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Analytical model and simulations of closed-loop rebreather systems for Earth and Space applications

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Humans in extreme environments, regardless of whether in space or deep in the oceans of the Earth, rely on life support systems to be kept alive and perform their exploration missions. Diving is similar to extravehicular activities in its duration and the need for human respiratory sustaining. This paper presents the development of an analytical rebreather model, which is the system that recirculates and conditions the air the diver is breathing during a dive. The capability of simulating rebreather performance is currently lacking in the diving commercial and military industry. We believe that the advantages of having such a model are multi-fold: it can be used for mission planning, evaluating the impact of adding a new technology or modifying existing parameters or operational regime on an hardware configuration without performing expensive and time consuming hardware tests. An analytical model, like the one developed in this paper, can also be used in complement with hardware testing to fine tune systems and increase resource endurance through the application of different electronic control strategies. The developed Matlab/Simulink model of this rebreather is modular and can be generalized to study open, semi-closed or closed circuits, in which the breathing gas used is air, oxygen, nitrox or heliox. The system's operational environment can be the ocean's surface (1 atmosphere), space (less than 1 atmosphere pressure) or deep underwater (more than 1 atmosphere pressure). After introducing the analytical modeling process for the rebreather, this paper goes on to explore the model's applications for the study of different oxygen control strategies in order to maximize the oxygen lifetime during a dive, as well as the model's applicability as an aid in accident investigations. We aim to determine what is the maximum endurance of a rebreather system, given a particular, set configuration of components, as well as to study the reverse problem: if we set a mission endurance, what architectures would be able to achieve this level? Additionally, we are interested in studying how the tradespace of diving depth versus the diving systems' endurance looks like and how more complex control methods can help in pushing the existent boundary toward higher endurance limits. We show that more complex control algorithms can extend the duration of the oxygen tanks in a rebreather by a factor of 6.35, and, when given a set endurance level, control can help lower the tank sizes by a factor of 4.

Nomenclature

ABS = Acrylonitrile, Butadiene, Styrene
ADS = Advanced Diving System
BCD = Buoyancy Control Device
BOL = Beginning of Life
BOV = Bail-Out Valve

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BSAC = British Sub-Aqua Club
CCOUBA = Closed-Circuit Oxygen Underwater Breathing Apparatus CCR Closed-Circuit Rebreather
CMI = Constant Mass Injection
CO_2 = Carbon Dioxide
DAN = Divers Alert Network
DCS = Decompression Sickness
eCCR = Electronically-controlled Closed Circuit Rebreather
EOL = End of Life
FMECA = Failure Modes, Effects and Criticality Analysis
fsw = feet seawater
LED = Light - Emitting Diode
mCCR = Mechanically-controlled Closed Circuit Rebreather
msw = meters sea water
NAUI = National Association of Underwater Instructors
NOAA = National Oceanic and Atmospheric Administration
OC = Open Circuit
OPM = Object Process Methodology
PADI = Professional Association of Underwater Dive Instructors
PID = Proportional – Integral – Derivative
RBW = Rebreather World
RMV = Respiratory Minute Volume
RQ = Respiratory Quotient
SAC = Surface Air Consumption
SCR = Semi-Closed Circuit Rebreathers
SITS = Scientist in the Sea Program
SPECWAR = Naval Special Warfare Forces
STPA = System Theoretic Process Analysis
UBA = Underwater Breathing Apparatus

I. Introduction

Underwater - the other two thirds of our world - is a fascinating place. There are a lot of activities to do under the water, from photography, seeing fish and other critters up close, exploring new places or seeing historic shipwrecks [1] to rescue/salvage missions, submarine repair missions or diving research/development [2]. Diving equipment allows us to visit the underwater world by making it possible to breathe, see and move comfortably under the surface. Gear helps us transform from land-dwellers to somewhat similar to aquatic beings, even for a short while. A mask helps us see clearly, the scuba regulator and gas tanks provide the air we need, fins allow us to swim efficiently and the wetsuit helps maintain our body temperature and stay warm. The diving equipment varies in function of the environment we want to dive in: tropical scuba equipment is for warm water temperature (24°C/75°F and up), temperate scuba equipment is for diving in moderate temperature (cooler than 24°C/75°F), cold water scuba equipment covers water temperatures cooler than 15°C/60°F and the technical diving equipment, used by very experienced, highly trained divers to visit environments beyond the limits of recreational diving [3].

We can classify diving equipment in two broad categories, depending on how the breathing gas is used: open circuit and rebreather systems. Typically, for recreational purposes and when the diver is not a technical diver, (s)he will use an open circuit scuba equipment. The diver inhales gas from the tanks and exhales it to the surrounding environment (shown in Figure 1) [3].

Figure 1 – Open circuit scuba system: the diver inhales air from the tanks and exhales it into the surrounding environment

Rebreathers reuse the gas we exhale by recycling the good part and replenishing it from the gas tanks for the next breath. The system also features a carbon dioxide scrubber for removing the carbon dioxide from the diver's exhaled breath. This means that the gas supply is significantly smaller than the one we would carry in open circuit diving, a huge benefit that allows longer dives. Another advantage is the quiet factor: since rebreathers do not vent gas to the environment, no bubbles are formed during diving so we can approach marine animals that would normally shy away from bubble noise. Additionally, because we breathe gas that has been warmed by us and by the recycling process (specifically the absorption of carbon dioxide in the scrubber), rebreather diving keeps us warm, which is a bonus in cool water [4]. There are two rebreather types:

- Closed-Circuit Rebreathers (CCRs) - these systems recycle all the air we exhale, only a few bubbles escape during ascent to release the expanding gas. These systems require two gas supplies: a diluent (air, nitrogen or helium, depending on the depth we are diving at) and oxygen
- Semi-Closed Rebreathers (SCRs).

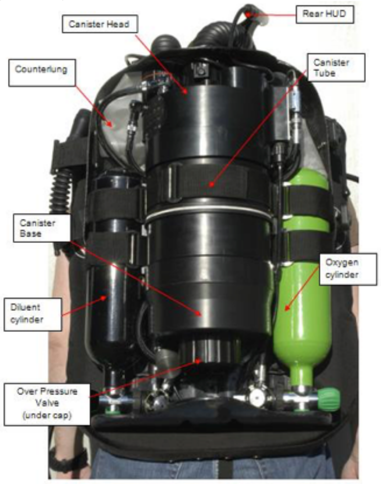


Figure 2 – A rebreather system and its components

Figure 2 shows a rebreather system components:

- Counterlung, also called a diaphragm, is an expandable bag that expands when we inhale
- Valves, that direct the air circulation from our mouth through the breathing assembly (also called the exhalation mouthpiece) to the carbon dioxide scrubber, to the diaphragm for gas makeup and then back through the inhalation part of the breathing assembly (also called the inhalation mouthpiece)
- Mouthpiece - closes and also connects the Bail-Out Valve (BOV), which is an open-circuit second stage regulator connected to a cylinder with breathing gas for emergencies
- Gas supply - feeds into the gas flow to replenish the oxygen we consume during diving and also increases the diaphragm volume as needed for buoyancy
- Oxygen sensor(s) and control system - located behind the gas cylinder; measure the oxygen partial pressure in the breathing gas, which is then fed to the electronic control system. This system calculates what the necessary oxygen partial pressure we need for the depth and metabolic rate that we are at and adjusts in accordingly
- Head-up display (HUD) - displays the consumables states in the system (battery state, gas tank pressure, carbon scrubber duration).

A. Paper objective

The focus of this paper is on life support systems, specifically air revitalization loops that keep humans alive in harsh environments, such as deep in the oceans or far out in the unwelcoming vacuum of space. Both astronauts and rebreather divers need a specific gas combination at set pressures in order to explore and work in those environments and we use similar systems for these tasks. We have chosen to study, model and simulate a unified system configuration that can be used both for Earth and Space applications. We briefly presented its configuration in Figure 2. In the spacesuit, the rebreather is integrated in the life support equipment backpack, shown in Figure 3.

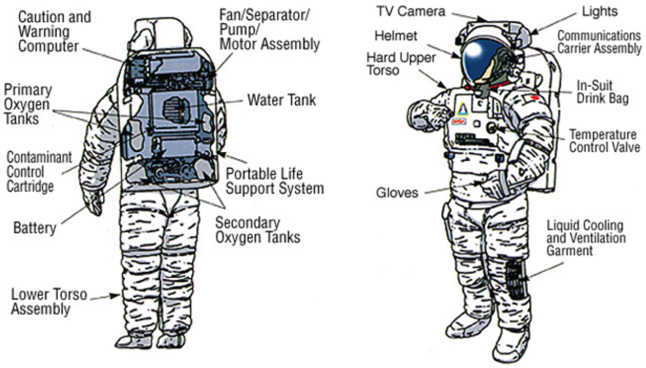


Figure 3 – A spacesuit – the rebreather system is integrated in the backpack

On Earth, as well as in space, we would like to extend our exploration time and keep the equipment weight to a minimum, because the effort to carry it would be a burden to the explorer. In order to do this, we need to study other methods to control the oxygen gas addition in the diaphragm. On one hand, there are systems that automatically dose the oxygen, irrespective of the diver's activity. These systems have been shown as being wasteful in their oxygen dosage, and the diver is exposed to oxygen toxicity issues or hypoxia [52]. On the other hand, there are manual oxygen dosing systems, that are heavily prone to failures. Therefore, systems that automatically dose the oxygen and adapt this dosage as a function of diver activity are preferred and these are what we are studying here.

The research questions we are looking into are the following:

1. What configurations of air revitalization systems (gas tank sizes and gas control methods) can achieve a specified mission endurance, where endurance is defined as the lifetime of the mission?
2. What is the maximum endurance that an air revitalization architecture can achieve, given a set gas tank size?
3. How does the tradespace of mission endurance versus the gas control method look like?

We aim to answer the questions above using an analytical model of an air revitalization systems, particularly a rebreather, that we will model and simulate in this paper.

B. Motivation and background

Although diving or traveling into space does not need a long motivation list, we need to be aware of the risks and dangers that the human body might face when performing these activities. This keeps us safe and helps us enjoy these activities even more!

The model that will be built in the next sections is that of a generic ventilation and air revitalization loop for space and Earth applications, but in the next sections we will use the unified term of a rebreather model to describe it, while keeping in mind its generality and multiple application range.

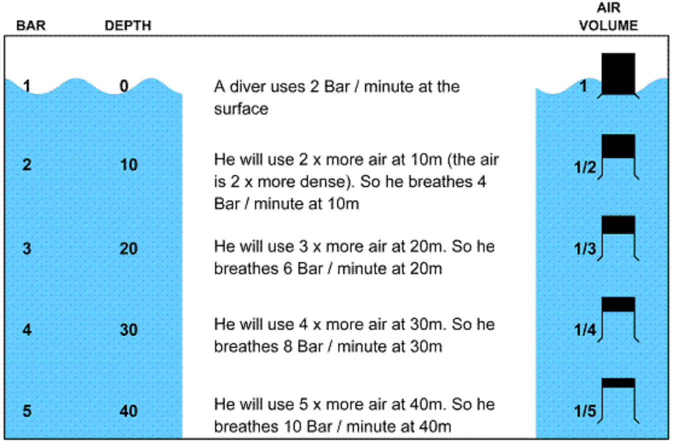
A few basic diving concepts are presented in the following:

- Pressure increases with depth: the deeper a diver descends, the more pressure the water above him/her exerts on the diver's body. The pressure a diver experiences is a sum of the pressures above him, both of the water and the air. Most pressures in scuba diving are given in a unit called ata, used in place of the atmosphere (atm) to indicate that the pressure shown is the total ambient pressure on the system. For example, an underwater pressure of 3.1 ata would mean that the 1 atm of the air above the water is included in this value. Additionally, every 10 meters (33 feet) of salt water exert 1 ata of pressure [7]. Air compresses according to Boyle's law:

  $$pV = \text{constant}$$

  This law states that as the pressure changes, the volume of the gas in the diver's body and soft equipment changes too. As the diver descends, the increase in pressure causes the air in the body's air spaces to compress. The air spaces are for example the ears, mask and lungs feel like vacuum because the compressing air creates a negative pressure. This can cause delicate membranes, like the ear drum, to be sucked into these air spaces causing pain and injuries. On ascent, the reverse happens: the air spaces expand as a consequence of decreased pressure. The air spaces in the lungs and the ears of the diver experience a positive pressure as they become overfull of air. If the diver does not breathe properly under water, this process could burst his/her ears or lungs. In order to prevent a pressure related injury, divers must equalize the pressure in their body's air spaces to the pressure around them. To equalize the pressure in the diver's air spaces on descent, the diver adds air to his body airspaces to counteract the "vacuum" effect (this can be done by breathing normally, as this adds air to the lungs, adding air to the mask by breathing out of his nose or adding air to the ears and sinuses using pressure equalization techniques). During ascent, the diver needs to release air from his airspaces, because the ambient pressure decrease causes the air to feel as if it has too much volume (this can also be done by breathing normally and allowing the body to eliminate the extra air from the lungs and by ascending slowly and allowing the extra air in the diver's ears, sinuses and mask to bubble out on their own).

- Buoyancy is an object's tendency to float. In scuba diving, the term is used not only to describe the diver's tendency to float, but also the tendency to sink or do neither in the water.
- The Air Consumption Rate – This is the mass flow rate at which the diver uses his/her air. Air consumption rates are given in the amount of air the diver breathes in one minute at the surface. There are two different methods of measuring air consumption in scuba diving: the Surface Air Consumption (SAC) rate and the Respiratory Minute Volume (RMV) rate [10]. It is useful to know the air consumption rate in scuba diving for three main reasons:
    - Dive planning – the diver can use his/her air consumption rate to calculate how much time he can stay underwater at the planned depth and to determine if (s)he has enough gas to make the return trip. The air consumption rate is also useful in determining the required reserve tank pressure for a dive. It is often surprising for divers to see that the calculations indicate that more than the standard 700-1000 psi of reserve pressure may be required to get a buddy team safely to the surface. When decompression stops are made, the air consumption rate is critical in determining how much gas to carry for these stops [10]
    - Determining stress or comfort level – the air consumption rate is a great tool to gauge the diver's stress or comfort level: if, during a dive at 45 feet the diver notices that (s)he used 500 psi when the typical air consumption rate for 5 minutes of diving at that depth is 200 psi, then this is an indication that something is wrong
    - Identifying equipment problems – a diver who has a major leak may notice that the air consumption rate is unusually high although his/her breathing rate is normal. For example, an increased air consumption rate may be an indication that a diver's regulator may require servicing, as the breathing resistance (and so the air consumption rate) increases when a regulator requires servicing. In short, an example of different air usage at various depths is illustrated in Figure 4.



A diver uses 2 Bar / minute at the surface

He will use 2 x more air at 10m (the air is 2 x more dense). So he breathes 4 Bar / minute at 10m

He will use 3 x more air at 20m. So he breathes 6 Bar / minute at 20m

He will use 4 x more air at 30m. So he breathes 8 Bar / minute at 30m

He will use 5 x more air at 40m. So he breathes 10 Bar / minute at 40m

Figure 4 – Example of an air consumptions scenario during a dive

Like any experiences, diving has its risks. The extreme pressures involved can take a toll on the fragile human body if not handled appropriately. The most important medical conditions involved with diving are decompression sickness and nitrogen narcosis. Decompression sickness, also known as the bends or Caisson disease, is an illness that can affect divers or miners – in general people who are exposed to rapid external pressure changes. It is caused by the build up of nitrogen in the body: as we breathe, we inhale about 79% of nitrogen in the air. As a diver descends, this nitrogen is absorbed in the body tissues. The problem is when the diver ascends and the lowered pressure causes the nitrogen in the blood to come out of the solution. If the diver ascends too quickly, the nitrogen escapes at a fast rate and can cause bubbles, that can transfer to the arterial blood circulation and block the blood flow to areas like the lungs, brain and other essential areas of the body. This is why divers must ascend slowly, to give their bodies time to eliminate this nitrogen through normal breathing. If the diver spent a longer time under the water than the no-decompression limit (this is a time limit that refers to the maximum time a diver can spend underwater and ascend to the surface without decompression stops along the way), then decompression stops are mandatory. The treatment for DCS consists of oxygen administered on site and first aid, followed as soon as possible by recompression treatment in a recompression chamber.

Nitrogen narcosis is a state of altered awareness caused by breathing a high partial pressure (or concentration) of nitrogen. The deeper a diver goes, the higher the partial pressure of nitrogen and other gases will be. This will usually limit the depth a diver can go to. Once the nitrogen narcosis sets in, the diver should ascend at a safe rate in order to reduce the partial pressure of nitrogen in the air (s)he is breathing. Thirty meters is the average depth at which divers start to experience nitrogen narcosis. Narcosis has been called "the rapture of the deep" and many divers compare it to a pleasant state of drunkenness. Some divers use the "Martini rule" to roughly estimate the effects of narcosis during a dive: this rule states that for every increment of 18 meters depth, a diver experiences the narcotic effect of drinking one Martini. At depths of 60 meters divers are likely to experience severe narcosis and even unconsciousness.

Hyperoxia is the result of breathing an excessively high partial pressure of oxygen. Oxygen toxicity is a catastrophic hazard in diving, as the seizures caused can result in near death by drowning [12]. The seizures occur suddenly and with no warning signals. As there is an increased risk of oxygen toxicity on deep dives, long dives or dives in which oxygen-rich gases are used, divers calculate a maximum operating depth for the air mixture that they are using, and the cylinders used are clearly marked with this maximum depth [13]. Diving below 56 meters on air only would expose the diver to oxygen toxicity, as the partial pressure of oxygen exceeds 1.4 bar, so a gas mixture has to be used that contains less than 21% oxygen. Augmenting the nitrogen content of the gas is not a good solution, because it would lead to nitrogen narcosis. This problem is solved by adding helium, which is not narcotic - nitrogen can be completely replaced with helium, and the resulting mix is called heliox, or by replacing a part of nitrogen with helium, and the resulting mix is called trimix.

The opposite of hyperoxia is hypoxia - a condition when there is not enough oxygen in the diver's ventilation circuit to meet metabolic requirements. If oxygen is not added to the ventilation loop, the existing oxygen will be consumed in 2- 5 minutes and the remaining gas mixture is not capable to sustain life [14]. The maximum oxygen partial pressure at sea level is 120 kPa and the minimum is 10 kPa.

Hypercapnia is a condition caused by abnormally elevated levels of carbon dioxide in the blood. This can be prevented by scrubbing the carbon dioxide from the air in the rebreather system. However, monitoring the partial pressure of CO_2 is important to make sure that the CO_2 scrubbing equipment is functioning normally. The maximum carbon dioxide partial pressure is 2.93 kPa, the equivalent of 22 mm Hg [1].

C. Life support systems for diving

A rebreather is a breathing device that absorbs the carbon dioxide from the humans exhaled breath and allows the rebreathing (recirculating) of the unused oxygen in the air. Oxygen is added to this loop to replenish the amount metabolized by the user. The rebreather is different from the open-circuit breathing apparatus, in which the exhaled gas is passed to the environment (described in Section 1.2). Rebreather technology is used in a wide variety of areas: in space (when the oxygen supply is limited and the external environment (vacuum) is not able to support life), in firefighting (where the environment is toxic) or in hospitals (where it is used by a patient under anesthesia to supply the concentrated gas to the patient without contaminating the air the medical staff breathes). A typical rebreather configuration is shown in Figure 5. The diver exhales into a bag (called a 'counterlung' (4)). A scrubber (3) removes the carbon dioxide and fresh gas is added to replace the metabolized oxygen (11). This recycled gas in inhaled again by the diver. In the case of a pure oxygen rebreather, the breathing gas contains mainly oxygen, and the partial pressure of oxygen in the circuit is dependent on the ambient pressure. This rebreather type maximized the efficiency of gas usage and provides a bubble-free, silent diving capability useful in military applications. Due to the fact that the carbon dioxide absorption in the scrubber (3) is an exothermic reaction, the air is warmed by the heat and so the diver breathes warm, humid gas. The presently recommended oxygen partial pressures for maintaining life range from 0.1 bar (10.1 kPa) to 1.6 bar (162 kPa). A partial pressure above this upper limit may lead to acute oxygen toxicity, manifested by epilepsy-like convulsions, which is fatal underwater. A ppO_2 under this limit will lead to unconsciousness.

Rebreathers are classified into either semi-closed circuit rebreathers (SCR) or manually or electronically controlled closed-circuit rebreathers (mCCR or eCCR). In a SCR, oxygen enriched gas is pumped through a constant flow injector into the circuit, typically at 6-12 bar L/min to substitute the metabolized oxygen. Excess gas in the circuit is vented through an overpressure valve. The maximum depth that a diver can reach using this circuit is limited by the percentage of oxygen in the supply gas. In a CCR, the partial pressure of oxygen is kept at a constant level. In mixed-gas diving, the breathing gas in the CCR contains nitrogen or helium. To maintain a constant oxygen partial pressure, a control loop is needed. This loop contains electrochemical oxygen sensors, whose output is proportional to the partial pressure of oxygen; these are the sensor elements. In a mCCR, the diver manually adjusts the oxygen partial pressure by adjusting the oxygen injection valve or adding oxygen manually. In an eCCR, this task is usually performed automatically, by a microcontroller actuating a solenoid valve [43].

Both types of closed rebreather systems have many advantages:

- Gas efficiency: open circuit scuba diving has a gas efficiency of less than 5% on the surface, to below 0.5% at 100 msw depth. In a CCR, because the gas is recycled, this gas efficiency reaches almost 100% and so the design of these rebreathers is smaller and light-weight than OC scuba, but more complex
- Silence: the CCR allows bubble-free operation, only during the ascent phase is gas vented from the circuit
- Warm, humidified breathing gas is provided to the diver, due to the exothermal carbon dioxide absorption reaction in the scrubber. Cold breathing gas in cold water may lead to the regulator freezing; in a CCR this is avoided. The downside to operating a CCR in cold water is that the scrubber efficiency can be impaired.

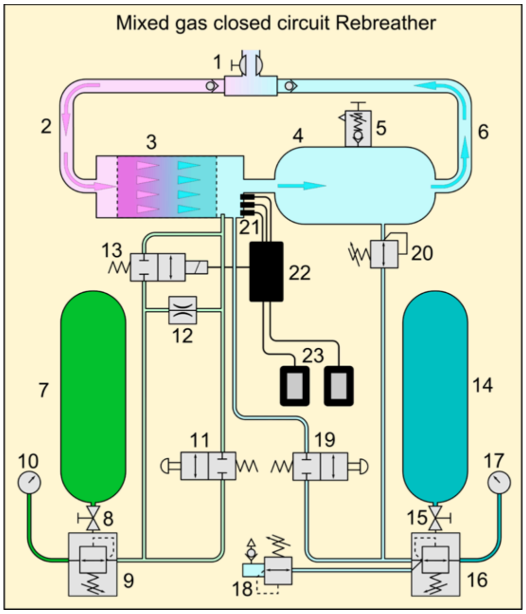


Figure 5 - Schematic diagram of electronically controlled closed circuit mixed gas rebreather (1) Dive/surface valve and loop non-return valves (i.e. ’mouthpiece’) (2) Exhaust hose (3) Scrubber (axial flow) (4) Counterlung (5) Overpressure valve (6) Inhalation valve (7) Diluent cylinder (8) Diluent cylinder valve (9) Absolute pressure diluent regulator (10) Diluent submersible pressure gauge (11) Diluent manual bypass valve (12) Diluent constant mass flow metering orifice (13) Electronically controlled solenoid operated oxygen injection valve (14) Oxygen cylinder (15) Oxygen cylinder valve (16) Oxygen regulator (17) Oxygen submersible pressure gauge (18) Bailout demand valve (19) Manual oxygen bypass valve (20) Automatic oxygen valve (21) Electronic control and monitoring circuits (22) Primary and (23) secondary display units [20]

Presently, we can only attain certain diving depths and system endurances, as shown in Figure 6. The fronts shown in red, orange, green and blue show the limits of a current capability. The goal of this research, as mentioned before, is to push these frontiers towards more endurance (as the purple arrow points to in Figure 6). We are investigating here if we can achieve this fact by increasing the oxygen tank control complexity.

At the end of this paper, we will revisit this graph and show how much control can help extend the endurance envelope for closed circuit rebreather systems.

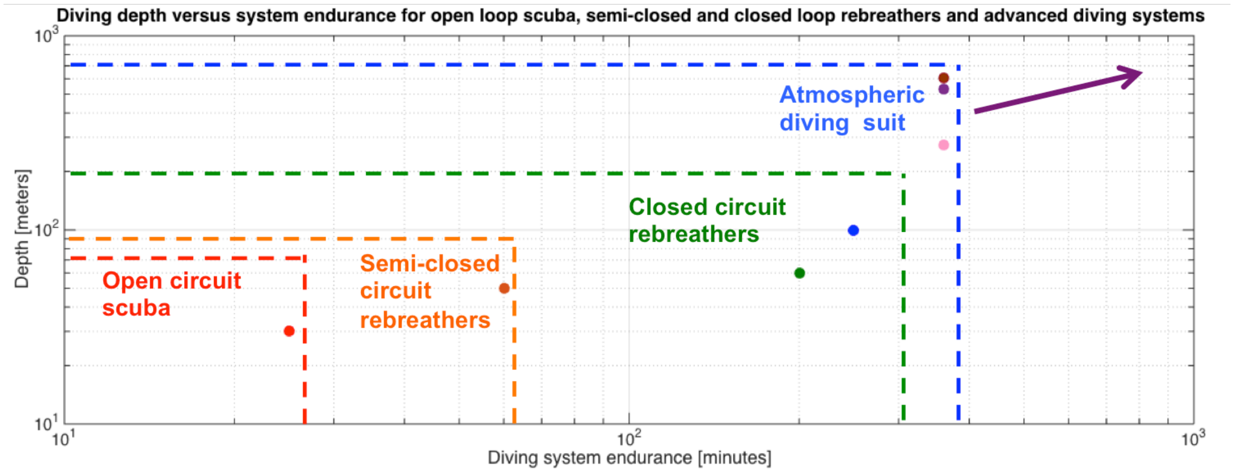


Figure 6 – Double logarithmic plot of diving depth versus dive system endurance for open circuit scuba, semi-closed and closed circuit rebreathers, and Advanced Diving Systems (ADS)

II. Rebreather model development

There are numerous reasons for which analytical models of hardware should be constructed. Software models allow for preliminary calculations and tests to be simulated before the diving mission, can also be used to estimate how many resources we would need for a mission and so aid the mission planning process. These models can also be used to study the impact of different technologies on the mission endurance, for example what advantages could we obtain through the use of breathing hoses with a larger diameter or a different carbon scrubber technology. The most gain is obtained when the analytical model is used in conjunction with the hardware development process: in the beginning of this process, a generic analytical model can be constructed in order to calculate basic parameters such as how much capacity should the carbon dioxide scrubber have given multiple diver metabolic processes or depths or how big the gas tanks need to be and what is their gas composition. As the hardware development progresses, the analytical model can be refined to include the corrugated breathing hoses and calculate the breathing resistance the diver would have to face. This decision can be fed back to the hardware developers, who can adjust the diameter and length of the hoses to pose minimum resistance and so maximum efficiency. Additionally, such a model can help understand if the hoses are the unique factors influencing this breathing resistance and if not, how the other elements (like for example the filters at the input and output of the carbon dioxide scrubber) influence each other and, in turn, influence the breathing resistance. Breathing resistance is only one of the areas the model-hardware conjunction can be useful in. Another would be, for example, the carbon dioxide scrubber. If the model's carbon dioxide scrubber would be refined to represent various types of scrubbers available, like Sodasorb or Metox, then simulations can show which of these would be more effective and under which diving circumstances.

Overall, an analytical model can help reduce the hardware development time, improve the quality of the hardware coming out of this process through the possibility of simulating design decisions before they are made and so creating a welcoming environment for design changes and feedback loops, so that in the end the optimal decision is implemented in the final hardware. It also contributes to time and cost savings of the overall design process (when we compare the analytical model aided design process with the traditional build and test approach).

This section presents the methodology for the construction of the analytical model of the rebreather. All model components existent in this model are common to all types of rebreathers and their sizes can be configured as we wish, therefore we are developing a generalized rebreather model. Furthermore, we can also add or remove additional modules to create and simulate different diving configurations. The first piece to be coded and simulated is the human breathing process, shown in Section II.A. After the model of human breathing was built, we simulated its functionality with a 10 liter breathing bag and checked the results with what would be expected for a human breathing in a fixed volume of gas with no gas additions and no carbon dioxide scrubbing. We then added the oxygen and nitrogen tanks into the code, as well as the electronic circuits to control the addition of these gases. Afterwards, we split the single breathing volume mentioned above into two smaller breathing volumes, called mouthpiece exhalation and mouthpiece inhalation. We did this in preparation for the model's specialization to the MK16 rebreather, which features two separate breathing volumes for inhalation and exhalation.

The gas species of the human breath are based on the values indicated in the Bioastronautics Databook [22], as well as the shape of the breathing signal and its amplitude.

A. Human breathing modeling

Respiration is the process in which air is moved in and out of the lungs, during which the tissue enzymes oxidate, using oxygen and producing carbon dioxide. Breathing is one of the physiological respiration processes needed to sustain life, delivering oxygen to the body and removing carbon dioxide. Once oxygen reaches the blood, it is moved throughout the body by the circulatory system. In addition to carbon dioxide, breathing also results in a loss of water from the body. Exhaled air has a relative humidity of 100% because of water diffusing across the moist surface of breathing passages and alveoli.

There are four sub-processes of respiration:

- Breathing or ventilation, detailed in this Section.
- External respiration, which is the exchange of gases (oxygen and carbon dioxide) between the inhaled air and the blood
- Internal respiration, which is the exchange of gases between the blood and tissue fluids
- Cellular respiration.

In addition to these main processes, the respiratory system also serves to:

- Regulate the blood's pH (this process occurs in coordination with the kidneys) to a nominal value of 7.35 - 7.45.
- Defense against microbes (bacteria and viruses)
- Control of body temperature with the help of evaporative loss during exhalation.

For the purpose of this paper, only the first process of ventilation will be detailed and modeled, since we are interested in the interaction between human breathing and the environment (an external process) and not in the internal processes of respiration and cellular respiration.

Ventilation is the exchange of air between the external environment and the alveoli. Air moves in the respiratory system from high pressure to low pressure, with the help of the diaphragm (a schematic of the respiratory system is shown in Figure 8): when the abdomen is relaxed, the volume of the body expands, causing a pressure drop in the thorax, leading to an expansion of the lungs. When the diaphragm relaxes, the air leaves the lungs due to their elasticity. This is relaxed breathing and needs little energy. When the need increases, the abdominal muscles resist expansion, the increased abdominal pressure then tilts the diaphragm and ribcage upwards with an increase in volume and the entry of air. Expiration follows the relaxation of the diaphragm and the abdominal muscles, and can be increased by the downward action of the abdominal muscles and rib cage up to the maximum tidal volume as an upper limit.

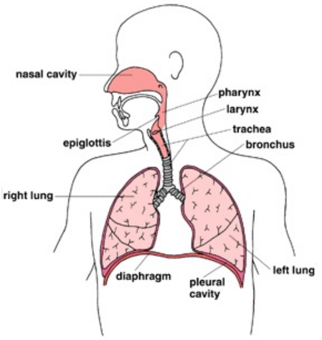


Figure 7 – Schematic view of the human respiratory system

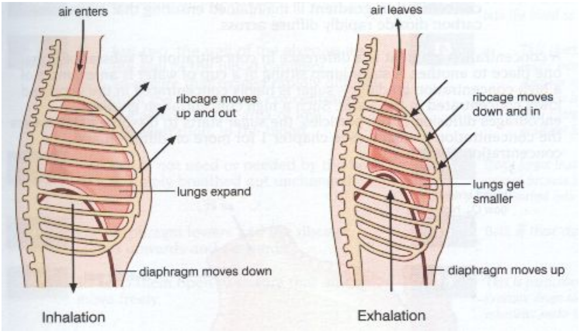


Figure 8 – Schematic view of the inhalation and exhalation processes

The main aspects of the human model for this research project are driven by the project requirements: to build a model of the human breathing such that we can observe the tend of metabolic gases in the diaphragm. Specifically, the model shall predict the oxygen, carbon dioxide and water concentrations in the air the human is breathing, together with its pressure and temperature. In the model developed here, we have been approximated as a sinusoidal waveform, with a frequency of 20 breaths/minute at rest [22]. The breathing amplitude is a parameter in the model and we can adjust it to represent different work intensities. This amplitude of this wave is given by the tidal volume of the lungs (defined as the volume of gas inspired or expired during each respiratory cycle). After combining all the information presented above, we constructed a human breathing model. The signals that trigger the inhalation and exhalation processes are sinusoidal. The composition of the inhaled and exhaled air, as well as its amplitude, is changing according to a simplified model of the human metabolism, as follows:

1. The carbon dioxide output amount is determined by the oxygen input, multiplied by the respiratory quotient (RQ), considered to be 0.85
2. The oxygen amount exhaled is (1 − RQ) multiplied by the amount of oxygen inhaled
3. The amount of nitrogen exhaled is 0.94 of the amount of nitrogen inhaled
4. The exhalation breath is considered to be saturated with water vapor (100% relative humidity).

All the numbers above are based on the following Table, detailing the exhaled air composition of a human based on what (s)he inhaled [22].

Table 1 - Inhalation and exhalation components by partial pressure in a 760mmHg total pressure [22]

Activity	**Oxygen (mmHg)**	**Carbon Dioxide (mmHg)**	**Water (mmHg)**	**Nitrogen (mmHg)**	**Total (mmHg)**
Inhalation	158	0.3	5.7	596	760
Exhalation	116	32	47	565	760

The implementation of the human module is shown in Figure 9. We start with a sinusoidal signal that gives the frequency and amplitude of the breathing signal. The equation for this signal is:

$$B(t) = \left(\frac{A}{2}\right) sin\left[\left(\frac{2\pi}{3}\right)\cdot t\right]$$

where *B(t)* is the breathing signal, *A* is the amplitude of the inhalation and *t* is time. Then signal is routed to the breathing signal generator, which takes this trigger signal and splits it into a breathing out and breathing in waveforms. These are then fed to the human model, which, in function of the RQ and the gas composition in the mouthpiece inhalation, creates the exhaled gas and determines its composition, using Table 1. The human model needs memory as well, because the human exhalation composition is based on what was previously inhaled, so we need to remember this and feed it back to the human module to correctly calculate the exhalation gas composition.

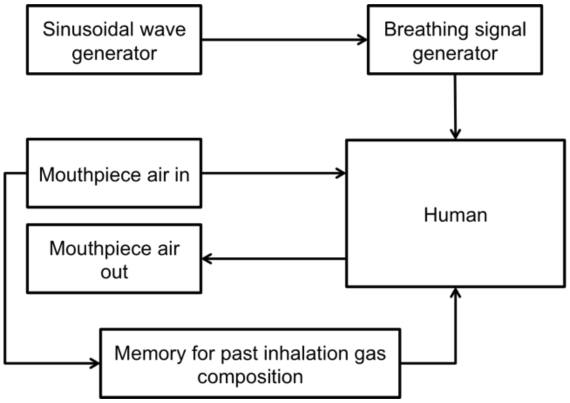


Figure 9 – The human module implemented in the rebreather analytical model

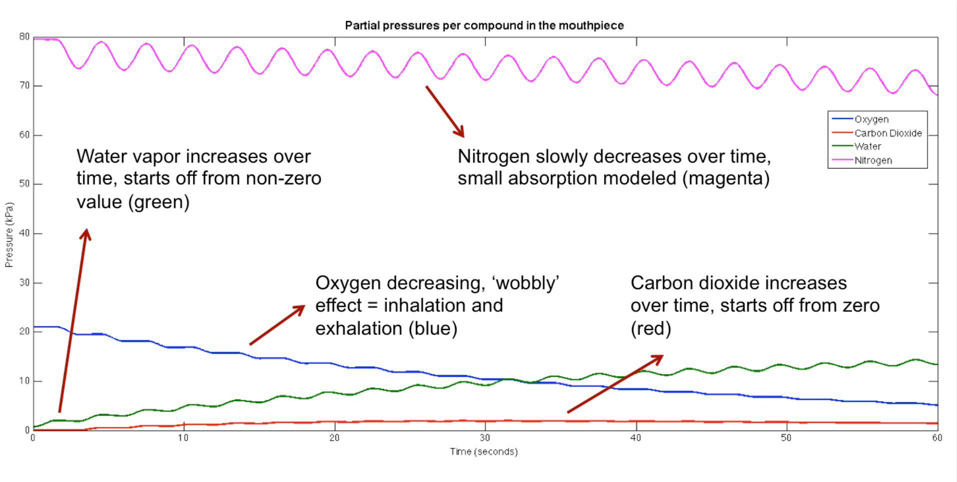
Other characteristics of the human breathing model we implemented in the analytical model are as follows:

- The output air composition and rate is adapted to the inhaled air composition and rate
- The sinusoidal breathing pattern is used to trigger the inhalation and exhalation processes, the rates and compositions are calculated as previously explained
- At each point, the model 'remembers' what was previously inhaled - the model has memory
- The human inhales air from the volume he lives in - the inhalation air composition is modified as the air in the mouthpiece inhalation is modified.

In order to validate that the calculations and the human breathing model, we conducted the following test: we modeled a mouthpiece of 10 liters and assumed the human breathes in it. No gas tanks or carbon dioxide scrubber were implemented at this point. This situation is illustrative for example of the case when a human breathes in a pocket of air under a capsized boat or of a human breathing in a similar air pocket that formed as a result of a mine disaster or avalanche. The air parameters monitored in the model are:

1. Air pressure [kPa]
2. Air temperature [degrees K]
3. Air mass flow rate [kg/second]
4. Air density [kg/m^3]
5. Air mass [kg]
6. Oxygen mass fraction [-]
7. Carbon dioxide mass fraction [-]
8. Water mass fraction [-]
9. Nitrogen mass fraction [-]

The air in the mouthpiece exhalation is assumed to be 78.24% nitrogen, 20.78% oxygen, 0.0394% carbon dioxide and 0.75% water ([22], Table 11-2). If we superimpose the limits presented in Section I.B, we obtain Figure 11. From this we can conclude that the biggest threat for humans living off a limited pocket of air is that they will quickly run out of oxygen, before the carbon dioxide concentration rises to a toxic level.



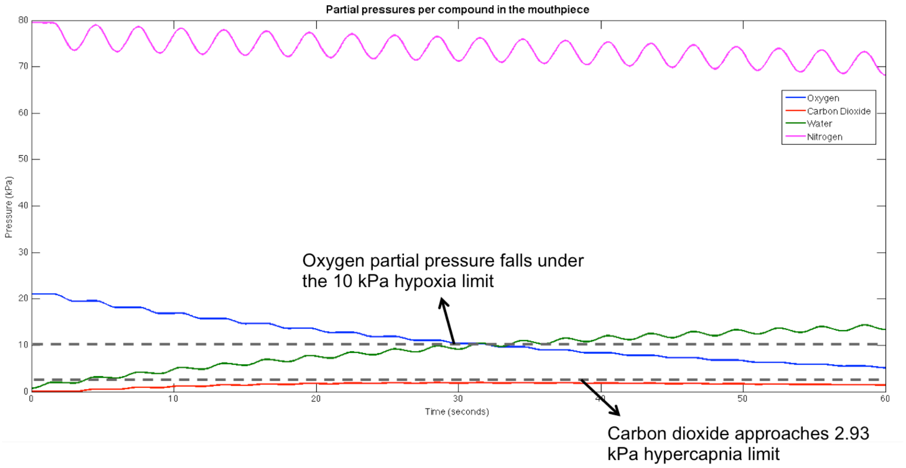
Water vapor increases over time, starts off from non-zero value (green)

Nitrogen slowly decreases over time, small absorption modeled (magenta)

Oxygen decreasing, 'wobbly' effect = inhalation and exhalation (blue)

Carbon dioxide increases over time, starts off from zero (red)

Figure 10 – The pressure evolution in the mouthpiece for different gas components for constant volume breathing in a fixed volume bag



Oxygen partial pressure falls under the 10 kPa hypoxia limit

Carbon dioxide approaches 2.93 kPa hypercapnia limit

Figure 11 – Pressure trend in the mouthpiece for a 10 liter confined volume: figure shows the hypoxia and hypercapnia limits

B. Analytical model structure

The computer model is constructed as a connection of modules, as shown in Figure 12, based on the rebreather system composition from the US Navy diver's manual [14] for the MK16 MOD0. The requirements of this analytical model are the following:

- The model shall monitor O_2, CO_2, H_2O concentrations in the air the human is breathing
- The model shall monitor air pressure and temperature
- Modular design (each system shall be represented in code as a block) in order to reconfigure the model as desired
- A block or module shall be added or removed in less than 1 minute
- Fast simulation time and data processing - for the purpose of this model, a dive time of 4 hours shall be simulated in less than 15 minutes and the results shall be ready for further processing in under 10 minutes
- The model shall capture the system dynamics within 20% of accuracy compared to empirical time series data.

The programing language was chosen as Matlab Simulink, due to Simulink's ability to satisfy all the above requirements. The Simulink representation of the MK16 rebreather is shown in Figure 12. The human breathing model is enclosed in the human block, its output and input are air compositions for the exhaled, respectively the inhaled air. The exhaled air (shown with a red arrow) is forwarded to the mouthpiece. The carbon dioxide-laden air is then passed through the carbon dioxide scrubber, which absorbs the carbon dioxide from the air and then transmits it to the diaphragm. This is the point at which the air is reconditioned: the partial pressures of the oxygen and nitrogen are computed and maintained at healthy limits for the diver, according to the depth at which (s)he is operating. From here, the air is inhaled from the diver and, after the metabolic processes have been completed, this air is converted into exhaled air and this process is repeated. The model also contains gas tanks, which store the oxygen and nitrogen gases used to makeup the air for the diver. The electronic control and monitoring circuits contain sensors for measuring the partial pressures of oxygen and nitrogen, control circuits for determining how much gas mass has to be added/subtracted in order for the required partial pressure of gas to be reached or maintained, and valves that are actuated by these circuits for physical addition of the gas from their corresponding tanks. The diluent considered in this analytical model is nitrogen. Nitrox is used for diving up to 40 meters depth, so the analytical model can simulate only these scenarios. For depths greater than 40 meters, the diluent gas used is heliox (helium and oxygen).

The orange and red arrows in Figure 12 represents an airway. The red arrows indicate air rich with carbon dioxide, typically coming from the diver's exhalation, and the orange arrows represent conditioned air, ready for the diver to inhale. Each such air way is encoded in Matlab Simulink as a vector, with the following components:

1. Air temperature [K]
2. Air pressure [kPa]
3. Air density [kg/m^3]
4. Air mass [kg]
5. Oxygen mass fraction [-]
6. Carbon dioxide mass fraction [-]
7. Nitrogen mass fraction [-]
8. Water mass fraction [-]
9. Air volume [m^3]

Note that the composition of the air vector contains redundant elements. This is used to check the conservation of mass throughout the simulation through sum of pressures, sum of masses and sum of mass fractions.

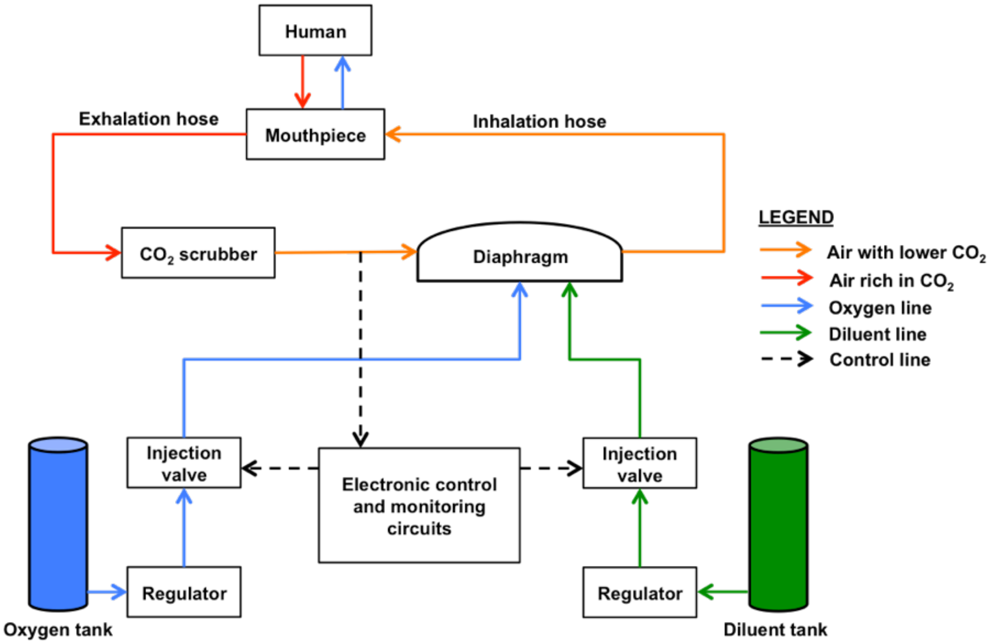


Figure 12 - The structure of the US Navy MK16 MOD0 rebreather analytical model

The problems we are exploring are:
1. How to maximize the oxygen tank endurance?
2. What configurations of gas tank sizes and gas control strategies can be used to meet that duration?

In previous literature and in real life, the objective is usually to maximize the endurance of the oxygen tank, so this is the gas that we will consider to maximize in this problem. The diluent consumption rate is relatively small compared to the oxygen consumption rate and mostly used during the descent stage.

The first problem can be formulated as a discrete optimization problem: the objective function *J(x,p)* is the endurance of the oxygen tank, the parameters *p* are the diver's body type (height, weight, age and metabolic rate and diving depth). The design variable *x* is the control type applied to the rebreather and this is a discrete variable. The constraint *g(x,p)* of this problem are the available oxygen mass in the tank at the beginning of the dive. The mathematical formulation of the problem is given below.

$$\begin{gathered}\min J(x,p)\\ \text{such that } g(x, p) \leq m_{\text{initial oxygen}}\\ \text{where } x = \text{control type, design variable and } p \text{ model parameters}\end{gathered}$$

where the control type can be : open loop, Constant Mass Injection (CMI), bang- bang, Proportional - Integral - Derivative (PID) and the model parameters are height, weight, age, metabolic rate and diving depth.

The second problem is derived from the first problem: we set a specified mission duration and explore the hardware combinations that can satisfy this.

III. Baseline case study – the MK16 rebreather

The MK16 MOD0 is a constant partial pressure, closed circuit mixed gas under- water breathing apparatus (UBA), used by the Naval Special Warfare (SPECWAR) forces. This UBA combines the advantages of a free-swimming diver with the depth advantages given by mixed gas. The maximum working limits for the MK16 MOD0 UBA are 150 feet seawater (fsw) (45.72 meters) when N_2O_2 (air) is used as a diluent, and 200 fsw (60.96 meters) when 84/16 HeO_2 mix is used [14]. Figure 13 shows the assembled UBA [14]. The purpose of this research project is to build an analytical model of the system shown in Figure 13, so we will decompose this system according to its subsystems and the functions that they perform. Figure 13 shows an Object Process Methodology (OPM) [21] decomposition of the UBA. OPM is a language similar to SysML, which allows the functions and the systems that perform those functions to be represented in the same environment [58]. The functions are represented by ellipses with a blue contour, and systems by rectangles with a green contour. The black triangle indicates a decomposition, while the lollipop-looking symbol represents a system that enables a particular function.

Figure 13 – The MK16 MOD0 closed circuit rebreather

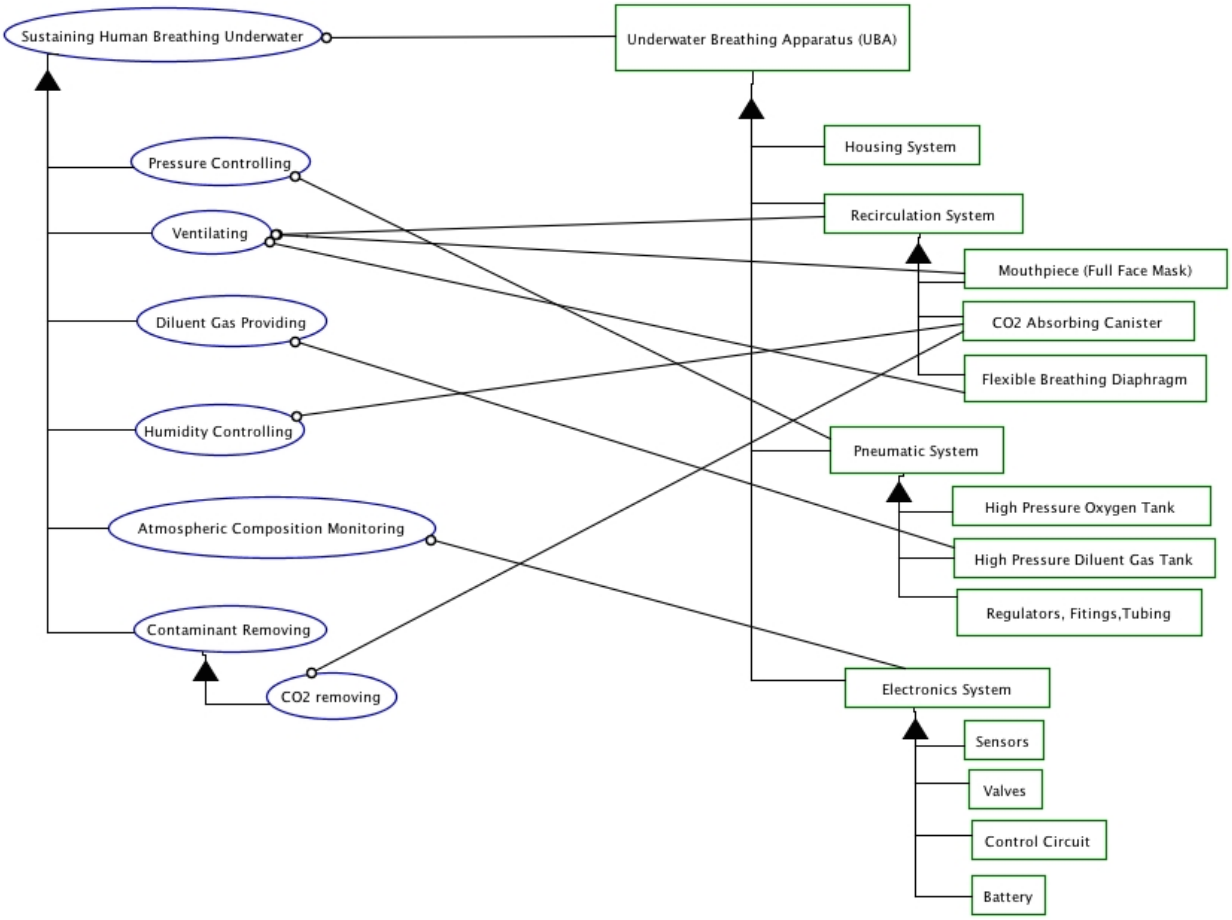


Figure 14 – Object Process Methodology (OPM) functional decomposition of the MK16 MOD0 closed circuit rebreather

The assumptions for this model are the following:

- The breathable air is made up of an inert gas (nitrogen in this case) and oxygen
- The breathing gases behave as a perfect mixture and their volumes and partial pressures are computed using the ideal gas law
- the recipient internal pressure instantaneously equals the external pressure
- The respiration is modeled by a partial oxygen subtraction from the collapsible bag (counterlung)

- The carbon dioxide scrubber behaves ideally, capturing all produced carbon dioxide [54]
- The temperature of the breathing gas is not influenced by the water temperature the diver is in (this is because the carbon dioxide reaction is exothermal and it warms up the breathing gas)
- The metabolic rate and depth profiles for a typical dive are shown in Figure 15 and the simulations of the rebreather using these profiles are shown in the next section.
- The mouthpiece total volume is 110 mL, this is split evenly between the mouthpiece for exhalation and inhalation [14]
- The diaphragm volume is 10 liters [14]
- The gas tanks have a wet volume of 175 cubic inches, are rated for 3015 psia [14]
- The breathing hoses volumes are included in the diaphragm volume and they are not considered to be corrugated
- The diameter of the breathing hoses is not considered in the model. This can be easily adapted by adding another module to the analytical model to represent the hoses and then subtract their volume from the diaphragm
- The diaphragm is modeled as having variable volume, dependent on the gas it contains. In the real rebreather implementation, the diaphragm is modeled as elastic in order to reduce the breathing resistance.

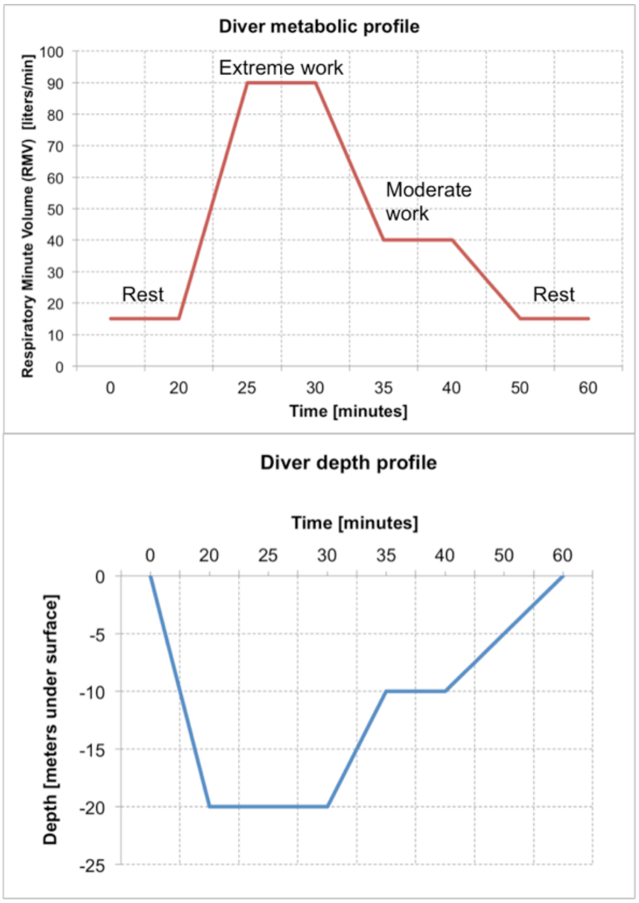


Figure 15 – The metabolic rate and depth profiles for a typical dive

Strategies for maximizing oxygen duration

Maximizing the endurance of oxygen is the key to longer dives. Possible solutions to this problem, studied since the 1980s, are best described in a paper by Nuckols, Gavin and Finlayson [52], and are presented in this section. There are four main design approaches for the control of the injection of oxygen in the breathing loop:

1. **Constant Mass Injection (CMI) systems** – use one or more critical flow orifices to maintain a constant rate of makeup gas into the breathing loop. The rate of gases injected corresponds to the deepest point the diver would get to and at max activity levels. The advantages of this method are that it conserves breathing gas better than open circuits and, if it is used for inert gas injection, it makes deeper dives possible. However, due to the high injection rate and the fact that it is not coupled to the diver's breathing rate, the circuit may experience considerable variations in oxygen partial pressures and in inert gas pressures, leading to risk of hypoxia at great depths and oxygen toxicity at shallow depths.
2. **Depth dependent injection systems (DDI)** – these systems vary the gas injection rate based on depth, so it counteracts great variation in gases and results in safe breathing mixtures at any depth. On the flipside, due to the fact that the gas consumption increases at depth, this leads to a high injection rate in those conditions.
3. **Constant volume injection systems (CVI)** – these contain a breathing circuit that alternates between closed and, during fresh gas injection and flushing, semi-closed. The advantage of these systems is that they are coupled to the diver's respiratory rate: the diver exhales, fills a bellow, which is then vented and refilled with fresh gas.
4. **Variable volume exhaust systems (VVE)** – couples the injection of fresh gas with the diver's respiration rate (based on the fact that a diver's respiration rate is coupled with his metabolic oxygen requirement). Uses a mechanical coupling device with the counter-lung to control the rate at which gas is exhausted from the loop. The circuit volume is controlled by dumping carbon dioxide as the diver exhales. Oxygen is added only when the gas volume decreases and is not enough to fill the diver's lungs. The advantages of these systems are that the rate of gas dumped is correlated with the diver's respiratory rate, this coupled injection system reduces oxygen level variability as diver activity levels change and that this is the most desirable injection levels for oxygen from the four evaluated systems. On the downside, using this system leads to a high oxygen consumption rate.

From these four systems we chose to implement the first one, Constant Mass Injection (CMI) in our model and compare its performance with three other control types, detailed in the following:

- **Open loop control** - this type of control is typical of scuba diving equipment, where the diver inhales air from the tank and expels it in the surrounding environment
- **Constant Mass Injection** - detailed above
- **Bang-bang control** - also known as an on-off or a hysteresis controller, this is a feedback controller that switches abruptly between two states. For example, most residential temperature control systems contain bang-bang controllers. In optimal control problems, it is sometimes the case that a control is restricted between a lower and an upper bound, like in the case of the rebreather. They are often implemented because of their simplicity and convenience. If the process value is greater than the set point + hysteresis then the controller output is set to 1 (ON state). If the process value is less than the set point - hysteresis then the controller is set to 0 (OFF state). If the process value is between the (set point + hysteresis) and the (set point - hysteresis) then the controller output is maintained at the same level as before.
- **Proportional - Integral – Derivative (PID) control** - this type of control is a feedback loop controller widely used in industrial systems. A PID controller calculates the error between the process value and a desired set point, and then attempts to minimize the error by adjusting the process through a set of manipulated variables. This control algorithm involves three separate control parameters: P, the proportional term which depends on the present error, I, the integral term, depending on the accumulation of past errors, and D is a predictive term of future errors, based on the current rate of change. The weighted sum of these three actions is used to adjust the process via the control element such as a gas valve for the rebreather case.

We studied the implementation of these various control methods and how they impact the endurance of the oxygen tank. We fixed the depth of the dive to surface level and the diver metabolic rate to extreme work (we will vary these in Section IV), and observed how varying the control type affects the duration of the oxygen tank. The results of this analysis are shown in Figure 16. The endurance of the oxygen canister given in this graph is 40 minutes because the diver metabolic rate is extreme work only. In a normal dive, the profile of the diver's activity is a combination of rest, moderate and extreme work, not only extreme work, therefore we expect that in a normal dive, the oxygen canister will last longer than this, this case is a worst case usage scenario. Also, when the depth profile is varied as shown in Figure 15, nitrogen is used as a diluent and this fact will additionally extend the duration of the oxygen tank.

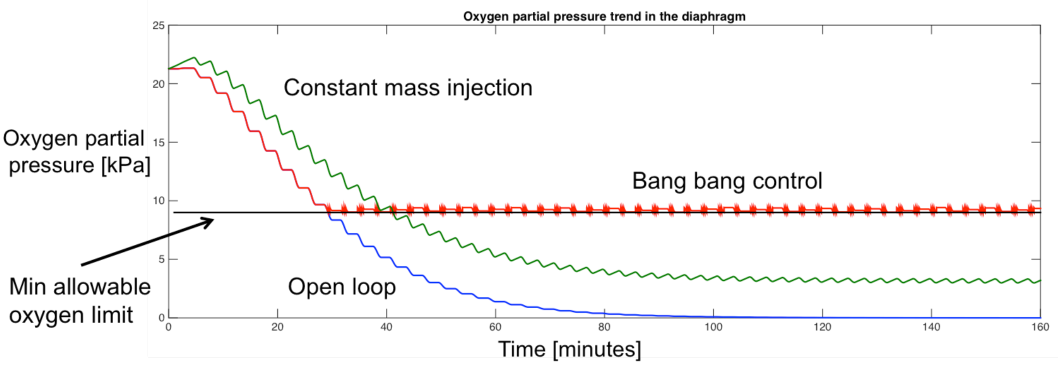


Figure 16 – Comparison of open-loop, bang-bang (electronic) and constant mass injection (mechanical) control on a surface level, extreme work metabolic rate dive

We can conclude from Figure 16 that fixed mass or volume injection systems will consume the oxygen tank faster than systems that are coupled with the diver's breathing rate, represented in Figure 16 by the bang-bang control (the oxygen amount that will be replenished is equal to what was metabolically consumed by the diver). Therefore, we will focus next on systems that adapt the amount of oxygen replenished on the diver's metabolic consumption. We considered the implementation of a PID controller and compared its performance with the bang-bang control. The schematic of the control setup is shown in Figure 17.

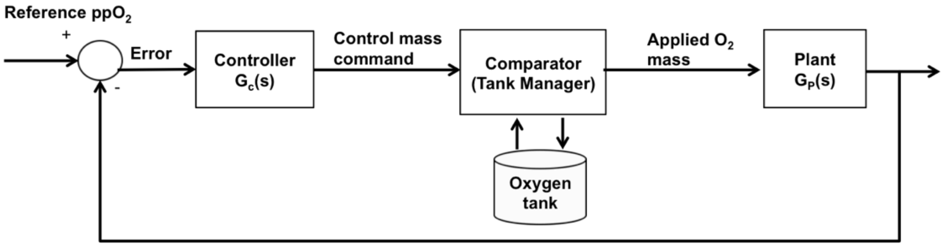


Figure 17 – The schematic of the partial pressure of oxygen in the diaphragm control problem

The PID controlled plant has a duration of the oxygen tanks of 18 hours. This high value is explained by the fact that the PID controller assumes a very good model of the plant and leads to an optimal control type, which realistically is not the case. There is a lot of uncertainty in the real human breathing model that has not been captured in this mid-fidelity analysis of the system.

Figure 18 presents a classification of control types of the human breathing model in function of the controller complexity. In the past, people used to implement bang-bang or PID controllers because the hardware at that time did not allow more complicated control systems to be implemented. But with the growth of computer systems, any control policy is now implementable [59]. Therefore, control complexity has become a crucial issue in control theory research [60]. Complexity is defined as the degree to which a process is difficult to analyze, explain or understand. Cardoso [61] defines it as the number and intricacy of activity interfaces, transitions, conditional and parallel branches, as well as the existence of loops. In order to measure this controller complexity, we have two choices:

- Count and sum the number of branch splits [61], also called McCabe's cyclomatic complexity [62]
- The order of the controller [63]. In this paper, we use the order of the controller as a measure of controller complexity.

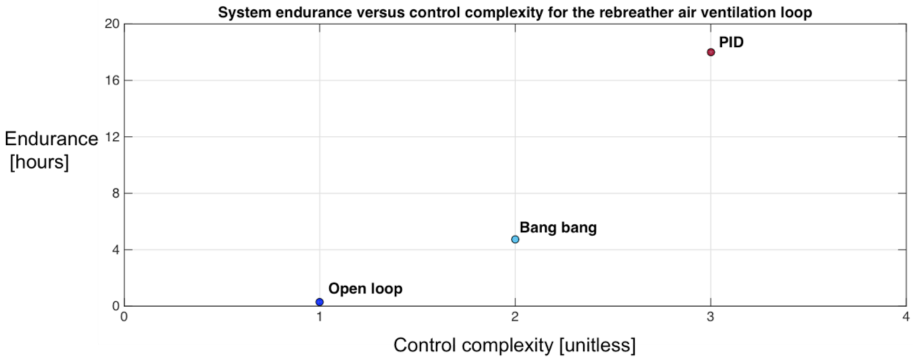


Figure 18 - Oxygen tank endurance for various control types applied to the human breathing model

If we refer back to the problem described in section II, we have solved the first problem: how to maximize the oxygen tank duration. Figure 18 shows the endurance of the oxygen tank in function of different controller types, classified in function of their complexity, as explained above. Out of all the control types implemented, we see the PID controller maximizes the oxygen tank endurance. This result is also shown in Figure 20. We can see that better control on the oxygen tank can improve the system's endurance from a typical 170 minutes to 1000 minutes.

In order to solve the second problem, we will set the endurance of the oxygen tank to 12 hours and, given the two control options we have implemented, derive a combination of control strategies and tank sizes that would enable the rebreather system to attain that set endurance value.

We see that we have two such combinations: we can choose PID control and use an oxygen tank of 1.3 kg or bang-bang control and a tank of 5.1 kg. With PID control, we have achieved a 74% reduction in tank size for the same mission endurance. This goes to demonstrate the fact that, using adequate electronic control, the resource endurance can be extended far beyond what mechanical control (constant mass injection systems, see Figure 16) can.

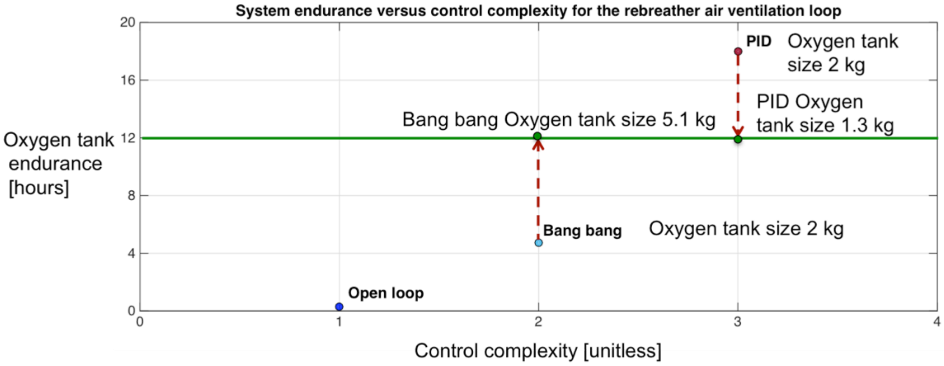


Figure 19 – Oxygen tank endurance for various control types applied to the human breathing model

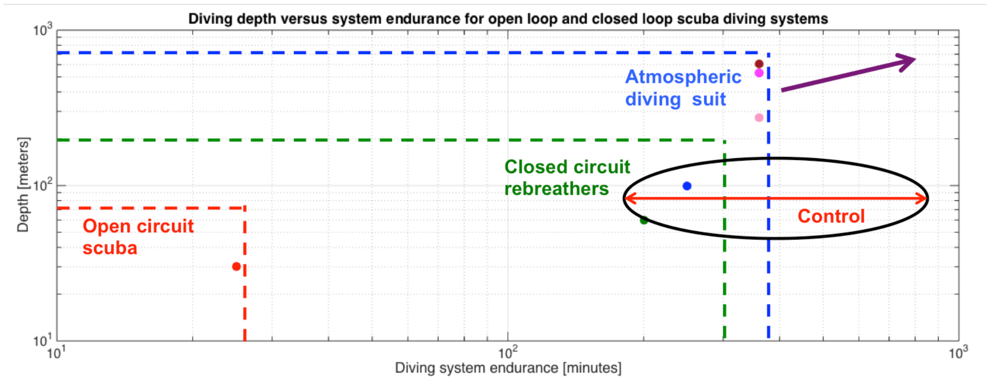


Figure 20 – Control can extend the endurance of closed-circuit rebreather systems: 1000 minutes instead of the typical 170 minutes

IV. Advanced applications of the MK16 rebreather model

We can now use the model constructed in the previous sections to simulate realistic dive scenarios. This is useful to understand how much gas will be consumed during a dive and will help plan it. We will first simulate a surface-level depth and vary the diver's metabolic profile between rest and extreme work, and then also vary the depth of the dive and observe the simulation results.

A. Variable depth and metabolic rate

The first step to generalize the model built in the previous sections is to incorporate and simulate a variable metabolic rate of the diver. We do this by varying the amplitude of the respiratory signal and keeping the respiratory frequency constant.

Figure 21 shows the effect of varying the diver metabolic rate on the oxygen tank pressure and the way the total diaphragm pressure behaves as a result of this variation: on the top of this Figure we can see how the amplitude of the breathing signal changes to emulate a rest metabolic rate (for time from 0 to 20 minutes), then progress to moderate work and then extreme work (from 25 to 35 minutes) and then reversing this profile until rest (from 50 to 60 minutes). The middle graph shows the pressure of the oxygen tank. We can see that the slope of the pressure changes, indicating a higher depletion rate at extreme work regimes and a smaller one when the diver is resting. The total diaphragm pressure is not affected by this process and this is the result of control on the oxygen and nitrogen partial pressures. The decomposition of the total diaphragm pressure shown in the last graph in Figure 21 is presented in Figure 22. The oxygen controller kicks in when the oxygen partial pressure drops below 10 kPa and the diver would be in hypoxic danger. The nitrogen controller is active all the time and keeps the nitrogen partial pressure around 80 kPa, which is nominal atmospheric level and is adequate for the depth level at which we are performing this simulation (surface depth). We can observe the transients of nitrogen in the last graph in Figure 22, and they quickly die down and the controller maintains the nominal gas partial pressure.

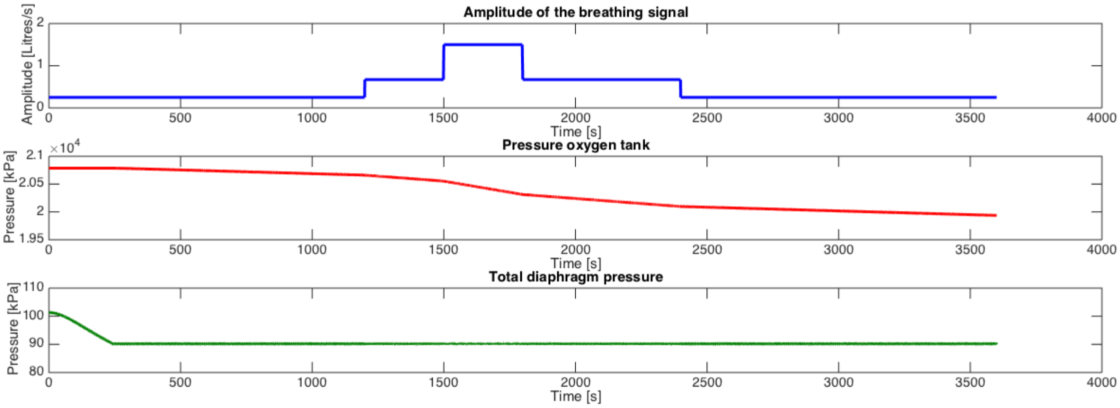


Figure 21 – Effect of varying the diver metabolic rate on the oxygen tank pressure (we see changes in slope: steepest slope corresponds to extreme work regime, shallowest to rest) and the overall diaphragm pressure (maintained despite metabolic rate variations)

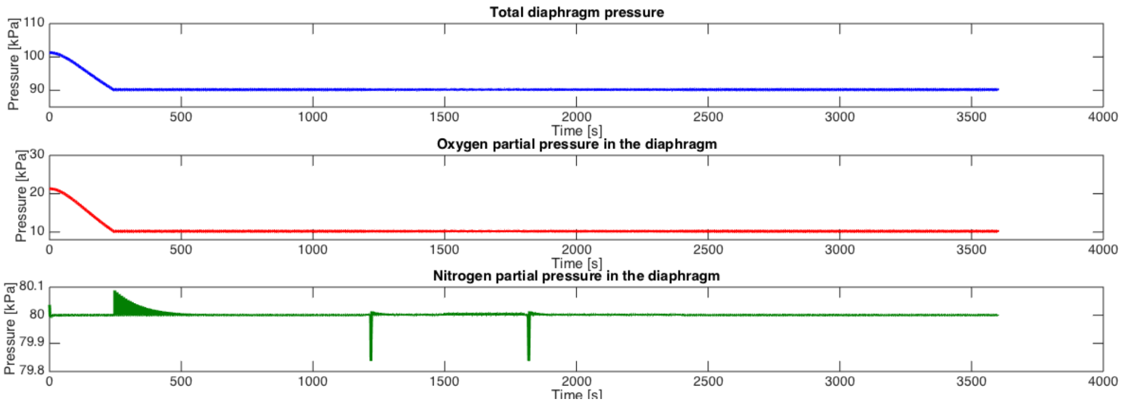


Figure 22 – Decomposition of total diaphragm pressure from Figure 21 in major constituent partial pressures: oxygen and nitrogen

The next simulation we performed was varying the depth profile, while keeping the metabolic rate at a moderate work metabolic level. Both the oxygen and the nitrogen partial pressures in the diaphragm are controlled with a bang-bang control type: the partial pressure for oxygen is limited at 20% the total diaphragm pressure, and for nitrogen this limit is 70%. The bang-bang controllers are set to maintain these limits, with a tolerance of +/- 10%.

While the oxygen tank is depleting constantly, to replenish the oxygen metabolized by the diver, the nitrogen is used much sparingly and typically during the descent phase of the dive. It is for these reasons that the nitrogen tank is usually smaller than the oxygen tank. If, however, the tanks are the same capacity, then it is expected that the nitrogen tank will last several dives. In most diver rebreather accidents, there are cases in which the divers have endangered their lives by not monitoring the nitrogen tank pressure and performing several descents and ascents.

Figure 22 shows the pressure adaptation by the rebreather during a constant metabolic rate dive (set at moderate work), for a variable diving depth (shown in Figure 15).

When we keep the metabolic rate fixed and vary the depth of the dive, we get the results shown in Figure 23. The controllers are tasked to maintain the required partial pressures of gases in the diaphragm, according to the external pressure and the control laws described above. The break-down of the diaphragm total pressure in oxygen and nitrogen partial pressures is illustrated in Figure 23.

Figure 24 shows how the oxygen and nitrogen tanks are used as a function of the depth of the dive. We can see that, as the external pressure increases, the oxygen tank depletion slope increases, showing an increased oxygen flow rate to the diaphragm. The nitrogen tank is used to make up the rest of the required atmosphere composition for that specific depth and it is only used during the descent phase of the dive.

After simulating the model with a variable breathing rate and then, separately, at a varied dive profile, we unify these and presents the model's adaptation to a variable metabolic rate and at a variable depth profile. The amplitude of the breathing signal and the external pressure at that depth are shown in the top part of Figure 25, while the bottom part of this Figure illustrates the total diaphragm pressure and its decomposition in the oxygen and nitrogen partial pressures.

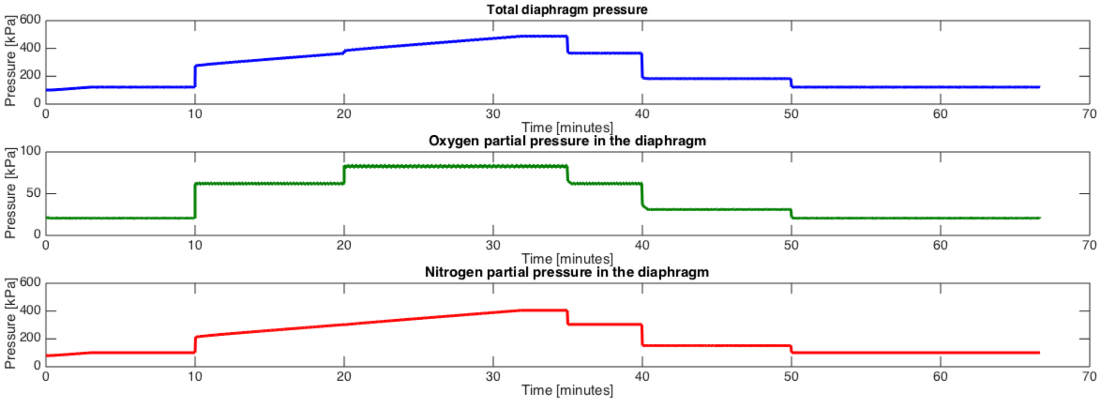


Figure 23 - The decomposition of diaphragm total pressure in oxygen and nitrogen partial pressures during the dive profile shown in Figure 15

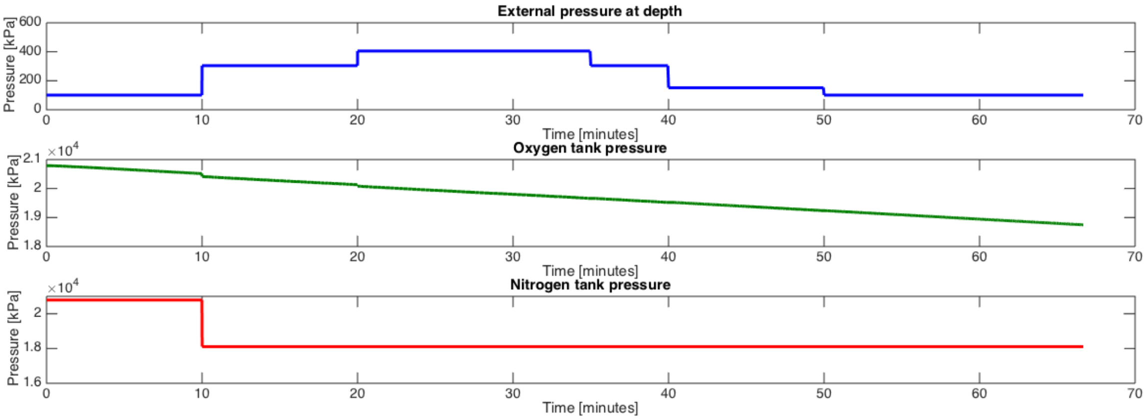


Figure 24 – The usage of the oxygen and nitrogen tanks as a function of the depth of the dive

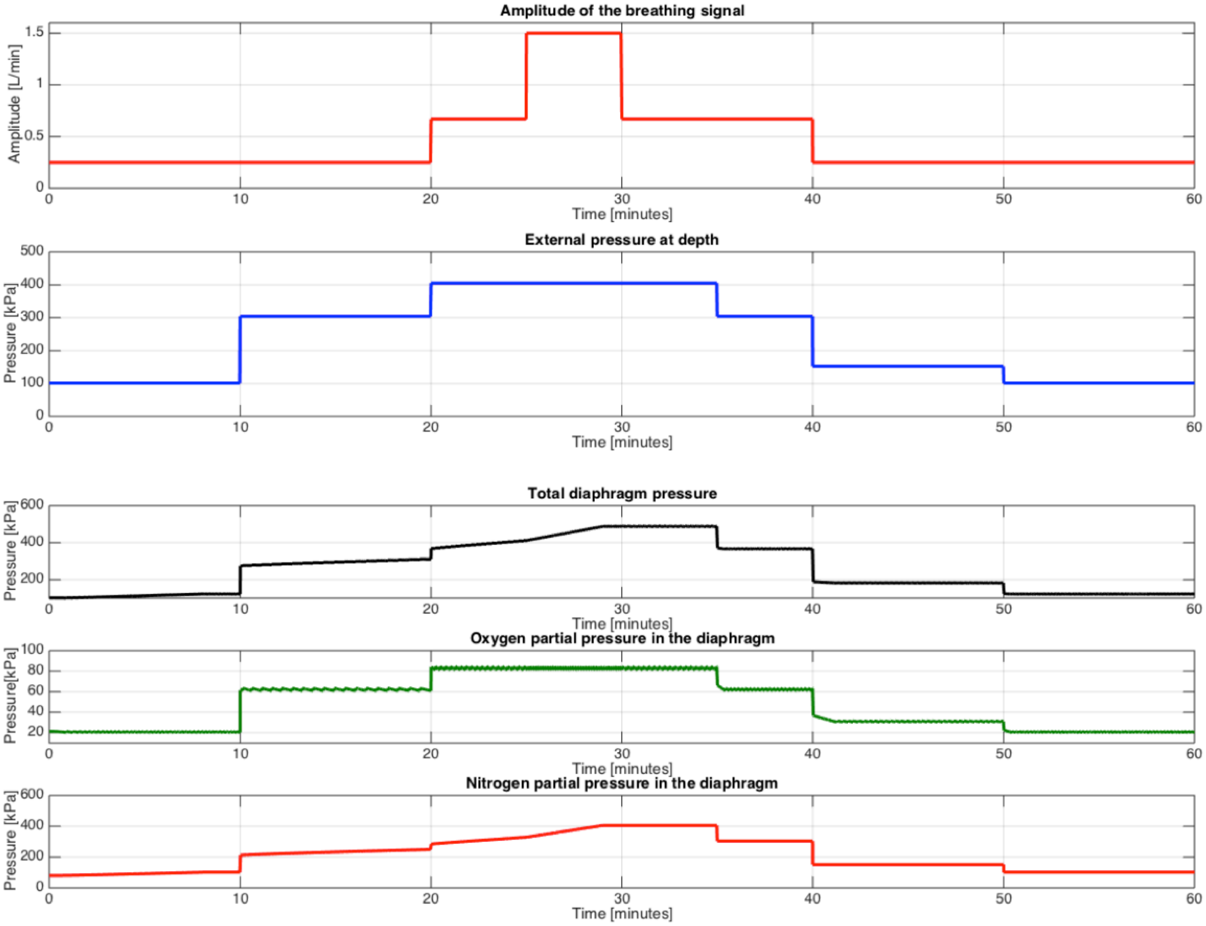


Figure 25 - Diaphragm pressure adaptation shown in a typical diving profile: variable metabolic rate and variable depth

B. Simulations of typical rebreather accidents

Closed-circuit rebreathers (CCRs) are, due to their complexity, more prone to accidents than open-circuit (OC) scuba. In his analysis of mechanical failure risk in the Wakulla Springs project, Stone calculated that the risks of purely mechanical failures for CCR result in a theoretical risk increase of failure of 23 times compared to OC scuba configurations [39]. Therefore, the assumption that the CCRs are less mechanically reliable than OC scubas is considered true and this is why most CCR divers carry OC scuba cylinders for ’bailout’ during their dives, in case of CCR failure. When the presence of a redundant scuba is included in the failure risk calculations and compared to an OC scuba diver conducting a decompression dive with two decompression gasses, the overall risk of mission critical equipment failure becomes similar.

Between 1998 and 2010, 181 deaths were recorded in the Deeplife database, associated with rebreather accidents. There was a peak of 24 deaths in 2005, prior to that deaths have averaged 8 per year, after that they averaged 20 per year, as seen in Figure 26. Of the total of 181 deaths, 31.5% had insufficient data to form any conclusions, 44% were attributed to equipment-related problems, 24% to diving-related problems and the remainder were a mixture of problems such as acute myocardial infarction, loss of consciousness from diabetes mellitus etc. In the BSAC mortality data for OC diving, 13 cases were caused by equipment failure and in 36 cases the divers ran out of gas. Despite the perceived relative simplicity of the OC equipment, almost 9% of deaths were caused by equipment failure. The Deeplife database indicates that this number increases to 30% in the case of CCR.

This section shows how the developed analytical model presented in section II can be used to simulate the rebreather’s response to common rebreather accidents. We have identified three main failure modes that lead to rebreather accidents:

- **Oxygen supply failure** – the reasons for this would be oxygen valves not turned on, control electronics not turned on, oxygen sensors incorrectly installed, oxy- gen valve partly blocked or damaged sensors
- **Running out of air** – there are two separate scenarios here worth considering:
    - Beginning the dive with a low starting pressure or smaller gas cylinders than required for the dive
    - Diver overexertion due to rough seas, trauma, buoyancy issues, inexperience, diving deeper than usual
- **Carbon dioxide scrubber faults** - this can lead to the scrubber entirely or partly not working, for example when the active ingredient is almost consumed, the scrubber’s efficiency drops dramatically or when the scrubber material is not properly packed or sealed.

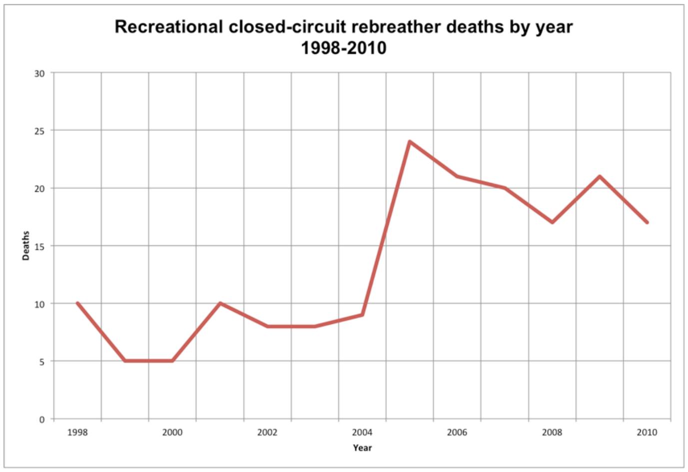


Figure 26 – Recreational closed-circuit rebreather deaths by year 1998-2010

1. Oxygen supply failure

Mass from the oxygen tank is introduced into the diaphragm when the oxygen partial pressure drops under 10 kPa, as indicated in section I. The nominal pressure of the oxygen tank is 3015 psi and it contains 0.8 kg of gas. Figure 27 shows the normal functionality of the oxygen tank: it injects mass to the diaphragm to maintain the oxygen partial pressure to the set limit. The top graph of Figure 27 shows the decrease of oxygen tank pressure as mass is consumed from it, the middle part of this Figure gives the mass of oxygen that is injected from the tank (it is constant because the injection system is making up for the oxygen that the diver consumes through his metabolism, which is constant since the work regime is extreme work and the depth is constant).

Figure 28 shows a tank failure 3 minutes into the dive. This is consistent with accident reports, which describe cases of dives where the problems occur due to equipment related failure: failure to monitor oxygen gauge to see that the partial pressure in the diaphragm was decreasing, hoses disconnection, leaky tank, valve failure, bad oxygen sensor, non-tight hose seal or inexperienced diver [36, 37, 38]. The time the diver has to react to correct the problem decreases as the depth dive increases, Figure 28 shows a tank failure at surface depth, in which situation the diver has less than 30 seconds to attempt a rescue.

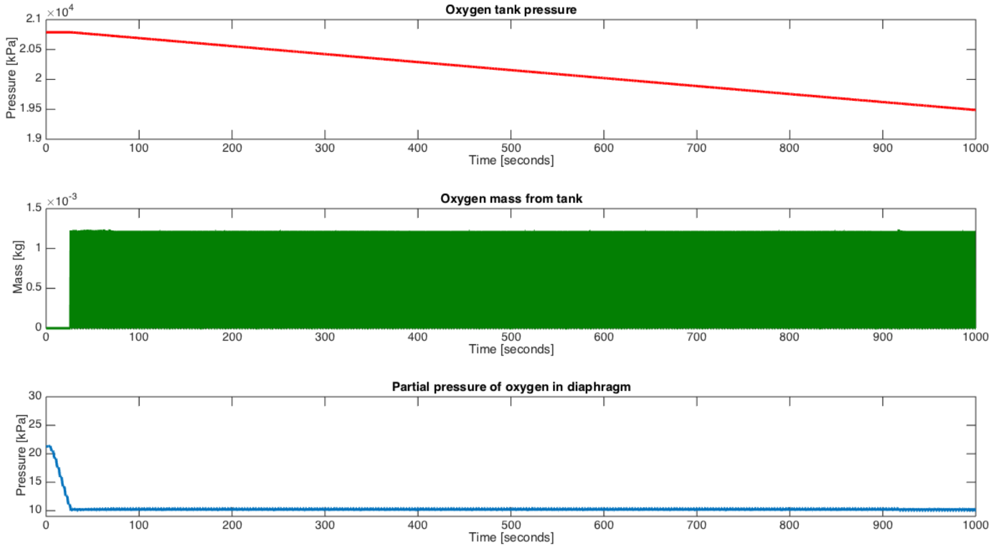


Figure 27 – Oxygen tank nominal functionality: it injects mass into the diaphragm to maintain the oxygen partial pressure to the set limit

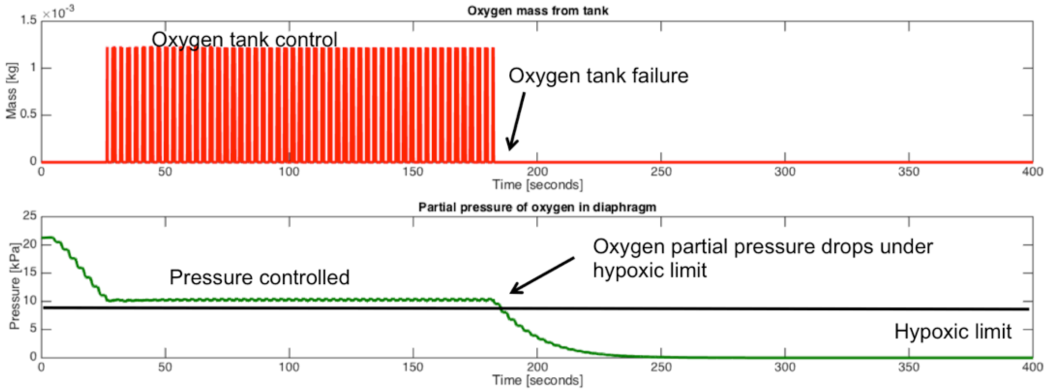


Figure 28 – Oxygen tank failure

2. Insufficient amount of air

The main factor that leads to diving accidents when the air was not enough for a dive is that the diver had a low tank pressure at the start of the dive and failed to notice this. The analytical tool we developed in this paper can simulate this scenario: Figure 29 shows the simulation results comparing a dive with a nominal tank starting pressure and one when the diver started with a tank at one-tenth of the nominal pressure. We can see that the partial pressure in the diaphragm is maintained for roughly ten times longer when the oxygen tank is at the nominal pressure (graph on top) and the tank discharge (on the bottom).

Another cause for accidents of this type are successive descent - ascent trips with- out oxygen pressure monitoring or the diver working at a higher metabolic rate then originally planned for the mission. The graphs shown in Figure 29 are generated when the diver was performing at extreme working conditions throughout the mission and therefore illustrate a worst case scenario in which the air quickly becomes insufficient. If the diver observes the low oxygen partial pressure and reduces his/her work regime, the time it would take to run out of air would be extended to a point where, hopefully, the diver has had enough time to correct this issue.

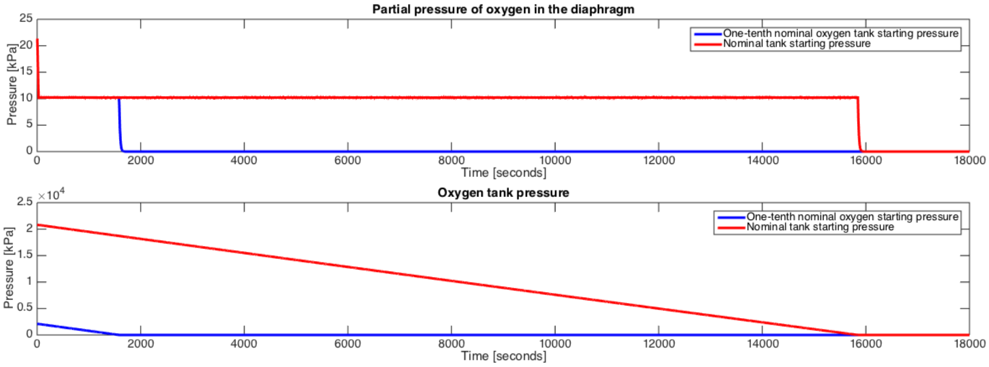


Figure 29 – Low oxygen tank starting pressure

3. *Carbon dioxide scrubber faults*

The carbon dioxide scrubber is a another important failure point in the rebreather configuration. Loose connections to or from the scrubber, a lower operating temperature than expected, the scrubber not being replaced or serviced on time can all lead to failures of the carbon dioxide scrubber. We have modeled two possible failures for the scrubber: Figure 30 shows total scrubber failure, in which the air in the scrubber has the same carbon dioxide concentration as the air entering the scrubber and no carbon dioxide is removed. We can see that the hypercapnia limit is quickly exceeded, causing the diver to experience symptoms ranging from headaches, confusion, lethargy to disorientation, panic, hyperventilation, convulsions, unconsciousness and eventually death. When the scrubber is not working, the carbon dioxide concentration in the diaphragm increases up to about 90 kPa, when it starts to level off. The reason for this is that the amount of oxygen in the diaphragm is controlled to 10 kPa and so the diver, through metabolic processes, will exhale a constant carbon dioxide amount, equal to (1-RQ) times the inhaled amount of oxygen, which is kept constant. The concentration of carbon dioxide in the diaphragm does not keep on increasing because the human was modeled as inhaling a fixed volume of air from the diaphragm. When the partial pressure of carbon dioxide in the diaphragm increases, the breaths that the human inhales contain increasing amounts of carbon dioxide which dissolve in the blood and are also exhaled as a result of metabolic processes. This leads in the end to a leveling of carbon dioxide in the diaphragm air contents. In a nutshell, the human acts as a carbon dioxide filter up to the point when the carbon dioxide partial pressure in the diaphragm is maintained constant (and much above the hypercapnia limit, so if this situation were to occur during a real dive, the diver would be severely incapacitated after maximum one minute in the dive).

The second scenario we modeled in the simulation was a scrubber working at 30% of its capacity. The results of this are shown in Figure 31. After transient effects when the carbon dioxide partial pressure in the diaphragm increases to a level at which very mild hypercapnia effects can be observed, the scrubber manages to create a stable operating level for the partial pressure of carbon dioxide and maintain it at an elevated but not dangerous level of 1 kPa (hypercapnia begins to be observed at 2.63 kPa). The spikes in the middle Figure are transient effects and show the efforts of the carbon dioxide scrubber's attempts to stabilize the output carbon dioxide mass, once a stable operating regime has been attained, the amplitude of these spikes is dramatically decreased.

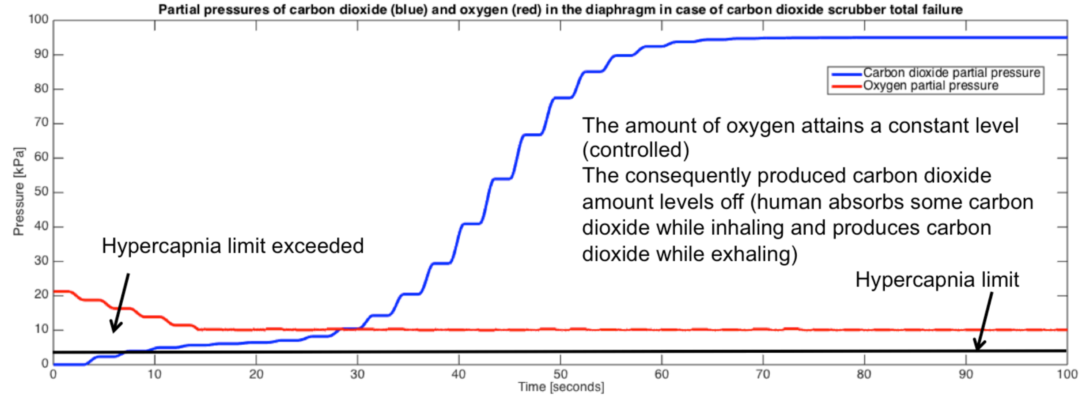


Figure 30 – Scrubber failure

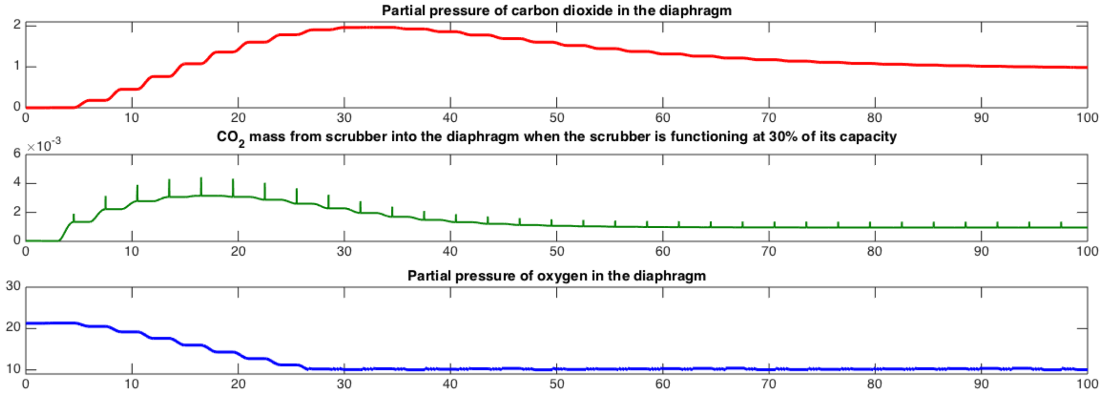


Figure 31 – Carbon dioxide scrubber low yield: simulated at 30%

V. Conclusions and future work

This paper presented the development of an analytical rebreather model, which is the system that recirculates and conditions the air the diver is breathing during a dive. The capability of simulating rebreather performance is currently lacking in the diving commercial and military industry. We believe that the advantages of having such a model are multi-fold: it can be used for mission planning, evaluating the impact of adding a new technology or modifying an existing parameters or operational regime on an hardware configuration without performing expensive and time consuming hardware tests. An analytical model, like the one developed in this paper, can also be used in complement with hardware testing to fine tune systems and increase resource endurance through the application of different electronic control strategies.

The developed Matlab/Simulink model of this rebreather is modular and can be generalized to study open, semi-closed or closed circuits, in which the breathing gas used is air, oxygen, nitrox or heliox. The system's operational environment can be the ocean's surface (1 atmosphere), space (less than 1 atmosphere pressure) or deep underwater (more than 1 atmosphere pressure). After introducing the analytical modeling process for the rebreather, this paper goes on to explore the model's applications for the study of different oxygen control strategies in order to maximize the oxygen lifetime during a dive, as well as the model's applicability as an aid in accident investigations.

We aim to determine what is the maximum endurance of a rebreather system, given a particular, set configuration of components, as well as to study the reverse problem: if we set a mission endurance, what architectures would be able to achieve this level? Additionally, we are interested in studying how the tradespace of diving depth versus the diving systems' endurance looks like and how more complex control methods can help in pushing the existent boundary toward higher endurance limits.

We show that more complex control algorithms can extend the duration of the oxygen tanks in a rebreather by a factor of 6.35, and, when given a set endurance level, control can help lower the tank sizes by a factor of 4.

There are two main avenues that we would like to explore in the future: the first one is related to model validation and the second one to the accident investigations. We would like to obtain diving data, specifically about the oxygen and nitrogen levels during a typical dive, and compare them with our model and use that data to calibrate it. Additionally, as explained in the introduction, the true value of an analytical model comes not when it stands alone, but when it is used in conjunction with hardware testing. Therefore, we would like to use the model to guide future hardware development, particularly in minimizing the breathing resistance and maximizing the endurance of the gas tanks.

The accident investigation brief analysis shown here is only one of the numerous available frameworks for accident investigation. An example of such a framework is System Theoretic Process Analysis (STPA) [65], which is a hazard analysis technique designed to take into account potential system failure modes that result from systems interactions and which are not captured by traditional failure analyses, like Failure Modes, Effects and Criticality Analysis (FMECA). We would like to expand the accident analyses capabilities of this model by incorporating notions from STPA into the simulations and so provide a more complete tool for rebreather simulation.

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