed for linear systems. The prevalence of linear IMC strategies is due to two reasons. First, there are well established methods for the development of linear models from input-output data [62]

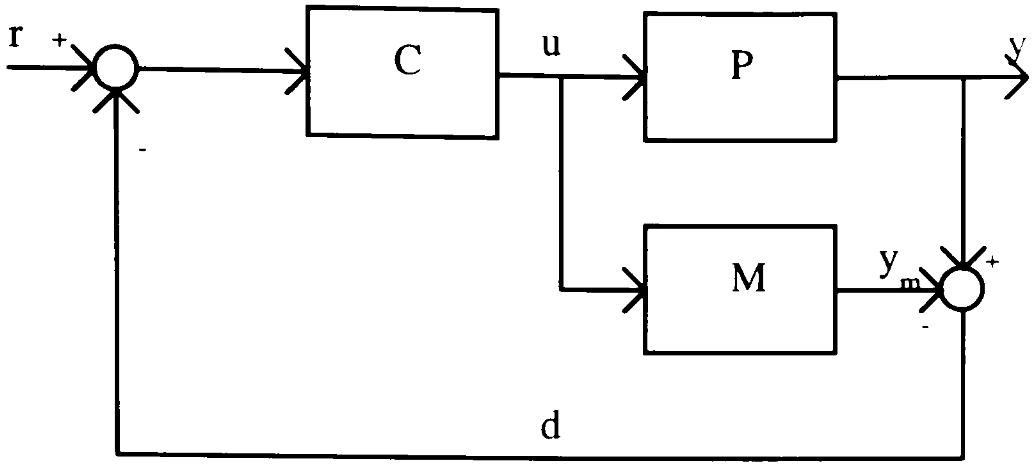


Figure 3.1: The Basic Structure of IMC

while practical identification techniques for nonlinear models are still being developed. Secondly, controller design techniques for nonlinear models are considerably more difficult than for those of linear models.

Unfortunately, most physical systems are nonlinear. IMC schemes for nonlinear systems usually require linearization and linear controller design based on the linearized model. However, this approach results in often poor approximation of the real systems and may not be successful when the process is highly nonlinear. Therefore, the performance is often severely degraded from the desired performance.

In this work, a NIMC strategy will be proposed to deal with nonlinear systems in a general manner. Toward this objective, the general framework for a nonlinear internal model control (NIMC) structure will be presented in the next section.

3.2 Internal Model Control Structure for Nonlinear Systems

In this section, the framework to be introduced is presented in a general normed linear vector space setting with the control systems described by nonlinear operators. The nonlinear IMC structure is shown in Figure 3.2.

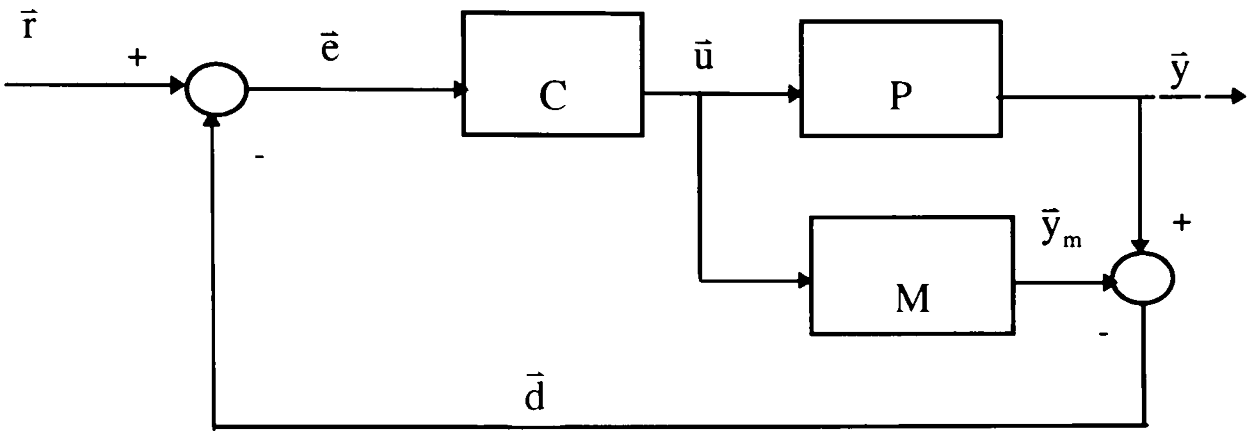


Figure 3.2: The Nonlinear IMC Structure

The nonlinear operators M, and C represent the plant model and the controller respectively. The plant is represented by a nonlinear operator, P, which maps $\vec{u}$ to $\vec{y}$ through the relation

$$\vec{f}(\vec{y}, \dot{\vec{y}}, \ddot{\vec{y}}, \ldots, \vec{y}^{(n)}, \vec{u}) = \vec{0}, \qquad (3.1)$$

where $\vec{f}$ = a real continuous vector function

$$= \begin{bmatrix} f_1(\vec{y}, \dot{\vec{y}}, \ddot{\vec{y}}, \ldots, \vec{y}^{(n)}, \vec{u}) \\ \cdot \\ \cdot \\ \cdot \\ f_1(\vec{y}, \dot{\vec{y}}, \ddot{\vec{y}}, \ldots, \vec{y}^{(n)}, \vec{u}) \end{bmatrix},$$

$\vec{y}$ = a output vector

$$= \begin{bmatrix} y_1(t) \\ \cdot \\ \cdot \\ \cdot \\ y_q(t) \end{bmatrix} \text{ and}$$

$\vec{u}$ = a input vector

$$= \begin{bmatrix} u_1(t) \\ \cdot \\ \cdot \\ \cdot \\ u_l(t) \end{bmatrix}.$$

In real applications, the plant has a maximum input signal and a maximum output signal which should not be exceeded. Thus, it is also reasonable to assume that the magnitude of each signal in the control system is bounded. Here, each signal in the control system is considered as a vector in the uniform norm vector space with bounded norm, i.e., $\|\cdot\|_\infty < \infty$.

From the block diagram, the following relationships can be derived:

$$\begin{aligned} \vec{e} &= \vec{r} - \vec{y} + \vec{y}_m , \\ \vec{y}_m &= MC(\vec{e}) , \\ \vec{d} &= (P - M)C(\vec{e}). \end{aligned} \tag{3.2}$$

From these relations, the attractive properties of the NIMC can be proved as follows [35]:

Property 1: (Stability) when the model is perfect, i.e., P=M, the stability of both controller and plant is sufficient for overall system stability.

Proof When P=M, there is no signal feed through the feedback path. i.e., $\vec{d}$ =0. As a result, if the open-loop system is input-output stable, then the closed-loop system is also input-output stable.

Property 2: (Perfect Control) If there are C such that C is equal to M^{-1} and there are P equal to M, then perfect control is achieved, i.e., $\vec{y} = \vec{r}$.

Proof Since $C = M^{-1}$, The result is

$$\vec{y}_m = MC(\vec{e}) \\ = \vec{e}.$$

Thus, from $\vec{e} = \vec{r} - \vec{y} + \vec{y}_m$, we get

$$\vec{y} = \vec{r}. \tag{3.3}$$

Property 3: (Zero steady state error) If the controller is equal to the inverse of the plant model for $t \to \infty$, i.e., $C_\infty = M_\infty^{-1}$ for $t \to \infty$, and the closed-loop system is stable, then steady state error will be zero for a constant reference signal.

Proof Giving that $\vec{y}_m = MC(\vec{e})$ when t reaches to infinity, the relation can be written as

$$\vec{y}_{m\infty} = M_\infty C_\infty(\vec{e}_\infty).$$

Since $C_\infty = M_\infty^{-1}$, the result is

$$\vec{y}_{m\infty} = M_\infty(M_\infty^{-1})\vec{e}_\infty \\ = \vec{e}_\infty .$$

It follows directly from $\vec{e} = \vec{r} - \vec{y} + \vec{y}_m$ that

$$\vec{y}_\infty = \vec{r}. \tag{3.4}$$

These properties are based on ideal conditions. In practice, the perfect model assumption is rarely satisfied. There is a signal feeding through the feedback path. The closed-loop system is possibly unstable, although the open-loop system is stable. A technique to stabilize the closed-loop system when the perfect model could not be achieved will be presented later.

According to property 2, the IMC structure can achieve perfect control. However, perfect control usually requires large control actions. This is undesirable in practice. To solve this problem, a filter, F, is used in series with the controller C as shown in Figure 3.3 [22]. Here, $\vec{d}$ is noise that is caused by modeling mismatch; P is the part of the plant that can be modeled by M, i.e., P=M; C is the controller; F is the filter.

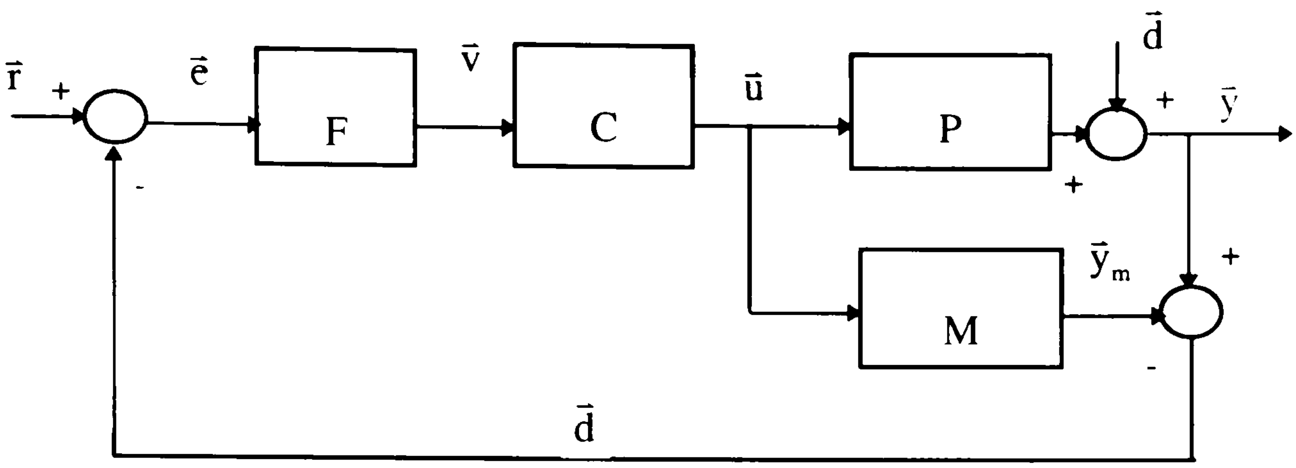


Figure 3.3: Completed Nonlinear IMC Structure

The relationship can be derived directly from the block diagram as follows:

$$\vec{y} = P[I + C(P-M)]^{-1}CF(\vec{r}-\vec{d}) + \vec{d}. \tag{3.5}$$

Provided that P is equal to M, the relationship is reduced to

$$\begin{aligned} \vec{y} &= PCF(\vec{r}-\vec{d}) + \vec{d} \\ &= PCF(\vec{r}) + (I - PCF)(\vec{d}), \end{aligned} \tag{3.6}$$

where I is the identity operator.

Specially, if the controller is assumed to be the model inverse of the plant, i.e., $C=P^{-1}$, then the relationship is expressed as

$$\vec{y} = F(\vec{r}) + (I - F)(\vec{d}). \tag{3.7}$$

From this relationship, the filter could reduce the effect of the disturbance, $\vec{d}$, to the output. In addition, the filter could be used to limit the control action, $\vec{u}$. The filter design issue will be addressed later.

The above analyses are based on the two assumptions: P=M and $C = M^{-1} = P^{-1}$. To implement the NIMC structure in Figure 3.3, both the plant model and the inverse of the plant model must be constructed. However, constructing the perfect model inverse of the plant model can not be achieved in practice. As a result, according to property 3, the steady state error could not be eliminated.

In this work, an alternative implementation will be proposed to deal with the steady state error problem. Here, the controller C is implemented directly from the plant and the plant model M is not required to be implemented. In the NIMC structure in Figure 3.3, the plant model M is required to give $\vec{y}_m$ when the input is $\vec{u}$. Theoretically, $\vec{y}_m$ must equal $\vec{v}$ because the controller C is the model inverse of the plant model M. In the previous works, this could not be achieved in practice. When the controller C is implemented directly from the plant, $\vec{y}_m$ equal to $\vec{v}$ is simply achieved by directly connecting $\vec{y}_m$ to $\vec{v}$, which is the input to the controller. Therefore, the plant model M is not required. The new structure is shown as Figure 3.4.

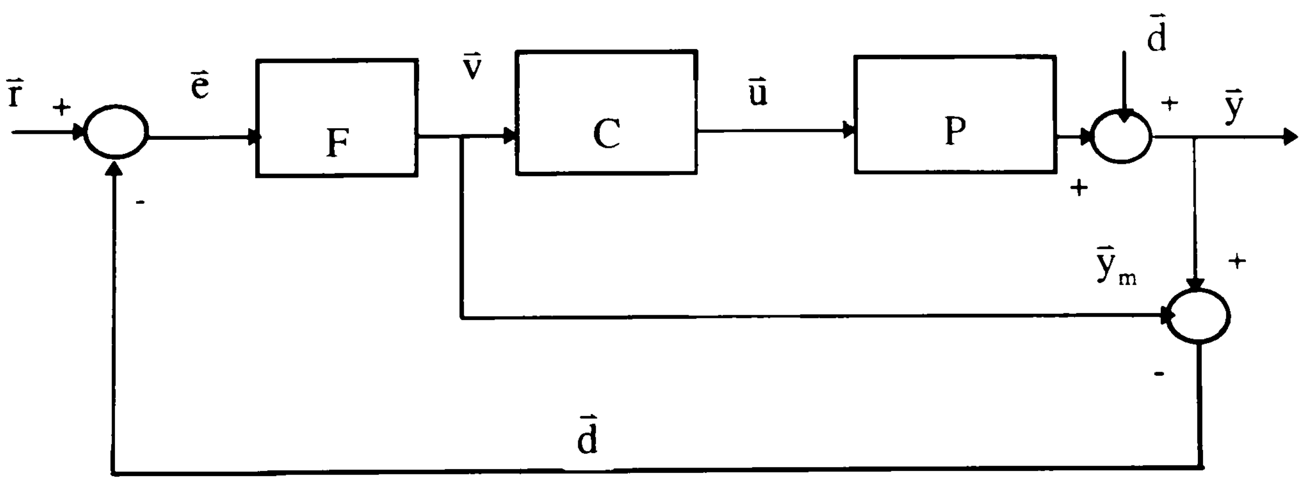


Figure 3.4: Modified Nonlinear IMC Structure

As expected, the previous relations, which were derived for the previous IMC structure, are still valid for the new IMC structure. The new structure is capable of eliminating the offset problem caused by the imperfect model inverse of the plant model. Moreover, the new structure is less complicated than the previous structure.

From the above analysis, the controller, which is the model inverse of the plant, plays a crucial role in the control performance. Thus, a study of the theoretical aspects of invertibility will be presented in the next section.

3.3 Invertibility of Systems

Here, a controlled plant is presented by a nonlinear operator which maps an input vector, $\vec{u}$, to an output vector, $\vec{y}$, through the relations

$$\vec{f}(\vec{y}, \dot{\vec{y}}, \ddot{\vec{y}}, \ldots, \vec{y}^{(n)}, \vec{u}) = \vec{0}$$

Based on this relations, the model inverse of the plant can be expressed as

$$\vec{u} = \vec{\varphi}(\vec{y}, \dot{\vec{y}}, \ddot{\vec{y}}, \ldots, \vec{y}^{(n)}) \ . \tag{3.8}$$

In general, the existence and uniqueness of $\vec{u}$ is not guaranteed, unless some restrictions are imposed on $\vec{f}(.)$. The restrictions on $\vec{f}(.)$ that ensure the existence and uniqueness of $\vec{u}$, will be addressed by the implicit function theorem.

Theorem: Implicit Function Theorem [100]:

Let a system of n equations

$$F_i(x_1, x_2, \ldots, x_n, u_1, u_2, \ldots, u_l) = 0 \quad i = 1, 2, \ldots, l \tag{3.9}$$

in l+n variables $x_1, x_2, \ldots, x_n, u_1, \ldots, u_l$ have a real solution $x_1^0, x_2^0, \ldots, x_n^0$,

$u_1^0, u_2^0, \ldots, u_l^0$, such that

$$F_i(x_1^0, x_2^0, \ldots, x_n^0, u_1^0, u_2^0, \ldots, u_l^0) = 0 \qquad i = 1, 2, \ldots, l.$$

If the functions,

$$F_i(x_1, x_2, \ldots, x_n, u_1, u_2, \ldots, u_l) \quad i = 1, 2, 3, \ldots, l,$$

are continuous and have continuous first partial derivatives in some region R of the (l+n) dimensional space enclosing the point P^0, whose coordinates are $(x_1^0, x_2^0, \ldots, x_n^0, u_1^0, u_2^0, \ldots, u_l^0)$ and if the Jacobian

$$\begin{bmatrix} \frac{\partial F_1}{\partial u_1} & \frac{\partial F_1}{\partial u_2} & \cdots & \frac{\partial F_1}{\partial u_n} \\ \frac{\partial F_2}{\partial u_1} & \frac{\partial F_2}{\partial u_2} & \cdots & \frac{\partial F_2}{\partial u_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F_l}{\partial u_1} & \frac{\partial F_l}{\partial u_2} & \cdots & \frac{\partial F_l}{\partial u_n} \end{bmatrix} \tag{3.10}$$

is different from zero at the point P^0, then the inverse of Eq(3.9) can be expressed as

$$u_i = \varphi_i(x_1, x_2, \ldots, x_n) \quad i = 1, 2, \ldots, l \tag{3.11}$$

in the vicinity of the point $(x_1^0, x_2^0, \ldots, x_n^0)$ in such a way that

$$u_i^0 = \varphi_i(x_1^0, x_2^0, \ldots, x_n^0) \quad i = 1, 2, \ldots, l. \tag{3.12}$$

Moreover, the set of solutions is unique and the functions u_i are continuous together with their first partial derivatives in some neighborhood of the point $(x_1^0, x_2^0, \ldots, x_n^0)$.

The theorem is presented in a general manner. To be more specific, the implicit function theorem for a single input-output system is shown as :

Theorem: Implicit Function Theorem for Single Input-Output Systems [100]:

Let $f(x_1, x_2, \ldots, x_n, u) = 0$, where $x_1 = y$, $x_2 = \dot{y}$, ..., $x_n = y^{(n)}$, has a real solution $x_1^0, x_2^0, \ldots, x_n^0, u^0$ such that $f(x_1^0, x_2^0, \ldots, x_n^0, u^0) = 0$. If $f(x_1^0, x_2^0, \ldots, x_n^0, u^0)$ have continuous first partial derivatives in the neighborhood of $(x_1^0, x_2^0, \ldots, x_n^0, u^0)$, and $\frac{\partial f}{\partial u} \neq 0$ at $(x_1^0, x_2^0, \ldots, x_n^0, u^0)$, then the inverse of f(.) can be expressed as

$$u = \varphi(x_1, x_2, \ldots, x_n) \tag{3.13}$$

in the vicinity of the point $(x_1^0, x_2^0, \ldots, x_n^0, u^0)$ in such a way that

$$u = \varphi(x_1^0, x_2^0, \ldots, x_n^0).$$

Moreover, u is unique and $\varphi(.)$ is continuous function in the neighborhood of the point.

This theorem provides the conditions for existence and uniqueness of the inverse model of the plant, i.e.,

$$\vec{u} = \vec{\varphi}(\vec{y}, \dot{\vec{y}}, \ddot{\vec{y}}, \ldots, \vec{y}^{(n)}),$$

where $\vec{\varphi}(\cdot)$ is a continuous vector function. In this case, the invertibility of a plant is equivalent to the reachability property of the plant [55]. The reachability is defined as follows:

Definition [130]: Reachability

A plant is said to be reachable, if there exists an input, u, that will drive the state of the plant at time t_0, i.e., $\vec{y}_0^{(n)}(t_0), \vec{y}_0^{(n-1)}(t_0), \ldots, \dot{\vec{y}}_0(t_0), \vec{y}_0(t_0)$ to any other state in a finite time, i.e., $\vec{y}_1^{(n)}(t_1), \vec{y}_1^{(n-1)}(t_1), \ldots, \dot{\vec{y}}_1(t_1), \vec{y}_1(t_1)$ where $t_1 < \infty$.

Therefore, if there exist $\vec{u} = \vec{\varphi}(\vec{y}, \dot{\vec{y}}, \ddot{\vec{y}}, \ldots, \vec{y}^{(n)})$, then it guarantees that the state of the system can be driven to any other state in the operating region.

The sufficient and necessary conditions for systems to have an inverse have been addressed by the implicit function theorem. However, the theorem does not mention how to find the solution or how to construct an inverse model. This issue will be presented later.

3.4 The Stability of NIMC Structure

So far, the stability of the NIMC structure was presented only under the assumption that a perfect model can be achieved. The perfect model assumption is an artificial assumption. In practice, there is always model mismatch. Therefore, it is unacceptable to rely on this assumption for stability.

A powerful theorem for studying the stability of nonlinear systems is the Lyapunov stability theorem which is based on the state space approach [113]. In order to determine system stability, a Lyapunov function must be constructed. However, the method suffers from the difficulty of determining an appropriate Lyapunov function for a given system. Since there is no general effective approach for determining Lyapunov functions, one has to use trial and error, and experience to search for appropriate Lyapunov functions [106].

Here, an alternative stability approach that is based on an input-output approach to the stabilization of a nonlinear feedback system is attempted. Given that the open-loop system is stable, this method is proposed to keep the closed-loop system stable in the bounded-input bounded output (BIBO) sense. The BIBO stability is defined as follows:

Definition: BIBO stability [3]:

Let S be a nonlinear system and $\gamma(S)$ be the gain of the system, which is defined as

$$\gamma(S) = \sup_{\substack{\bar{u} \\ \bar{u} \neq 0}} \frac{\|S\bar{u}\|_\infty}{\|\bar{u}\|_\infty}, \tag{3.14}$$

where $\|\cdot\|_\infty$ is the uniform norm and $\|\bar{u}\|_\infty < \infty$. Then, the system is called bounded-input bounded-output (BIBO) stable if the system has bounded gain, i.e., $\gamma(S) < \infty$.

A sufficient condition for the closed-loop stability of a feedback system is addressed by the small gain theorem.

Theorem: The small gain theorem [26]:

Consider the system in Figure 3.5.

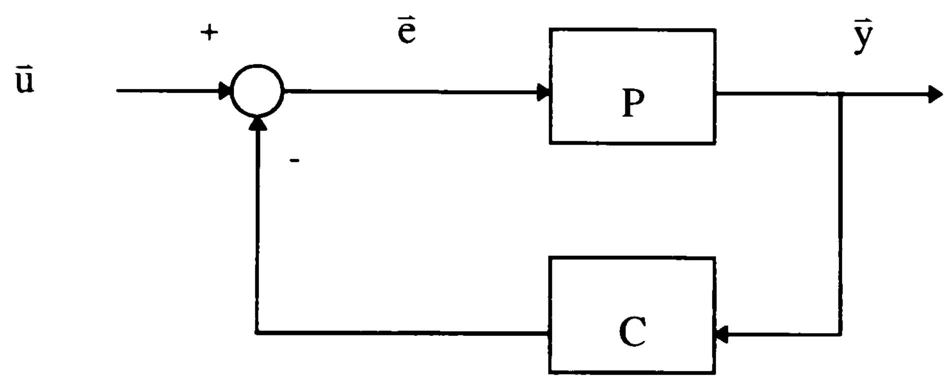


Figure 3.5: Feedback Control structure

Let P be a nonlinear operator that represents a given plant and C be a nonlinear operator which represents a controller. In addition, the operators P, C have bounded gain, i.e., $\|P\|_\infty < \infty$ and $\|C\|_\infty < \infty$. Then, the closed-loop system is BIBO stable if the closed loop gain is less than one, i.e., $\|P\|_\infty \|C\|_\infty < 1$.

To apply the small gain theorem to the NIMC structure, the structure is further modified and redrawn as in Figure 3.6.

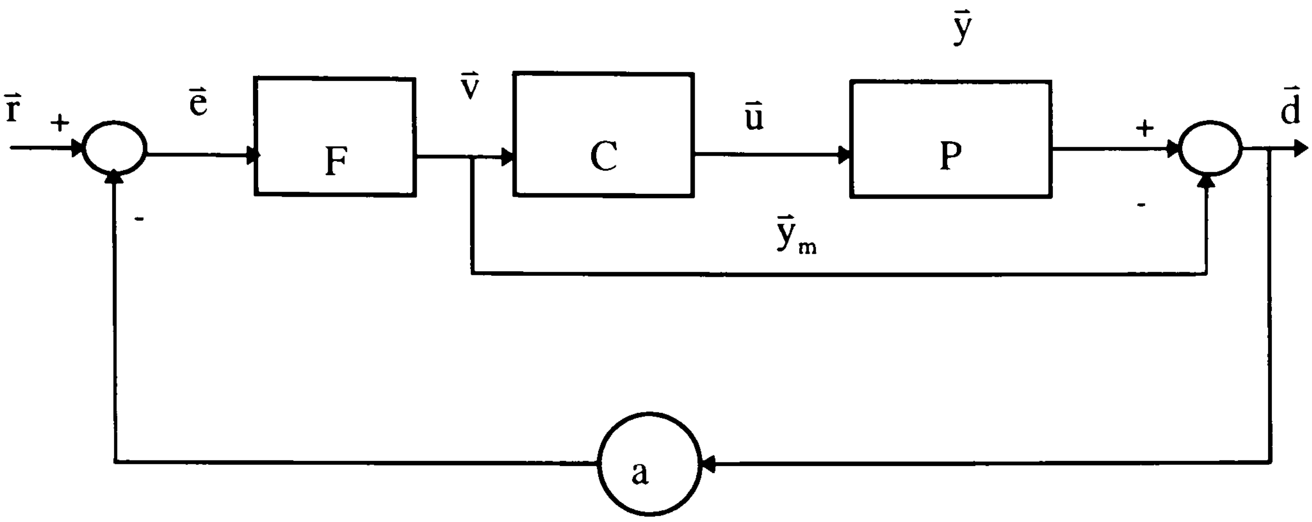


Figure 3.6: The NIMC Structure

Here, a system with gain "a" is added to the feedback path. From the block diagram, it also can be directly derived that the input-output relation is

$$\vec{y} = F(\vec{r}) + (I - aF)(\vec{d}), . \tag{3.15}$$

and the stability condition for the NIMC structure is

$$a\|(P_d C - I)F\|_\infty < 1 . \tag{3.16}$$

The Eq(3.16) is true under the assumption that the opened loop system is BIBO stable, i.e.,

$$\|(P_d C - I)F\|_\infty < \infty . \tag{3.17}$$

By using the properties of an operator norm,

$$\|AB\| \le \|A\|\|B\| \text{ and}$$
$$\|A + B\| \le \|A\| + \|B\|,$$

it follows that

$$\|(P_d C - I)F\|_\infty \le \left(\|P_d C\|_\infty + \|I\|_\infty\right)\|F\|_\infty = \left(\|P_d C\|_\infty + 1\right)\|F\|_\infty .$$

Since the filter, F, is always designed to have finite gains, i.e., $\|F\|_\infty < \infty$, the sufficient condition of Eq(3.17) can be written as

$$\|P_d C\|_\infty < \infty. \tag{3.18}$$

According to Eq(3.18), the proposed stability criterion does not have to require the plant, P_d, to be stable, provided that $P_d C$ is stable.

To exam the stability condition, the gains of the operators must be computed. Here, a computing method will be discussed. Let a system in Figure 3.7 is represented by an operator H. The gain of the system is defined as

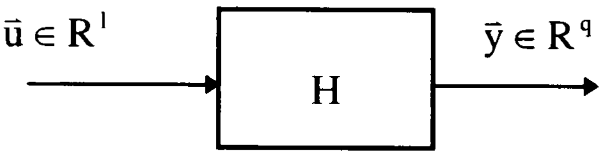


Figure 3.7: A system is represented by an operator H

$$\begin{aligned} \text{Gain} = \gamma(H) = \|H\|_\infty &= \sup_{\substack{u \\ u \neq 0}} \frac{\|H(\vec{u})\|_\infty}{\|\vec{u}\|_\infty} \quad \text{for } \|\vec{u}\|_\infty < \infty \\ &= \sup_{\substack{u \\ u \neq 0}} \frac{\|\vec{y}\|_\infty}{\|\vec{u}\|_\infty} \quad \text{for } \|\vec{u}\|_\infty < \infty, \end{aligned} \tag{3.19}$$

where $\|\vec{u}\|_\infty = \sup_{0 \le \tau \le \infty} \left\{ \max_{1 \le i \le l} |u_i(\tau)| \right\}$ and $\|\vec{y}\|_\infty = \sup_{0 \le \tau \le \infty} \left\{ \max_{1 \le i \le q} |y_i(\tau)| \right\}$.

Frequently, computing the gain of nonlinear systems involves searching over all inputs in the entire input vector space that give the maximum ratio of output norm to input norm. This is not feasible in general.

In this work, instead of the exact gain, a pseudo gain for nonlinear systems will be proposed. If the system H is a nonlinear system, then the pseudo gain of the system H at time t is defined as

$$
\begin{aligned}
\|H\|_{(ps)} &= \frac{\|\vec{y}^*(\tau)\|}{\|\vec{u}^*(\tau)\|} \quad \text{for } 0 \le \tau \le t \text{ and } \vec{u}^* \ne 0 \\
&= \frac{\sup\limits_{0\le\tau<t}\left\{\max\limits_{1\le i\le q}\left|y_i^*(\tau)\right|\right\}}{\sup\limits_{0\le\tau<t}\left\{\max\limits_{1\le i\le l}\left|u_i^*(\tau)\right|\right\}}, \quad \vec{u}^* \ne 0,
\end{aligned}
\tag{3.20}
$$

where $\vec{u}^*$ is a particular input vector function and $\vec{y}^*$ is an output response function due to $\vec{u}^*$. Thus, the pseudo gain at time t is considered to be the gain of the system due to the input $\vec{u}^*$.

By using the pseudo gain defined in Eq(3.20), a new stability criterion can be expressed as

$$
a\,\|(P_dC - I)F\|_{(ps)} < 1, \tag{3.21}
$$

where

$$
\|(P_dC - I)F\|_{(ps)} = \frac{\sup\limits_{0\le\tau<t}\left\{\max\limits_{1\le i\le q}\left|d_i(\tau)\right|\right\}}{\sup\limits_{0\le\tau<t}\left\{\max\limits_{1\le i\le l}\left|e_i(\tau)\right|\right\}}. \tag{3.22}
$$

Here, the gain of the nonlinear operators in the previous stability criterion in Eq(3.16) is replaced by the pseudo gain.

Since the pseudo gain at time t is the gain of the system due to the current input, $\vec{u}^*$, at time t, the closed-loop gain that is computed from the pseudo gain is considered as the current closed loop gain of the system due to the current input, $\vec{u}^*$, at time t. If the magnitude of the current input vector, $\vec{u}^*$, is bounded and the current closed-loop gain for the current input is kept less than one at each time step, this is also sufficient enough to maintain the closed loop stability. However, the value of the pseudo gain must be inevitably evaluated at each time step. This requires an on-line adaptation algorithm. The algorithm will be presented later in the NIMC strategy section.

3.5 Fuzzy Logic Systems for NIMC

Currently, fuzzy logic systems for control are getting a lot of attention. There have been many control structures which are proposed for implementing fuzzy logic techniques. Here, a fuzzy logic system is proposed as the controller in the NIMC structure. The control architecture is shown in Figure 3.8. In Figure 3.8, Fuzzy 1 is designed as the model inverse of the plant P.

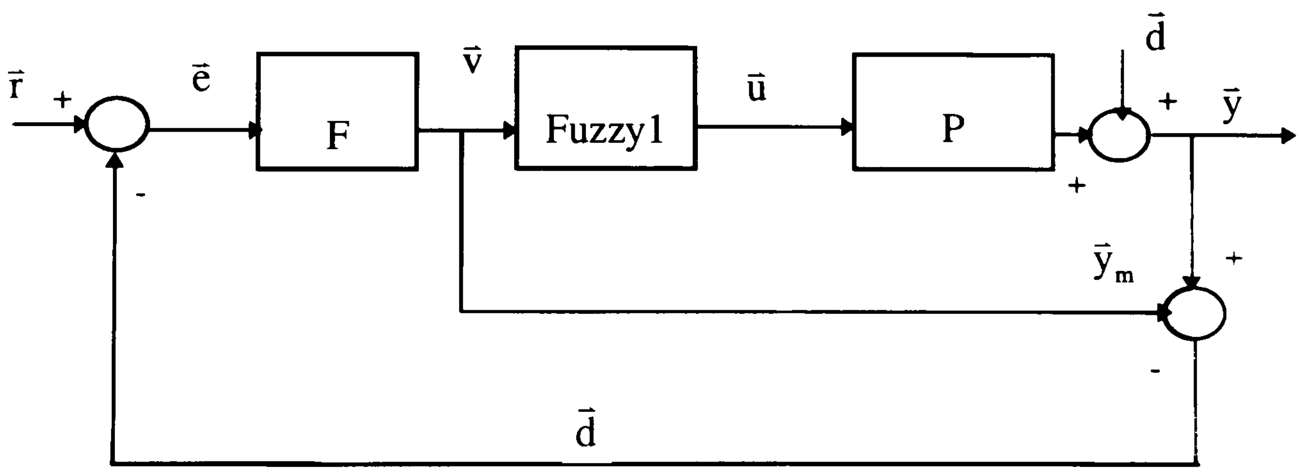


Figure 3.8: Fuzzy NIMC Structure

In this work, fuzzy systems as fuzzy basis function expansions are proposed to model nonlinear systems. Fuzzy basis functions, introduced by Wang [115], were addressed in section 2.4. The fuzzy systems as fuzzy basic function expansions are one class of a few fuzzy systems that have been theoretically proved to be a universal approximator [115]. Finally, since FBF expansions are linear in the parameters, linear identification methods, which are usually less complex than nonlinear identification methods, can be used to obtain the parameters.

3.6 Modeling The Plant Inverse

According to section 3.2, the plant is described through the relations:

$$\vec{f}(\vec{y}, \dot{\vec{y}}, \ddot{\vec{y}}, \ldots, \vec{y}^{(n)}, \vec{u}) = \vec{0} \tag{3.23}$$

where $\vec{f}(.)$ is a continuous vector function in the operating region. It is assumed that the plant has known order, i.e., the highest derivative order of the output, n, in the $\vec{f}(.)$. However, the dynamics of the plant (equivalently the relationship of $\vec{y}, \dot{\vec{y}}, \ddot{\vec{y}}, \ldots, \vec{y}^{(n)}, \vec{u}$) are partially or completely unknown.

According to the plant described in Eq(3.23), the inverse of the plant can be written as

$$\vec{u} = \vec{\varphi}(\vec{y}, \dot{\vec{y}}, \ddot{\vec{y}}, \ldots, \vec{y}^{(n)}), \tag{3.24}$$

where $\vec{\varphi}(.)$ is a continuous vector function. In the NIMC structure, the input of the model inverse(the controller) is connected to the output of the filter, $\vec{v}$. Thus, the relations can be written as

$$\vec{u} = \vec{\varphi}(\vec{v}, \dot{\vec{y}}, \ddot{\vec{y}}, \ldots, \vec{y}^{(n)}). \tag{3.25}$$

while the model inverse of the plant in the Eq(3.25) is in continuous time form, it is implemented digitally. That is the values of the variable are known only at discrete time intervals. Thus, the continuous time variables in the Eq(3.25) are replaced by discrete time variables. The Eq(3.25) can be written as

$$\bar{u}_t = \bar{\varphi}(\bar{v}_t, \dot{\bar{y}}_t, \ddot{\bar{y}}_t, \ldots, \bar{y}^{(n)}_t), \tag{3.26}$$

where $\dot{\bar{y}}_t$, $\ddot{\bar{y}}_t$, ..., and $\bar{y}^{(n)}_t$ are estimated by using the backward Euler approximation as followings:

$$\dot{\bar{y}}_t = \frac{\bar{v}_t - \bar{y}_{t-\Delta t}}{\Delta t},$$

$$\ddot{\bar{y}}_t = \frac{\dot{\bar{y}}_t - \dot{\bar{y}}_{t-\Delta t}}{\Delta t} = \frac{\bar{v}_t - 2\bar{y}_{t-\Delta t} + \bar{y}_{t-2\Delta t}}{(\Delta t)^2},$$

and so on for $\dddot{\bar{y}}_t$, ..., $\bar{y}^{(n)}_t$.

An approach for the modeling of the plant inverse chooses the structure of the FBF expansions to be the same as that of inverse model described through the relation in Eq(3.26). Thus, $\bar{v}_t$, $\dot{\bar{y}}_t$, $\ddot{\bar{y}}_t$, ...,and $\bar{y}^{(n)}_t$ are the inputs of the fuzzy model and $\bar{u}_t$ is the output of the fuzzy model.

Initially, the parameters of the fuzzy model are obtained by using the off-line version of the least square algorithm. Then, the parameters of the fuzzy model can also be adjusted in on-line fashion by using the recursive least square with forgetting factor algorithm described in section 2.5. A proposed on-line training architecture is shown in Figure 3.9. The architecture in the Figure 3.9. is known as the general learning architecture [95].

3.7 Filter Design

From Eq(3.7), the relationship between an output, $\bar{y}$, and the reference signal,

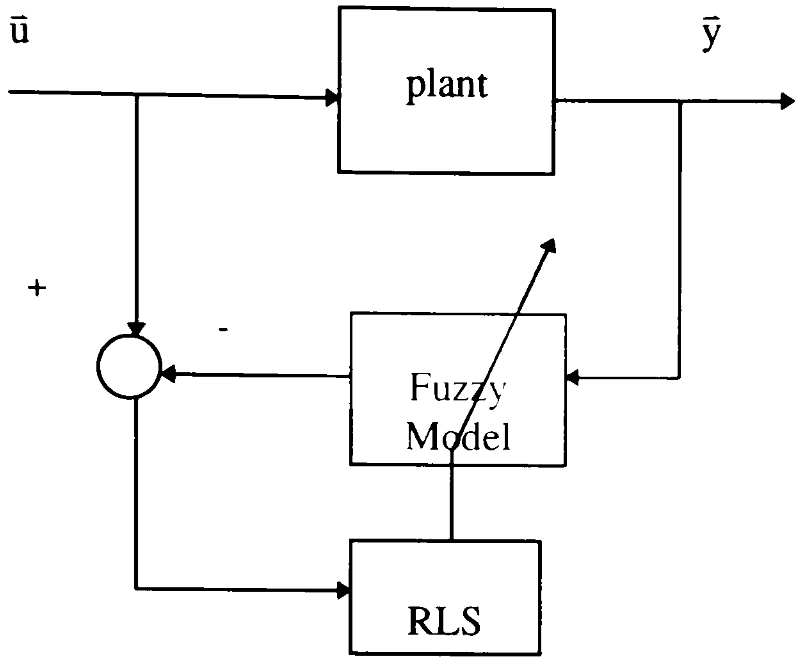


Figure 3.9: On-line Identification Architecture

$\bar{r}$, can be rewritten as

$$\bar{y} = F(\bar{r}) + (I - F)(\vec{d}), \tag{3.27}$$

where F is the filter and $\vec{d}$ is the disturbance. From the equation, if F=I (identity operator), then the control is perfect, i.e., $\bar{y} = \bar{r}$. However, this requires large control actions, which are undesirable in practice. Thus, a suitable filter should gradually reduce the disturbance effect and behaves as an identity operator at steady state.

Here, the filter F is proposed to have the following transfer function:

$$F : F_j(s) = \frac{b_{0,j}}{s^n + b_{n-1,j}s^{n-1} + b_{n-2,j}s^{n-2} + \ldots + b_{0,j}} = \frac{V_j(s)}{E_j(s)}, \tag{3.28}$$

where the $b_{i,j}$'s are the constants and j=1, 2, ..., m. The equation also can be expressed as

$$v_j^n + b_{n-1,j}v_j^{n-1} + b_{n-2,j}v_j^{n-2} + \ldots + v_j = b_{0,j}e_j, \; j = 1, \ldots, q, \tag{3.29}$$

where v_j is the j^{th} filter output and e_j is the j^{th} filter input. According to the final value theorem, i. e.,

$$\lim_{t\to\infty} h = \lim_{s\to 0} sH(s)\,,$$

when time t goes to infinity, the unit step response of the filter is equal to one, i.e.,

$$\lim_{t\to\infty} v_j = 1,\ j=1,\ 2,\ldots,q\ . \tag{3.30}$$

This means that the gain of the filter equals one at steady state. Equivalently, the filter behaves as the identity operator at steady state.

From the relationship in Eq(3.27), the plant output follows the output of the filter. Thus, the shape of the plant output response can be designed by choosing an appropriate set of $b_{i,j}$'s. Since the filter is a linear system, the techniques of linear systems can be employed. As an example, to ensure the stability of the filter, all the roots of the characteristic equation must lie in left half of the s-plane. Furthermore, in order to completely control the plant, the order of the filter should be equal to the order of the plant.

Finally, the transfer function of the filter, F, in the Eq(3.28) is in continuous time form. To implement the filter, F, digitally, it is desirable to transform the transfer function in continuous time form into discrete time form. In other words, the transfer function, F, in the s-domain needs to be transformed to the z-domain. Here, based on the backward Euler approximation [93], the transformed transfer function of the filter in the z-domain is

$$F : F_j(z) = F_j(s)\Big|_{s=\frac{1-z^{-1}}{\Delta t}}\ j=1,\ 2,\ \ldots,\ q \tag{3.31}$$

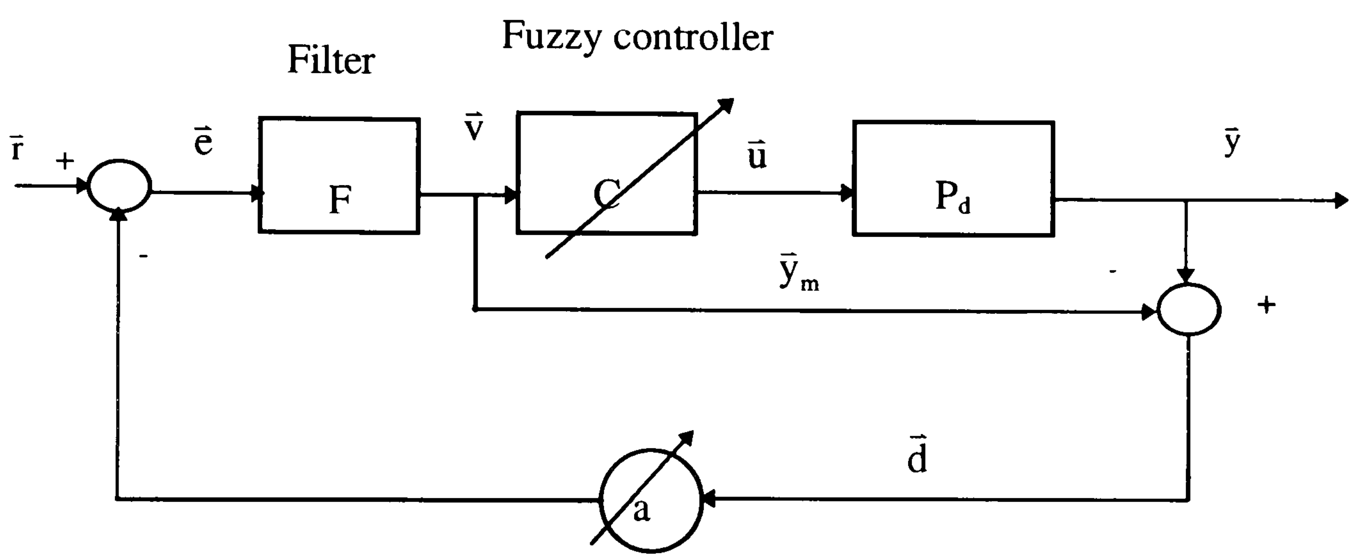
where Δt is a sampling period.

3.8 Adaptive Fuzzy NIMC Strategy

An adaptive fuzzy NIMC strategy is proposed for a nonlinear system that can be represented by a continuous function in the form of

$$\vec{f}(\vec{y}, \dot{\vec{y}}, \ddot{\vec{y}}, \ldots, \vec{y}^{(n)}, \vec{u}) = \vec{0},$$

where $\vec{u}$ is the input, $\vec{y}$ is the output, $\dot{\vec{y}}$ is the first derivative of the output and similarly for $\ddot{\vec{y}}, \ldots, \vec{y}^{(n)}$. The block diagram of the adaptive fuzzy NIMC structure is shown in Figure 3.10. The flow chart of the control strategy is also shown in Figure 3.11.



$0 \le a \le 1$

Figure 3.10: Block Diagram of The Adaptive Fuzzy IMC Structure

From the flow chart, the control strategy is divided into two major phases: control phase and adaptation phase. In the control phase, the plant output $\vec{y}$ tracks the reference signal, $\vec{r}$. The relationship between $\vec{y}$ and $\vec{r}$ are described by

$$\vec{y} = F(\vec{r}) + (I - aF)(\vec{d}). \qquad (3.32)$$

Then, controller parameters and "a" may be adjusted according to two criteria: stability criteria and large error criteria. This is called the adaptation phase. The rest of this section will discuss the adaptation procedure.

First, the stability criterion is checked. The criterion is

$$a\left\|(P_d C - I)F\right\|_{(ps)} K < 1. \tag{3.33}$$

where K is a real number that is equal or greater than one, i.e., $K \in R$ and $K \geq 1$. The criterion in the Eq(3.33) is modified from the criterion in Eq(3.21). The reason to add K in this criterion is to increase the sensivity to detect the instability of the closed-loop system. For K=1, the criterion is activated when the closed-loop gain is greater than or equal to one. According to the small gain theorem, the closed loop gain of a stable system is required to be less than one. However, when the closed-loop gain is greater than one, the closed-loop system may already be unstable which will severely degrade the control performance. Thus, it is desirable to activate the criterion before the closed-loop gain reaches one. Equivalently, K is set to have a value greater than one. Typically, K has a value slightly greater than one.

From the Eq(3.32), the disturbance, $\vec{d}$, will be completely rejected from the output when F is an identity operator and "a", which has a value between zero to one, is equal to one. From the filter design section, the requirement for the filter, F, is simply achieved at steady state. Here, "a" is designed to keep the system stable. Normally, "a" is set to be one. However, when the stability criterion is not satisfied, "a" must be reduced. The new "a" is determined from

$$a = \frac{1}{\left\|(P_d C - I)F\right\|_{ps} K}. \tag{3.34}$$

As mentioned before, small values of "a" reduce the ability of the system to reject the

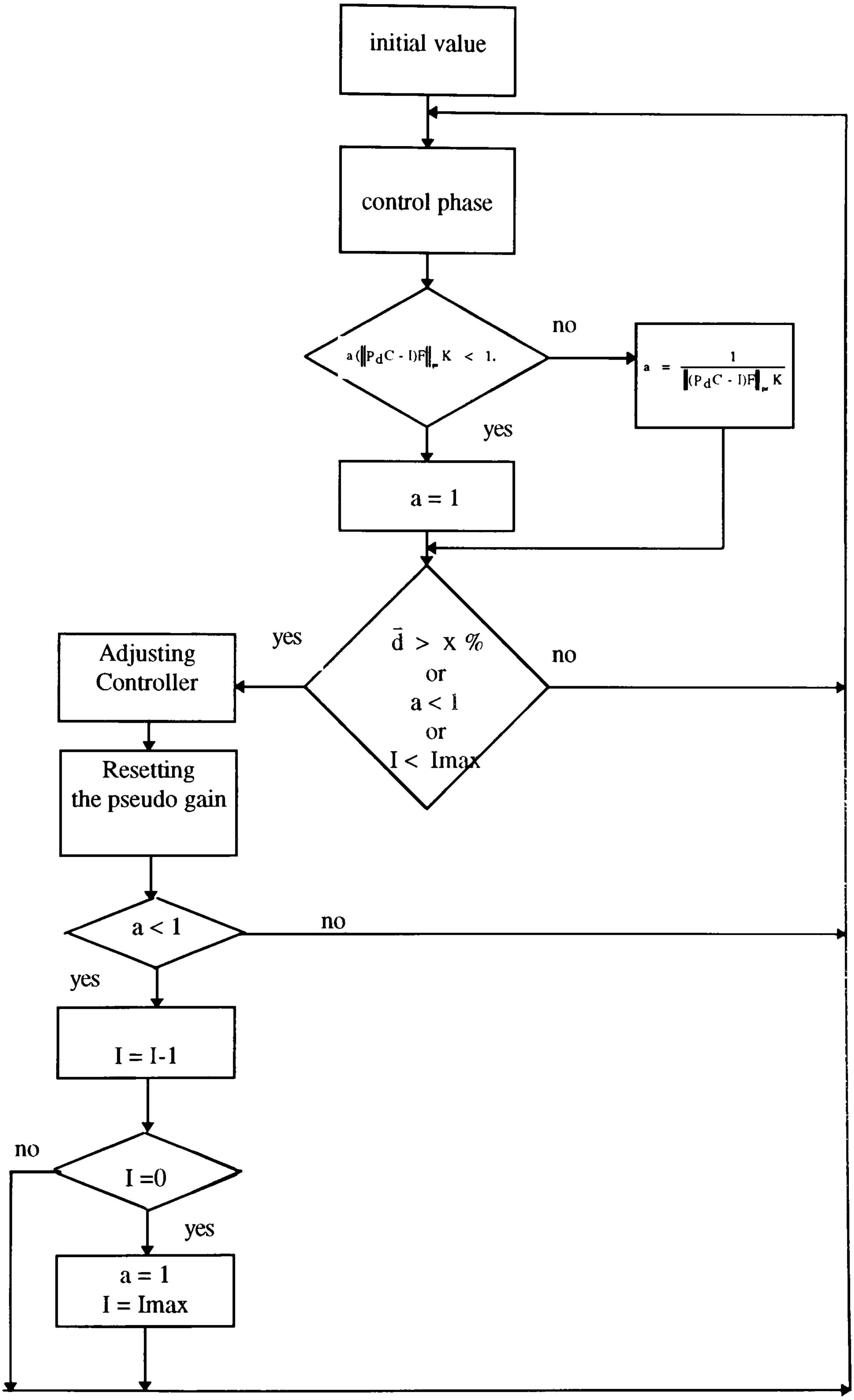


Figure 3.11: The Flow Chart of the Adaptive NIMC Strategy

disturbance, $\vec{d}$, from the output. According to the stability criterion in Eq(3.33), another way to reduce the closed loop gain is adjusting the fuzzy controller, which is the imperfect model inverse of the plant. However, adjusting the fuzzy controller requires more computing effort than adjusting the "a." Moreover, the learning process of the fuzzy controller is usually slow. Thus using only this technique may not keep the closed-loop system stable once an instability is detected.

Therefore, the combination of both techniques is proposed in the strategy. First, once the closed loop instability is detected, the value of "a" is reduced and stays the same for a number of consecutive time steps. While the value of "a" stays less than one, the parameters of the fuzzy controller are being adjusted. The learning period continues for a number of consecutive steps. Then, the value of "a" is reset to one again.

It can be seen from the relations in Eq(3.32) that the disturbance, $\vec{d}$, still has an effect on the output $\vec{y}$ during the transient states. This may cause undesirable transient performance when $\vec{d}$ is large. As the second criterion for adaptation, a strategy is proposed to reduce the effect of the disturbance, $\vec{d}$. Here, the disturbance, $\vec{d}$, is caused by the imperfect model inverse of the plant. Thus, to reduce the disturbance, the parameters of the controller, which is the model inverse of the plant, is adjusted.

However, adjusting the controller requires intensive computing effort. It may not be suitable to adjust the parameters at every step. Thus, the strategy is proposed to adjust the parameters only when the disturbance, $\vec{d}$, is larger than a threshold value.

Finally, since the strategy involves the pseudo gain,

$$\|(P_d C - I)F\|_{(ps)} = pcf = \frac{\sup\limits_{0 \le \tau < t}\left\{\max\limits_{1 \le i \le q}|d_i(\tau)|\right\}}{\sup\limits_{0 \le \tau < t}\left\{\max\limits_{1 \le i \le l}|e_i(\tau)|\right\}} = \frac{\text{maxd}(t)}{\text{maxe}(t)}, \tag{3.35}$$

the on-line searching for the pseudo gain is also required.

Initially, maxd(0), maxe(0) and pcf are assigned small values. Then, during control, maxd(t) and maxe(t) are the maximum values of $\max\limits_{1 \le i \le q}|d_i(\tau)|$ and $\max\limits_{1 \le i \le l}|e_i(\tau)|$ respectively for τ between zero to t. The pcf is the ratio of maxd(τ) to maxe(τ) for τ between zero and t.

The strategy is also involved in the adaptation of the fuzzy controller. This has an effect on the searching technique. Since each maxd(t), maxe(t) and pcf is designed to keep only the maximum values, the maximum values of the previous model may dominate the maximum values of the new model. The result is that the pseudo gain will not correspond to the new fuzzy controller. To solve this problem, maxd(t), maxe(t) and pcf are reset to a small value again when the parameters of the fuzzy controller are changed.

3.9 Summary

This chapter describes the NIMC scheme. First, some attractive properties of the input-output relationship of the NIMC structure are presented. Then, the modified NIMC structure is introduced. In addition, the stability criterion based on the small gain theorem is proposed. Then, the fuzzy basis function (FBF) expansion is proposed as the controller in the modified NIMC structure. Finally, based on the analytical results from the previous section, an adaptive fuzzy NIMC strategy is proposed. The strategy will be applied to control nonlinear systems in the next chapter.

CHAPTER 4

SIMULATION STUDIES

In this chapter, the adaptive fuzzy NIMC strategy developed in the last chapter is applied to the control of four nonlinear systems. The first application is the control of an inverted pendulum. The second application is the control of a pendulum model. The third application is the control of the forced Van der Pol equation. Finally, the fourth application is the control of a two-link robot manipulator, which is a multiple input-multiple output system.

4.1 Inverted Pendulum model

An inverted pendulum system is shown in Figure 4.1.

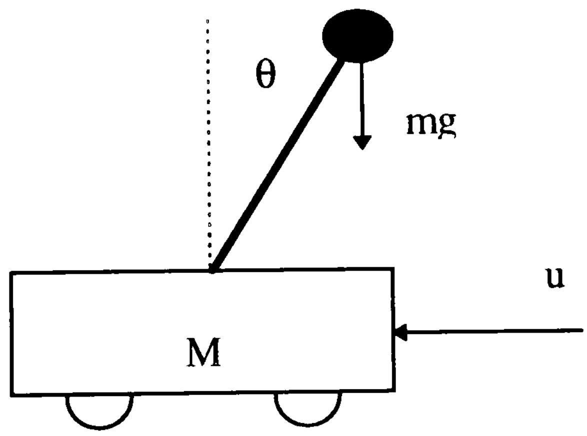


Figure 4.1: Inverted Pendulum

where M is the mass of the cart (kg),

m is the mass of the pendulum (kg),

l is the length of the pole (m),

g is the gravitational constant (9.8 m/s^2),

θ is the pole's angle from the vertical (rad).

If it is assumed that the pole is a narrow, uniform rod, with no friction at the pivot and the cart travels in one direction along a frictionless track, the equation of motion is

$$\ddot{\theta} = \frac{-m \cdot l \cdot \sin(\theta) \cdot \cos(\theta) \cdot \dot{\theta}^2 + (m+M) \cdot g \cdot \sin(\theta) - u \cdot \cos(\theta)}{l\left[M + m \cdot \sin^2(\theta)\right]} . \tag{4.1}$$

The derivation of the equation is shown by Henders [35].

The NIMC structure is shown in Figure 4.2.

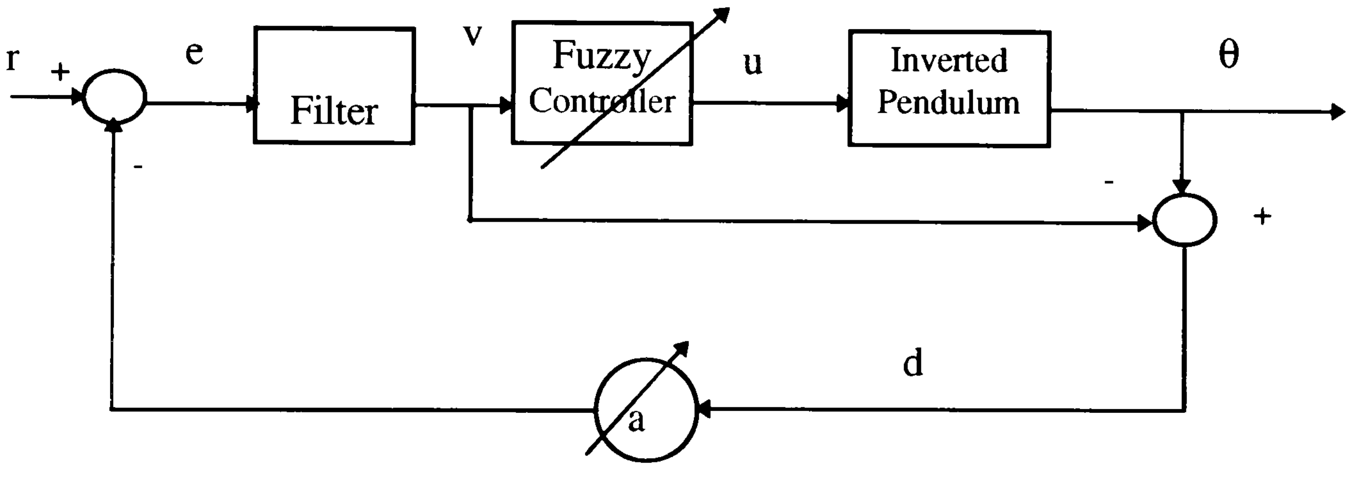


Figure 4.2: The NIMC Structure

The control objective is to make the output response of the system follow a desired setpoint in a predescribed manner. Here, the output response is required to meet two performance criteria. The first criterion is settling time, t_s. The settling time is the time required for the step response to settle within 2% of its final value. The second criterion is that the output response is overdamped, i.e., there is no overshoot. In this example, the output response is required to have the following specifications:

$$t_s \leq 1.5 \text{ sec}, \qquad \xi \geq 0.9, \tag{4.2}$$

where ξ is the damping ratio. Since the output of the controlled system follows the output of the filter, the requirement of the output response can be simply achieved by designing the filter to meet the requirement.

To meet the requirements of the output response, the transfer function of the filter is

$$F(s) = \frac{8.778}{s^2+5.33s+8.778} \ . \tag{4.3}$$

The step response of the filter is shown in Figure 4.3.

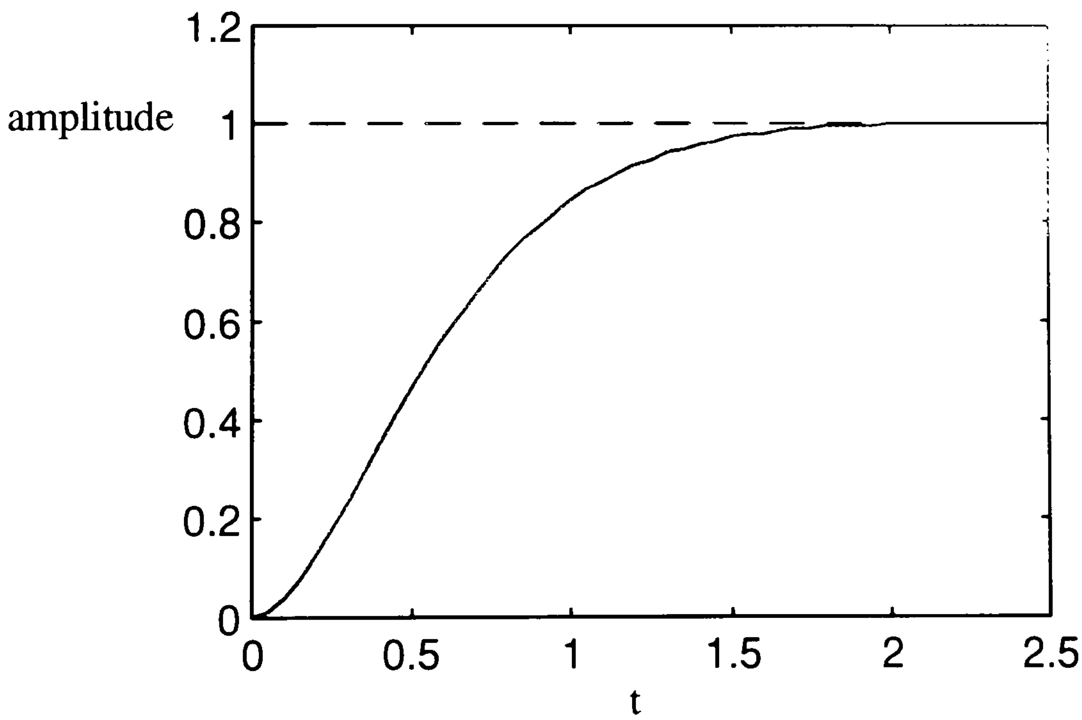


Figure 4.3: The step response of the filter

4.1.1 Off-line modeling of the fuzzy controller

From the input-output relation of the plant, the inverse of the plant is a function of θ and its derivatives, i.e.,

$$u = \varphi(\theta_t, \dot{\theta}_t, \ddot{\theta}_t). \tag{4.4}$$

where $\varphi(\cdot)$ is a continuous function. To model the inverse, the structure of the FBF expansion is chosen to be the same as the inverse of the plant. Here, each θ_t, $\dot{\theta}_t$, $\ddot{\theta}_t$ is described by five fuzzy variables: "positive big" (PB), "positive small" (PS), "zero" (ZE), "negative small" (SM), "negative big" (SB). In addition, the membership function of each fuzzy variable is the gaussian function. The membership functions of the fuzzy variables defined on a normalized domain [-6, 6] are shown in Figure 4.4.

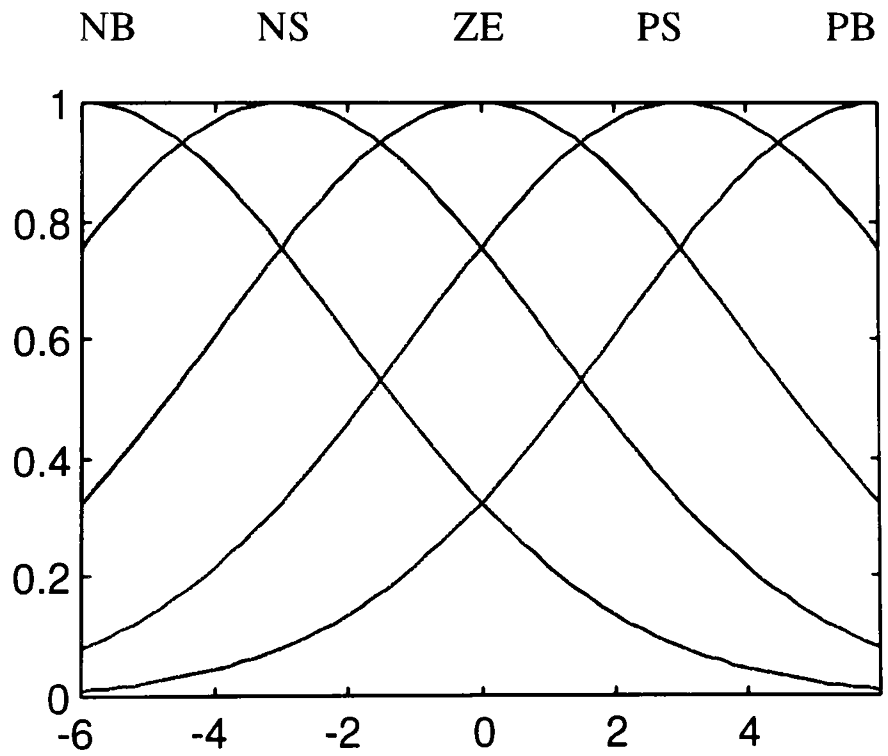


Figure 4.4: The membership functions of the fuzzy variables defined on a normalized domain [-6, 6].

The normalized values of each θ_t, $\dot{\theta}_t$, $\ddot{\theta}_t$, u use the same membership function distribution, shown in Figure 4.4.

Since each θ_t, $\dot{\theta}_t$, $\ddot{\theta}_t$ is described by the five fuzzy sets, the fuzzy rule base consists of 125 rules. The "if" parts of the fuzzy rules can be written as

$$R^j : \text{IF } \theta_t \text{ is } A^j_\theta \text{ and } \dot{\theta}_t \text{ is } A^j_{\dot{\theta}} \text{ and } \ddot{\theta}_t \text{ is } A^j_{\ddot{\theta}} \quad , \; j = 1, \ldots, 125 \tag{4.5}$$

where A^j_θ is a fuzzy set of θ in the j^{th} rule and similarly for $A^j_{\dot{\theta}}$ and $A^j_{\ddot{\theta}}$. Therefore, the fuzzy basic function expansion is formed as

$$u = \sum_{j=1}^{125} \alpha_j p_j \; . \tag{4.6}$$

The fuzzy basis functions, p_j's, are calculated from the "if" parts of the fuzzy rules as follows:

$$p_j = \frac{\mu_{A^j_\theta}(\theta_t) \cdot \mu_{A^j_{\dot{\theta}}}(\dot{\theta}_t) \cdot \mu_{A^j_{\ddot{\theta}}}(\ddot{\theta}_t)}{\sum_{j=1}^{125} \left(\mu_{A^j_\theta}(\theta_t) \cdot \mu_{A^j_{\dot{\theta}}}(\dot{\theta}_t) \cdot \mu_{A^j_{\ddot{\theta}}}(\ddot{\theta}_t) \right)} \qquad j= 1, \ldots, 125. \tag{4.7}$$

By using the least square algorithm, the parameters of the fuzzy model , α_js, are identified from 300 input-output data of the plant model that has the following parameters:

m=0.1 kg, M=1 kg and l=0.25 m.

Then, the fuzzy model is used as the controller in the NIMC structure. The simulations are conducted under three situations. First, the controller is the approximated model of the model inverse of the plant. Secondly, the controller is not the perfect model inverse of the plant, i.e., there is modeling error. Third, the imperfect model inverse of the plant causes the closed-loop system to be unstable. The simulation results are shown in the section of the simulation results.

4.2 Pendulum

A pendulum model, shown in Figure 4.5, is used for the simulation. The equation of motion [71] is

$$\ddot{\theta} + \frac{k}{l}\dot{\theta} + \frac{g}{l}\theta = \frac{1}{m \cdot l}u(t), \tag{4.8}$$

where m is the point mass (kg), l is the length of the pole (m), g is the gravitational constant (9.8 m/s^2) , k is the friction coefficient at the pivot (kgm^2/s).

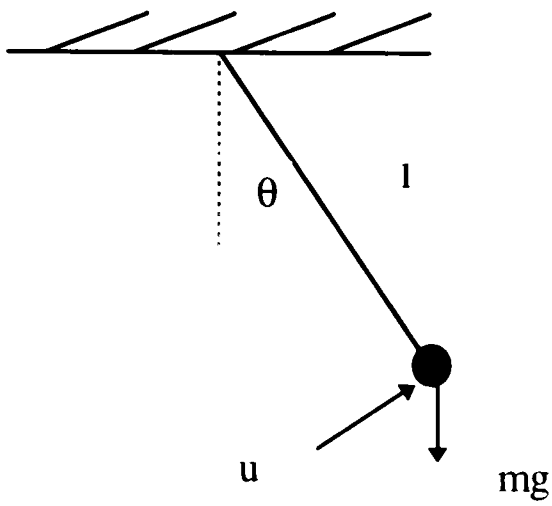


Figure 4.5: Pendulum

The NIMC structure is similar to that of the inverted pendulum. Here, the control objective and the requirements on the output response are also the same in the previous example. As a result, the transfer function of the filter is the same as Eq(4.3).

4.2.1 Off-line modeling of the fuzzy controller

From the input-output relation of the plant, the inverse of the plant is a function of θ and its derivatives, i.e.,

$$u = \varphi(\theta_t, \dot{\theta}_t, \ddot{\theta}_t). \tag{4.9}$$

where $\varphi(\cdot)$ is a continuous function. Thus, the structure of the FBF expansion is chosen to be the same as the Eq(4.9). Here, each input and output variable are described by five fuzzy variables: "positive big" (PB), "positive small" (PS), "zero" (ZE), "negative small" (SM), "negative big" (SB). The membership functions of the fuzzy variables, defined on a normalized domain [-6, 6], are shown in Figure 4.4.

The FBF expansion is formed as the Eq(4.6) and Eq(4.7). The parameters of the fuzzy model are identified from 300 input-output data of the plant model that has the following parameters:

$$m=0.3 \text{ kg}, k=0.01 \text{ kgm}^2/\text{s and } l=0.25 \text{ m}.$$

Then, the fuzzy model is used as the controller in the NIMC structure. Like the previous example, the simulations are conducted under three situations. The simulation results are shown in the section of the simulation results.

4.3 Forced Van Der Pol Equation

The forced Van der Pol equation is shown in Eq(4.10):

$$\ddot{y} + 2a(y^2 - 1)\dot{y} + by = cu(t), \tag{4.10}$$

where a, b and c are positive constants. Originally, the equation was introduced to study the oscillations in a vacuum tube circuit in 1927 [9]. Later, the equation has been used as an example to study the self-excited oscillation of nonlinear systems.

When u=0, the forced Van der Pol equation displays oscillation of fixed amplitude and fixed period. This oscillation is called limit cycle. The plot of the Van der Pol oscillation, when a=1, b=1, c=1, $y(0) = 0$ and $\dot{y}(0) = 2$, is shown in Figure 4.6. The well-known phase plane plot of the Van der Pol oscillation is also shown in Figure 4.7.

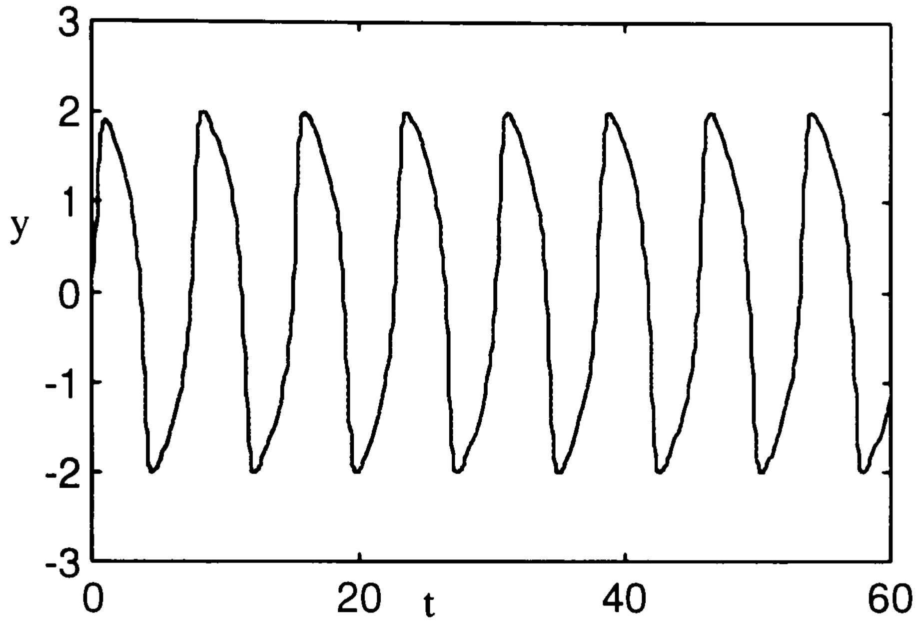


Figure 4.6: The plot of the Van der Pol oscillation

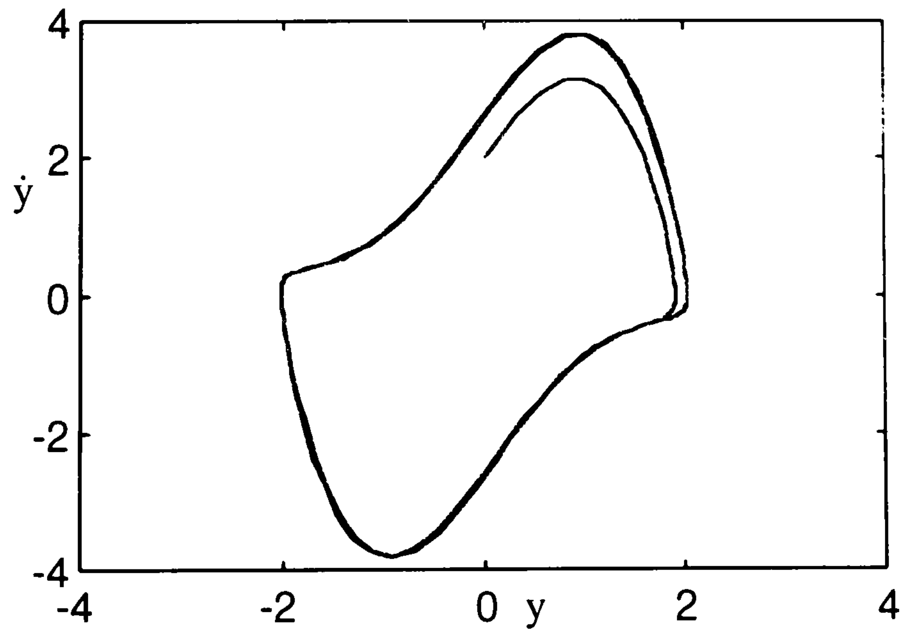


Figure 4.7: Phase plane plot of the Van der Pol oscillation

Here, the limit cycle phenomenon is undesirable. Thus, the control objective is to avoid the limit cycle phenomenon. In addition, the output response is required to be the same as the previous two examples.

Since the order of the system and the requirements on the output response are the same as the previous examples, the NIMC structure and the transfer function of the filter are also the same as the previous examples. The structure of the FBF expansion, which is the fuzzy controller, is chosen to be $u = \varphi(y,\ \dot{y},\ \ddot{y})$. Like, the previous examples. five fuzzy variables are used to described each input-output variable of the fuzzy model. Then a set of 300 input-output data of the plant model with a= 1, b= 1, c= 4, are used for the off-line identification of the parameters of the fuzzy model. The simulation results under the three situations are shown in the section of the simulation results.

4.4 A Two-Link Cylindrical Robot Manipulator

A two-link cylindrical robot manipulator is shown in Figure 4.8. From Fig 4.8, the base link rotates about z-axis by a revolute joint and the second link can change the length of the link by a prismatic joint.

The motion of the cylindrical robot manipulator can be described by a mathematical equation which is called the equation of motion. The equation of motion for the system can be obtained from the law of Lagrangian mechanics [46]. In this work, assuming that the links are rigid body, all masses are lumped at discrete point and the effects of actuator dynamics are ignored, the equation of motion for the system can be described as

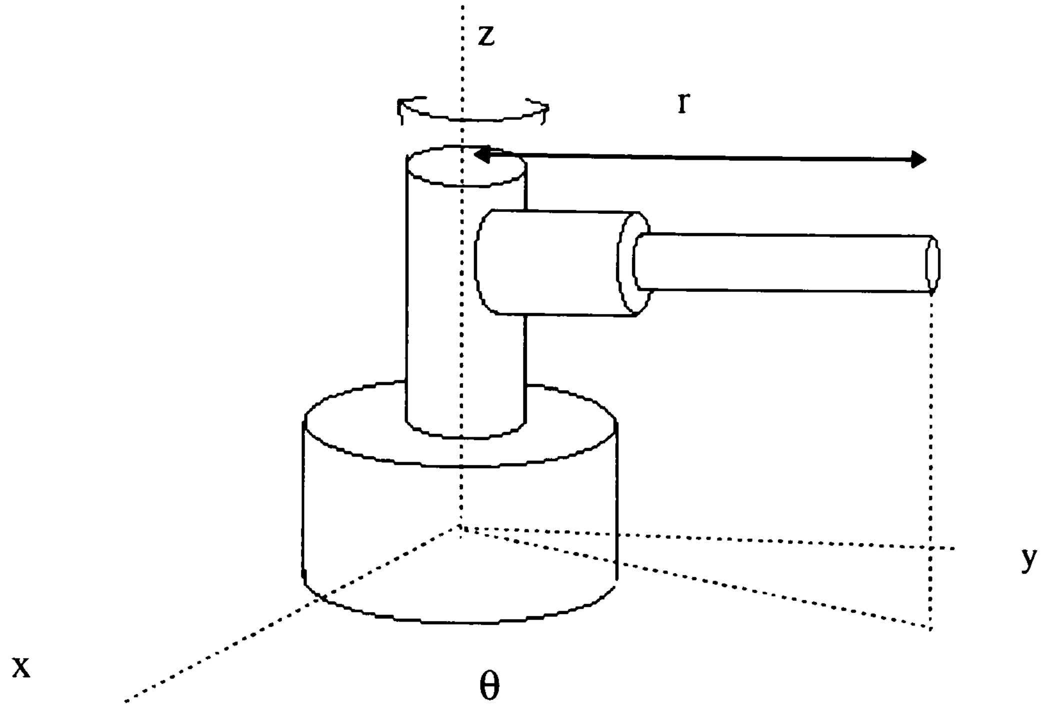


Figure 4.8: A Two-Link Cylindrical Robot Manipulator

$$\begin{bmatrix} I_\theta + m_2 r^2 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{r} \end{bmatrix} + \begin{bmatrix} 2r\dot{r}\dot{\theta}m_2 \\ -m_2 r\dot{\theta}^2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \qquad (4.11)$$

where I_θ is the inertia of the base link (kgm^2),

m2 are the point mass of link2 (kg),

r is the length of link 2 (m),

τ_1 , τ_2 are the required torque for link 1 and link 2 respectively (Nm),

θ is the angular position of link 2 (rad).

The derivation of the equation is given by Koivo [46] and Lewis [57].

The NIMC structure is shown in Figure 4.9. A basic problem in controlling the robot is to make each output response follow each desired setpoint in a predescribed manner. Like the previous examples, each output response of the robot is desired to meet two performance criteria: the overdamped output response and the settling time (ts). Here, each output response of the robot is required to have the following specifications:

$$t_s \le 1 \text{ sec}$$
$$\xi \ge 0.9.$$

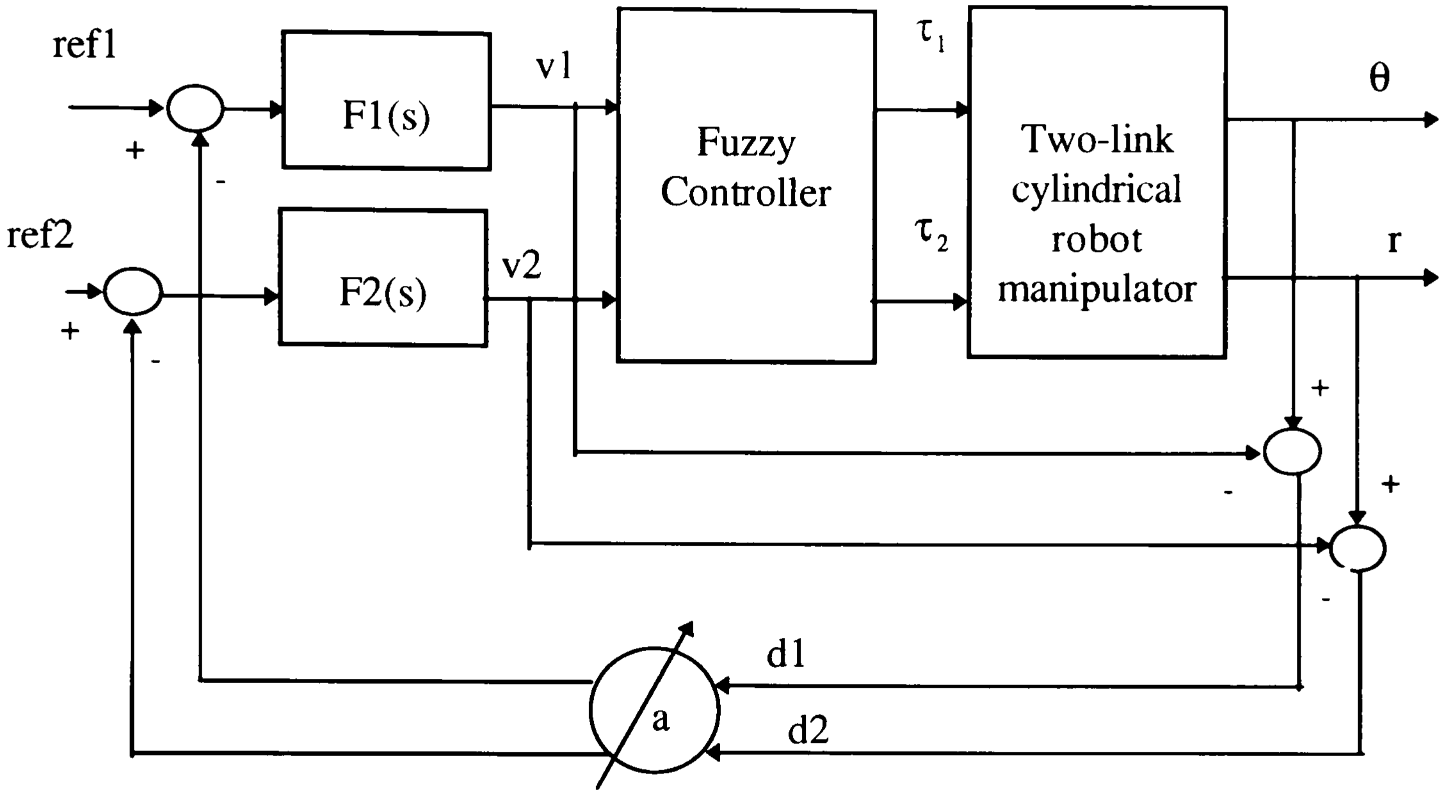


Figure 4.9: The NIMC Structure of The Two-link Cylindrical Robot Manipulator

Based on these criteria, the transfer function of each filter is given by

$$F1(s) = F2(s) = \frac{20.25}{s^2 + 8.1s + 20.25} \quad .$$

<u>4.4.1 Off-line modeling of the fuzzy controller</u>

Based on the equation of motion in the Eq(4.11), the inverse of the robot can be written as a function of output variables and their derivative, i.e.,

$$\begin{aligned} \tau_1 &= \varphi_1(r, \dot{r}, \ddot{r}, \dot{\theta}, \ddot{\theta}) \text{ and} \\ \tau_2 &= \varphi_2(r, \ddot{r}, \dot{\theta}), \end{aligned} \tag{4.12}$$

where $\varphi_1(\cdot)$ and $\varphi_2(\cdot)$ are continuous functions. Here, two FBF expansions are required to model the inverse of the robot: the first FBF expansion and the second FBF expansion are used to model $\varphi_1(\cdot)$ and $\varphi_2(\cdot)$, respectively. Like the previous examples, each input and output variable of both FBF expansions are described by five fuzzy variables and the membership function of each fuzzy variable are shown in Figure 4.4.

Then the parameters of the two fuzzy models are identified from the input-output data of the plant model with the following parameters:

$$I_\theta = 0.2\,\text{kgm}^2,\ m_2 = 2\,\text{kg}.$$

These fuzzy models are used as the controller in The NIMC structure. Similar to the previous examples, the simulations are conducted under three situations. The results are shown in the section of simulation results.

4.5 Simulation Results

In this section, the simulation results of the four nonlinear systems are presented.The simulations are conducted under three situations: First, fuzzy models are the approximated models of the model inverses. Second, fuzzy models are not the perfect models of the model inverses. Third, The modeling mismatch of the fuzzy controller results in the closed-loop system instability. The parameters of each nonlinear system under the three simulation situations are shown in Table 4.1, Table 4.2, Table 4.3. In addition, the tables also provide mean square errors (MSEs) of output responses from desired responses. For comparison purpose, a MSE, when each plant is controlled by the non-adaptive fuzzy NIMC strategy, is also shown in these tables. Furthermore, the plots of control results of each nonlinear system under the three situations are indicated in the tables as well. The explanations of the simulations under each situation are given as following:

Table 4.1 Fuzzy models are the approximated models of the model inverses

Nonlinear System		MSE of nonadaptive NIMC	MSE of adaptive NIMC	Figure
Inverted Pendulum m=0.1, M=1, l=0.25		0.0013	0.00085	Fig. 4.10, 4.10
Pendulum m=0.3, l=0.25, k=0.1		0.00095	0.00023	Fig. 4.12, 4.13
Forced Van der Pol a=1, b=1, c=4		0.0036	0.0021	Fig. 4.14, 4.15
two-link robot $I_\theta = 0.2,\ m_2 = 2$	r	0.0162	0.0033	Fig. 4.16 c and d Fig. 4.17 c and d
	θ	0.0030	0.0013	Fig. 4.16 a and b Fig. 4.17 a and b

Table 4.2 Fuzzy models are not the perfect models of the model inverses

Nonlinear System		MSE of nonadaptive NIMC	MSE of adaptive NIMC	Figure
Inverted Pendulum m=1, M=3, l=0.25		0.0104	0.0037	Fig. 4.18, 4.19
Pendulum m=1, l=0.5, k=0		0.0189	0.0014	Fig. 4.20, 4.21
Forced Van der Pol a=3, b=5, c=1.7		0.0247	0.0095	Fig. 4.22, 4.23
two-link robot $I_\theta = 0.5,\ m_2 = 4$	r	0.0169	0.0044	Fig. 4.24 c and d Fig. 4.25 c and d
	θ	0.015	0.0051	Fig. 4.24 a and b Fig. 4.25 a and b

Table 4.3 The modeling mismatch of the fuzzy controller results in the closed-loop system instability

Nonlinear System		MSE of nonadaptive NIMC	MSE of adaptive NIMC	Figure
Inverted Pendulum m=3, M=10, l=0.25		-	0.0023	Fig. 4.26
Pendulum m=3, l=1.5, k=0		-	0.0009	Fig. 4.27
Forced Van der Pol a=0.3, b=0.3, c=0.2		-	0.0218	Fig. 4.28
two-link robot $I_\theta = 3$, $m_2 = 4$	r	-	0.0049	Fig. 4.29 c and d
	θ	-	0.0024	Fig. 4.29 a and b

4.5.1 Fuzzy models are the approximated models of the model inverse

To test how well fuzzy models, obtained from the off-line identification, represent the model inverses, each fuzzy model is used as the controller in the NIMC structure of each nonlinear system. For comparison purposes, the plots of the control results also show the desired response of each nonlinear system.

From the simulation results of each nonlinear system, it is seen that there are no differences between the desired response and the output response. In addition, the plots of v and y for each nonlinear system shows that v and y appears the same. This means that the fuzzy models well approximate the model inverses of the plants. Finally, it can be seen from the plots that the plant output of each nonlinear system completely follows the output of the filter when the controller is the perfect model inverse. This confirms the theoretical results from the previous chapter.

4.5.2 Fuzzy controllers are not the perfect model inverses of the plants

The previous experiments show that the fuzzy models, obtained from the off-line identification, well represent the model inverses of the plants. However, during operation, the parameters of the plants may change. As a result, the fuzzy models are not the model inverse of the new plants. Therefore, the ability of the adaptive NIMC strategy in dealing with modeling error will be studied.

Here, each fuzzy model which is designed for each plant in the section 4.5.1 is used as the controller for controlling the plant indicated in Table 4.2. Both non-adaptive fuzzy NIMC and adaptive fuzzy NIMC strategy are applied to control each nonlinear plant.

From the results of each nonlinear system, there are no significant differences between the output response and the desired response, although the fuzzy model is not the exact model inverse of the plant. It can also be seen that the steady state error of each control system is eliminated in spite of the large modeling error at steady state.

However, during transient phase in each control system, the modeling error, d, has an effect on the output response. As a result, there are approximately 1%, 3%, 4% and 2% differences between the output response and the desired response during transient phase in the inverted pendulum, the pendulum, the forced Van der Pol equation and the two-link robot manipulator, respectively. As indicated in the plots, the effects of the modeling error, d, on the output response of each control system are insignificant for the case that the adaptive fuzzy NIMC strategy is applied. This is the case because the strategy allows the parameters of each fuzzy model to be adjusted in on-line fashion. Thus, the output response of each control system that is controlled by the proposed strategy appears similar to the desired response. This indicates that the adaptive fuzzy NIMC strategy well performs in reducing the modeling error.

4.5.3 The modeling mismatch of fuzzy controllers results in the closed-loop system instability

Stability is always the primary concern in the control systems. Sometimes, the varying of plant parameters results in closed-loop system instability. Therefore, the ability of the proposed strategy to maintain closed-loop system stability is necessary. Here, the simulations assume that each plant has the parameters shown in Table 4.3 and the initial fuzzy controller of each control system is set up from input-output data of the plant in Table 4.1.

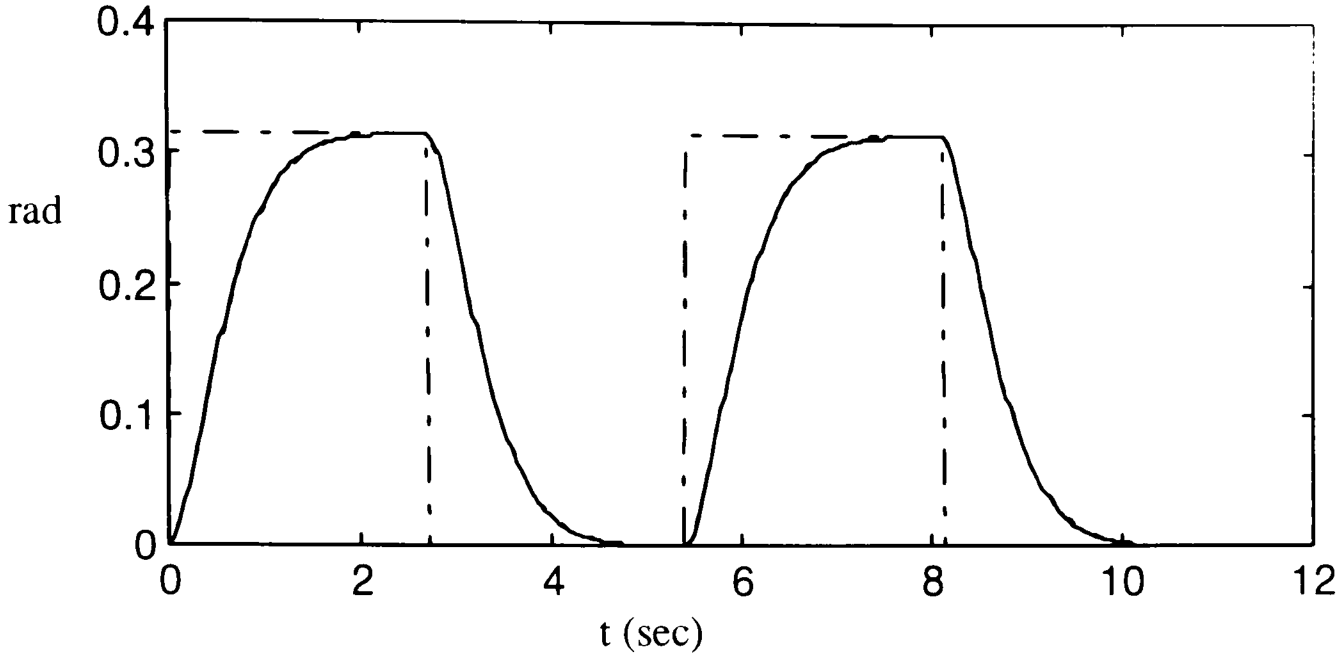
Without the adaptive fuzzy NIMC strategy, the control systems are not stable as shown in Figures 4.26, 4.27, 4.28 and 4.29. Then, the adaptive fuzzy NIMC strategy is applied to control each plant. The results show that the proposed strategy can maintain the closed-loop system stability of each control system. This clearly demonstrates the effectiveness of the adaptive fuzzy NIMC strategy in keeping the control systems stable.

4.6 Summary

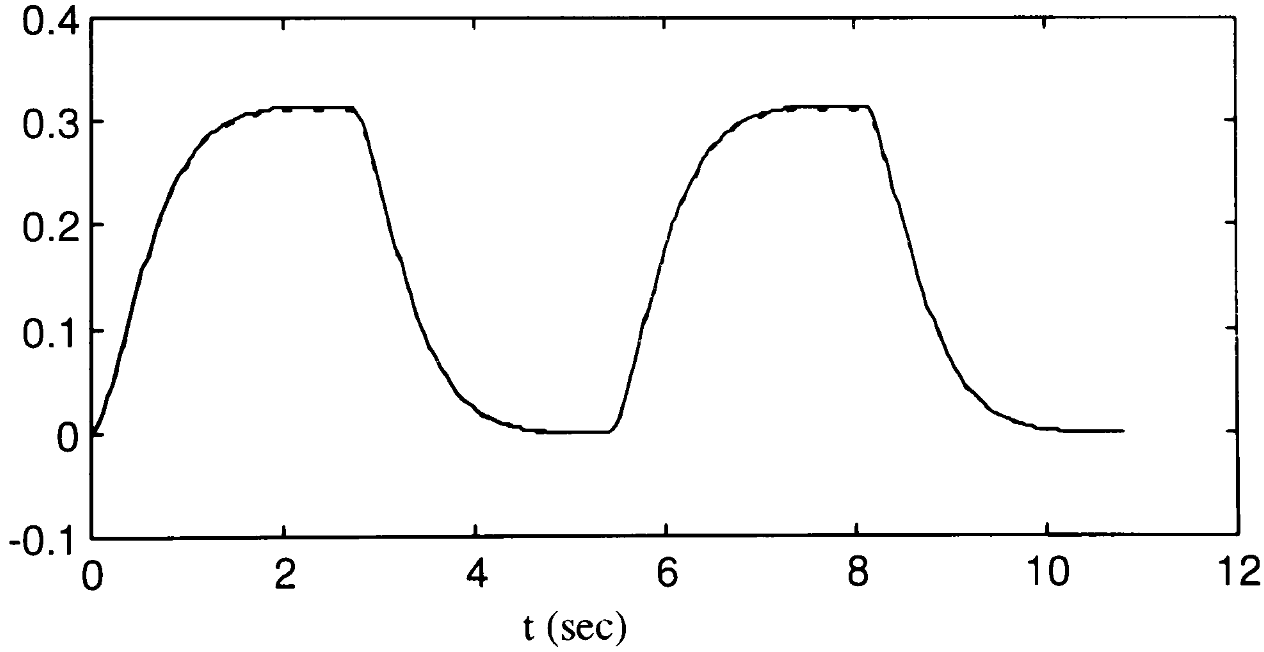
The adaptive fuzzy NIMC strategy was tested via simulations. Here, the strategy was applied to control four nonlinear systems. First, each fuzzy model was trained in off-line to capture the inverse dynamics of each plant and was used as the controller. It was shown that every fuzzy model well-represented the model inverse of its plant. In addition, the output response of each plant was the same as the desired response

Then, the simulations were also conducted under two situations. First, the initial fuzzy controllers were not the perfect model inverses of the plants. For the first situation, the adaptive NIMC strategy successfully reduced the modeling error. As a result, the output response of each plant did not severely degrade from the desired response.

Secondly, the closed-loop system was unstable. For the second situation, it was shown that the adaptive fuzzy NIMC strategy not only effectively kept the closed loop stable, but also improved the system performance. It can be concluded that the simulation results of controlling nonlinear systems are promising.

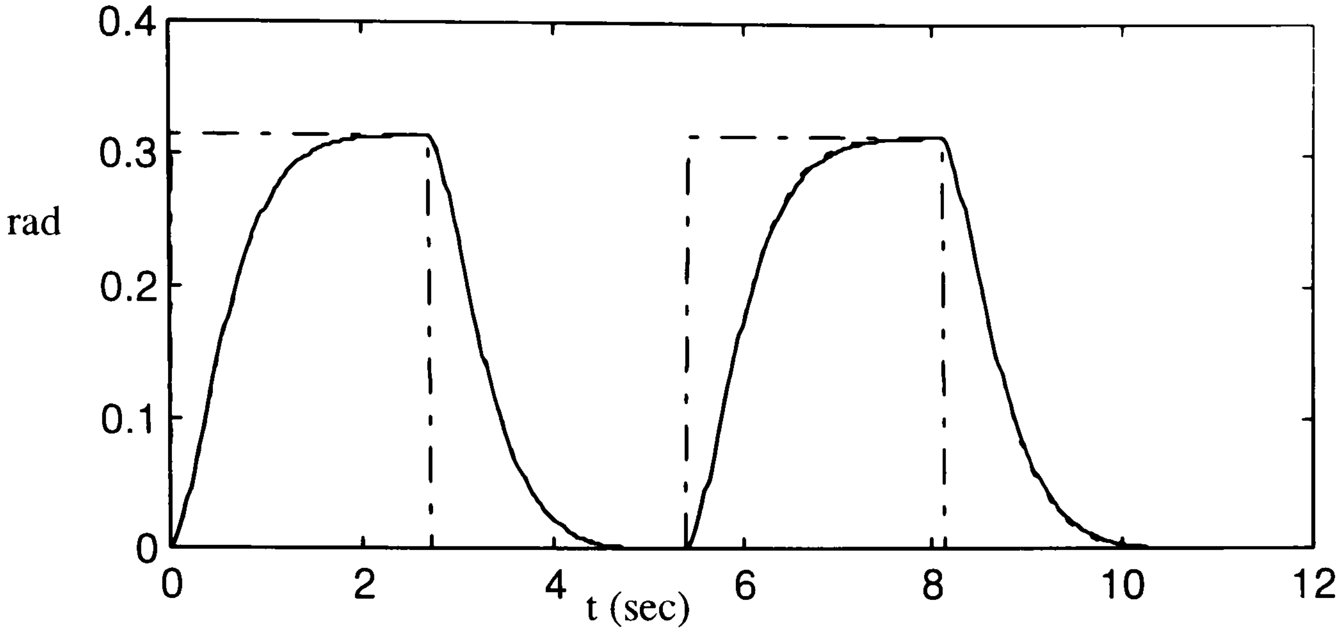


(a) The plots of reference signal ($-\ \cdot$), r, the output (-), θ and the desired response (--), y_d

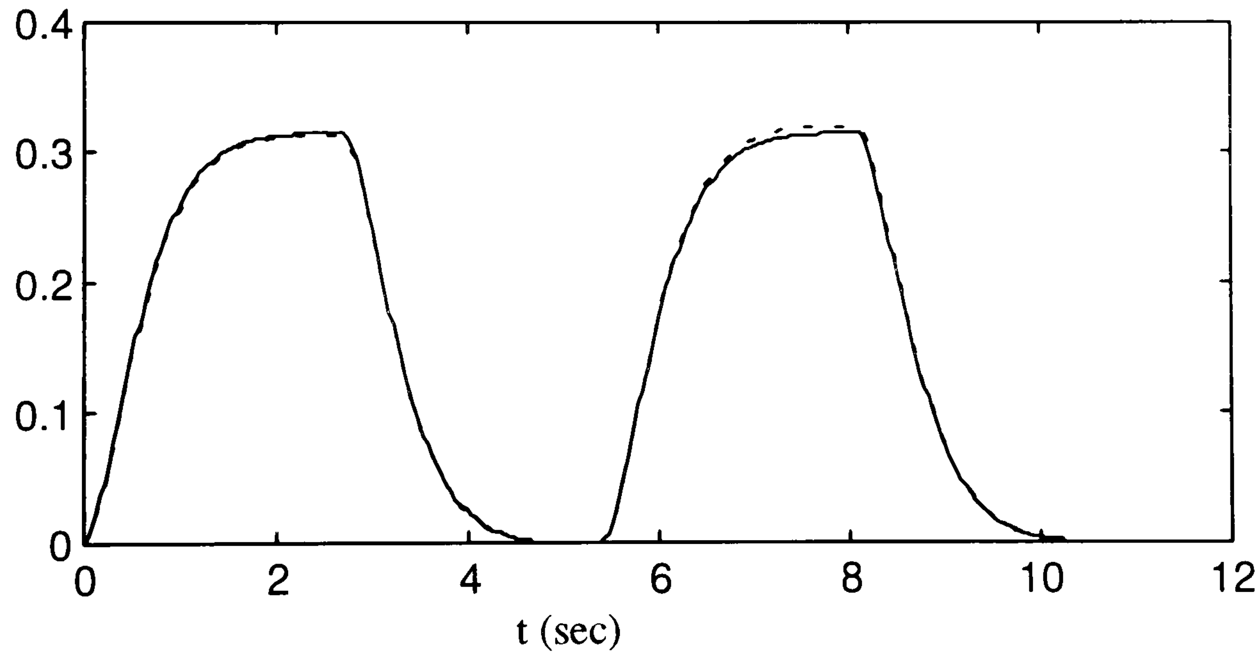


(b) The plots of the output of the filter (--), v and the output (-), θ

Figure 4.10: The control of the inverted pendulum by the non-adaptive fuzzy NIMC strategy when the fuzzy model is an approximation of the model inverse of the plant

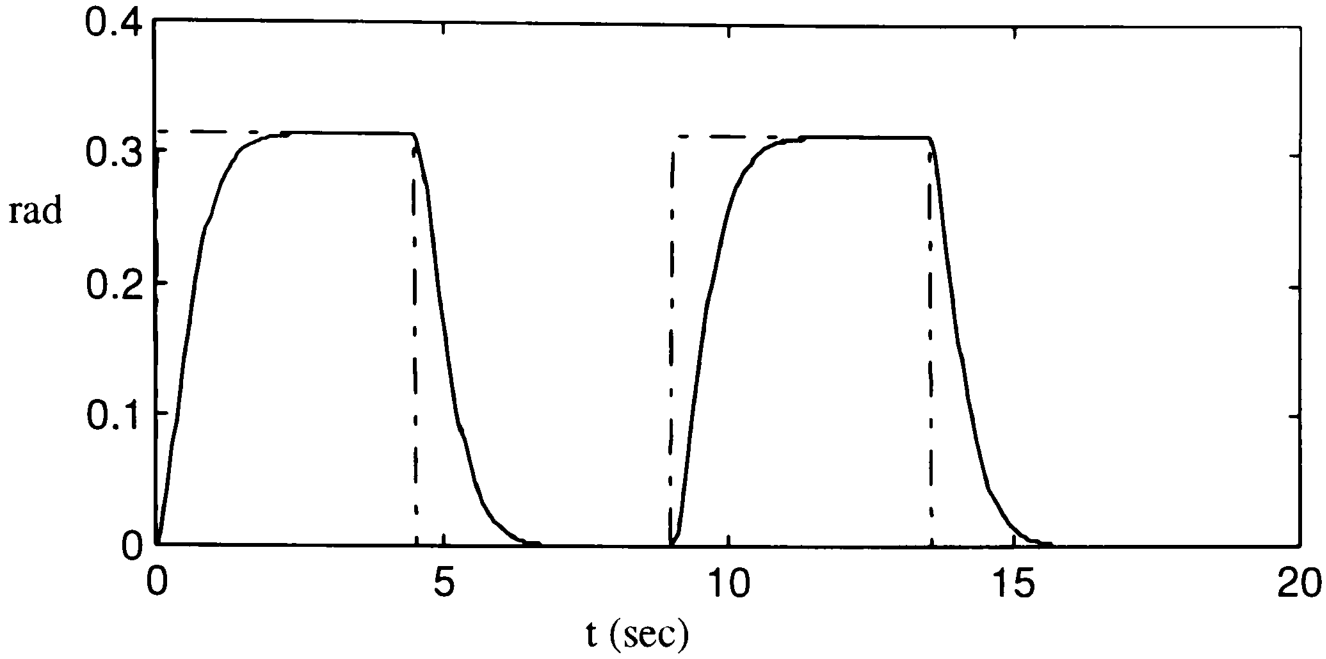


(a) The plots of reference signal (– ·), r, the output (-), θ
and the desired response (--), y_d

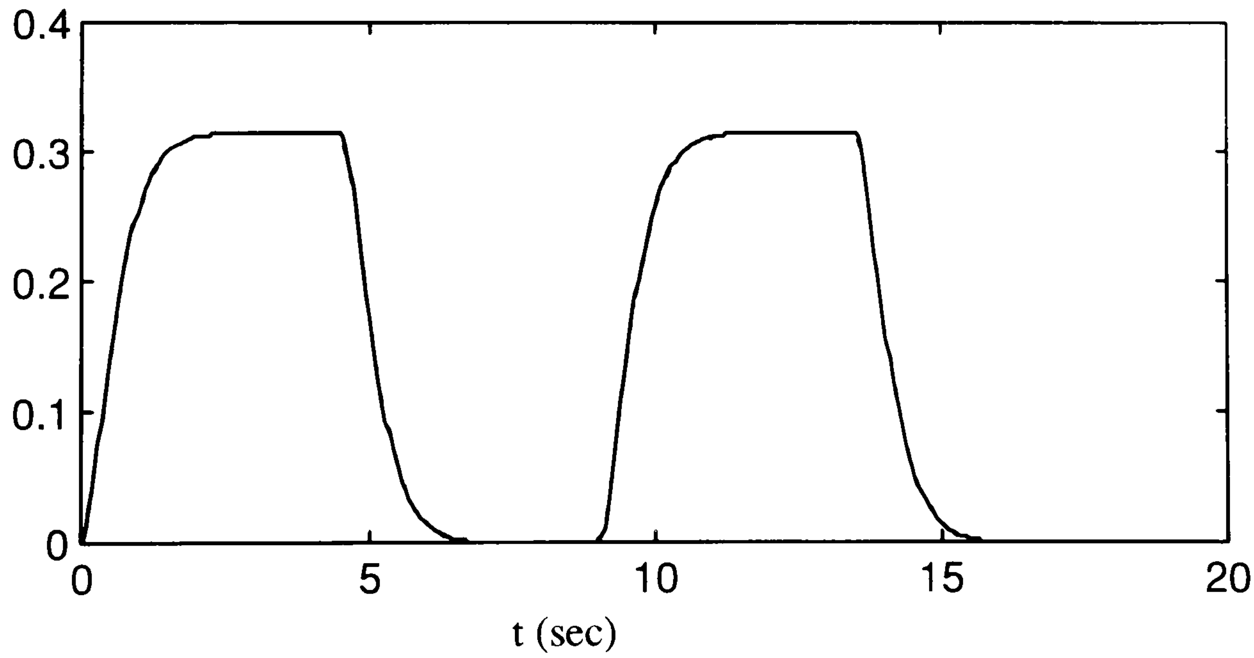


(b) The plots of the output of the filter (--), v and the output (-), θ

Figure 4.11: The control of the inverted pendulum by the adaptive fuzzy NIMC strategy when the fuzzy model is an approximation of the model inverse of the plant

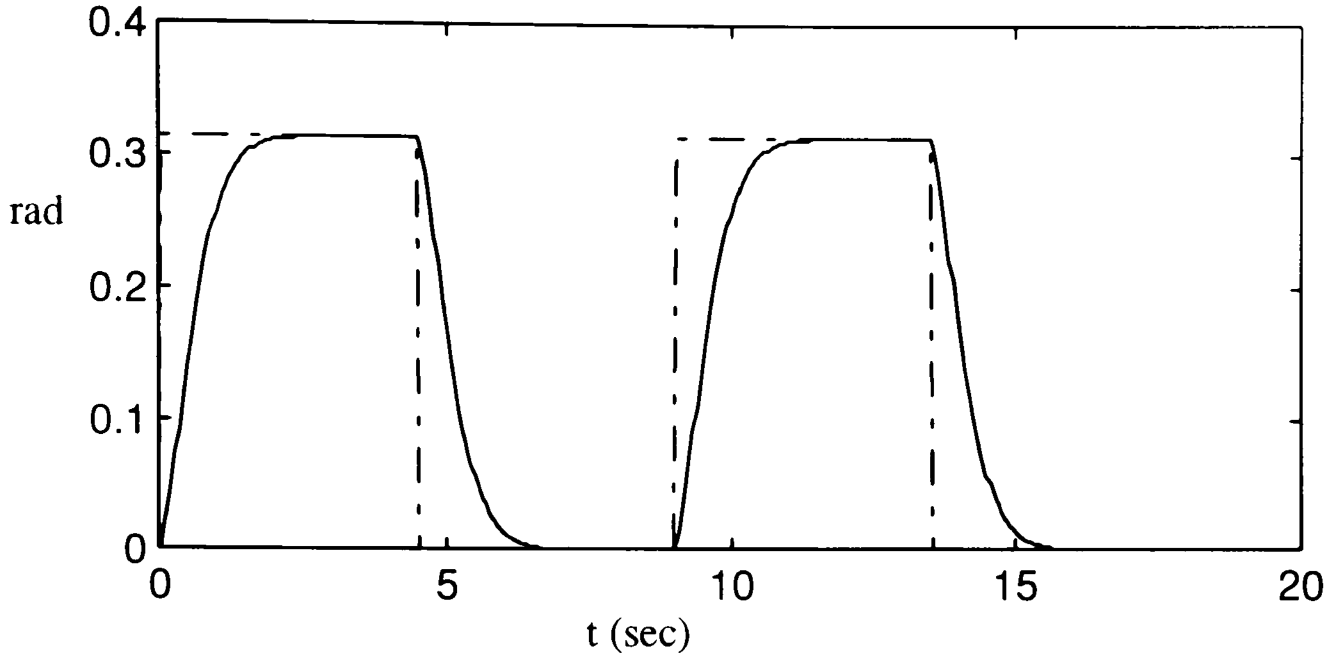


(a) The plots of reference signal (– ·), r, the output (-), θ and the desired response (--), y_d

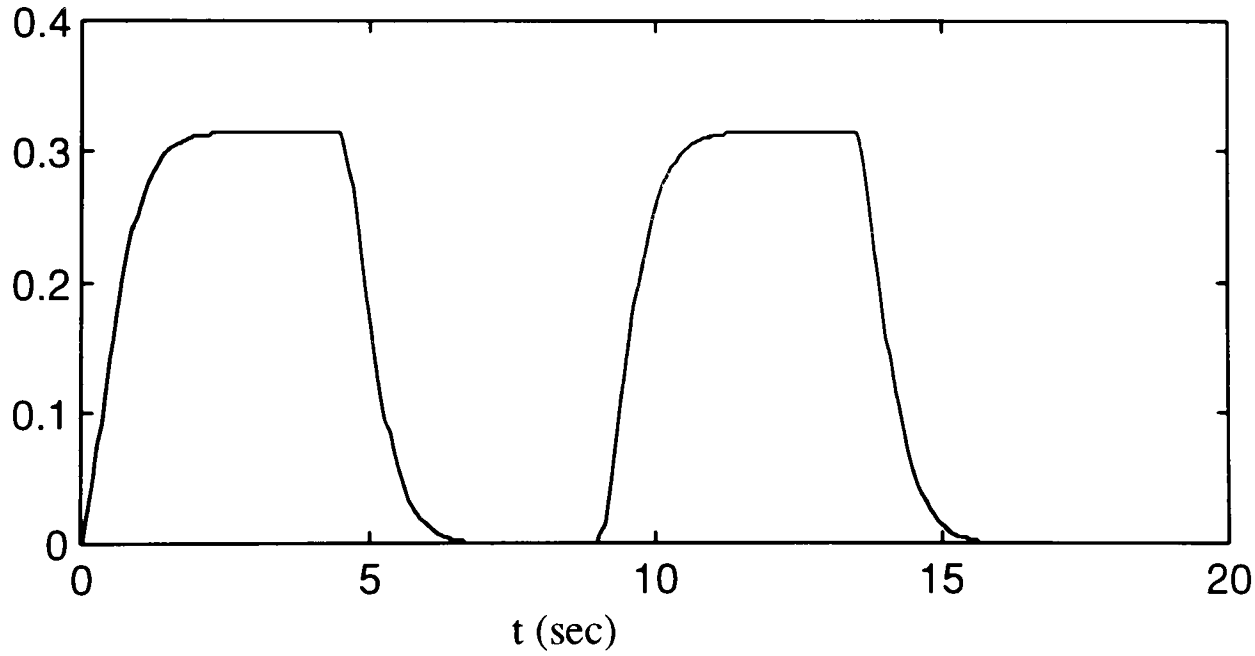


(b) The plots of the output of the filter (--), v and the output (-), θ

Figure 4.12: The control of the pendulum by the non-adaptive fuzzy NIMC strategy when the fuzzy model is an approximation of the model inverse of the plant

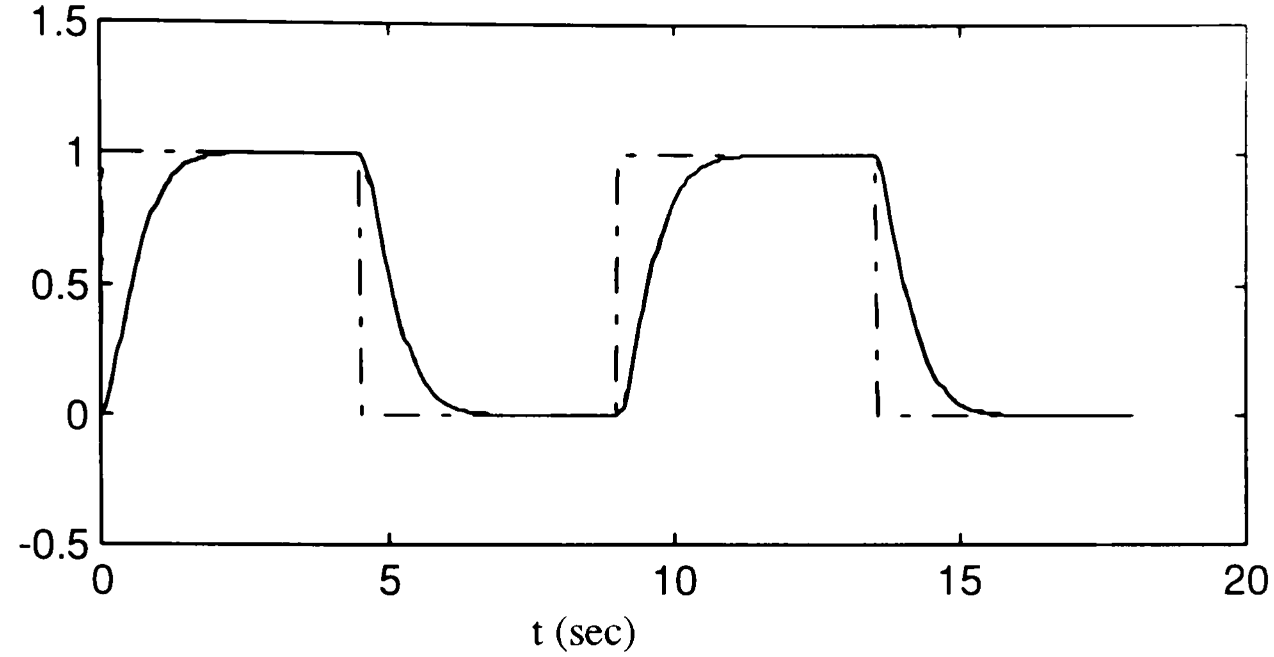


(a) The plots of reference signal (– ·), r, the output (-), θ
and the desired response (--), y_d

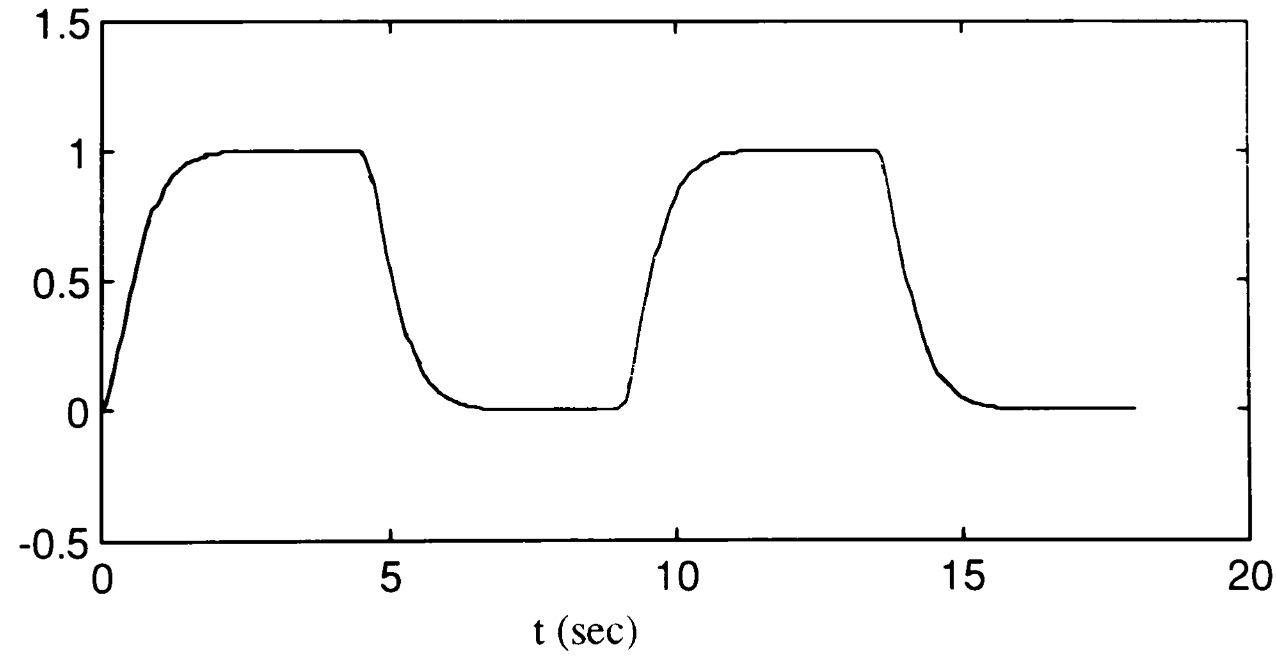


(b) The plots of the output of the filter (--), v and the output (-), θ

Figure 4.13: The control of the pendulum by the adaptive fuzzy NIMC strategy when the fuzzy model is an approximation of the model inverse of the plant

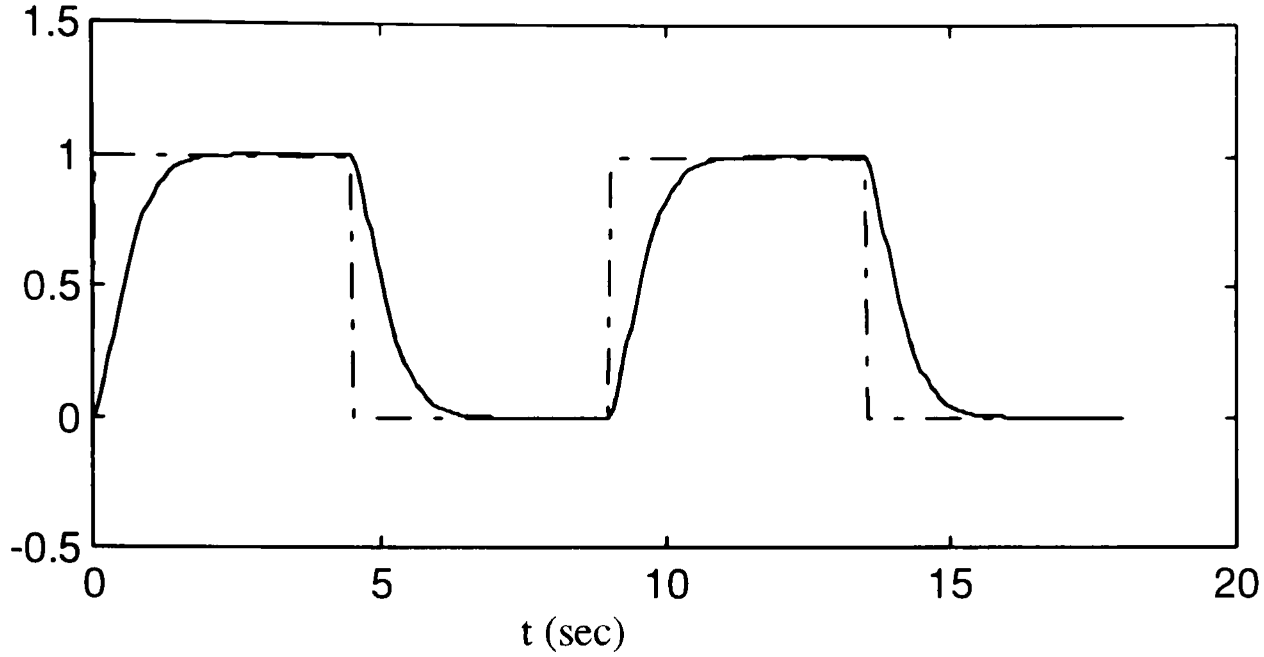


(a) The plots of reference signal (– ·), r, the output (-), y and the desired response (--), y_d

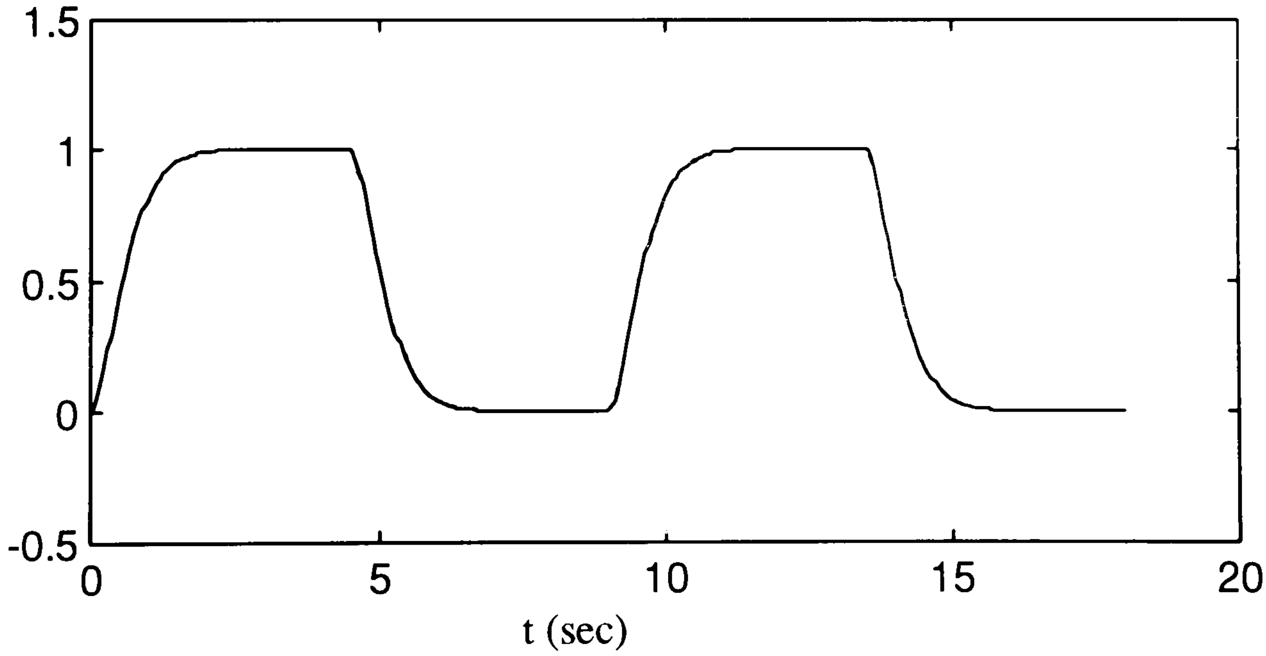


(b) The plots of the output of the filter (--), v and the output (-), y

Figure 4.14: The control of the forced Van der Pol equation by the non-adaptive fuzzy NIMC strategy when the fuzzy model is an approximation of the model inverse of the plant

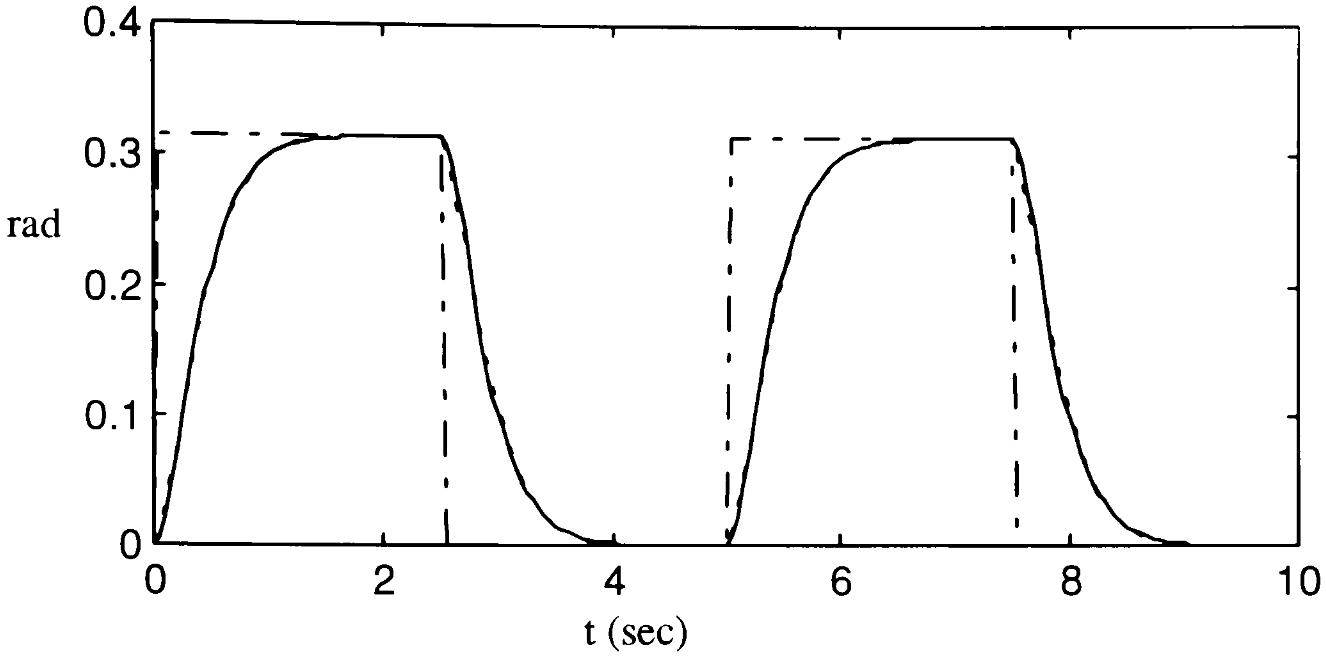


(a) The plots of reference signal (– ·), r, the output (-), y
and the desired response (--), y_d

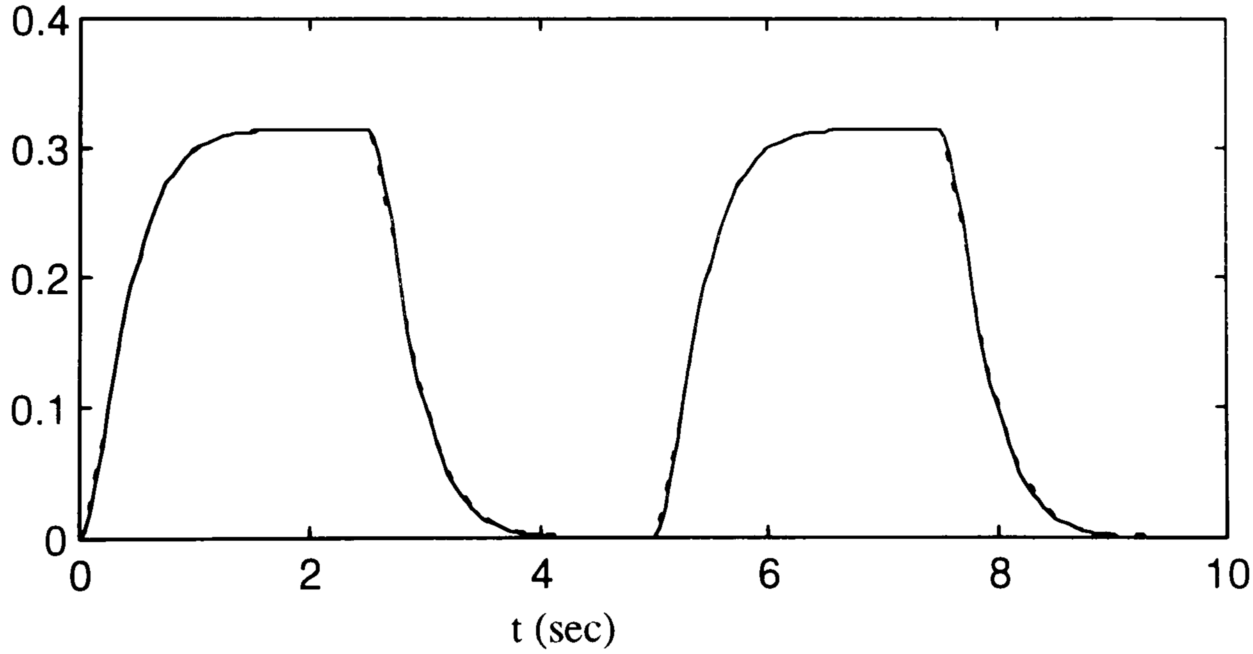


(b) The plots of the output of the filter (--), v and the output (-), y

Figure 4.15: The control of the forced Van der Pol equation by the adaptive fuzzy NIMC strategy when the fuzzy model is an approximation of the model inverse of the plant

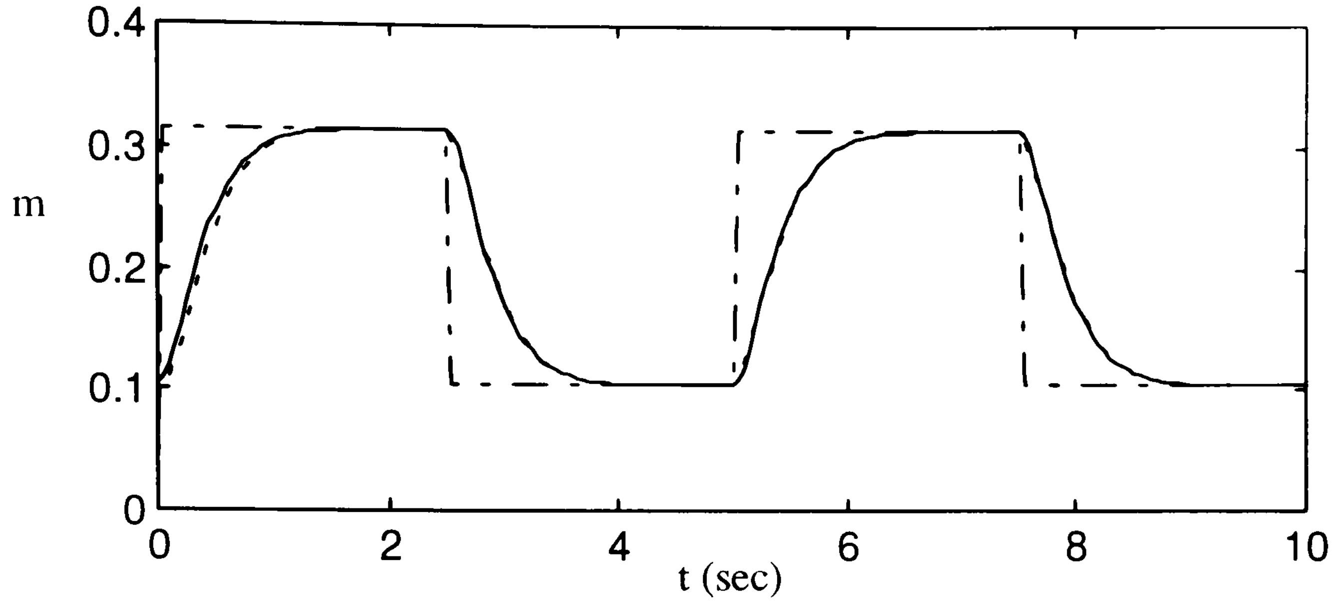


(a) The plots of reference signal (– ·), ref1, the first output (-), θ and the desired response (--), y_d1

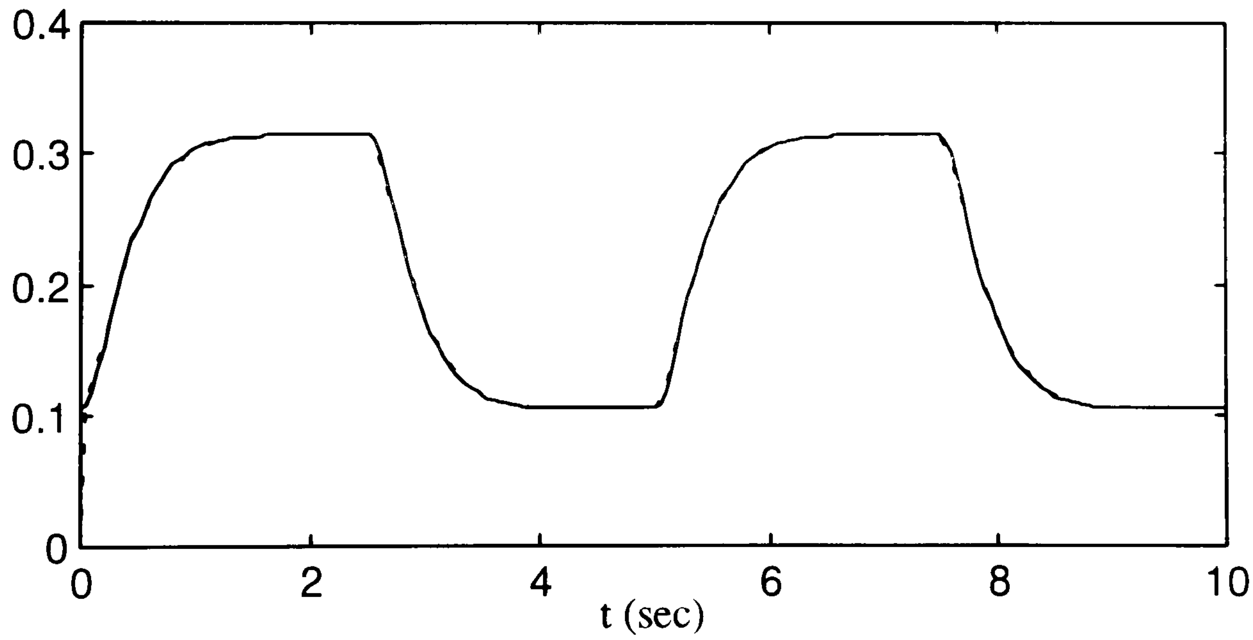


(b) The plots of the output of the filter (--), v1 and the first output (-), θ

Figure 4.16: The control of the two-link cylindrical robot manipulator by the non-adaptive fuzzy NIMC strategy when the fuzzy model is an approximation of the model inverse of the plant

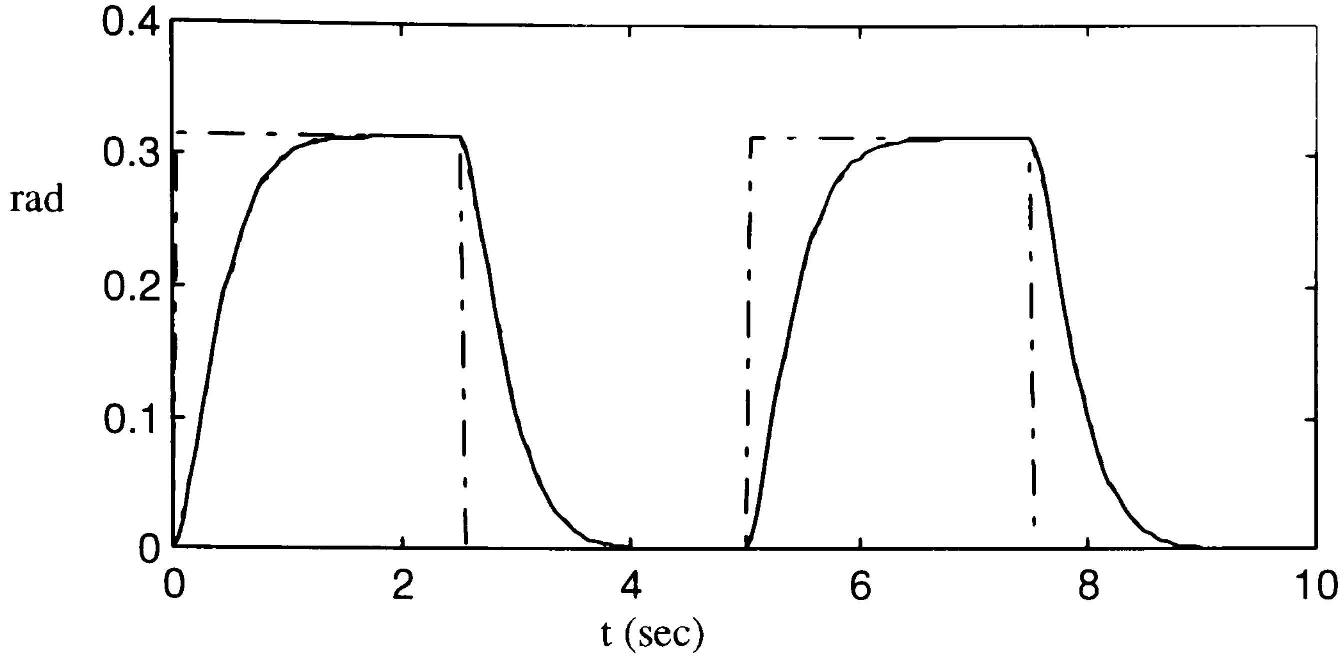


(c) The plots of reference signal (– ·), ref2, the second output (-), r
and the desired response (--), $y_d 2$

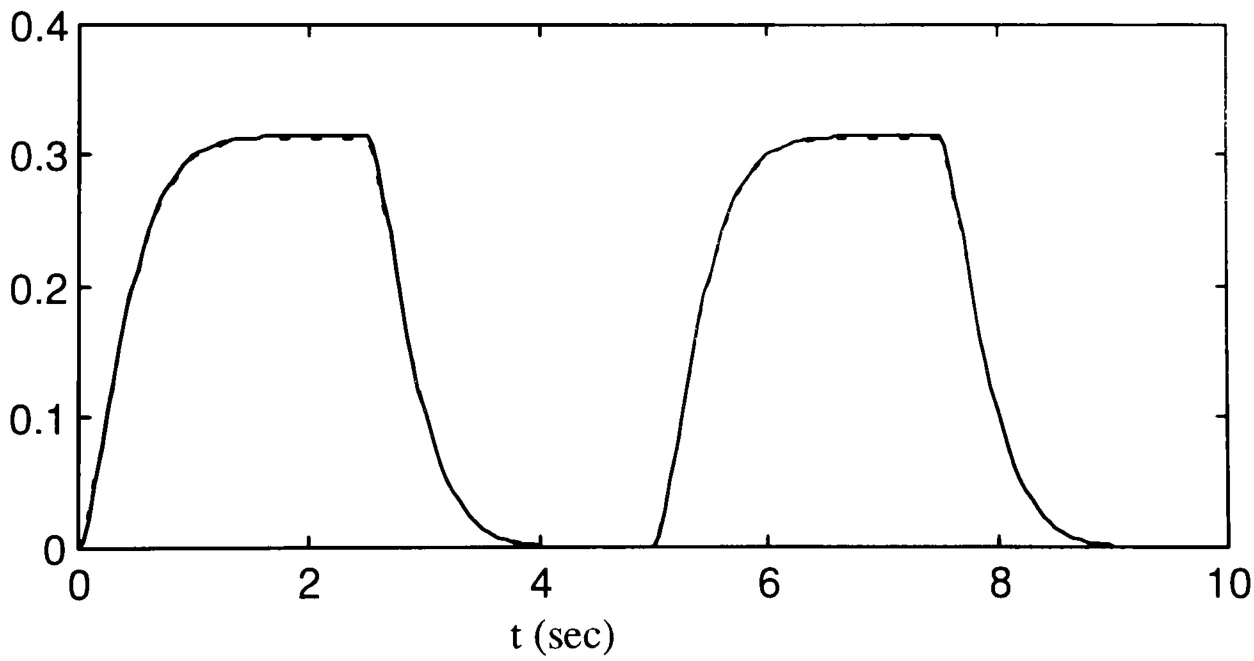


(d) The plots of the output of the filter (--), v2 and the second output (-), r

Figure 4.16: (continued)

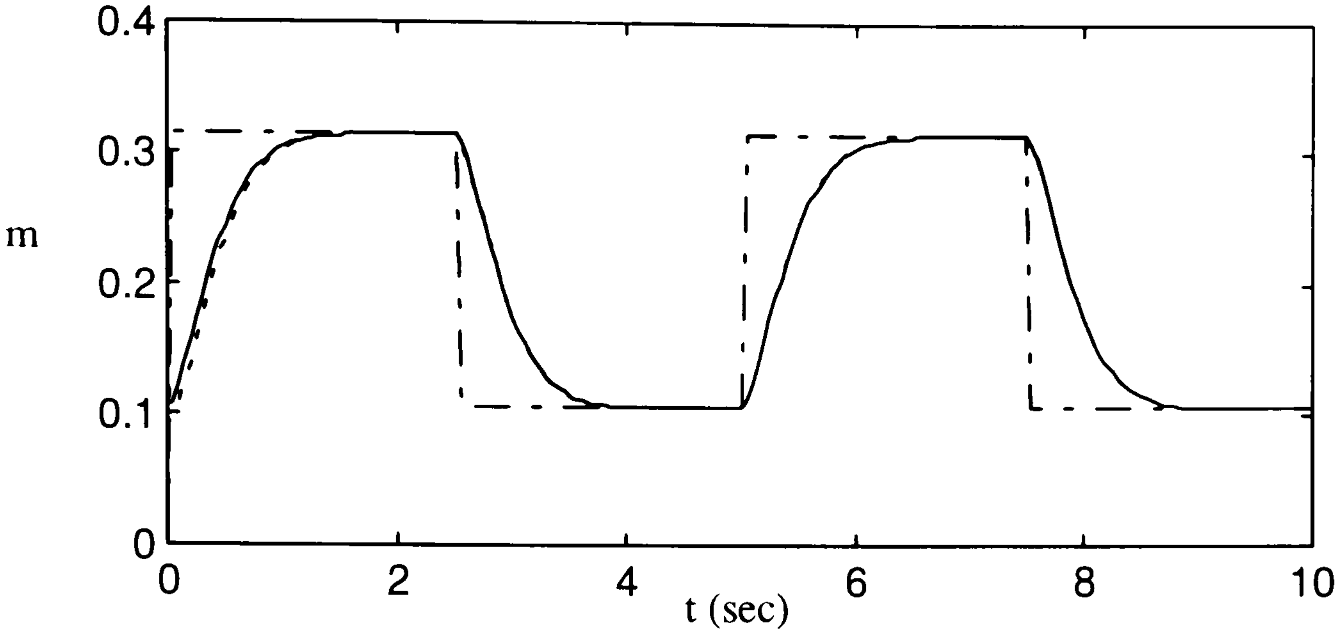


(a) The plots of reference signal (– ·), ref1, the first output (-), θ and the desired response (--), y_d1

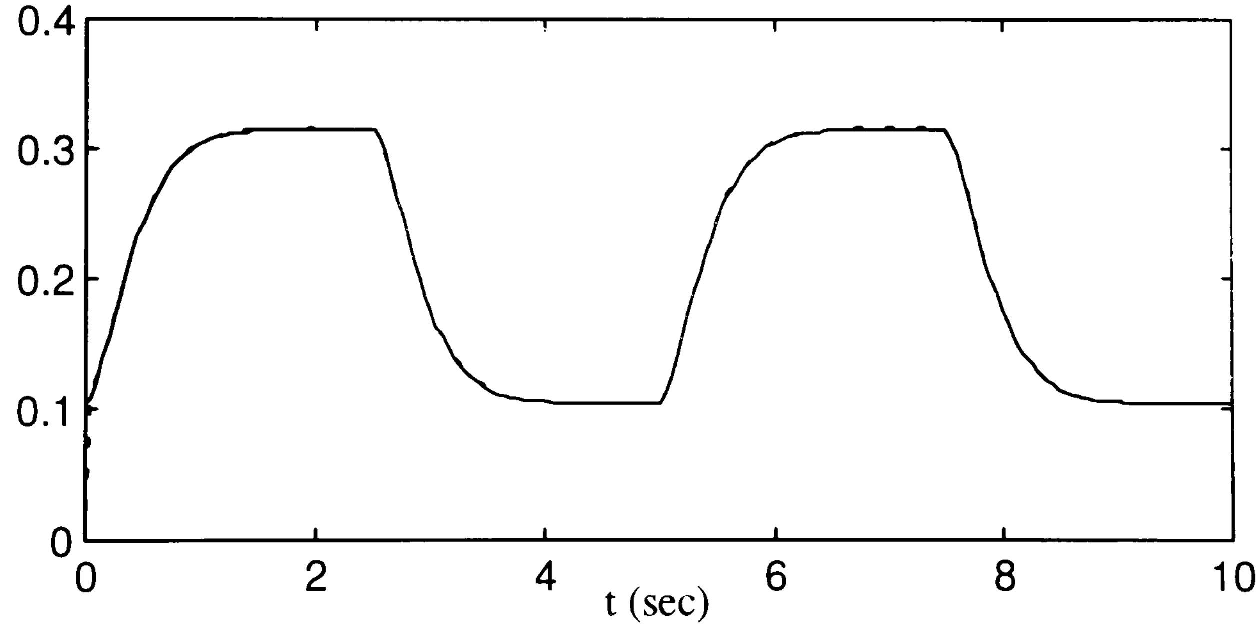


(b) The plots of the output of the filter (--), v1 and the first output (-), θ

Figure 4.17: The control of the two-link cylindrical robot manipulator by the adaptive fuzzy NIMC strategy when the fuzzy model is an approximation of the model inverse of the plant

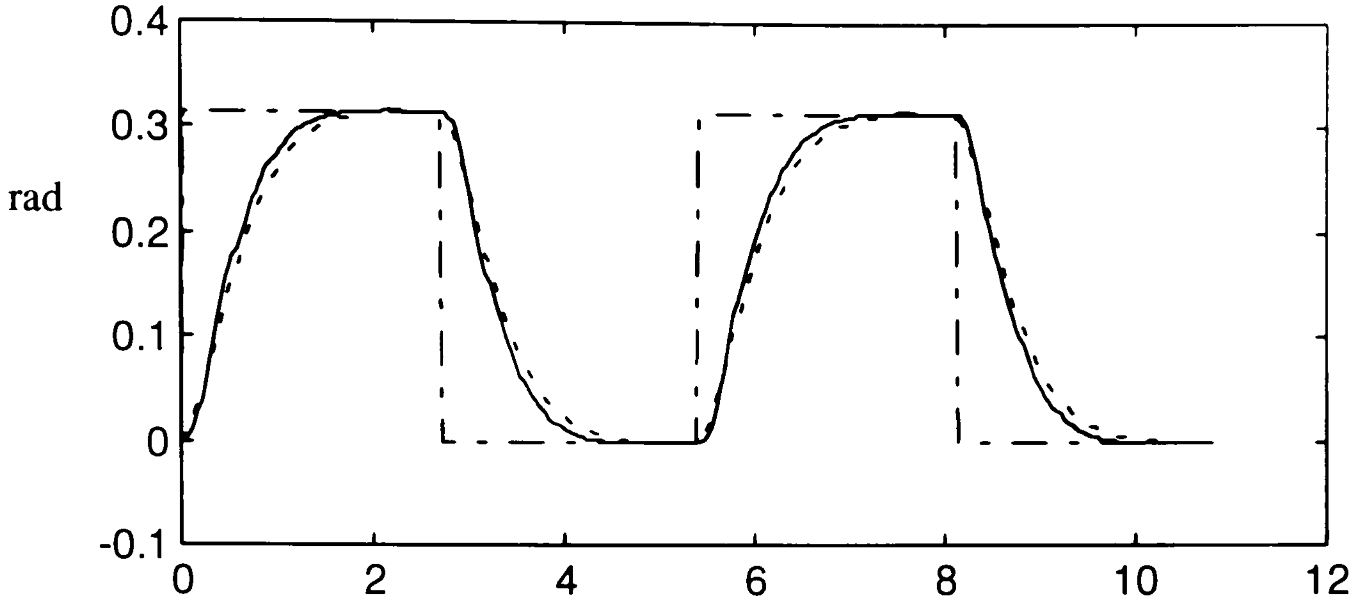


(c) The plots of reference signal (– ·), ref2, the second output (-), r and the desired response (--), y_d2

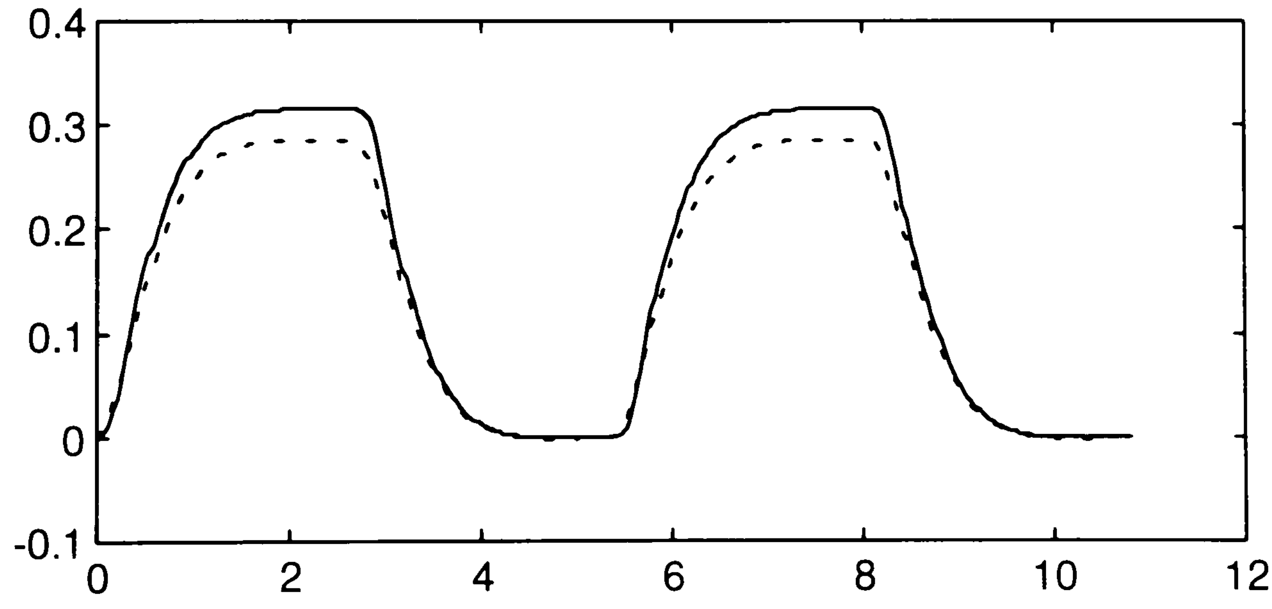


(d) The plots of the output of the filter (--), v2 and the second output (-), r

Figure 4.17: (continued)

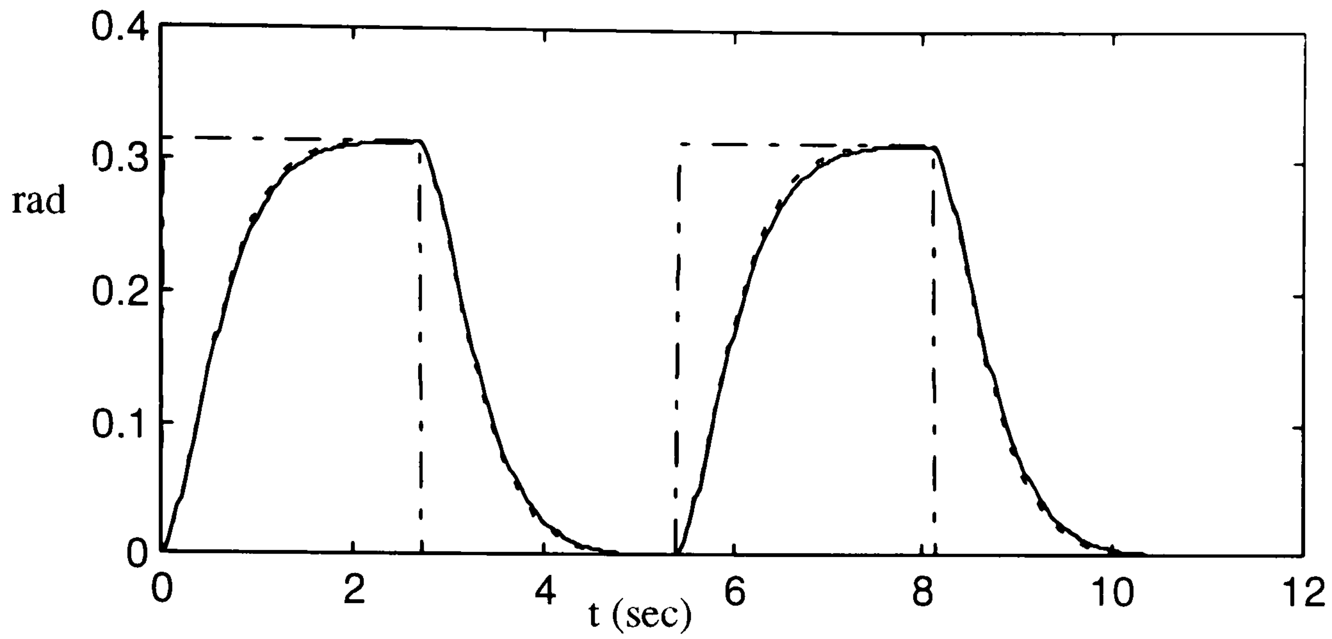


(a) The plots of reference signal (– ·), r, the output (-), θ and the desired response (--), y_d

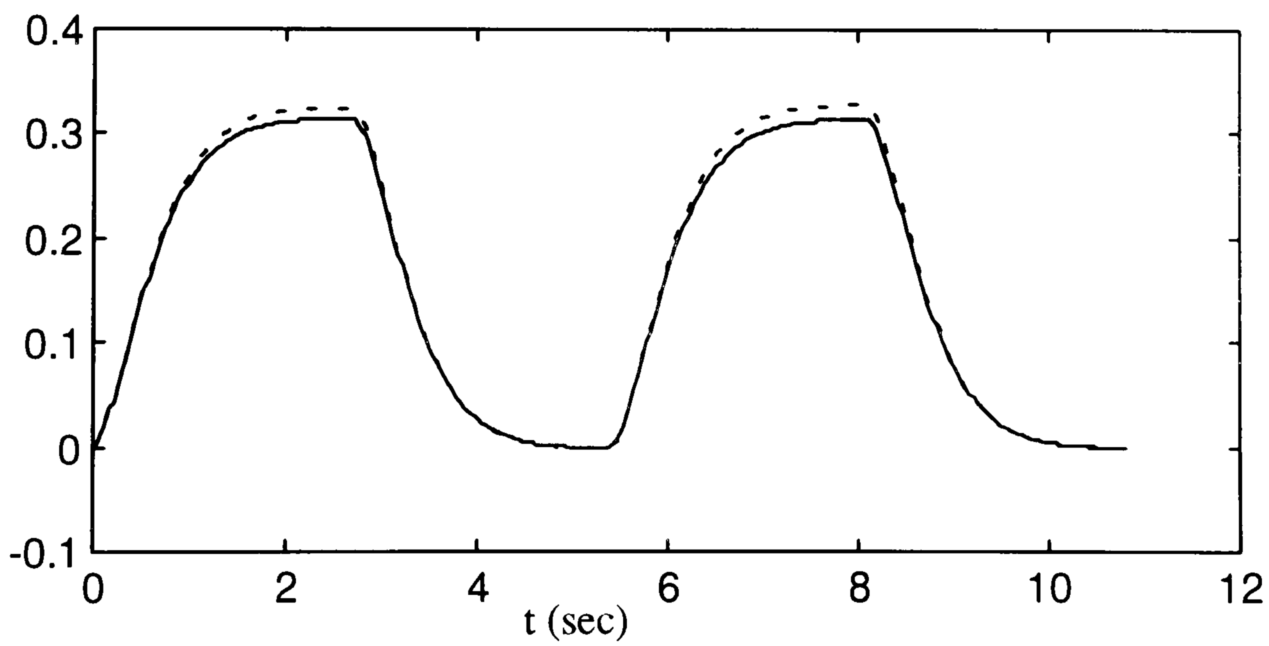


(b) The plots of the output of the filter (--), v and the output (-), θ

Figure 4.18: The control of the inverted pendulum by the non-adaptive fuzzy NIMC strategy when the fuzzy controller is not the perfect model inverse of the plant

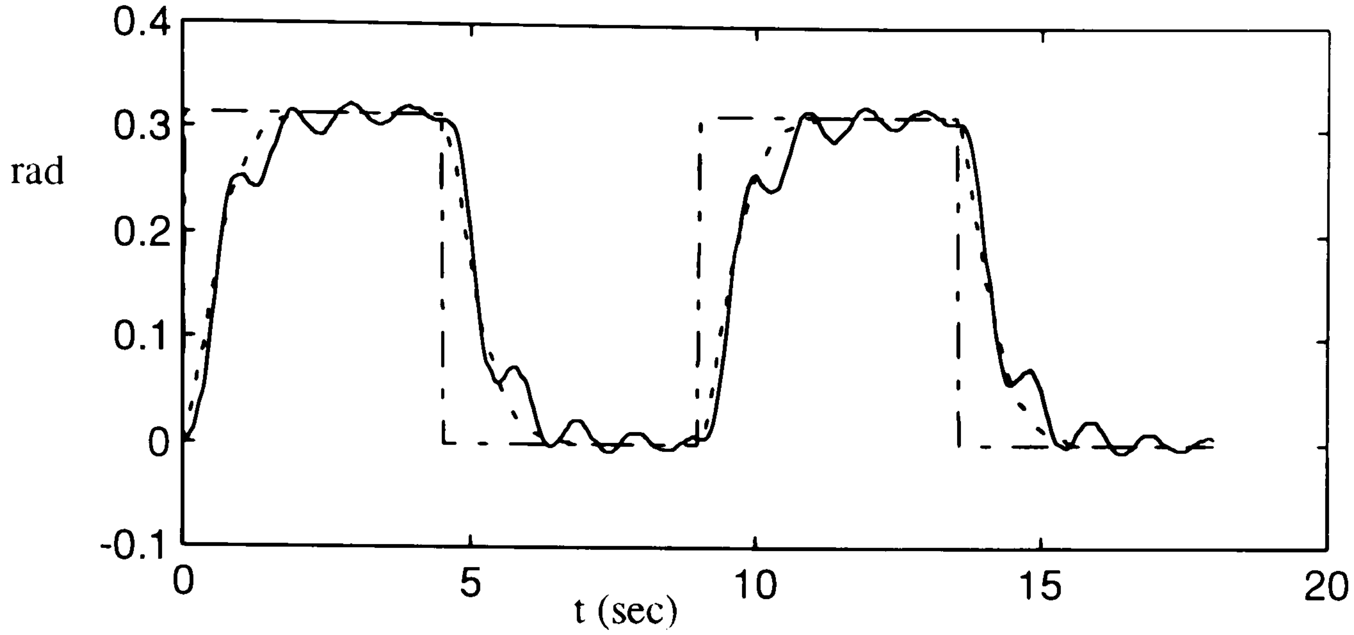


(a) The plots of reference signal (– ·), r, the output (-), θ
and the desired response (--), y_d

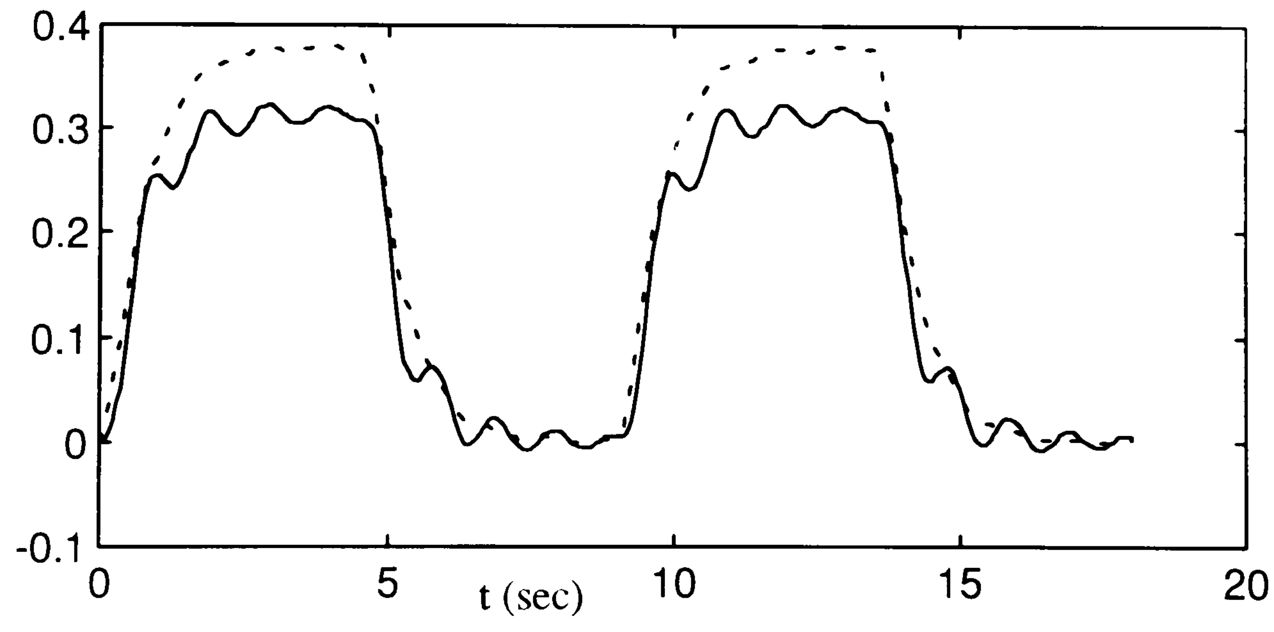


(b) The plots of the output of the filter (--), v and the output (-), θ

Figure 4.19: The control of the inverted pendulum by the adaptive fuzzy NIMC strategy when the fuzzy controller is not the perfect model inverse of the plant

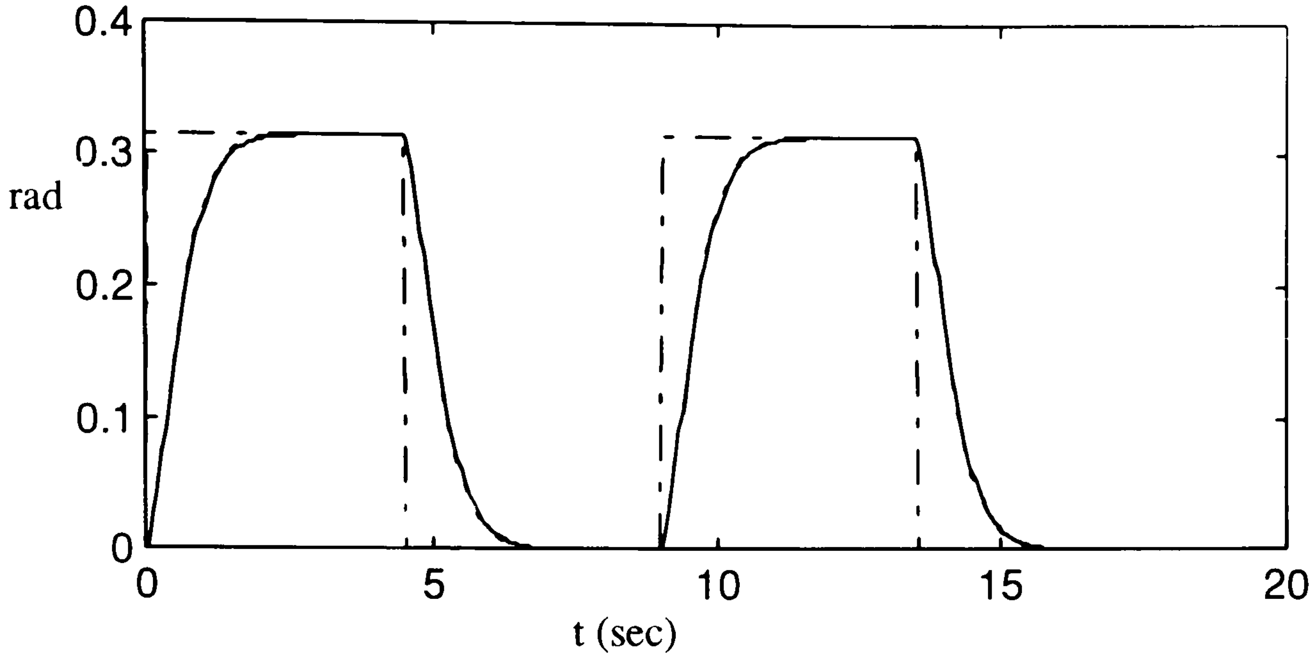


(a) The plots of reference signal (– ·), r, the output (-), θ
and the desired response (--), y_d

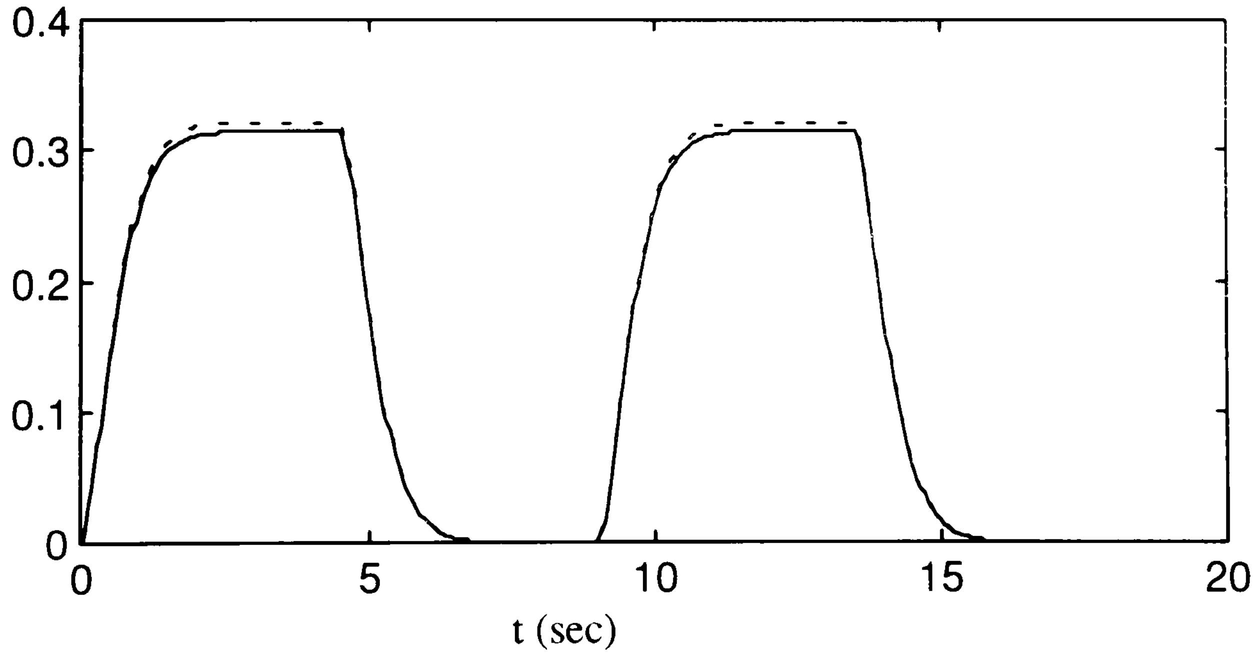


(b) The plots of the output of the filter (--), v and the output (-), θ

Figure 4.20: The control of the pendulum by the non-adaptive fuzzy NIMC strategy when the fuzzy controller is not the perfect model inverse of the plant

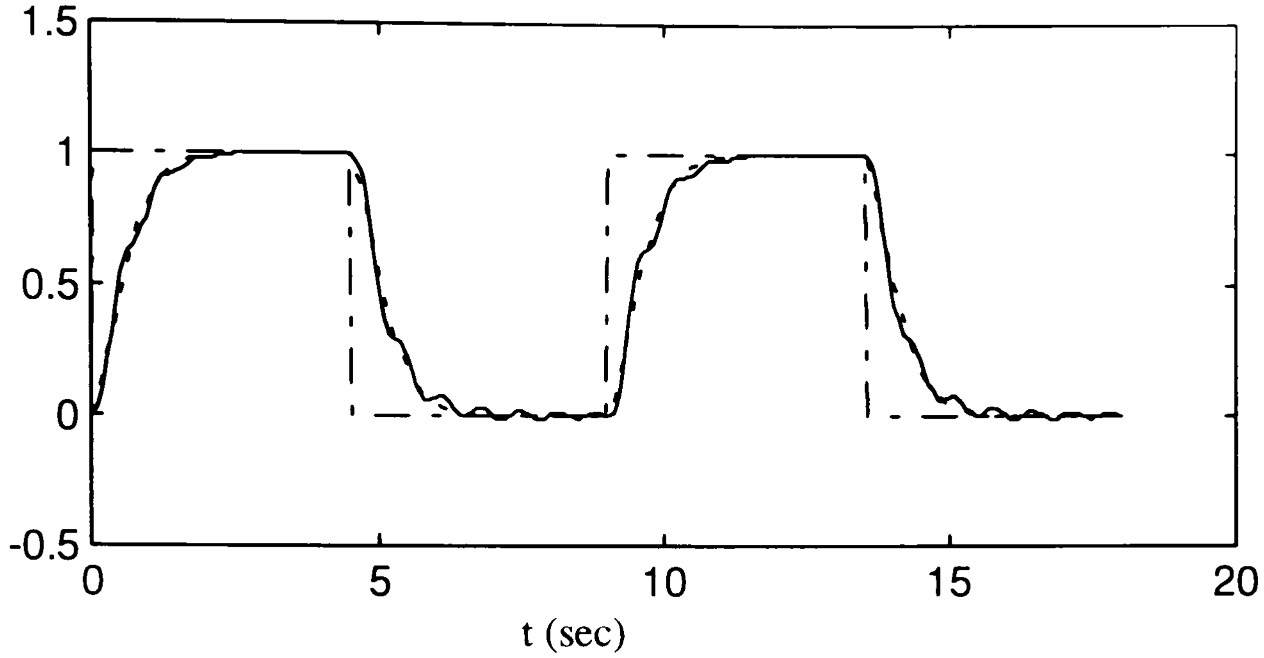


(a) The plots of reference signal (– ·), r, the output (-), θ and the desired response (--), y_d

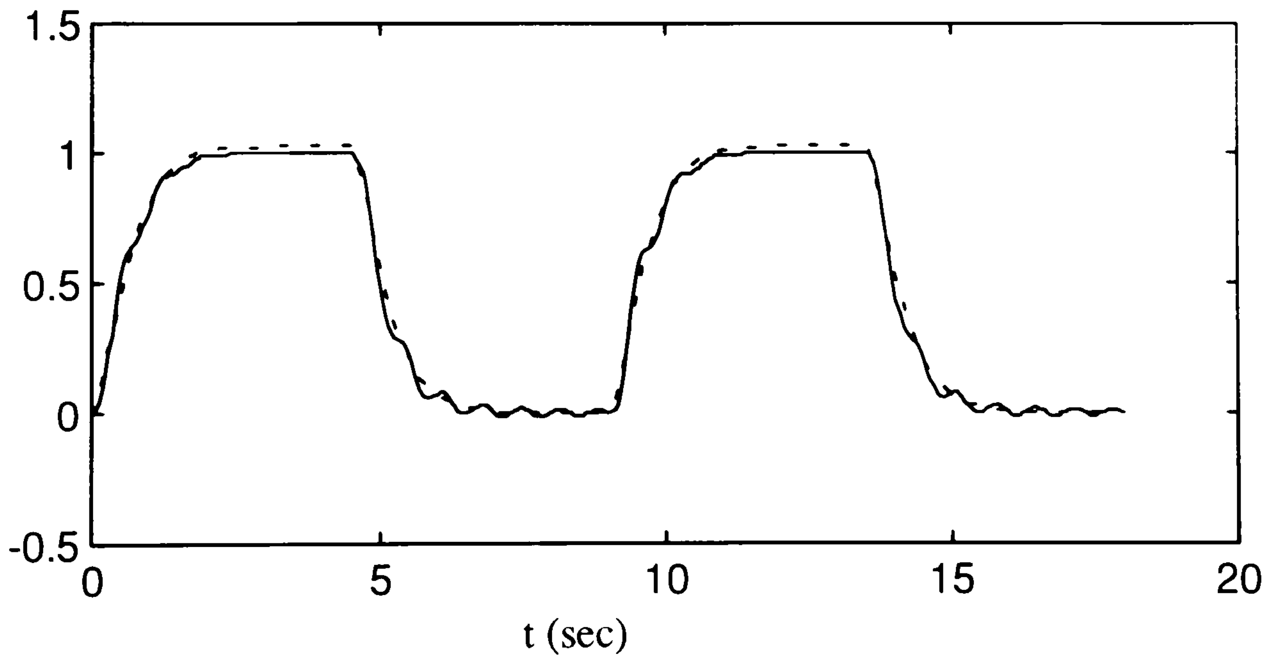


(b) The plots of the output of the filter (--), v and the output (-), θ

Figure 4.21: The control of the pendulum by the adaptive fuzzy NIMC strategy when the fuzzy controller is not the perfect model inverse of the plant

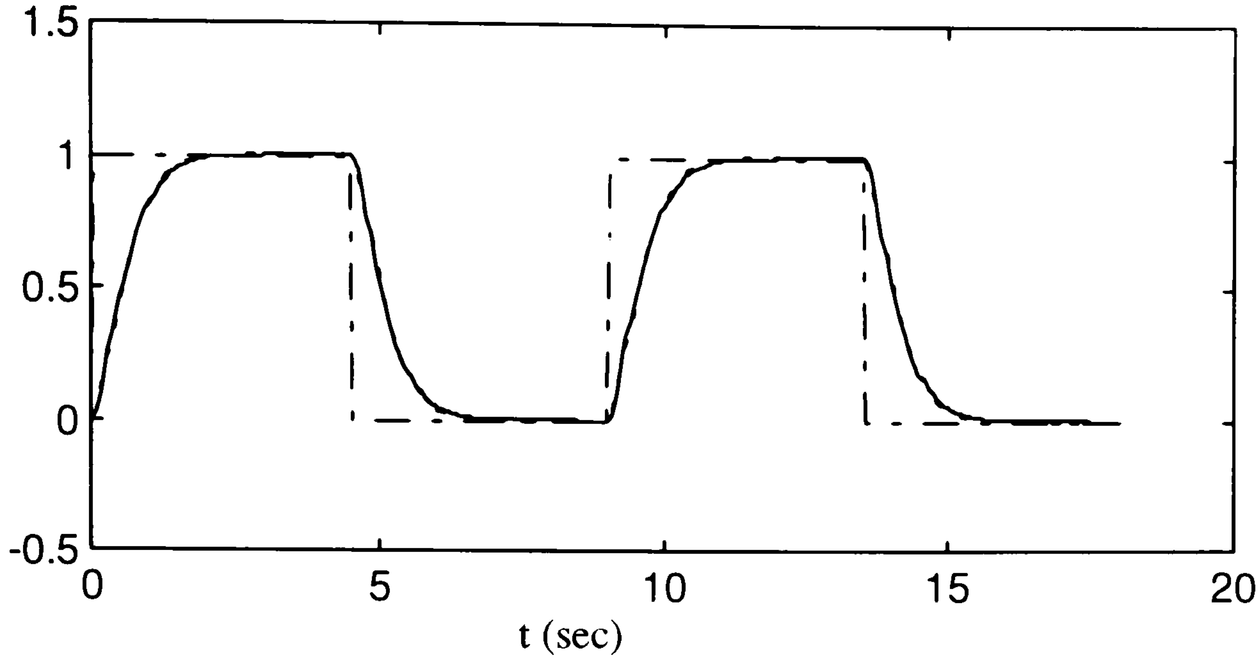


(a) The plots of reference signal (– ·), r, the output (-), y and the desired response (--), y_d

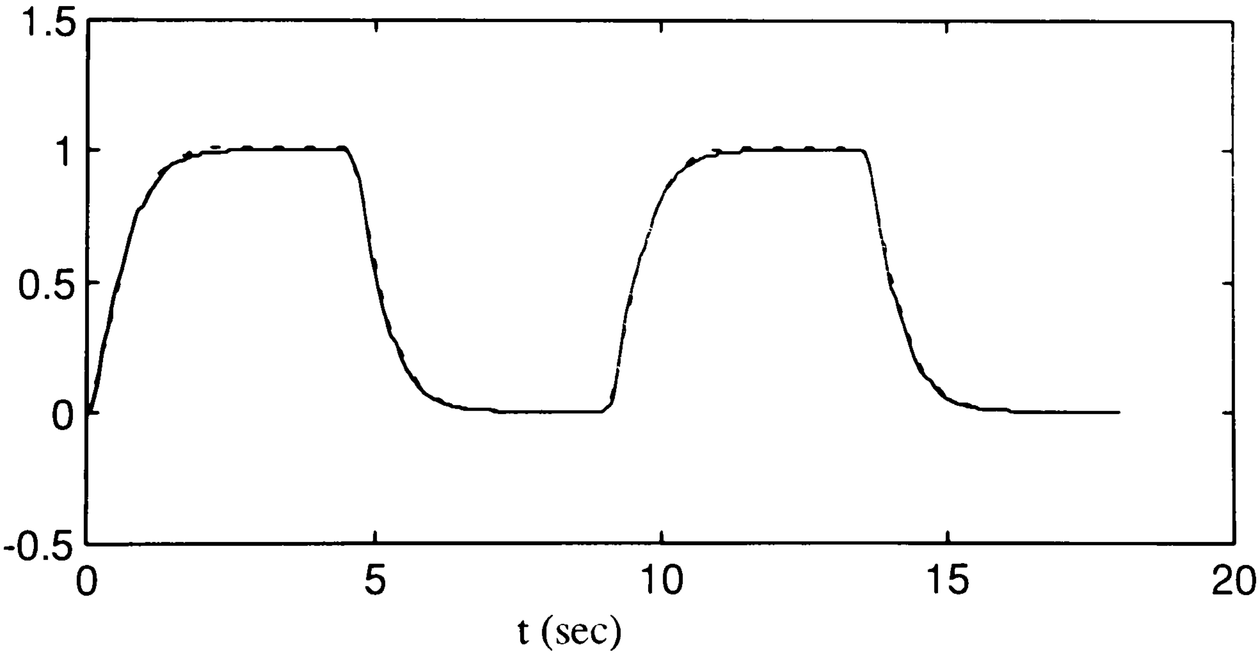


(b) The plots of the output of the filter (--), v and the output (-), y

Figure 4.22: The control of the forced Van der Pol equation by the non-adaptive fuzzy NIMC strategy when the fuzzy controller is not the perfect model inverse of the plant

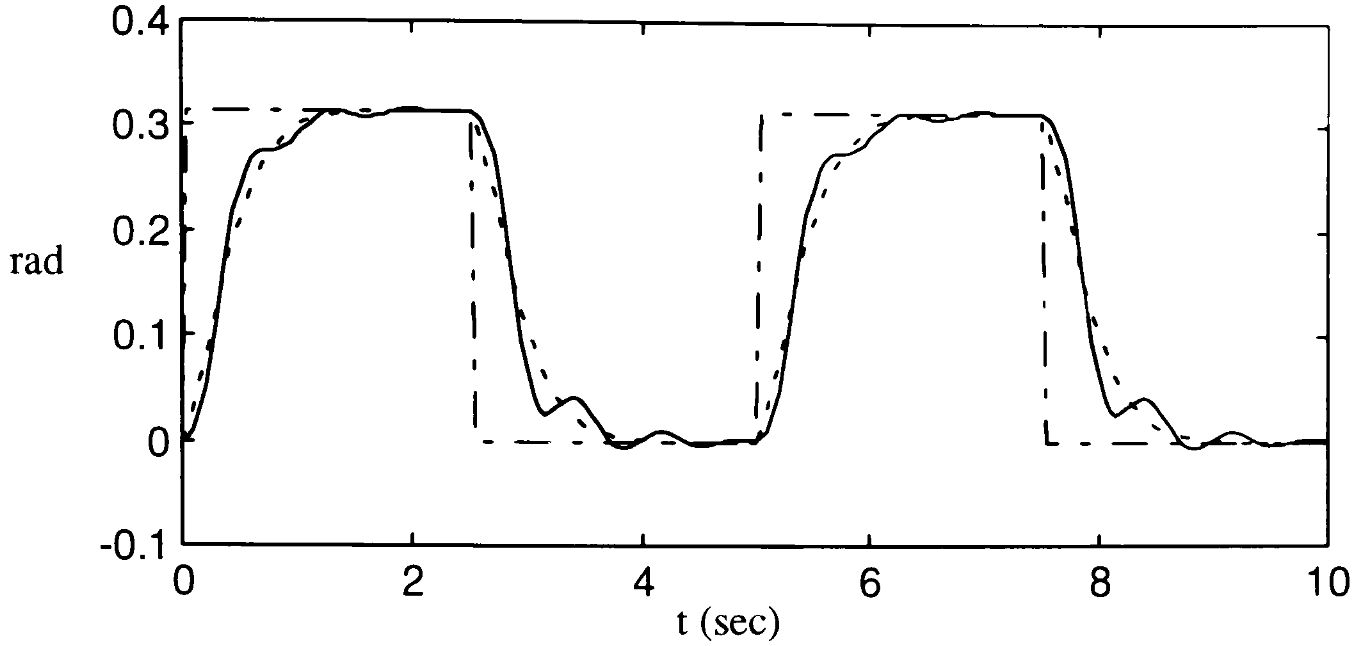


(a) The plots of reference signal (— ·), r, the output (-), y
and the desired response (--), y_d

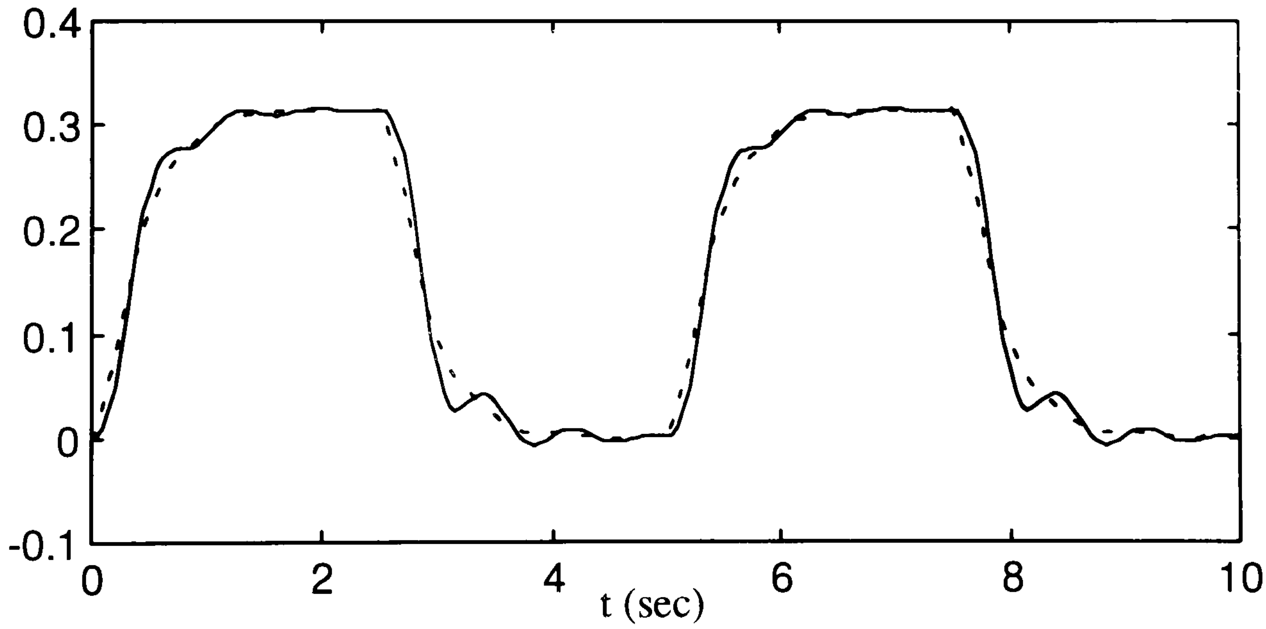


(b) The plots of the output of the filter (--), v and the output (-), y

Figure 4.23: The control of the forced Van der Pol equation by the adaptive fuzzy NIMC strategy when the fuzzy controller is not the perfect model inverse of the plant

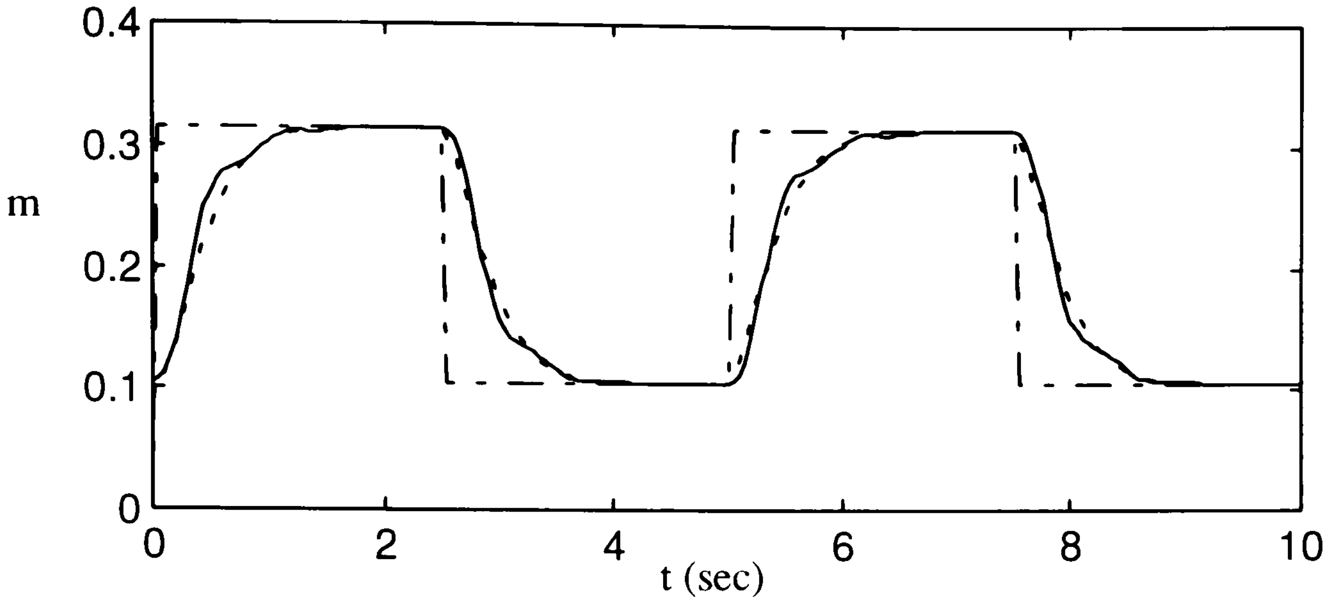


(a) The plots of reference signal (– ·), ref1, the first output (-), θ and the desired response (--), y_d1

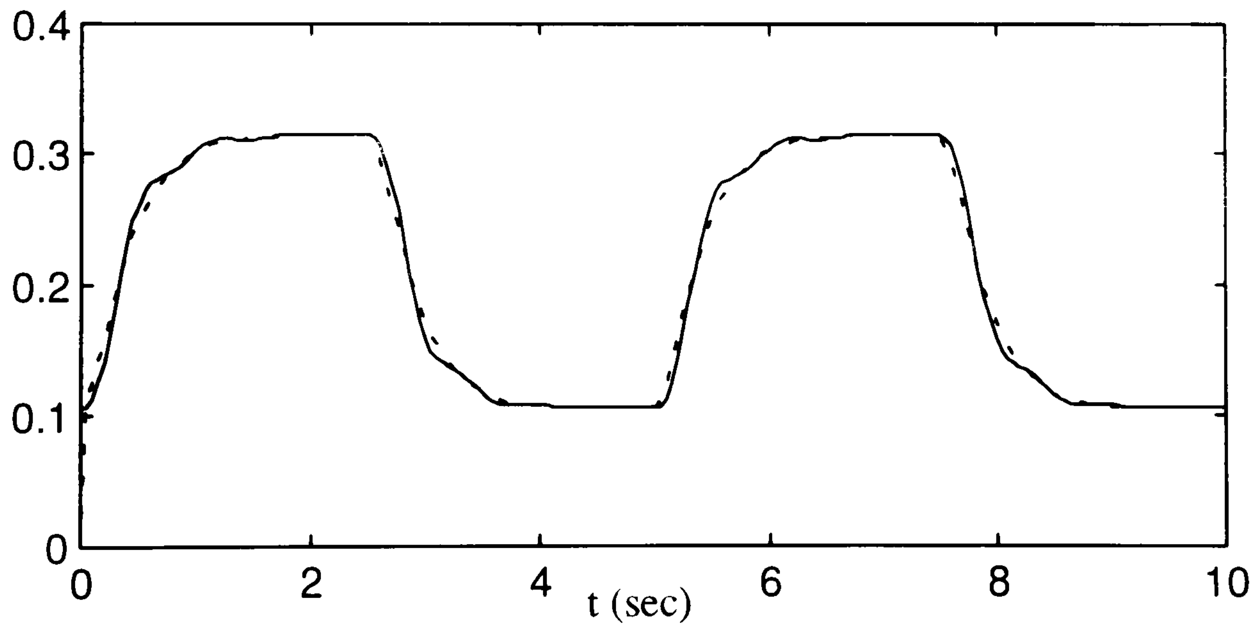


(b) The plots of the output of the filter (--), v1 and the first output (-), θ

Figure 4.24: The control of the two-link cylindrical robot manipulator by the non-adaptive fuzzy NIMC strategy when the fuzzy controller is not the perfect model inverse of the plant

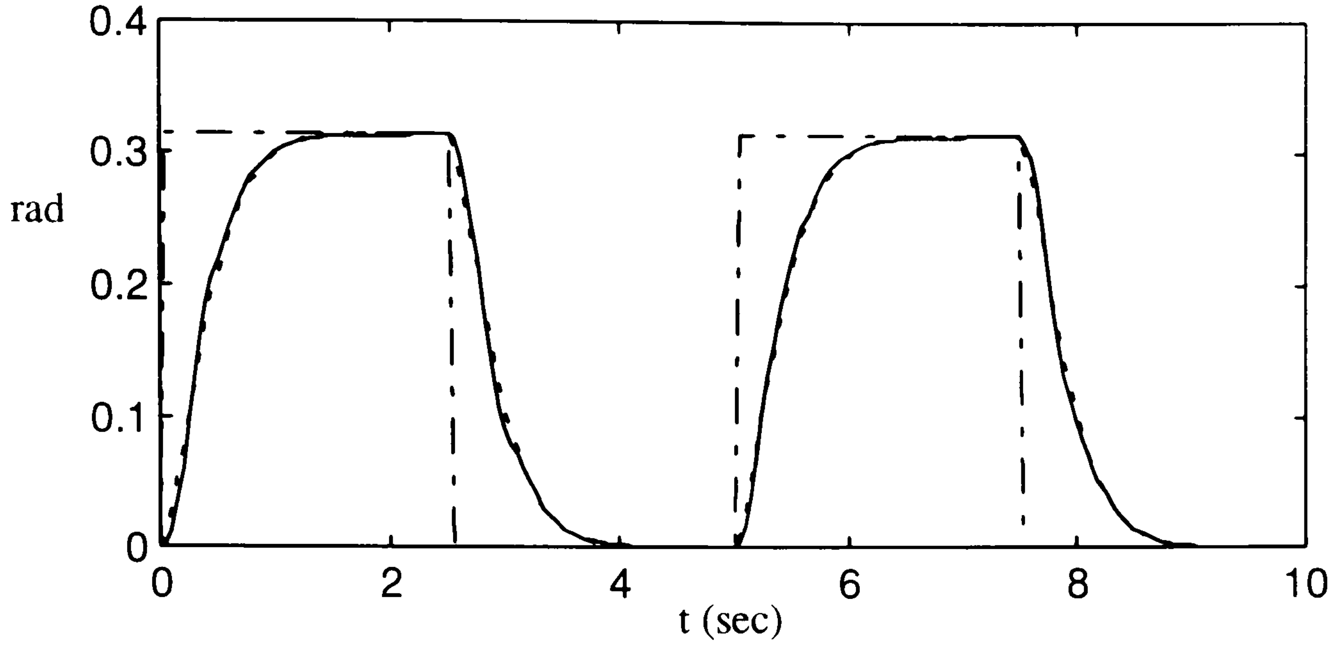


(c) The plots of reference signal (– ·), ref2, the second output (-), r and the desired response (--), y_d2

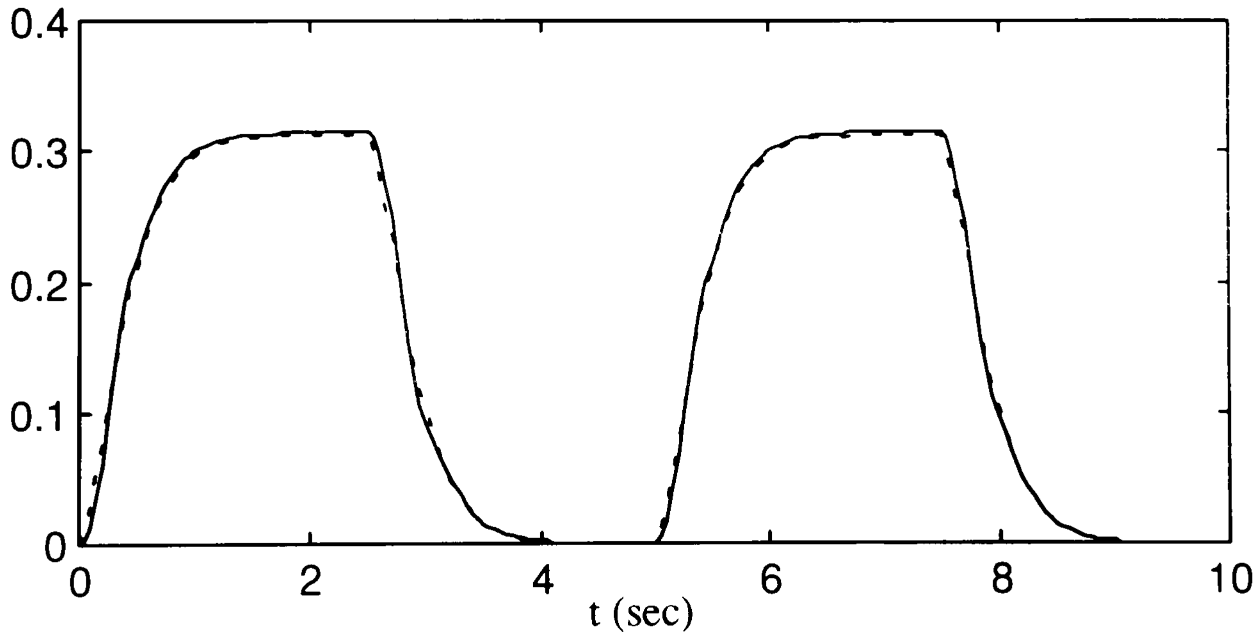


(d) The plots of the output of the filter (--), v2 and the second output (-), r

Figure 4.24: (continued)

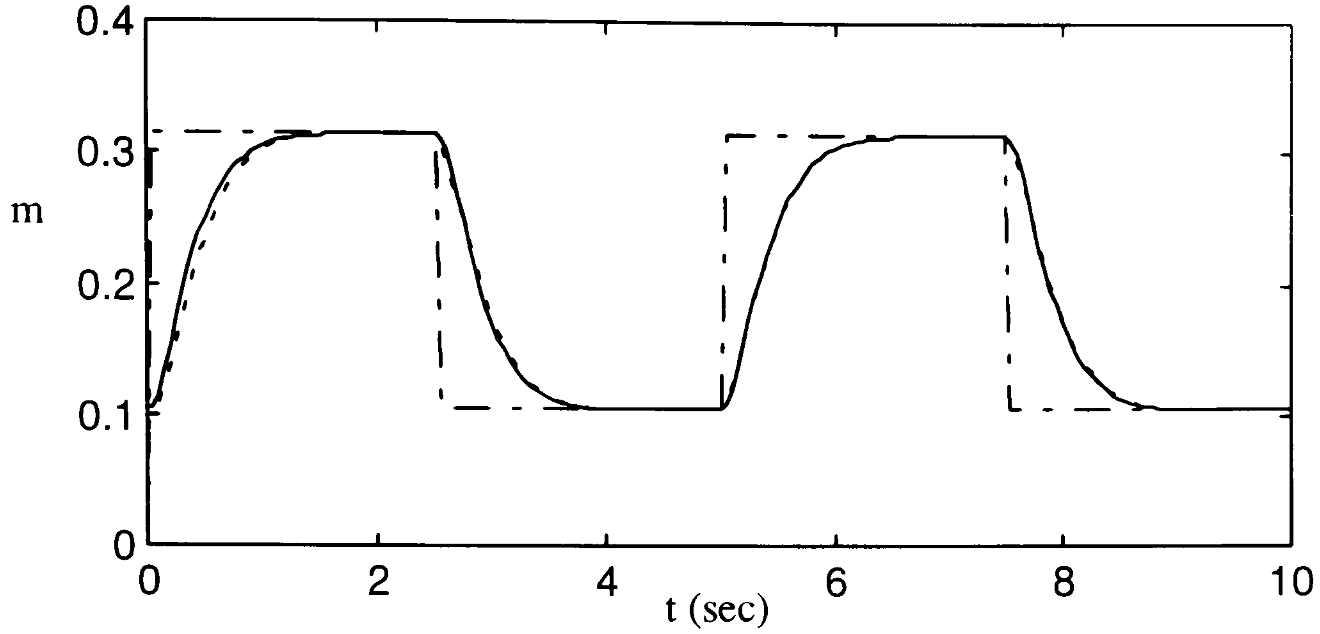


(a) The plots of reference signal (– ·), ref1, the first output (-), θ and the desired response (--), y_d1

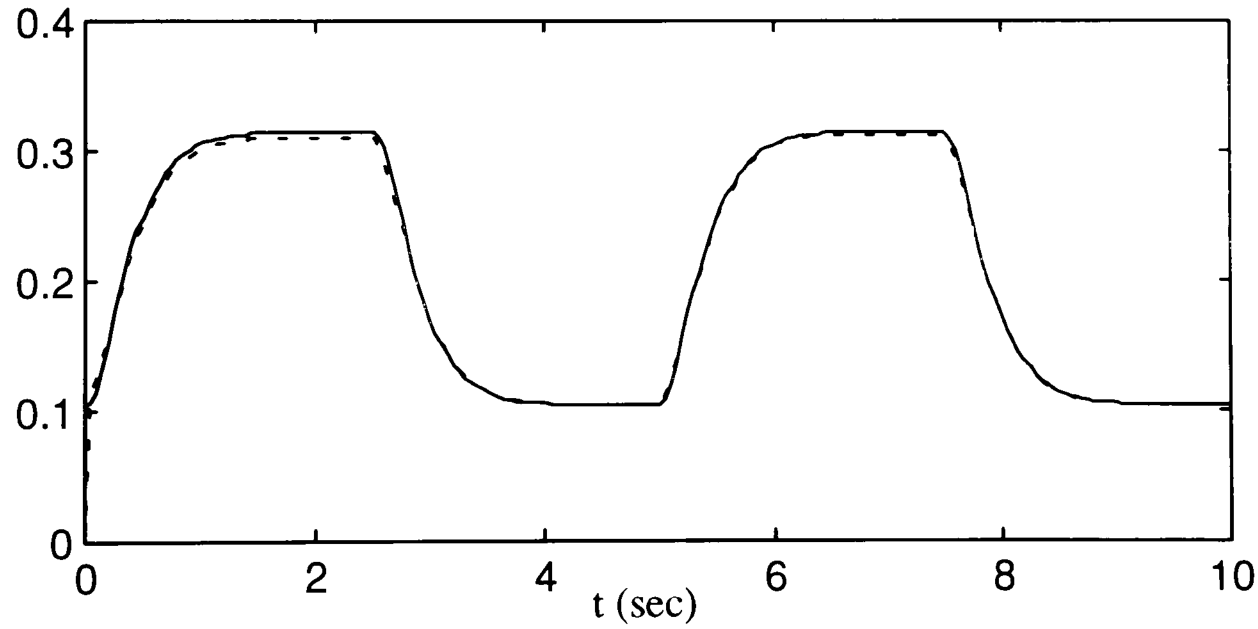


(b) The plots of the output of the filter (--), v1 and the first output (-), θ

Figure 4.25: The control of the two-link cylindrical robot manipulator by the adaptive fuzzy NIMC strategy when the fuzzy controller is not the perfect model inverse of the plant

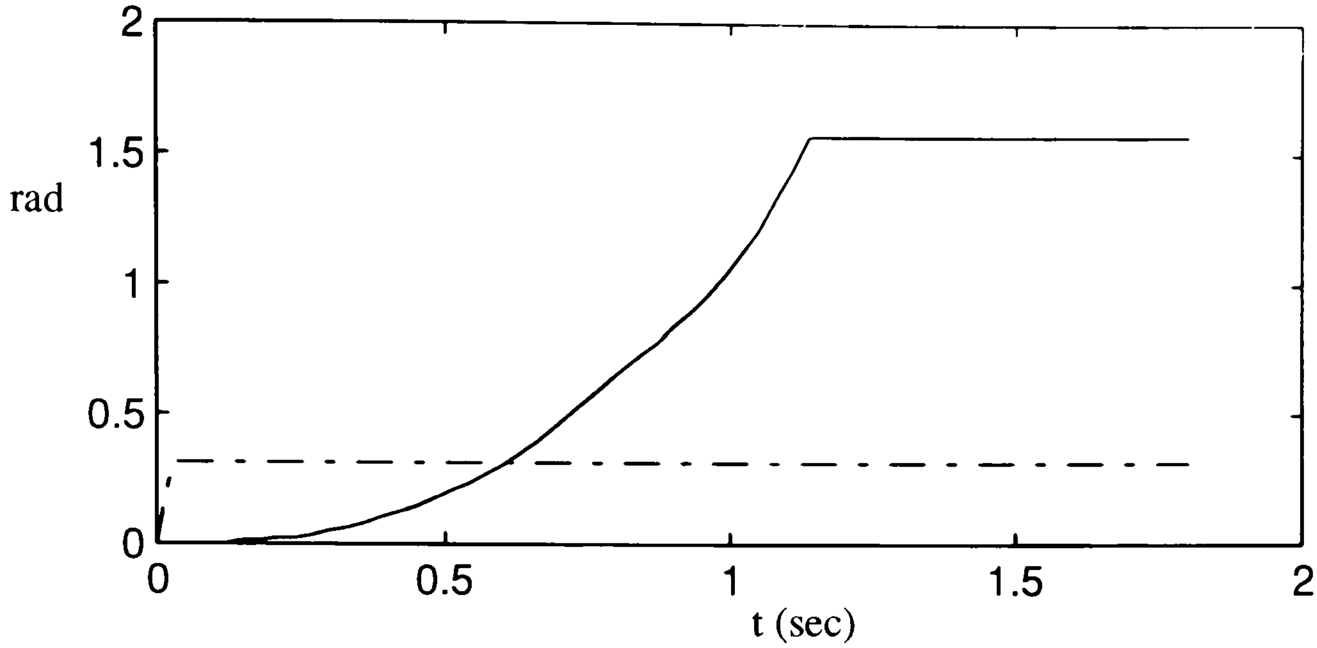


(c) The plots of reference signal (– ·), ref2, the second output (-), r and the desired response (--), y_d2

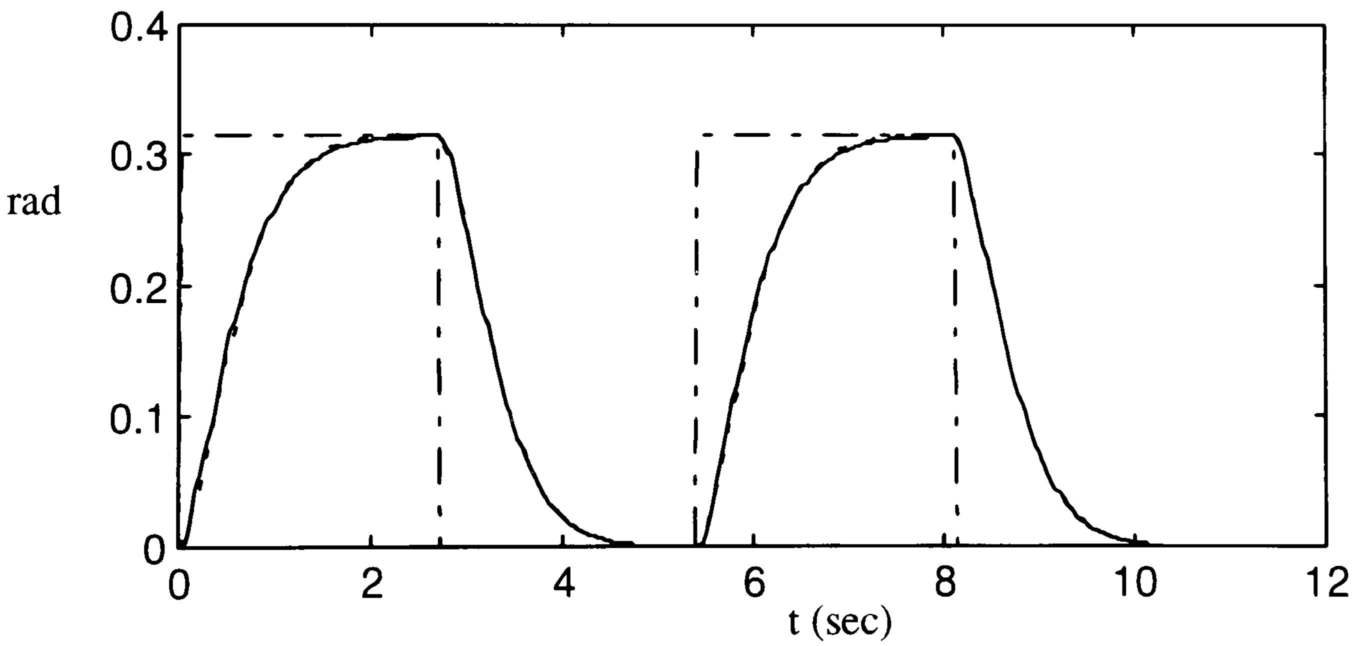


(d) The plots of the output of the filter (--), v2 and the second output (-), r

Figure 4.25: (continued)

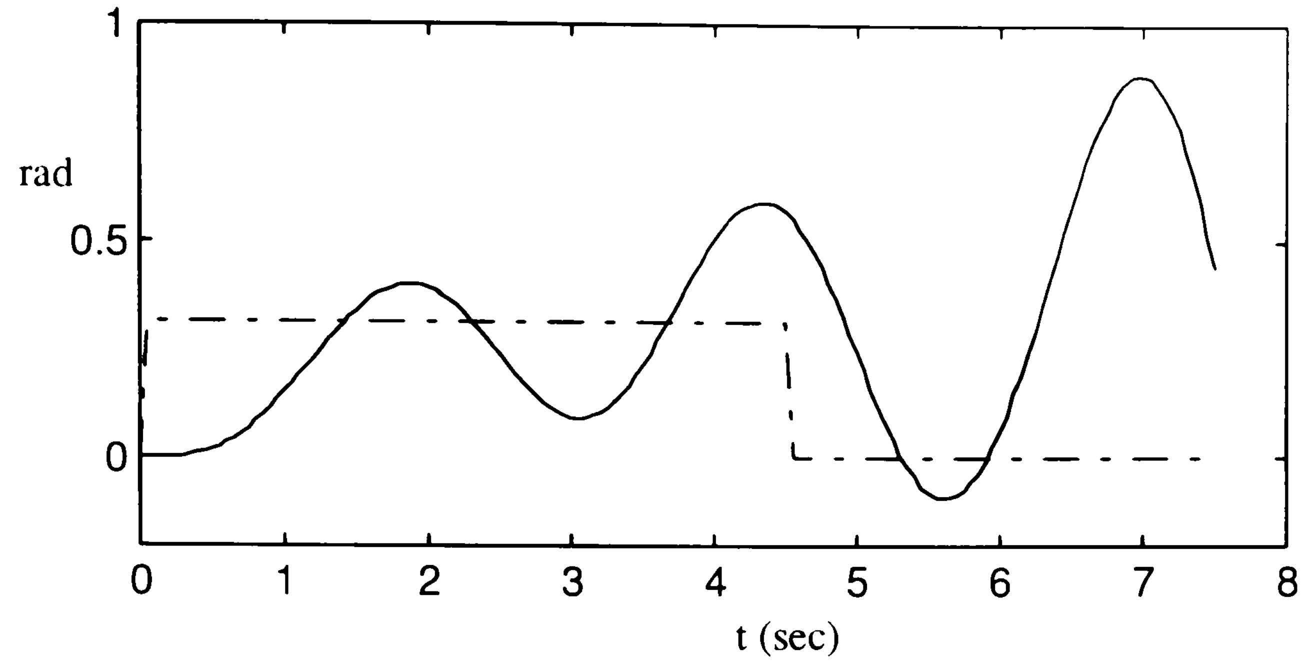


(a) The plots of reference signal (– ·), r, the output (-), θ , when the control strategy is the non-adaptive fuzzy NIMC strategy

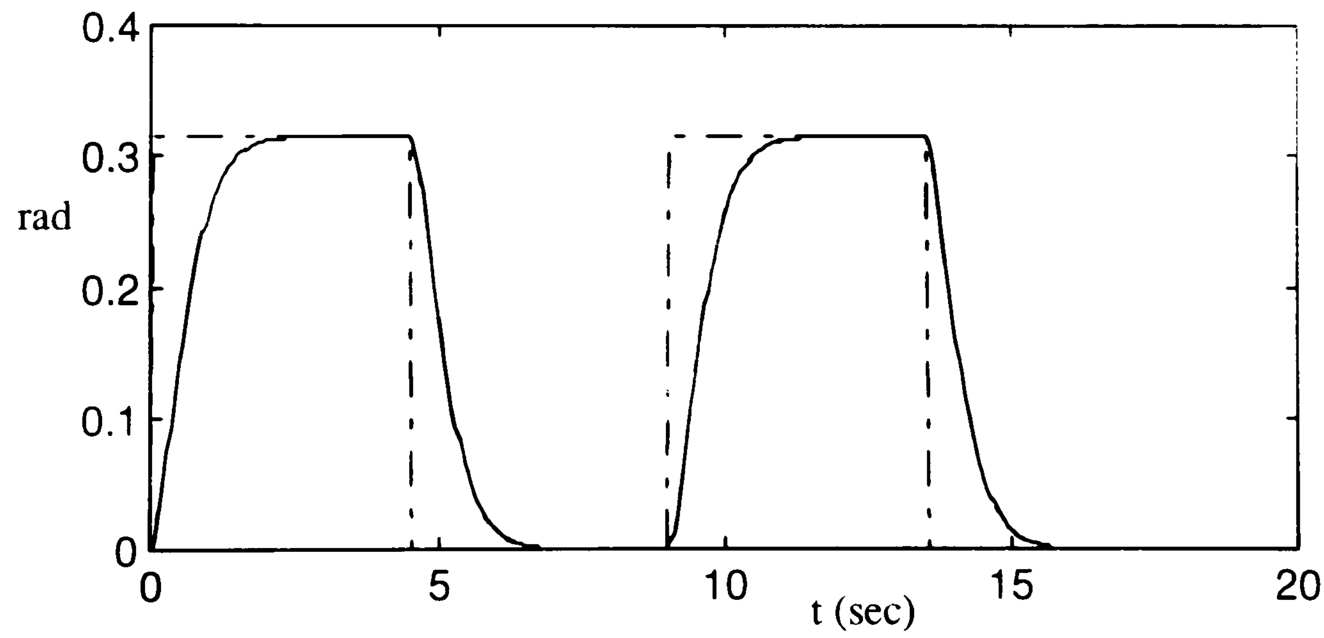


(b) The plots of reference signal (– ·), r, the desired response (--), y_d and the output (-), θ , when the control strategy is the adaptive fuzzy NIMC strategy

Figure 4.26: The control of the inverted pendulum when the modeling mismatch of the fuzzy controller results in the instability

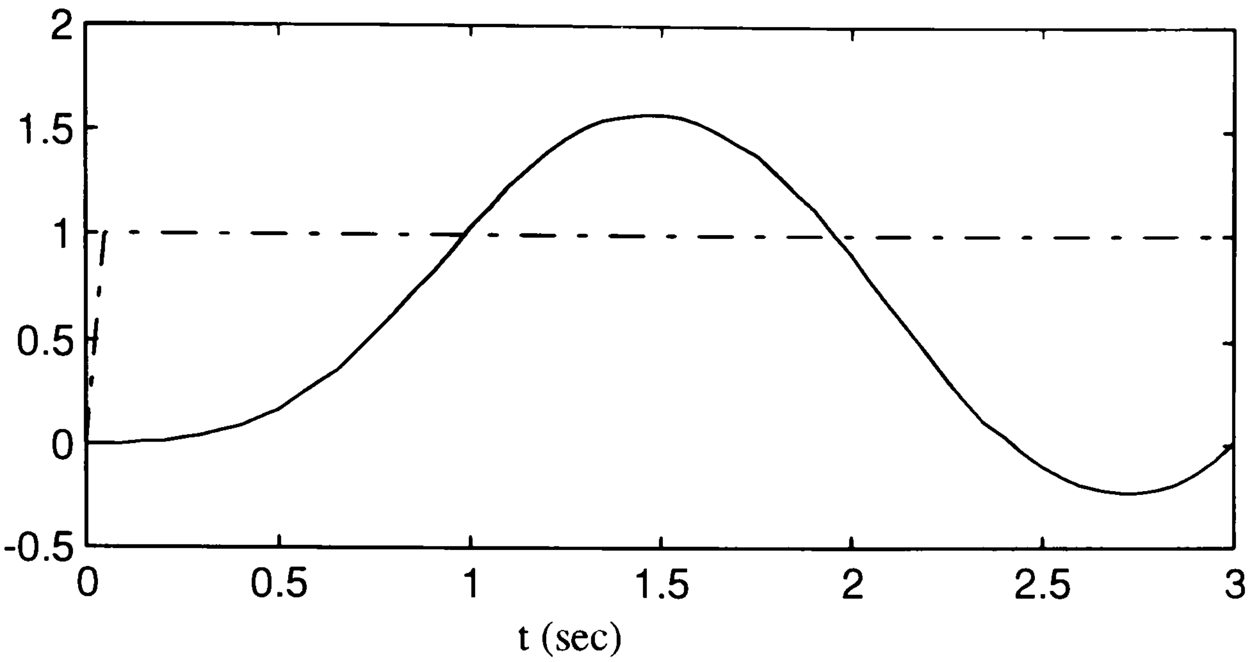


(a) The plots of reference signal ($-\ \cdot$), r, the output (-), θ, when the control strategy is the non-adaptive fuzzy NIMC strategy

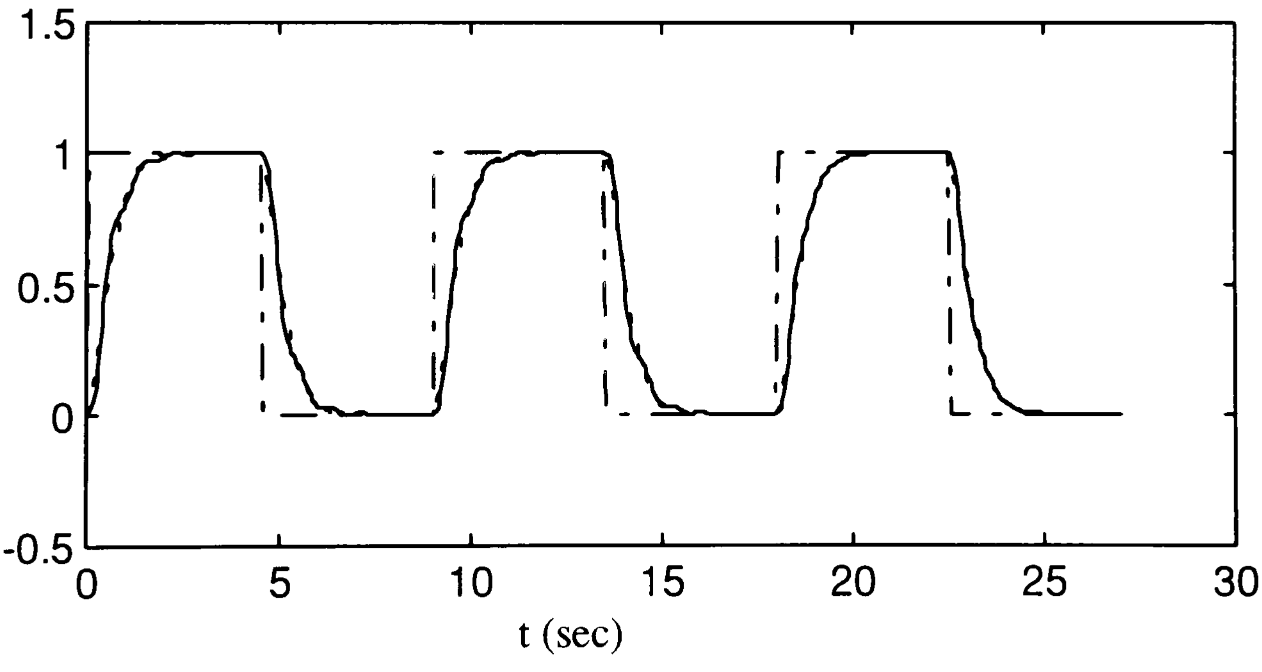


(b) The plots of reference signal ($-\ \cdot$), r, the desired response (--), y_d and the output (-), θ, when the control strategy is the adaptive fuzzy NIMC strategy

Figure 4.27: The control of the pendulum when the modeling mismatch of the fuzzy controller results in the instability

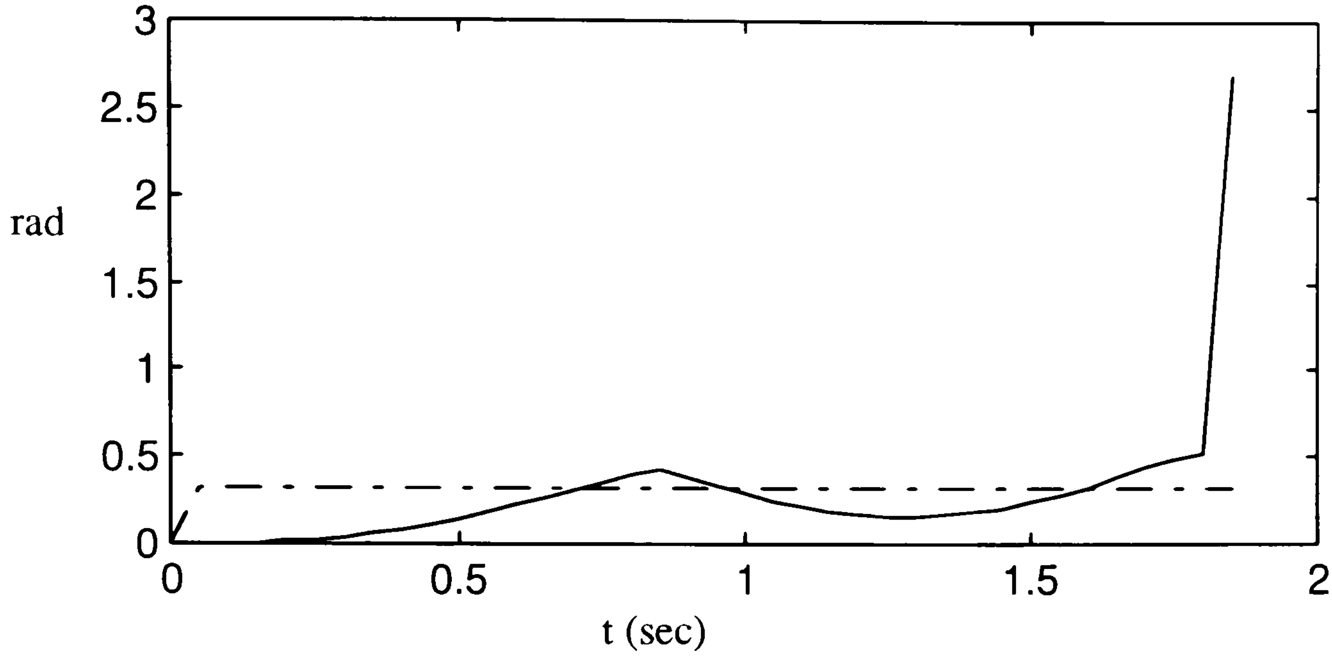


(a) The plots of reference signal ($-\ \cdot$), r, the output (-), y, when the control strategy is the non-adaptive fuzzy NIMC strategy

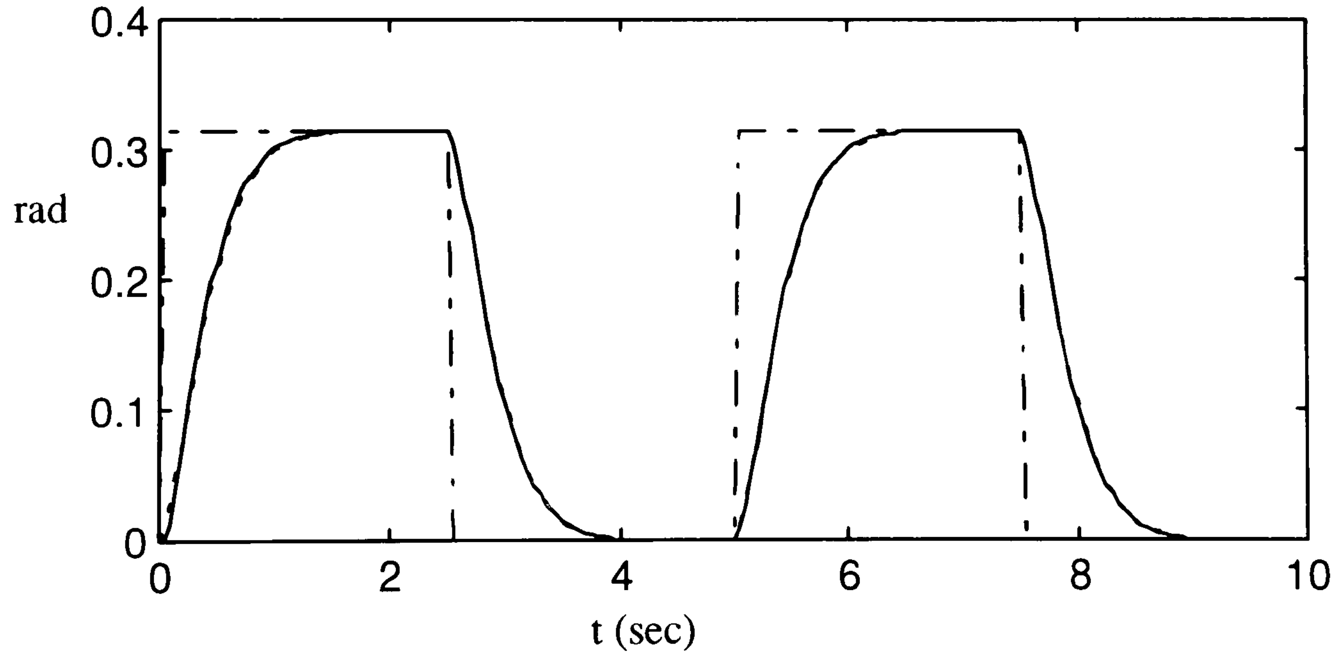


(b) The plots of reference signal ($-\ \cdot$), r, the desired response (--), y_d and the output (-), y, when the control strategy is the adaptive fuzzy NIMC strategy

Figure 4.28: The control of the forced Van der Pol equation when the modeling mismatch of the fuzzy controller results in the instability

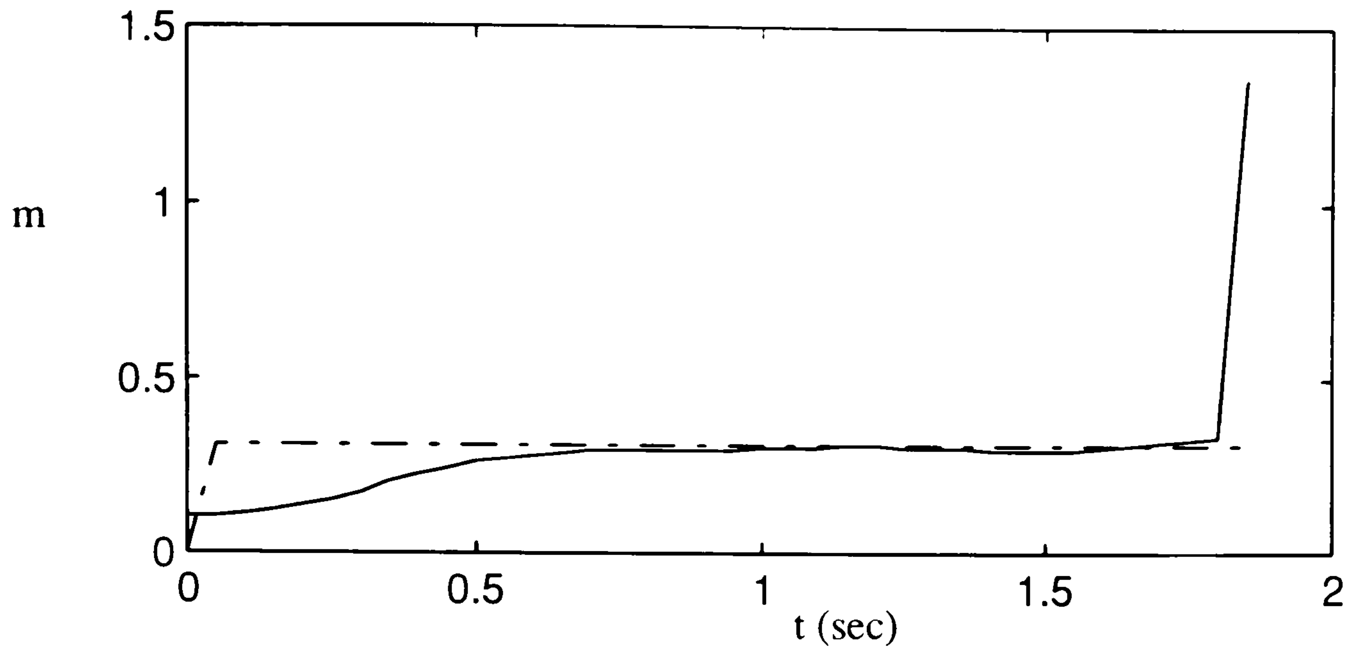


(a) The plots of reference signal ($- \cdot$), ref1, the first output (-), θ, when the control strategy is the non-adaptive fuzzy NIMC strategy

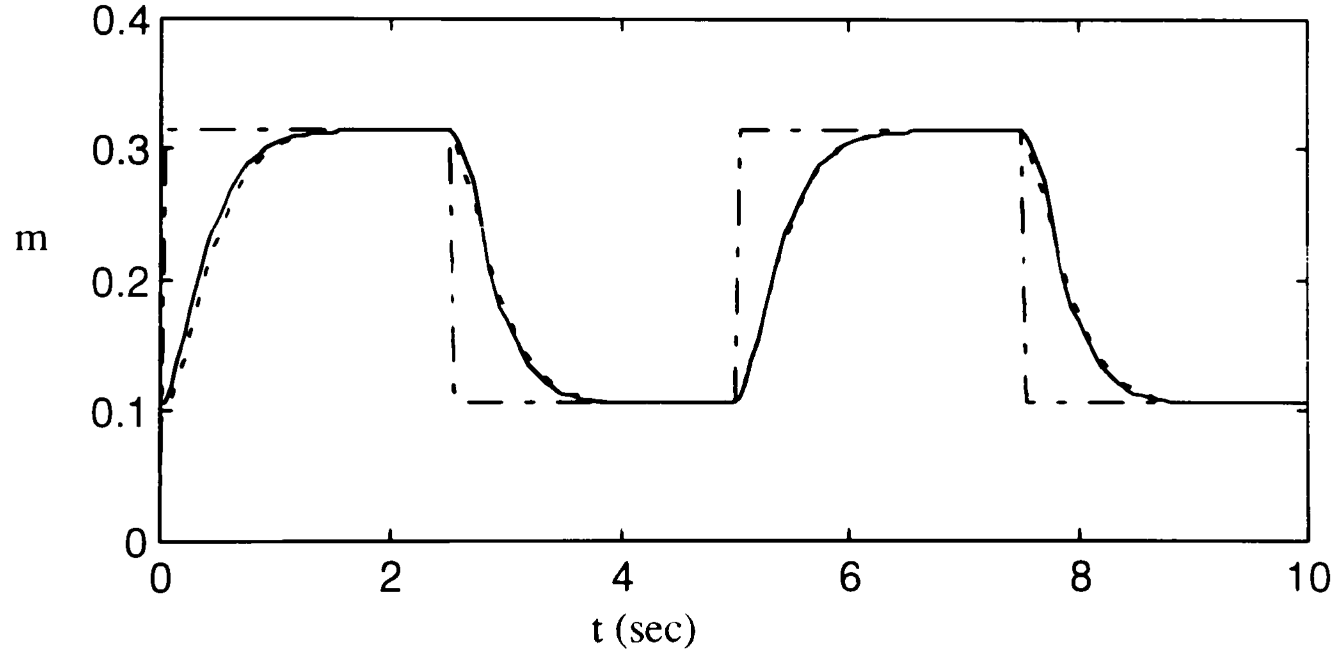


(b) The plots of reference signal ($- \cdot$), ref1, the desired response (--), y_d1 and the first output (-), θ, when the control strategy is the adaptive fuzzy NIMC strategy

Figure 4.29: The control of the two-link cylindrical robot manipulator when the modeling mismatch of the fuzzy controller results in the instability



(c) The plots of reference signal (– ·), ref2, the second output (-), r, when the control strategy is the non-adaptive fuzzy NIMC strategy



(d) The plots of reference signal (– ·), ref2, the desired response (--), y_d2 and the second output (-), r, when the control strategy is the adaptive fuzzy NIMC strategy

Figure 4.29: (continued)

CHAPTER 5

CONCLUSION

Recently, as alternative to conventional control, fuzzy control has been used for a variety of nonlinear control problems. Typically, fuzzy logic systems are used as nonlinear controllers that are called Proportional-Integral Derivative like Fuzzy Logic Controllers (PID-FLCs). The success of PID-FLCs in controlling nonlinear systems has been reported in several applications. However, there are no systematic and effective techniques to design PID-FLCs. A well-designed PID-FLC usually takes a lot of tuning, which usually relies on trial and error. Particularly, it is very difficult to design a PID-FLC such that the control system meets specific achievable requirements on control performance.

In this work, the nonlinear internal model control (NIMC) structure and the adaptive fuzzy NIMC strategy have been proposed to alleviate the problems of PID-FLCs. Unlike the previous work, a modified NIMC structure is used in this work to minimize error due to the modeling error of the model inverse of the plant. Basically, the modified structure consists of two major parts:

1. the controller which is the model inverse of the plant.
2. the linear filter which is used to shape the output response of the plant.

This structure has the following attractive properties:

1. the control structure has the ability to eliminate steady state error.
2. The relations between the filter and the output response can be found explicitly.

Thus, this allows designers to systematically and effectively construct the control structure to achieve certain specific control performances.

3. The stability criterion of the control structure can be found explicitly. Thus the closed-loop stability can be maintained by adjusting the parameters of the NIMC structure, which relate to the stability criterion, such that the stability criterion is satisfied.

Then, an adaptive fuzzy NIMC strategy has been proposed. The proposed strategy has three attractive features. First, the strategy provides an on-line adaptation to improve control performances of a plant over a wide range of operating conditions. Secondly, there are systematic techniques to keep the closed-loop system stable in an on-line fashion. Third, the fuzzy basis function (FBF) expansion is used to implement the controller. The use of the FBF expansion as the controller of the NIMC structure has the following advantages:

1. Accurate mathematical models of nonlinear plants are not required for constructing the FBF expansion. This can significantly enhance the ability of the strategy to control practical nonlinear systems whose exact mathematical models are quite difficult to obtain.

2. It is more convenient to construct the FBF expansion than other ordinary kinds of fuzzy systems. This is the case because the parameters of the FBF expansion are directly identified from input-output data. Thus, unsystematic tuning of fuzzy rules that is usually based on the experiences of designers is not required for constructing the FBF expansion.

3. A linear identification algorithm, which generally requires less computation than a nonlinear identification algorithm, can be applied to obtain the parameters of the FBF

expansion because the FBF expansion is linear in its adjustable parameters.

As examples of applying the proposed strategy to control nonlinear systems, simulation studies of controlling four nonlinear systems (e.g., a pendulum, an inverted pendulum, a forced Van der Pol equation, and a two-link cylindrical robot manipulator) have been conducted. The simulation results show that the proposed strategy can successfully control the four nonlinear systems. Particularly, the output response of each of the four nonlinear systems is the same as its desired response. This confirms that the control scheme can be designed to meet specific achievable requirements on control performances.

It should be mentioned that other control strategies may also provide similar control results as the proposed strategy if the control problems are addressed and solved appropriately. However, what sets the proposed strategy apart from other control strategies is that the control system that is controlled by the proposed strategy can be conveniently and systematically designed to meet desired performances.

This study opens a variety of future research opportunities. First, the strategy can be actually implemented in hardware for controlling real physical nonlinear systems in order to further test and verify the proposed strategy.

Secondly, for the early works of fuzzy control, the requirements on control performances of a fuzzy control system are usually defined in qualitative and unspecified manner. This is the case because it is considerably difficult to design a fuzzy control system to meet specific requirements on control performances. Currently, research is continuing in the direction of finding a design technique or a control strategy such that a fuzzy control system can be designed to meet specified requirements on control performance. This study has presented one way to achieve this. Other ways to achieve this type of requirements are very active areas of research. Thus, this issue could be further studied.

Thirdly, in this study, the fuzzy basis function expansion is proposed to model the inverse of a nonlinear plant. Although the exact mathematical model of the plant is not required for modeling the fuzzy model, the order of the plant (i.e., the highest differential order of the plant equation) is assumed to be known in order to construct the FBF expansion. At this stage, most techniques for constructing fuzzy models have been based on this assumption. In the future, it would be challenging to develop a technique for constructing fuzzy models such that the order of plants need not be known.

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APPENDIX

THE PARAMETER CONVERGENCE OF RLS WITH FORGETTING FACTOR

In this appendix, the parameter convergence of RLS with forgetting factor algorithm will be discussed. To analyze the algorithm, it is assumed that the data is generated from the system that is described by

$$y_t = P_t^T \theta^* . \tag{A-1}$$

Therefore, θ^* is the vector of true parameters. In addition, it is assumed that an input signal that is used to generate the data excites all the possible modes of the system. This is called persistently exciting [107]. The model with the recursive least square with forgetting factor algorithm can be written as

$$\begin{aligned}
y_t &= P_t^T \hat{\theta}_{t-1} , \\
\varepsilon_t &= y_t - \hat{y}_t , \\
\hat{\theta}_t &= \hat{\theta}_{t-1} + Q_t P_t \varepsilon_t , \\
Q_t &= \frac{1}{\lambda} \{ Q_{t-1} - \frac{Q_{t-1} P_t P_t^T Q_{t-1}}{\lambda + P_t^T Q_{t-1} P_t} \} .
\end{aligned} \tag{A-2}$$

Here, the stability will be analyzed by using the Lyapunov's direct method [63].

Let introduce $\tilde{\theta}_{t-1} = \hat{\theta}_{t-1} - \theta^*$.

it follows that

$$\begin{aligned}
\varepsilon_t &= y_t - \hat{y}_t \\
&= P_t^T \theta^* - P_t^T \hat{\theta}_{t-1} \\
&= - P_t^T (\hat{\theta}_{t-1} - \theta^*) \\
&= - P_t^T \tilde{\theta}_{t-1} .
\end{aligned} \tag{A-3}$$

Subtracting θ^* from both sides of the equation in Eq(A-2), the result is

$$\hat{\theta}_t - \theta^* = (\hat{\theta}_{t-1} - \theta^*) + Q_t P_t \varepsilon_t \tag{A-4}$$

or equivalently

$$\tilde{\theta}_t = \tilde{\theta}_{t-1} + Q_t P_t \varepsilon_t \tag{A-5}$$

Let a Lyapunov function be

$$V_t(\tilde{\theta}) = \tilde{\theta}_t Q_t^{-1} \tilde{\theta}_t \, . \tag{A-6}$$

Substituting Eq(A-5) into Eq(A-6), the results are

$$\begin{aligned} V_t(\tilde{\theta}) &= [\tilde{\theta}_{t-1} + Q_t P_t \varepsilon_t]^T Q_t^{-1} [\tilde{\theta}_{t-1} + Q_t P_t \varepsilon_t] \\ &= \tilde{\theta}_{t-1}^T Q_t^{-1} \tilde{\theta}_{t-1} + (P_t^T Q_t^T Q_t^{-1} \tilde{\theta}_{t-1}) \varepsilon_t + (\tilde{\theta}_{t-1}^T P_t) \varepsilon_t + P_t^T Q_t^T P_t \, \varepsilon_t^2 \, . \end{aligned} \tag{A-7}$$

From the section 2.5, we know

$$Q_t = Q_t^T \, .$$

Equation(A-7) then becomes

$$\begin{aligned} V_t(\tilde{\theta}) &= \tilde{\theta}_{t-1}^T Q_t^{-1} \tilde{\theta}_{t-1} + (P_t^T \tilde{\theta}_{t-1}) \varepsilon_t + (\tilde{\theta}_{t-1}^T P_t) \varepsilon_t + P_t^T Q_t^T P_t \, \varepsilon_t^2 \\ &= \tilde{\theta}_{t-1}^T [\, Q_{t-1}^{-1} + P_t P_t^T \,] \tilde{\theta}_{t-1} + (P_t^T \tilde{\theta}_{t-1}) \varepsilon_t + (\tilde{\theta}_{t-1}^T P_t) \varepsilon_t + P_t^T Q_t^T P_t \, \varepsilon_t^2 \\ &= V_{t-1}(\tilde{\theta}) + \tilde{\theta}_{t-1}^T P_t P_t^T \tilde{\theta}_{t-1} + (P_t^T \tilde{\theta}_{t-1}) \varepsilon_t + (\tilde{\theta}_{t-1}^T P_t) \varepsilon_t + P_t^T Q_t^T P_t \, \varepsilon_t^2 \, . \end{aligned} \tag{A-8}$$

The equation can be rewritten as

$$\begin{aligned} V_{t+1}(\tilde{\theta}) - V_t(\tilde{\theta}) &= \tilde{\theta}_t^T P_{t+1} P_{t+1}^T \tilde{\theta}_t + (P_{t+1}^T \tilde{\theta}_t) \varepsilon_{t+1} + (\tilde{\theta}_t^T P_{t+1}) \varepsilon_{t+1} \\ &\quad + P_{t+1}^T Q_{t+1}^T P_{t+1} \, \varepsilon_{t+1}^2 \, . \end{aligned} \tag{A-9}$$

Substituting Eq(3.69) into Eq(3.75), The results are

$$\begin{aligned} V_{t+1}(\tilde{\theta}) - V_t(\tilde{\theta}) &= \varepsilon_{t+1}^2 - \varepsilon_{t+1}^2 + P_{t+1}^T Q_{t+1}^T P_{t+1} \, \varepsilon_{t+1}^2 \\ &\quad + (-\varepsilon_{t+1}) \varepsilon_{t+1} \\ &= [\, P_{t+1}^T Q_{t+1}^T P_{t+1} - 1] \, \varepsilon_{t+1}^2 \, . \end{aligned} \tag{A-10}$$

From Eq(A-2), it follows that

$$
\begin{aligned}
P_{t+1}^T Q_{t+1}^T P_{t+1} &= P_{t+1}^T \frac{1}{\lambda} [Q_t - \frac{Q_t P_{t+1} P_{t+1}^T Q_t}{\lambda + P_{t+1}^T Q_t^T P_{t+1}}] P_{t+1} \\
&= P_{t+1}^T \{ \frac{Q_t P_{t+1}}{\lambda} [1 - \frac{P_{t+1}^T Q_t P_{t+1}}{\lambda + P_{t+1}^T Q_t^T P_{t+1}}]\} \qquad \text{(A-11)} \\
&= \frac{P_{t+1}^T Q_t P_{t+1}}{\lambda + P_{t+1}^T Q_t^T P_{t+1}} .
\end{aligned}
$$

Since $0 < \lambda \leq 1$ and Q_{t+1} is always positive definite, the result is

$$0 \leq P_{t+1}^T Q_{t+1}^T P_{t+1} = \frac{P_{t+1}^T Q_t P_{t+1}}{\lambda + P_{t+1}^T Q_t^T P_{t+1}} < 1 . \qquad \text{(A-12)}$$

From Eq(A-10), it follows that

$$\Delta V_t = V_{t+1}(\tilde{\theta}) - V_t(\tilde{\theta}) = [P_{t+1}^T Q_{t+1}^T P_{t+1} - 1] \varepsilon_{t+1}^2 < 0 . \qquad \text{(A-13)}$$

Since $V_t(\tilde{\theta})$ is positive definite, i.e.,

$$
\begin{aligned}
&V_t(0) = 0 \text{ and} \\
&V_t(\tilde{\theta}) > 0 \text{ for } \tilde{\theta} \neq 0,
\end{aligned}
\qquad \text{(A-14)}
$$

and $-\Delta V_t$ is also positive definite, it can be concluded that the estimated parameters can converge to the true parameters. Furthermore, $V_t(\tilde{\theta})$ in Eq(A-6) goes to infinity as $|\tilde{\theta}|$ goes to infinity, i.e., $V_t(\tilde{\theta}) \rightarrow \infty$ as $|\tilde{\theta}| \rightarrow \infty$. Thus, can be further concluded that for the entire parameter space, the estimated parameters can converge to true parameters.

A.1 The convergence rate of the RLS with forgetting factor

According to Eq(A-6),

$$V_t(\tilde{\theta}) = \tilde{\theta}_t Q_t^{-1} \tilde{\theta}_t .$$

Since Q_t^{-1} is positive definite, we can write [106]

$$\nu_{min}(Q_t^{-1}) \left\| \tilde{\theta}_t \right\|^2 \leq V_t(\tilde{\theta}) \leq \nu_{max}(Q_t^{-1}) \left\| \tilde{\theta}_t \right\|^2 \qquad \text{(A-16)}$$

where $\nu_{min}(Q_t^{-1})$ is the smallest eigenvalue of Q_t^{-1} and

$\nu_{max}(Q_t^{-1})$ is the largest eigenvalue of Q_t^{-1}.

Thus,

$$\alpha_1 \left\| \tilde{\theta}_t \right\|^2 \leq V_t(\tilde{\theta}) \leq \alpha_2 \left\| \tilde{\theta}_t \right\|^2 \qquad \text{(A-17)}$$

where α_1 and α_2 are positive constants.

From Eq(A-13), it can be written as

$$\Delta V_t = [\; P_{t+1}^T Q_{t+1}^T P_{t+1} - 1]\, \varepsilon_{t+1}^2 \;.$$

According to Eq(A-12),

$$0 \leq P_{t+1}^T Q_{t+1}^T P_{t+1} < 1 \,.$$

It follows that

$$\begin{aligned} \Delta V_t &= -\alpha_3\, \varepsilon_{t+1}^2 \,, \quad \alpha_3 \text{ is a positive constant} \\ &= -\alpha_3\, [\; \tilde{\theta}_t P_{t+1} P_{t+1}^T \tilde{\theta}_t] \qquad \text{(A-18)} \\ &\leq -\alpha_3\, \nu_{max}(P_{t+1} P_{t+1}^T) \left\| \tilde{\theta}_t \right\|^2 . \end{aligned}$$

From Eq(A-6), it follow that

$$\frac{\partial V_t(\tilde{\theta})}{\partial \tilde{\theta}} = 2 Q_t^{-1} \tilde{\theta}_t \,.$$

Thus,

$$\begin{aligned} \left\| \frac{\partial V_t(\tilde{\theta})}{\partial \tilde{\theta}} \right\| &\leq 2 \left\| Q_t^{-1} \right\| \left\| \tilde{\theta}_t \right\| \qquad \text{(A-19)} \\ &= 2\alpha_4 \left\| \tilde{\theta}_t \right\| , \quad \alpha_4 \text{ is a positive constant} \end{aligned}$$

From Eq(A-17), Eq(A-18), Eq(A-19), and Lyapunov stability theorem [113], it can conclude that the algorithm has exponentially convergence rate.