

Epistemicism and Vagueness

by

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ABSTRACT

Gradable vague predicates such as “tall,” “expensive,” “bald” etc. have raised serious problems in the philosophy of language and logic. A basic problem is the Sorites Paradox, which in turn gives rise to some other semantic, epistemological and psychological problems. In response, some philosophers have chosen an epistemic approach to vagueness. According to epistemicism, the extensions of vague predicates have exact boundaries, but we don’t know where those boundaries lie. Epistemicism is perhaps the most plausible solution to the Sorites Paradox, but no one has yet offered a precise and plausible explanation of our ignorance of cutoff points (the epistemological question) and also our inclination to reject their existence in the first place (the psychological question). This essay suggests a contextual proposal that does exactly that.

CHAPTER I

INTRODUCTION

Consider the following sentence:

(1) John is tall.

In ordinary contexts, if John's height is e.g. 6'8", we tend to say (1) is true and if his height is just 5'2", we tend to say (1) is false. What if John's height were 6'0"? In such a situation, we may or may not regard him as a tall person. Therefore, in such cases, determining the truth value of "John is tall" would be problematic. When we are trying to determine extensions and non-extensions of vague predicates such as "tall," "expensive," etc. we recognize that there are cases for which it is difficult or impossible to determine to which class they belong. Such cases are called *borderline* cases. What is the truth value of borderline cases is the main problem of vagueness.

Furthermore, it seems that vague predicates are the main source of another difficulty called *the Sorites Paradox*. The Paradox can be presented in numerous ways. In fact, for each vague predicate we have a different version of the Sorites Paradox. For instance, consider "expensive"¹: The following apparently valid premises, *P1* and *P2*, result in the unacceptable conclusion *C*:

P1. A car which costs \$50000 is expensive.

¹ In this paper, I mostly confine my discussions to the two gradable predicates: "tall" and "expensive"; however, expanding the results to the other gradable predicates is straightforward.

P2. Any car that costs just \$1 less than an expensive one is expensive.

By a recursive-like process, P1 and P2 imply that a car which costs \$49999 is still expensive, as is a car which costs \$49998, \$49997 and so on. Therefore:

C. Every car less than \$50000 is expensive.

Though, according to this conclusion, even a free car is still expensive.

How shall we resolve these difficulties? As I will discuss in section 2, it seems that the Sorites Paradox suggests that vague predicates have sharp boundaries. But, if the sharp boundary claim is true, then, as Delia Graff Fara² (2000) explains, we are faced with several other questions. These questions and some proposed responses are also alluded to in section 2. In sections 3 through 5, I will propose two context sensitive parameters that every contextual proposal to vagueness has to fulfill. Nevertheless, I will argue that the best available formal contextual theory, mostly defended by Delia Graff Fara³ (2000) and Christopher Kennedy (2006), does not adequately fulfill the second context sensitive parameter requirement. So, in section 6, I will criticize that theory and specifically its notion of “a significant difference”. Section 7 is devoted to my proposal and my alternative notion of “standard of accuracy” (the second context sensitive parameter). In the last three sections, I shall show the strength of my

² For name reference and citation, I follow her preferred instruction explained in:

<http://www.princeton.edu/~dfara/>

³ Although Fara’s theory is usually considered as a contextualist solution to the problem of vagueness (For example see (Sorenson, 2012)), she is reluctant to describe herself as a contextualist “because the context only has an indirect effect on the extension via the changes it makes to the speaker’s interest”. For this reason, Kennedy calls Fara’s theory a quasi-contextual theory. That worry seems unnecessary for the purposes of this paper, because I am merely concerned with her formal proposal here. Furthermore, direct or indirect, it seems that contexts affect on the extension of vague predicates in that theory.

proposal. I will argue that my notion of “standard of accuracy” doesn’t face those difficulties which "a significant difference" does and it provides a precise explanation to the epistemological and psychological questions.

CHAPTER II

A METAPHYSICAL QUESTION: IS THERE A CUTOFF POINT?

12:00pm is a moment of the day and 12:00am is a moment of the night. But, what about 5:50pm⁴? It seems that there are moments that do not clearly belong to the day or to the night. These borderline cases raise two basic questions, one metaphysical and one epistemological. The metaphysical question asks: is there any cutoff point (an exact moment) that divides the day from the night? The epistemological question asks: if there are cutoff points, why can we not know exactly where they are? I will give a positive response to the metaphysical question in this section. The epistemological question, however, shall be answered in section 9.

Although it might seem unintuitive to think cutoff points exist, there is a good argument in favor of them. We owe this compelling argument to the Sorites Paradox. Let's explain it in detail. A simple analysis of the Sorites Paradox⁵ shows that it is based on the following four apparently plausible premises:

(I) Fa (a is F)

(II) $(\forall x)(\forall y)(Fx \ \& \ Rxy \ \rightarrow \ Fy)$ (For every x and y , if x is F and x bears R to y (e.g. x is \$1 more expensive than y) then y is F too.)

(III) $\sim Fz$ (z is not F)

⁴ To be more precise, let's assume not in a place like the North Pole.

⁵ This analysis is due to Fara (Fara, 2000).

(IV) $(\exists b_1 \dots b_n) (Rab_1 \ \& \ Rb_1b_2 \ \& \ \dots \ \& \ Rb_nz)$ (There is a sequence of $n+2$ objects that starts with a and ends with z and every object in this sequence bears R to its immediate successor.)

If we hope to solve the Sorites Paradox, it is inevitable to reject at least one of these four premises. (I) and (III) seem to be completely safe, because they respectively just say that a is in the extension of F and z isn't. (IV) also seems to be unproblematic, since it merely says that there is a sequence of objects between a and z . Hence, it seems that the most suspicious assumption here is (II) and, indeed, a majority of philosophers agree on this. But, if (II) is false, by bivalence, its negation has to be true. This means that the following sentence is true:

$$(\exists x)(\exists y)(Fx \ \& \ Rxy \ \& \ \sim Fy)$$

I.e. there is an exact member (x) of the above sequence that is F , but its immediate successor (y) is *not* F . This means that there is a cutoff point between x and y . For example, assume that a and z are two cars, F stands for “expensive” and Rxy means that x is one dollar more expensive than y . In this interpretation, the above formula shall be interpreted as: there are two cars x and y , such that x is just 1 dollar more expensive than y and x is expensive but y is not.

If the true culprit of the Sorites Paradox is (II), which I think it is, then, we have to either reject bivalence or accept existence of cutoff points. I believe that giving up bivalent logic is too great a price to pay. Furthermore, those who have been ready to

pay this price have not been able to make any significant progress⁶. Therefore, it seems more plausible to accept that *there is* a cutoff point. Maybe we don't know where it exactly is, but there has to be one.

Denying the generalized sorites sentence (i.e. the assumption (II)) will block the Sorites Paradox, but it commits us to the existence of cutoff points and this raises other questions. Fara has posed three questions that must be answered by every theory which endorses the sharp boundaries claim. Kennedy summarizes Farra's questions as follows:

“1. The Semantic Question: If the inductive premise of a sorites argument is false, then is its classical negation — the sharp boundaries claim, that there is an adjacent pair in a sorites sequence such that one has the property named by the vague predicate and the other doesn't — true?

(a) If yes, how is this compatible with borderline cases?

(b) If no, what revision of classical logic and semantics must be made to accommodate?

2. The Epistemological Question: If the inductive premise is false, why are we unable to say which of its instances fail, even in the presence of (what we think is) complete knowledge of the facts relevant to judgments about the predicate?

3. The Psychological Question: If the inductive premise is false, why are we so inclined to accept it in the first place? What makes vague predicates tolerant in the

⁶ For example, see chapter 5 of Part I and chapter 9 of Part II in (Field, 2008).

relevant way? Why do they seem boundaryless?" (Kennedy, 2006; see also (Fara, 2000))

Epistemicism

The view which accepts the existence of cutoff points, but denies that we can know exactly where they lie is called *epistemicism*. Every theory which considers the sharp-boundary claim true is committed to respond the epistemological question. Therefore, I take epistemicism to be the view that considers the sharp-boundary claim true and, consequently, is committed to answer the epistemological question. Such a view has been supported by many philosophers with different approaches. Kit Fine (1975) has proposed a supervaluation semantic account, which claims borderline cases lead to truth-value gaps. Yet, Fine's theory implies that the Sorites sentence is false and sharp-boundaries claims are true. According to Fine's theory, there are x and y such that $R(x,y)$ and $F(x)$ are true but $F(y)$ is gappy. Therefore, $Fx \ \& \ Rxy \ \rightarrow \ Fy$ is also gappy and consequently $(\forall x)Fx \ \& \ Rxy \ \rightarrow \ Fy$ is false.

Timothy Williamson (1992, 1994, 1997) has proposed some epistemological arguments in favor of epistemicism. He believes that our ignorance of the cutoff points "is just what independently justified epistemic principles would lead one to expect" (1994, p:215). Williamson denies the existence of borderline cases which presumably are neither F nor non- F , where F is a vague predicate. Intuitively, there are cases (borderline cases) which are neither clearly thin nor clearly non-thin. But for Williamson, this doesn't show that they are neither thin not non-thin. Instead, it just shows that we are epistemologically incapable of *knowing* whether they are thin or

not. Suppose that Williamson is one of these borderline cases and P stands for the proposition: “Williamson is thin”. If we claim that P is neither true nor false:

$$\sim[T(\text{“}P\text{”}) \vee T(\text{“}\sim P\text{”})]$$

By Tarski’s T-schema:

$$T(\text{“}P\text{”}) \leftrightarrow P$$

$$T(\text{“}\sim P\text{”}) \leftrightarrow \sim P$$

By substituting equivalents for equivalents, it follows that:

$$\sim[P \vee \sim P]$$

or, equivalently,

$$\sim P \vee \sim \sim P$$

This is a contradiction. Hence, Williamson results that even borderline cases are either true or false and the sharp boundary claim is true.

Diana Raffman (1994, 1996) has offered a contextual solution. According to her theory, each total context is made of an internal and external context. The external context depends on external factors such as comparison class and domain of disclosure and the internal context is related to an individual’s psychological state. In this contextual approach, the Sorites sentence is false due to a category shifting that changes the extension of the predicate. Since the Sorites sentence is false, sharp

boundary claim is true. Consequently, she is also committed to the existence of cutoff points and answering the epistemological question.

Delia Graff Fara's theory (Fara, 2000) has some similarities to Raffman's theory. According to her view, vague predicates are interest relative. For instance, an object is expensive just in case it costs significantly more than some norm for being expensive, relative to our interests in that context. We will talk more about Graff's theory in the next sections.

There are of course many other theories which try to solve the problems of vagueness. It is not the objective of this paper, however, to explain and criticize each of these theories (except for Fara's formal proposal). This is done by other articles and discussion about them continues. Rather, this paper is going to offer and defend a new proposal.

Comparative predicates and vagueness

In this section, we will investigate some features of vague predicates to find some clues. These clues will help us in formulating a theory of vagueness.

A keen observer will recognize that a feature of vague predicates is that they all have a meaningful comparative form. For example, for a vague predicate like "tall", it makes sense to say:

(2) John is taller than Gary.

On the other hand, this is not the case for non-vague predicates. For example, consider the predicate "number". It even seems ungrammatical to say:

(3) 2 is numberer than 5.

This suggests that there is a relation between vague predicates and comparative predicates. Vague predicates are not innate properties. They are relational properties. In contrast with innate predicates, extensions of vague predicates are determinable relative to a group of related objects. For example, how could Adam (the person mentioned in the Bible) know whether or not he is a tall man? Since Adam was a unique being and there was no other man at the time, this question was unanswerable. Whether or not Adam was a tall man depends on the height of other men. This suggests that tallness is not an *innate* predicate, but rather a *relational-comparative* one. If it was innate, Adam in principle could answer that question merely by studying himself. But this seems to be vain. Therefore, vague sentences obtain their truth-value regarding a *comparison class* of related objects. This is our first clue.

If we reflect on these insights, we might obtain even more useful clues. One can correctly observe that *comparative forms of vague predicates are not vague*. For example, “tall” is vague, i.e. if x is a borderline case, it would be difficult to determine the truth value of “ x is tall”. But, the comparative form of that sentence, “ x is taller than y ”, is not vague. It doesn’t matter what margin of difference in height there is between x and y , we can always, in principle, determine the truth value of “ x is taller than y ”. Even if John is just one nanometer taller than Gary, then he is taller than Gary. *There is no borderline case for comparative form of gradable vague predicate*⁷. They are not vague. This second clue suggests that we might be able to propose a

⁷ Please notice that this is not the case for non-gradable vague predicates. A comparative usage of a non-gradable predicate like “nice” might still be vague, e.g. “John is nicer than Gary”.

semantic theory that enables us to determine the truth value of vague sentences using some appropriate non-vague comparative form of them.

In the next two sections, I will propose two context sensitive parameters which I think every adequate contextual theory of vagueness has to satisfy.

The first context sensitive parameter ($NORM(tall)(P)$)

Using the clues we obtained, we are ready to propose a contextual semantic theory for vagueness. If we determine a *comparison class* of objects related to an utterance of a vague sentence, we might be able to determine the truth value of that vague sentence using the comparative form of it in the following way:

(4) An utterance of “ x is tall” is true iff x is *taller than* the typical height of the members of a related *comparison class*.

For instance, “John is tall”, relative to the comparison class USA basketball players, is true, if and only if, John is taller than the typical height of USA basketball players. Following Delia Graff Fara (2000) and Christopher Kennedy (2006), we shall show “the typical height of in a related comparison class” by $NORM(tall)(P)$, where (P) stands for the type. ($NORM (tall))(P)$ presents our first context sensitive parameter.

The second context sensitive parameter ϵ

Using the first context sensitive parameter, ($NORM (tall))(P)$, we could propose (4) which is a step forward. But, I shall show that it is not enough. Let’s consider the following sentence:

(5) John is the same height as Gary.

Assume that the difference between John's height and Gary's height is very small. For example, assume that John is just one nanometer taller than Gary. Is (5) true? Well, in normal contexts we tend to say "yes". But, on the other hand, it seems that if John is even one nanometer taller than Gary, then John is taller than Gary. Therefore, John is *not* the same height as Gary and (5) is false. It seems that we have conflicting intuitions here: On the one hand, the entailment from "John is one nanometer taller than Gary" to "John is taller than Gary" seems sound. On the other hand, this means that (5) is false, as it doesn't matter how small is the difference between John's height and Gary's height. Since it is almost impossible to find two men with *exactly* the same height, utterances of sentences like (5) would almost always be false.

In order to solve this puzzle, we should ask ourselves: why do we tend to regard (5) as true, when the difference between John's height and Gary's height is very small? Well, this is not a hard question to answer. We usually don't care about very small differences. If the difference is too small we might even be unable to recognize it. Therefore, it seems that in each context we have a *standard of accuracy*. We are ready to ignore what can either be considered a trivial or unrecognizable difference in that context. For instance, when we want to buy a car, we don't care about the difference of \$1. In an ordinary utterance of (5), we *implicitly* have in mind that the difference between John's height and Gary's height is not recognizable to the naked eye. Still, there are other contexts that we *explicitly* state the amount of the standard of accuracy.

In scientific contexts, for instance, we explicitly state the standard of accuracy when we report the error of our experiments.

These considerations lead us to our second context sensitive parameter, namely *standard of accuracy*. Let's represent this parameter by ' ϵ '. In context C , every difference less than ϵ is simply ignored by an (explicit or implicit) stipulation between the speaker and the hearer. I think this is intuitively acceptable, since we always use sentences like (5) in our everyday usage of language without being worried about subatomic differences.

CHAPTER III

CRITIQUE OF FARA'S THEORY

Perhaps the best formal contextual theory available is a theory mostly defended by Delia Fara (Fara, 2000) and Christopher Kennedy (Kennedy, 2006). This formal proposal satisfies our first context sensitive parameter. But, instead of a “standard of accuracy” parameter, it uses a “significant difference” relation. I have found this notion problematic and, consequently, the theory and its analysis inadequate. So, I shall criticize that theory in the next three subsections.

Fara's analysis of vague sentences

Before going further, let's first see how Fara analyzes vague sentences. She says:

“I propose that

- That car is expensive

is to be analyzed as meaning:

- That car costs a lot,

which in turn is to be analyzed as meaning:

- That car costs significantly more than is typical.” (Fara, 2000, p: 22)

This analysis, I think, is limited to some very simple sentences like “that car is expensive”. It cannot successfully analyze more complicated vague sentences like (6):

(6) That car is a bit expensive.

By Fara’s analysis, it seems that we should understand (6) as meaning:

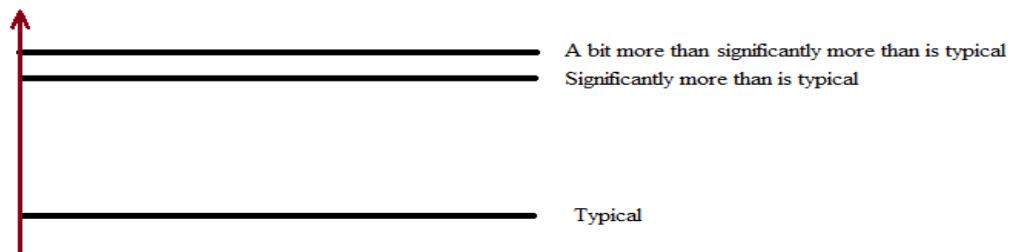
(7) That car costs a bit a lot.

Or:

(8) That car costs a bit significantly more than is typical.

As we see, neither (7) nor (8) make sense and they don’t even look well-formed.

Fara might respond that, in her theory, sentences like (6) have to be analyzed in a different way. She might suggest that “That car is a bit expensive” must be understood as “That car costs a bit more than is significantly more than is typical”. The structure of this sentence is still a bit confusing, but it makes sense. The following picture explains what it means.



But, is this really what we mean by “that car is a bit expensive”? This analysis seems unjustifiably complicated and far from our natural understanding. Intuitively, “that car is a bit expensive” merely means that “that car costs a bit more than is typical”, not “a bit more than is significantly more than is typical”. I think the problem actually demands a simpler analysis: A car that costs a bit more than typical is a bit

expensive. A car that costs a lot more than typical is too expensive. A car that costs more than typical at all is expensive. A car that has a typical cost is neither cheap nor expensive. In fact, in order to say that my car is neither cheap nor expensive, one can simply say “that car has a typical cost”.

Nevertheless, some people might have conflicting intuitions about what I just said. For example, one might say: “I don’t get the intuition. Just because a car is \$5 over the typical cost, it doesn’t seem expensive.” Such objections, however, are based on the background assumption that there is not an exact cutoff point there. In some sense, my opponent is right; it seems unintuitive to say \$5 more makes any difference. But she is right just because the idea of existence of cutoff points (epistemicism) itself is somehow unintuitive. Nevertheless, I have argued for existence of cutoff points in section 2. Such intuitions open doors to the Sorites Paradox. The Sorites Paradox shows that some of our intuitions are wrong and must be revised. In order to block that paradox, we have to recognize the false intuitions behind it. There are exact boundaries. Therefore, my respond to the objection is simple: in a context that the typical cost of a car is exactly \$6472, a car that costs \$5 over the typical is expensive. It is exactly \$5 more expensive.

To support the claim that these intuitions are based on the belief that there are not cutoff points, I should show that similar objections can be raised against other versions of epistemicism as well. For instance, consider Fara’s theory. Suppose that a car costs just \$1 *less than* significantly more than is typical. Since such a car costs less than significantly more than typical, Fara’s theory implies that it is not expensive. But this

seems unintuitive. If significantly more than typical for a specific type of car is \$6100, then a \$6100 car is expensive. Intuitively, a car that costs just 1\$ less than \$6100 is expensive too. However, Fara's theory implies that it isn't, because it costs \$1 less than what is significantly more than typical. However, as I mentioned above, epistemicism rejects this kind of intuitions.

Fara's notion of a significant difference

As I mentioned, for Fara, "That car is expensive" means "That car costs significantly more than is typical." Then, one might ask: *what if that car costs more than typical but not significantly more than typical?* Fara's theory implies that in this situation, that car would not be expensive (because it doesn't cost *significantly* more than typical). Therefore, the theory implies that, in such a situation, the following sentence will be true:

(9) That car costs more than is typical but it is not expensive.

However, it seems that a car that has a typical cost is neither cheap nor expensive and a car that costs more than typical, then, is expensive. It might be just a bit expensive, but, being even a bit expensive seems to entail being expensive. If a car costs one penny more than typical, then it is one penny more expensive. Fara's analysis entails that a car which costs more than a typical car, but not significantly so, is neither typical, cheap nor expensive: it is neither typical nor cheap, because, as was said, it costs *more* than a typical car; it is not expensive, because it does not cost significantly more than a typical car. This seems unintuitive to me. Suppose that you ask someone who has recently bought a car, "Was your car expensive?" and she

replies, “No.” You then ask, “Was it cheap?” She replies, “No.” You finally ask, “Did it have a typical cost?” and she again replies, “No.” I think such a set of answers would be unusual and strange.

Fara’s formal proposal

Fara and Kennedy analyze “tall” as follows:

$$\lambda P \lambda x [tall(x) ! > (NORM(tall))(P)]$$

(i.e. “ x is tall” iff x ’s height is significantly greater than the norm of height for the type P).

As the above formula shows, the notion of a significant difference is formalized by a *relational symbol* (i.e. $!>$), not a parameter. A relational symbol, unlike a parameter, cannot capture context sensitivity in formalizations. For instance, let’s see how this formal proposal analyzes “John is tall” in the two contexts: $C1$: [(P = USA basketball players); (Significant difference= 1cm)] and $C2$: [(P = USA basketball players); (Significant difference= 0.5cm)]. According to that formal proposal, “John is tall” will be formalized in the both contexts *by the same formula*, i.e.:

$$(10) \lambda(USA - Bask) \lambda John [tall(John) ! > (NORM(tall))(USA - Bask)]$$

The relational sign $!>$ has two different meanings in $C1$ and $C2$, i.e. in $C1$ it means more than 1cm, but in $C2$ it means more than 0.5cm. However, *they both are represented in (10) by the same formula*. So, this formalization does not capture the difference between the two contexts. It could not capture the context shifting from $C1$ to $C2$ due to a change in “significant difference “ from 1cm to 0.5cm.

CHAPTER IV

THE NEW PROPOSAL

I have suggested that a contextualist solution for vagueness should adequately account for the two context sensitive parameters: $(NORM(tall))(P)$ and ε . In this section, I offer a proposal which accounts for these parameters. First, I will introduce my analysis of vague sentences and then I will explain how it works.

According to my analysis, an utterance of:

(11) x is tall.

shall be understood as:

(12) $\lambda P \lambda x ([x]_{\varepsilon} > (NORM_{\varepsilon}(tall))(P))$.

(i.e. The x 's height rounded to ε is greater than the typical height rounded to ε .⁸)

For instance, “John is tall” is true in the context $[(P: \text{USA-Basketball players}); (\varepsilon: 1\text{cm})]$ if and only if John’s height measured and rounded to 1cm accuracy is greater than the typical height of USA-Basketball players rounded to 1cm. This proposal suggests the following algorithm for determining the truth value of “John is tall”:

(1): Determine standard of accuracy ε and type P ;

(2): Determine the height of John rounded to ε ;

⁸ The index ε in “ $[x]_{\varepsilon}$ ” and “ $NORM_{\varepsilon}$ ” indicates that their related amounts are rounded to ε . Also, please notice that in this formalization instead of a significant difference relation ($!>$), we use the usual “greater than” relation ($>$).

(3): Determine $(NORM_{\epsilon}(tall))(P)$ (i.e. NORM of tall for type P rounded to ϵ);

(4): If John's height rounded to ϵ is *greater than* $(NORM_{\epsilon}(tall))(P)$, he is tall, otherwise, he is not.

Notice that Fara's notion of a significant difference is merely applied to the boundaries, while standard of accuracy is applied to the whole picture, i.e. to every amount in the context (e.g. every measurement and calculation). ϵ determines the size of each pixel and, therefore, the resolution of our picture of the world. By taking smaller and smaller ϵ 's we can improve the resolution of our picture as much as we wish and, in principle, there is no restriction on it.

Furthermore, in contrast with the notion of a significant difference, the amount of *NORM* is not independent from the standard of accuracy (ϵ) in the context. This is why it is represented by index ϵ . For example, in context C : [(P : USA basketball players); (ϵ : 1cm)] the *NORM* might turn out to be, for example, 206cm. However, if we change ϵ from 1cm to 1mm, our new *NORM* will change to, for example, 205.8cm.

In Fara's approach, the amount of "a significant difference" is assumed to be completely independent of the amount of *NORM*. Her theory fails to recognize that without knowing what amounts we are ready to ignore (what amounts we consider non-significant in Fara's words), we cannot even know the amount of *NORM* in the context. We are epistemologically incapable of knowing the *exact* amount of the *NORM*, because we cannot know those quantities without any rounding error. In this way, in addition to type theory, my proposal benefits from mathematical error theory.

CHAPTER V

SOLVING DIFFICULTIES OF FARA'S THEORY

Fara's first difficulty

I argued that Fara's theory cannot successfully analyze sentence (6) i.e. "that car is a bit expensive". In this subsection I shall show that my proposal can successfully analyze this kind of sentence. In my proposal, "that car is expensive" in context *C* must be analyzed as:

(13) The cost of that car rounded to ϵ is more than the typical cost for a car in a related comparison class rounded to ϵ .

Therefore, (6), which was problematic for Fara's theory, shall be understood as:

(14) The cost of that car rounded to ϵ is a bit more than the typical cost for a car in the related comparison class rounded to ϵ .

As is evident, (14) makes complete sense and matches with our ordinary understanding.

Furthermore, in my proposal, "My car has a typical cost" simply means: The cost of my car rounded to ϵ is equal to the typical cost of a car rounded to ϵ . Consequently, when we round the cost of my car, as well as the typical cost of a car to ϵ , the price of my car will be either less than, equal to or more than typical. Therefore, in contrast with Fara's view, my proposal implies that each car is either expensive, cheap or has a typical cost. This is straightforward and matches with our ordinary understanding.

Fara's second difficulty

I argued that Fara's theory entails that (9) is true:

(9) That car costs more than is typical but it is not expensive.

This is unacceptable, because "that car costs more than typical" seems to entail "that car is expensive", as "that car has a typical cost" entails that that car is neither cheap nor expensive.

My proposal entails that (9) is false in every context of utterance. Let C be an arbitrary context. According to the first part of (9) (the left side of its conjunction): The cost of that car rounded to ϵ is more than $NORM_\epsilon$. According to the proposal, this implies that "that car is expensive" is true in C . This obviously contradicts with the right side of that conjunction which says "it is not expensive". Therefore, according to my proposal, (9) is a contradictory sentence and, therefore, it is always false.

Fara's third difficulty

We mentioned in subsection 6.4 that, Fara and Kennedy's formal proposal cannot capture the difference between the contexts $C1$ and $C2$ (See sentence (9) in section 6.) In my formulation, the standard of accuracy is presented by a parameter rather than a relation and can capture the difference between $C1$ and $C2$. According to my formulation:

$C1: \lambda(USA - Bask) \lambda John [John's height]_{1cm} > (NORM_{1cm}(tall))(USA - Bask)$

$C2: \lambda(USA - Bask)\lambda_{John} [John's height]_{0.5cm} > (NORM_{0.5cm}(tall))(USA - Bask)$

The difference between $C1$ and $C2$ is captured by the different indices for ε .

CHAPTER VI

THE EPISTEMOLOGICAL AND PSYCHOLOGICAL QUESTIONS

The epistemological question

In each context, we have a cutoff point for that context which is in fact its $NORM_\epsilon$. In this way, my proposal rejects premise (II) of the four premises of the Sorites Paradox and therefore blocks it. Consequently, my solution for the Sorites Paradox is committed to the existence of cutoff points. So, I have to respond Fara's epistemological question: why are we epistemologically incapable of determining the *exact* location of cutoff points?

For every continuous gradable vague predicate, there is a cutoff point (exactly one point in its mathematical notion) that divides extensions from non-extensions of that predicate. In each context, we know approximately where it is. It is *approximately* where the $NORM_\epsilon$ is. However, we cannot know where it *exactly* is, because we cannot know the *exact* amounts of continuous values. For knowing my *exact* height, we must be able to measure and determine my height *with its all decimals*. For determining the exact place of the cutoff point, we need to know the exact height of persons in the comparison class. In order to maximize accuracy, we must take $\epsilon=0$. But we cannot know the height of persons for *all infinite decimals* and consequently we cannot know the exact amount of the $NORM$ for its infinite decimals. We cannot because we are finite and they are infinite. This is why we are epistemologically

incapable of determining the exact place of cutoff points for continuous vague predicates.

For extensions of discrete vague predicates such as “is bald” and “is a heap”, my proposal offers a different response, because the vagueness of these extensions are not dependent on the decimal expansion of the number of hairs or grains of sand. Indeed, my view regarding discrete vague predicates is not an epistemicist view. Although the extensions of discrete vague predicates have exact boundaries just like continuous gradable vague predicates, in contrast with continuous vague predicates, we *can in principle* determine where exactly those cutoff points lie and in this sense I reject epistemicism in regard to discrete vague predicates. For instance, my view implies that we can take $\varepsilon = 1$ hair. In such a context, there is an exact number, i.e. $NORM_\varepsilon$, that every member of the related comparison class whose number of hairs is *even one hair* more than $NORM_\varepsilon$ is not bald. We have never been so accurate, but we *can in principle* do so. Other discrete vague predicates can be treated in the same way. Suppose, for example, that John wants to buy a car which typically costs \$6,000, and suppose that a dealer is selling such a car for \$6,010. Whether or not John wishes to buy the car from this dealer depends on how parsimonious he (that is, John) wants to be. Usually, we think that if a car which costs \$6,000 is not expensive, then neither is a car which costs \$6,010. But what if John were *extremely* parsimonious? For such a person, even differences of a penny count, and cutoff points for such people are vivid when it comes to financial issues.

The psychological question

“If the universally generalized sorites sentence is not true, why were we so inclined to accept it in the first place? In other words, what is it about vague predicates that makes them seem tolerant, and hence boundaryless to us?” (Fara, p:51, 2000) This is what Fara calls the psychological question⁹.

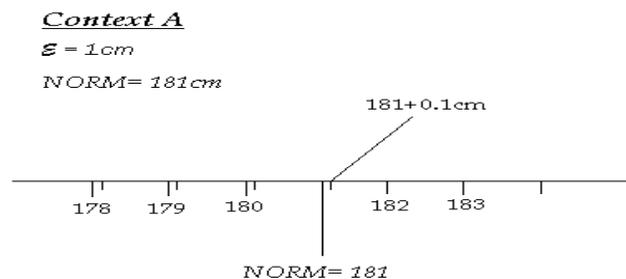
I think we are so inclined to accept the sorites sentence because we fall victim to a fallacy. According to my analysis, the fallacy is due to applying double standards in presumably one context; i.e. applying two standards of accuracy in the presumably same context.

I have emphasized that, in each context, every amount less than ϵ is unrecognizable or simply ignored. When we want to buy a car, our standard of accuracy might be \$100. we don't care about \$1 in that context, because it is less than our standard of accuracy. Whatever John's height is, it seems that adding just 1mm to his height doesn't make any difference in normal contexts, because it is not recognizable with the naked eye. The generalized sorites sentence uses this valid intuition but simultaneously inserts a new standard of accuracy into the context. Before explaining the fallacy behind the Sorites Paradox, let's first consider a version of the Sorites Paradox which seems to be a problem even for my view.

Sorites Argument: Suppose that in context C , the standard of accuracy is 1cm and $NORM_{1cm}$ is 181cm. Does adding just 1mm to the height of someone who is not tall in

⁹ I am not sure this is an appropriate name for the question. We are so inclined to accept the universally generalized sorites sentence, because we don't recognize the fallacy behind it. Nothing particularly psychological is going on here. We could simply ask: what is the fallacy behind the sorites paradox that deceives us and makes us so incline to believe that it is valid?

C, makes him tall? Since 1mm is less than the standard of accuracy in this context, the response must be no. Hence, someone whose height is 181.1cm is not tall and so is the person whose height is 181.2cm and so on. Eventually, someone whose height is 182cm or even more would not be considered tall. This obviously contradicts with our assumption (We assumed that the $NORM_{1cm}$ of the context is 181cm, which means a person whose height is 182cm is tall.)



The above diagram explains the paradox more precisely. The height of each person in this context is an *integer* and the minimum height for being considered as a tall man is 182cm. If someone's height is 181cm in this context, adding just 1mm to his height doesn't make any difference, because his new height, i.e. 181.1cm, is still less than 182cm. Now, we can continue this recursive process by adding 1mm to the new numbers (181.2cm, 181.3cm ...). Eventually, it will turn out that even a man whose height is 182cm or more is not tall, which contradicts our assumption.

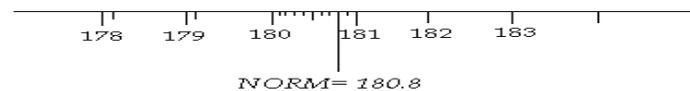
My Response to the Psychological Question: The above argument implies a double standard. The standard of accuracy of the data (181.1cm, 181.2cm ...) is 1mm, while the standard of accuracy for the $NORM_{1cm}$ is still 1cm (we assumed that it is still 181cm). If we wish to ask questions about 1mm differences, we have to change the

context (due to changing the standard of accuracy from 1cm to 1mm). If we want to know whether or not a man whose height is e.g. 181.1cm is tall, we have to apply this standard of accuracy (i.e. 1mm) on *every number* in the context, specifically on our $NORM_\epsilon$. The standard of accuracy for given numbers and the $NORM_\epsilon$ has to be the same; otherwise, we are mixing contexts. We should not use a double standard, and this is the fallacy behind the Sorites Paradox and the source of our inclination to think the sorites sentence is true. If we apply the new standard of accuracy (i.e. 1mm) *everywhere* in the above example, as I shall show below, we will see that adding 1mm *does* make a difference and the sorites sentence is wrong.

Context B

$$\epsilon = 0.1cm$$

$$NORM = 180.8cm$$



So, let's change our standard of accuracy from 1cm to 0.1cm everywhere in the context. Suppose that we do this and it turns out that our new $NORM_{0.1cm}$ is 180.8cm. In this context, someone whose height is 180.8cm is not tall (yet not greater than $NORM_{0.1cm}$), but adding 1mm to his height *does make a difference*. By adding just 1mm, his height will reach 180.9cm, which is more than $NORM_{0.1cm}$ and therefore, in this context, he must be considered a tall person. When someone asks: Does adding just 1mm to the height of someone who is not tall in *C* make him tall? I will respond: it might do so; however, in the context *C*, I cannot see it. I was inclined to say it does not in the first place, because I could not recognize it in *C*.

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