Design and Analysis of V-Groove Passive Cryogenic Radiators for Space-borne Telescopes & Instruments

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V-groove passive radiators are extremely efficient passive designs to thermally isolate a cryogenic telescope or instrument from warm environments in space, like the Sun, Earth, Moon or Spacecrafts. Compared to traditional multi-layer insulation blankets employed for thermal isolation, the V-Groove radiators provide as much as an order of magnitude improvement. The V-Groove design typically constitutes three low emissivity, lightweight and thin aluminized Mylar/Kapton sheets angled from each other by just few degrees. These successively reflect heat from warmer shields to space via their angular openings, thus minimizing heat flow into the cryogenic system. Thermal analysis of these V-Groove radiators is typically performed by sophisticated thermal software that uses hundreds of thousands of rays to simulate radiative heat flow between successive shields via reflections, which can be very time & resource intensive. We have arrived at very simple closed form equations to predict the thermal behavior of these radiators, that includes their radiative heat transfer factors and temperatures as a function of their basic thermo-optical properties and inter-shield angles. These predictions can be done by hand calculators or in spreadsheet tools like MS-Excel. They compare very well to those from sophisticated computer programs. The ease in the use of these simple equations allow for instantaneous predictions of cryogenic temperatures and their trends for design options and trade studies. A case study of this was utilized for the telescope project that is currently being designed for an all sky spectral survey of the universe. This paper will describe the derivations of these equations, their comparison with computer software results and their applicability for current and future cryogenic space telescope and instrument missions.

Nomenclature

F_{is}	=	view factor from inner shield's inner surface to space
F_{vv}	=	view factor from any v-groove shield to adjacent shield
F_{vs}	=	view factor from any v-groove shield to space (except inner shield's inner surface to space)
$\mathcal{F}_{vs}$	=	gray body radiation transfer factor from v-groove shield to space
$\mathcal{F}_{vv}$	=	gray body radiation transfer factor from v-groove shield to adjacent shield
ϵ_v	=	emissivity of v-groove shield
ϵ_{inner}	=	emissivity of inner surface of inner v-groove shield
T_v	=	temperature of V-groove shield
T_s	=	temperature of space
T_{inner}	=	temperature of inner shield
T_{middle}	=	temperature of middle shield
T_{outer}	=	temperature of outer shield
A_v	=	area of shield
SphereX	=	Spectro-Photometer for the History of the Universe, Epoch of Reionization and Ices Explorer

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I. Introduction

A typical V-groove radiator is depicted in Figure 1. It consists of two adjacent shields slightly angled from each other (just a few degrees is sufficient). Each shield is highly reflective (>95-98%) in the IR wavelengths, also very specular (mirror like). Heat radiated from shield 1 (the heat source) reaches adjacent shield 2 and is reflected from it to space (3). Due to the angle between them (and their specularity), almost all the reflected heat preferentially goes radially outwards to space instead of reflecting back to shield 1. Due to the low absorptivity of the shields, because of their high reflectivities, a very small fraction of the heat arriving from shield 1 is absorbed by shield 2. Hence, when multiple shields are layered back to back, as shown in Figure 2, heat radiated from shield 1successively gets reduced when reaching the next shield. Therefore, each shield combination stage successfully reduces the heat that finally reaches the last shield.

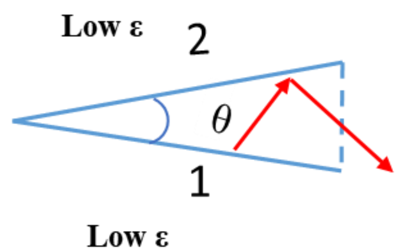

Figure 1. A Single pair of V-groove shields.

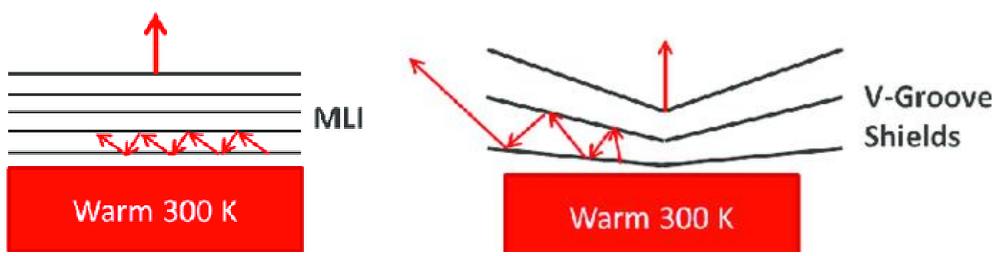

Figure 2. MLI shields vs. V-Groove Shields.

The standard Multi-Layer Insulation (MLI) shields perform very differently from V-Groove shields. MLI layers are essentially parallel to each other. Hence, the vast majority of radiation arriving at successively cooler downstream shields is reflected back to the warmer upstream one due to its high reflectivity. Therefore, no heat gets radiated sideways (radially) to space. This leads to the MLI shield not being very effective in reducing the radiation reaching the final downstream layer of the shield.

For MLI with a view factor F_{12} being 1 (parallel plates), and emissivities ε being small, then the radiation transfer factor between adjacent shields derived from elementary radiation heat transfer is $\Im_{12} = 1/[1/\varepsilon + 1/\varepsilon + 1] \cong \varepsilon/2$. Hence, when the two reflective surfaces only see each other and do not see the third black surface, the net coupling is ε/2. This is expected because an emitted photon reflects between surfaces until one side or the other absorbs it. If the emissivities of the two surfaces are the same, one expects a 50% probability that it will be absorbed by a given surface. Hence, only half of the emitted energy is transported between surfaces with the other half being reabsorbed by the emitting surface. Therefore, for multiple layers of shields, the effective radiative coupling for n shields is n(ε/2). The radiation transfer factor for V-groove shields is significantly smaller, by as much as an order of magnitude than for MLI - roughly equal to ε^2 for each pair - as will be shown later in this paper.

These V-groove shields were invented by Ray Garcia at JPL, analyzed by Helene Schember[1], and were first demonstrated in a development unit by Steven Bard and Walt Petric[2] at JPL who designed, built and tested (cryogenic/vibration) a flight demonstration unit in 1986 for Mars Observer (MO). The Flight unit was built by Lockheed Martin[3] for GRS instrument for the Mars Observer mission. Subsequently, they were successfully flown on the Planck Mission[4] where three shields achieved <40 K at the last shield while the spacecraft was at ~300 K (an almost 10 fold reduction in temperature). Figure 3 shows this telescope.

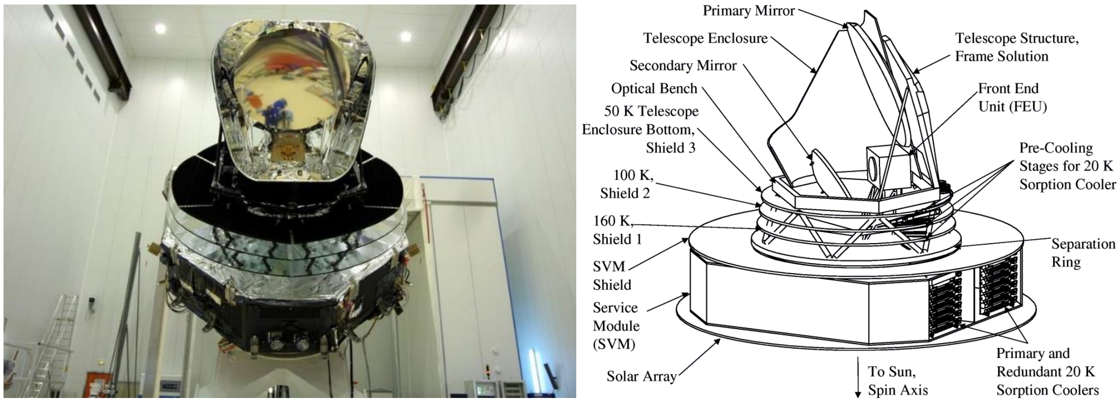

Figure 3. Planck Telescope Utilizing V-Groove Shields.

A very large-scale implementation of these V-Groove shields is being employed in the James Webb Space Telescope (JWST)[5] as shown in Figure 4. The sunshield has five layers of aluminized sheets to maximize their reflectivity. The kite-shaped sunshield about is 22 meters by 10 meters in size (about a football field in dimension).[5] In operation, it will receive about 300 kilowatts of solar radiation, but only pass 23 milli-watts to the other side.[5] It is a V-groove radiator and causes a temperature drop of 300 K[6] from front to back.[

Figure 4. JWST sunshield.

II. V-Groove Radiative Coupling Closed Form Equations

Typically, three V-groove shields suffice to produce a very large temperature reduction (e.g., 300 K to 40 K in Planck). Thermal analysis of these V-Groove shields is typically performed by sophisticated thermal software that uses hundreds of thousands of rays to simulate radiative heat flow between successive shields via reflections, which can be very time & resource intensive. We have arrived at very simple closed form equations to predict the thermal behavior of these radiators, that includes their radiative heat transfer factors and temperatures as a function of their basic thermo-optical properties and inter-shield angles. The simple analysis employs basic radiation heat transfer relationships for three gray body diffuse surfaces forming an enclosure as shown in Figure 5. The outer shield is the hottest one (heat source), the middle one is less hot and the inner one is the coldest.

The key assumptions employed for this simple thermal model are listed below and depicted in Figure 5:

- Surfaces are gray & diffuse (conservative)
- Uniform radiosity
- Surface properties are constant
- Isothermal surfaces
- View factors employed are average for entire surface
- Shields of equal areas

- Shield emissivity << 1, since it is typically ~0.05
- Space emissivity = 1
- View factor between adjacent shields, $F_{1\text{-}2} \simeq 1\text{-}\sin\left(\frac{\theta}{2}\right)$, assuming they are large planar surfaces
- No external heating from heavenly bodies
- Inner shield is the heat source, shields interact with adjacent shields & loses heat to space

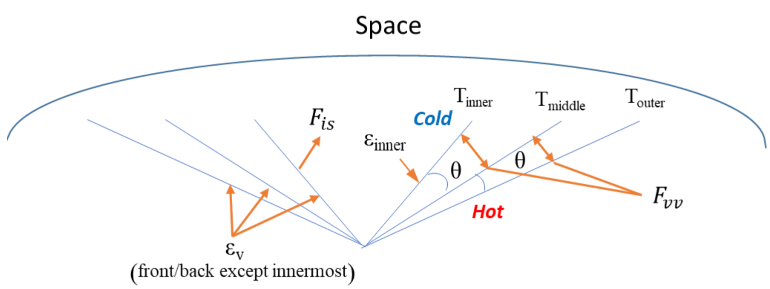

Figure 5. Schematic of three V-groove shields interacting with each other and with space

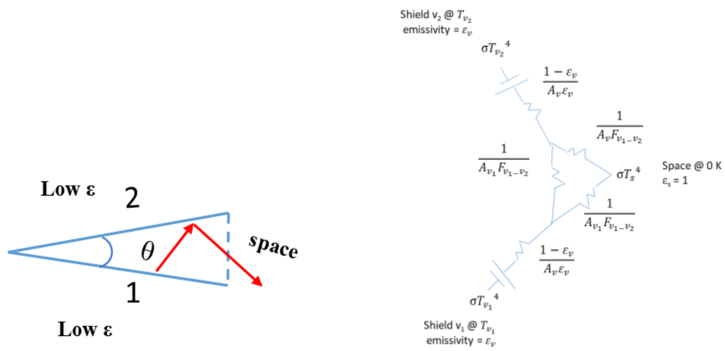

Figure 6. Oppenheim Radiation Network between three Gray Diffuse Surfaces

The derivation of the gray body radiation transfer factors utilizes the well-known Oppenheim radiation network using the following key properties of adjacent shields: view factors between them and to space, and their surface emissivities. The transformation of the network to gray body radiation transfer factors between the shields and to space then involves using the standard electrical analogy for transformation for Δ-Y and Y-Δ (Figure 7) that can be found in any heat transfer textbook along with algebraic manipulation. Simplifications utilizing the assumptions of low emissivities for the shields and black space along with view factors between large planar surfaces are employed to make the equations tractable.

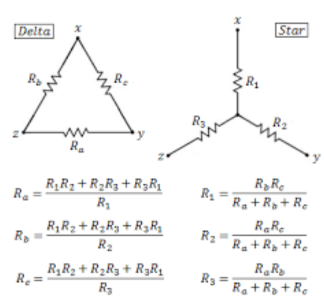

Figure 7. Delta-Star (Wye) Transformation

The resultant relationships for gray body radiation transfer factors ($\mathcal{F}_{12}$) between the shields and between any shield and space are shown below in equations 1 and 2. These show the transfer factors from v-groove shield to v-groove shield, $\mathcal{F}_{vv}$, and from either V-groove shield to space (radially out), $\mathcal{F}_{vs}$.

$$\mathcal{F}_{vv} = \frac{\left(\frac{F_{vv}}{1-F_{vv}{}^2}\right)}{\left(\frac{1}{\varepsilon_v} - 1 + \frac{1}{(1+F_{vv})}\right)^2 + 2\left(\frac{F_{vv}}{1-F_{vv}{}^2}\right)\left(\frac{1}{\varepsilon_v} - 1 + \frac{1}{(1+F_{vv})}\right)} \tag{1}$$

$$\mathcal{F}_{vs} = \mathcal{F}_{vv}\frac{\left(\frac{1}{\varepsilon_v} - 1 + \frac{1}{(1+F_{vv})}\right)}{\left(\frac{F_{vv}}{1-F_{vv}{}^2}\right)} \tag{2}$$

Additional manipulation of equations 1 & 2 leads to even simpler equations (3, 4) for the radiation transfer factors by utilizing the following approximations:

1) For relatively small angles (0-10°), $F_{vv} \approx 1$
2) $1/\varepsilon_v >> 1$

$$\mathcal{F}_{vv} = \frac{\epsilon_v^2}{2(1-F_{vv}+\epsilon_v)} \tag{3}$$

$$\mathcal{F}_{vs} = \frac{1}{\left(\frac{1}{\epsilon_v}+\frac{1}{1-F_v}\right)} \tag{4}$$

A very interesting observation from equation 3 is that the radiation transfer factor between adjacent shields ($\mathcal{F}_{vv}$) is proportional to ε_v^2 for V-groove shields, rather than $n(\varepsilon_v/2)$ for MLI shields. This is a very powerful demonstration of the advantage of V-groove shields since ε_v is typically quite low (on the order of 0.02 – 0.05), the squaring of it (ε_v^2) produces very low values coming to a fraction of it, $n\varepsilon_v/2$.

Knowing the transfer factors, the calculation of temperatures is done by simple heat balances between adjacent shields, and between any shield and space. The heat balance then yields their temperatures by assuming the temperature of the warmest (outer) shield. Equations 5 and 6 show two examples of heat balances, where equation 5 is for heat balance between outer and middle shield, and equation 6 is for heat balance between any two adjacent shields. Hence starting with the outer shield as a boundary temperature (e.g., 300 K for a spacecraft), one can sequentially and successively compute the temperature of the next shield, and so on for the following one.

$$\mathcal{F}_{vv}\left[T_{out}^4 - T_{mid}^4\right] = 2\mathcal{F}_{vs}\left[T_{mid}^4 - T_{space}^4\right] + \mathcal{F}_{vv}\left[T_{mid}^4 - T_{in}^4\right] \tag{5}$$

$$\mathcal{F}_{i\to i+1}\left[T_i^4 - T_{i+1}^4\right] = \mathcal{F}_{i+1\to i+2}\left[T_{i+1}^4 - T_{i+2}^4\right] + 2\mathcal{F}_{i+1\to s}\left[T_{i+1}^4 - T_s^4\right] \tag{6}$$

For a 3 shield case, solutions of the above equations leads to equations 7 & 8 for estimating the temperatures of the middle (less warm) and inner (coldest) shields with the knowledge of the outer (hottest) shield temperatures and the radiation transfer factors calculated by employing equations 1 & 2. Equation 8 assumes that the innermost shield's internal surface (looking straight to space) is also black in emissivity, unlike the other shields are. This is obviously a better option to reduce its temperature because it can reject that was intercepted by it from the warmer middle shield and can most effectively lose it to space where it has the largest view to space (except for view blockage from the telescope or other optical component that is being held at low temperatures).

$$T_{middle} = T_{outer}\left[\frac{1}{2\left(\frac{\mathcal{F}_{vs}}{\mathcal{F}_{vv}}\right)+1}\right]^{\frac{1}{4}} \tag{7}$$

$$T_{inner} = T_{middle}\left[\frac{\mathcal{F}_{vv}}{\mathcal{F}_{is}}\right]^{\frac{1}{4}} \tag{8}$$

Equation 9 pertains to an alternate case, which assumes that the innermost shield's internal surface (looking straight to space) is not black, like the other shields are, and has a very low emissivity. Obviously not a desirable situation but is provided here to provide its temperature if its view was significantly blocked.

$$T_{inner} = T_{middle}\left[\frac{\mathcal{F}_{vv}}{\mathcal{F}_{vv} + \mathcal{F}_{vs} + \varepsilon_v F_{is}}\right]^{\frac{1}{4}} \tag{9}$$

III. Numerical Examples

The calculations of temperatures of the V-groove shields can be easy performed on a hand held calculator knowing the inputs, angles between shields, their surface properties and the boundary temperature of the outer shield (heat source). As an example of calculations for a three V-groove shield system is shown below for the following set of parameters: $\theta = 6°$, $\varepsilon = 0.023$ for all shields, except inner shield surface looking straight out to space being black, and the temperature of the outer shield being 245 K (hottest, source temperature). In this particular example the inner shield's view factor directly looking out to space, from its inner surface, is assumed to be 0.5 to account for blockage from components like telescopes within its interior volume.

$F_{12} \simeq 1\text{-}\sin\left(\frac{\theta}{2}\right)$ yields the view factor between adjacent shields (F_{vv}) to be 0.948, from a shield to space (F_{vs}) of 0.052 (complement of shield-to-shield value, F_{vv}). Equations 3 & 4 then yield the gray body transfer factors between shields ($\mathcal{F}_{vv}$) of 0.0035, and between shields and space ($\mathcal{F}_{vs}$) of 0.0162. Equations 7 & 8 then yield the temperatures of the middle and inner shields of 134 K and 39 K. This shows the power of the V-groove shields in reducing the heat load on the cold innermost shield to reduce the temperature by an order of magnitude. Therefore, it is far superior to MLI in achieving it. Note that this case is with the assumption that the inner shield's surface looking straight out to space is black (not low emissivity).

Utilization of equation 9 yields the corresponding temperatures of 134 K and 78 K, respectively, when that same shield's surface looking straight out to space is low emissivity. Obviously, the black inner surface of the inner shield coupled with the large view to space reduces the temperature of the coldest shield enormously (by 40 K) because it can lose the heat intercepted by it very effectively. This is far superior to the case when the inner shield has a low emissivity inner surface.

It should be noted how a simple use of a hand calculator, by utilization of the above equations, with just a handful of input parameters, instantly yields the shield temperatures. Parametric cases can be done very simply by using a spreadsheet software like EXCEL. Hence this serves as a very useful design tool to quickly assess design parameters.

Example of variation of shield temperature as a function of shield emissivity is shown in Figure 8 for both a black inner shield's inner surface and a low emissivity one. It is obvious that the emissivity plays a very significant role in achieving low temperatures in the inner shield, with low emissivities yielding lower temperatures. This is due to the corresponding increase in the infrared reflectivity of the shields, which in turn improves the radial transfer of heat to space, hence reducing the transfer of heat to the adjacent cooler shield. Moreover, it is evident that the black inner shield's inner surface has a tremendous impact in reducing the temperature of the inner shield. As an example of this trend, a black inner shield's inner surface leads to a temperature of 46 K versus 86 K for a low emissivity one, for a given emissivity of shields of 0.03 for surfaces of shields facing each other.

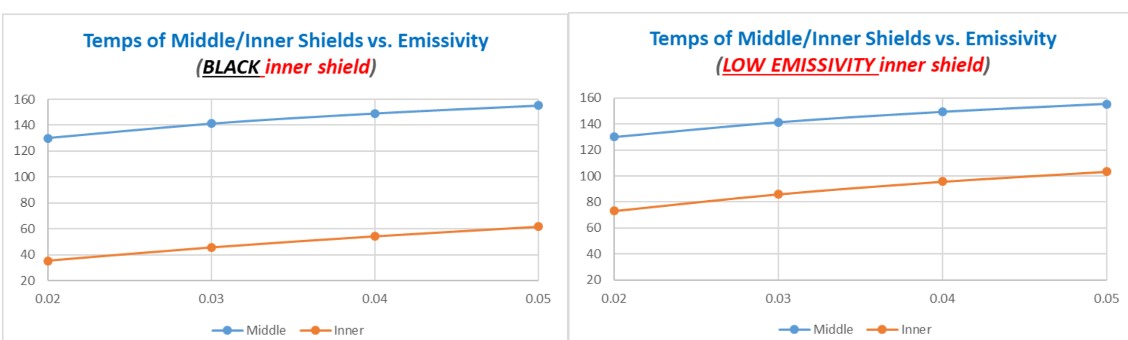

Figure 8. Temperatures of Middle/Inner Shields vs. Shield Emissivity for fixed shield angle of 6°.

Another example of variation of shield temperature as a function of inter-shield angle is shown in Figure 9 for both a black inner shield's inner surface and a low emissivity one. It is obvious that the shield angle plays a very significant role in achieving low temperatures in the inner shield, with large angles yielding lower temperatures. This is due to the higher view of the shields to space in the radial direction, which in turn improves the radial transfer of heat to space, hence reducing the transfer of heat to the adjacent cooler shield. Moreover, it is evident that the black inner shield's inner surface has a tremendous impact in reducing the temperature of the inner shield. As an example of this trend, a black inner shield's inner surface leads to a temperature of 39 K versus 78 K for a low emissivity one, for a given emissivity of shields of 0.023 for surfaces of shields facing each other.

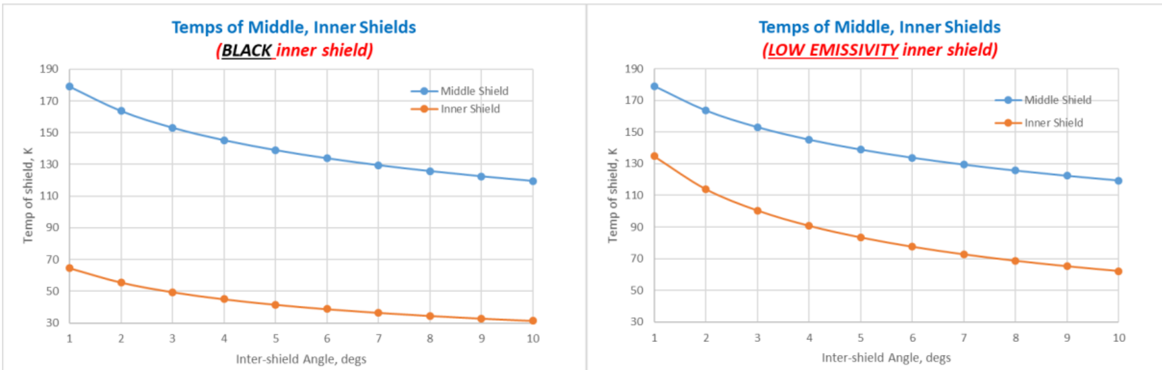

Figure 9. Temperatures of Middle/Inner Shields vs. Shield Angle for fixed shield emissivity of 0.023.

IV. Comparisons of Predictions with Sophisticated Programs

To compare the results from the simple approach presented in this paper, a one to one comparison was made with results from sophisticated software (Thermal Desktop) that uses tens of thousands of rays to compute the radiation transfer factors and the corresponding shield temperatures – this employed for an example Telescope Project, utilizing the same parameters. It also utilizes specular properties to account for reflections of heat from the shields to adjacent shields and space. Figure 10 shows a comparison between the two approaches and it is clear that the results compare remarkably well, in spite of the extreme simplicity of the approach delineated in this paper. For example, at an inter-shield angle of 5° (which is typical for V-groove shields), the simple model predicts 83 K, whereas the detailed computer model predicts 80 K - for a low emissivity inner shield's inner surface. For the middle shield, for the same case, the simple model predicted 139 K, whereas the detailed model predicted 138 K. These models were run parametrically for varying angles between shields to understand the effect of angles on the shield temperatures.

The obvious trend is that increasing the angles leads to lowering of the shield temperatures. Additionally, the effectiveness of the increased angles tends to slow down for larger angles indicating an optimization around 5-10°. Another trend in the comparison between the detailed and simplified approach is that angles < 5-6° the detailed model predicts lower temperatures, whereas they tend to be closer for larger angles. This can be attributed to modeling simplifications because the simple model assumes diffuse (non-specular) shields, whereas the detailed model accounts for specularity. At small angles, specularity helps because the radiation is preferentially directed to space in the radial direction due to specular reflections between adjacent shields, which is not the case for diffuse surfaces. In fact, the smaller this angle is, the closer the V-groove shields come to being MLI shields. However, in contrast, for larger angles, this radial opening is large enough to allow radiation to escape to space without help from specular reflections when compared to diffuse reflections due to the triangular shape of the radial opening.

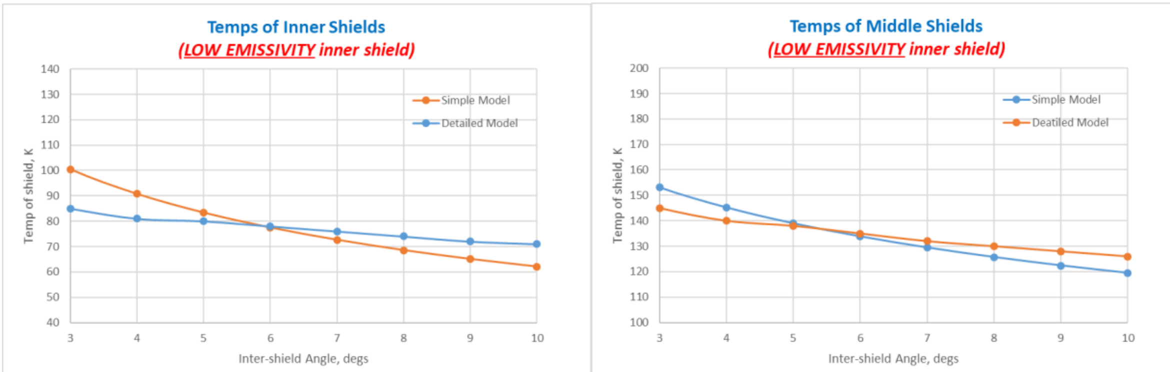

Figure 10. Comparison between Simple Hand Calculation Model and Sophisticated Computer Model

V. Conclusions

This paper provides a simple and powerful estimation approach for predicting V-groove shield temperatures by using simple hand calculations using the equations that were derived and presented here. Sample results from predictions utilizing sophisticated computer models shows a very good comparison between them and this simple approach, for the same set of input design parameters. This then provides a very quick and approximate way for the initial design of V-grooves. Of course, for a real flight design there are several more design and configuration factors involved, hence they have to be accounted for to accurately predict their temperatures. That prediction is not the purpose of this approach; rather it is an initial design that could be employed prior to detailed design.

Acknowledgments

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