

SIMULATION OF A MISCIBLE CARBON DIOXIDE FLOOD
USING A TRAPPED OIL FUNCTION

by

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NOMENCLATURE

<u>Symbol</u>		<u>Units (cgs)</u>
A	block horizontal surface area	cm^2
A_i	P_{i-1}^{n+1} coefficients	
B_i	P_i^{n+1} coefficient	
B	formation volume factor	cm^3/cm^3
C	Trapping constant-function of rock	
C_i	P_{i+1} coefficient	
C_A	areal coverage	
C_N	Displacement Efficiency	
C_v	vertical coverage	
C_{pv}	Pore volume compressibility	$1/\text{atm}^*$
C_f	Compressibility of fluid	$1/\text{atm}^*$
D_i	Known matrix quantities	atm^*
f	fraction of phase in the stream	
\bar{f}	average fraction of phase flowing in bank	
h	thickness of block	cm
K	absolute permeability	Darcies*
\bar{K}	mean permeability	Darcies*
\bar{K}_σ	permeability at 84.1% of the sample	Darcies*
K_r	relative permeability	
M	Mass of phase in the block	gms
μ	viscosity	cp
ϕ	porosity (fraction)	
P	Pressure	atm^*

P_i	Initial Pressure	atm*
Q_i	Injection Rate	cm ³ /sec
Q_m	mass flow rate of phase	gm/sec
Q_s	phase surface flow rate	cm ³ /sec
ρ_s	density of phase at surface	gm/cm ³
S	Phase Saturation (fraction)	
$\overline{S_{ob}}$	Average Saturation of oil in the oilbank	
S_{of}	Free oil saturation	
S_{or}	Residual Oil Saturation to waterflood	
S_{ot}	Trapped Oil saturation	
SW_i	Irreducible water saturation	
So_i	Initial Oil in place at start of imbibition process	
t	time	sec
V	Permeability variation	
V_{oil}	Volume of oil displaced by solvent front	cm ³
V_{water}	Volume of water displaced by solvent front	cm ³
x_f	length of the solvent front	cm
ΔX	length of the block	cm
Δy	width of the block	cm

* non-cgs units

Subscripts

f fluid
g gas
i denotes position
o oil
ob oil bank
s solvent
sol solvent
w water

Superscripts

(n) denotes known time level
(n+1) denotes unknown time level
* reduced saturation-corrected for irreducible water saturation

CHAPTER 1

INTRODUCTION

When analyzing a reservoir for possible tertiary development, several factors are needed to make an informed decision. These factors include predicted values of ultimate recovery, producing ratios, injection volumes, and pressure distributions. Following is a discussion and description of a simulator designed to provide these values.

The model is for one-dimensional, three-phase flow, two of the phases being miscible, and assumes piston-like displacement by the solvent. Effects of dispersion, diffusion and viscous fingering are omitted, and volumetric sweep efficiency is assumed to be 100 percent. Solvent is flowing exclusively behind the front, and miscibly displaces the oil, and the oil immiscibly displaces water. For the situation in which the field was previously waterflooded, water flows in front of the advancing oil front, and is the only fluid produced until oil breakthrough.

Relative permeabilities in the oil bank are determined using Corey's¹ equation for water relative permeability, and a correlation based on mobile oil saturations for the oil relative permeability. Relative permeabilities in the block containing the solvent front are calculated using Corey's¹ equation for the drainage curves. Ahead of the oil front, the reservoir is assumed to be at residual oil saturation t

waterflood. Behind the solvent front, the saturations are calculated using Land's trapped oil equations,² and relative permeability is calculated using a correlation based on mobile oil saturations. The pressure distributions obtained from the model are then examined, and oil-water production ratios are compared with the Buckley-Leverett results.

Reservoir Simulation Models

Reservoir simulation models are used to approximate fluid flow in a reservoir in order to determine the optimum production design. Effects of well spacing and producing mechanism can be calculated before actual implementation. Simulators are also employed to ascertain producing water-oil ratios, sweep efficiency, and other flow properties.

In devising a simulator, the reservoir is first divided into cells, each of which have their own rock and fluid properties. Cell size is partially determined by the complexity of the reservoir. It has been reported that adequate history matches and well performance have been predicted with grid sizes as large as 160 acres in some relatively homogeneous reservoirs.³

The reservoir complexity and the desired results also partially determine the grid dimension size of the model. For calculating the primary production performance in the swept area of a homogeneous reservoir, a one-dimensional model is normally adequate. Two-dimensional models are

frequently used in determining sweep efficiency, injection patterns, and bottom-hole pressure analysis.³ For heterogeneous reservoirs with multiple producing mechanisms, a three-dimensional model may be needed. However, because of economic considerations, it is often not practical to run a three-dimensional model. van Poolen and Breitenbach⁴ discuss a method by which a three-dimensional problem may be reduced to a two-dimensional grid system using pseudo-third dimensional features.

The production mechanism and the fluid composition are the criteria used when selecting the type of simulator. The three most common types are the black-oil, gas, and compositional models. The black-oil model is used when the composition of the produced oil is assumed to be constant. When only gas and water are present, the most economical model to run is the gas-water model.

Reservoir heterogeneity has complicated the process of assigning rock properties to cells. Dykstra and Parsons⁵ presented a correlation between waterflood recovery and permeability distribution, where the permeability variation, V is:

$$V = \frac{\bar{K} - K_{\sigma}}{\bar{K}} \quad (1-1)$$

where

\bar{K} = mean permeability

\bar{K}_0 = permeability at 84.1 percent of the cumulative sample.

J. D. Testerman⁶ relied on a statistical approach in order to determine reservoir properties in various zones.

Doepel and Sibley⁷ developed a finite difference equation for multilayered reservoirs which used dimensionless equations to express the solvent injected into each layer as a fraction of total injection. Laboratory measurements of displacement efficiency (C_M) were used to determine areal coverage (C_A) and vertical coverage (C_V). Recovery was proportional to the product of C_M , C_A , and C_V . Each layer was assigned the appropriate values of saturation, porosity, and permeability. Permeability variation (see Equation 1-1) was then used to determine production rate and recovery from each zone. It was noted that an increase in permeability variation adversely affected recovery. With 1.5 pore volumes of solvent injected, recovery decreased from 59% to 38% when the permeability variation was increased from .5 to .8.

Diffusion, Dispersion, and Viscous Fingering

Effects attributed to diffusion, dispersion and viscous fingering for a miscible flood have been modeled by many means. Dispersion (increased mixing caused by uneven fluid flow) appears to have a much larger impact than diffusion (mixing caused by the random motion of molecules). While there has been agreement on the importance of dispersion and

viscous fingering in a simulator model, there is conflict concerning the relevance of diffusion.^{8,9} J. S. Aronofsky developed a diffusion model for miscible fluids in porous media.⁸ In his experiments, fluid A, initially in place, would be displaced miscibly by fluid B. An effective diffusion coefficient was calculated based on porosity, fluid velocity of B and the concentration of B in the mixed zone. From his studies he developed a relationship among diffusion, displacement velocity and "mixing zone length," which he stated increased with increasing displacement velocity.

L. L. Handy presents a differing view on diffusion.⁹ Using two tracers with different diffusion coefficients, Handy studied the effects of diffusion on fluids with varying viscosities. He discovered that for adverse mobility ratios (greater than one), the effects of diffusion were negligible compared to dispersion and mechanical mixing.

Predicting viscous fingering patterns in a reservoir is another challenge to simulators. T. K. Perkins, et al.,¹⁰ developed a relationship between mean displacement and finger width and length. He also developed a correlation for suppression of the fingers by longitudinal dispersion. Handy also noted the effects of dispersion along the finger boundaries.⁹

Results of dispersion have been modeled by differing methods. Deans¹¹ did his work based on a mass transfer factor, stagnant volume fraction and tracer concentration,

while Goddard¹² based his on the attenuation and phase velocity of sinusoidal waves from gamma ray emissions.

Because of the stability problems encountered in models involving dispersion and diffusion, floating point simulators have been developed. A. O. Gardner, et al.,¹³ constructed a numerical method of moving reference points, at which positions and concentrations are computed each time step. It was reported to take into account dispersion, no matter how small the effect. Rachford¹⁴ also designed a moving point reference method, which resulted in the coordinate system moving with local characteristic velocity.

Todd and Longstaff¹⁵ developed an empirical model by which a three-phase simulator could be modified to simulate miscible flooding. The model used a "mixing parameter" to account for the effects of dispersion, fingering, and unstable frontal advances. This allowed for the use of a fairly coarse numerical grid-system without resulting instability. If the effects of dispersion were nearly negligible, the mixing parameter was set to zero, and the effective fluid properties were those of the oil and solvent. If, on the other hand, the rate of dispersion was large with respect to cell size, the components were assumed totally mixed, and the mixing parameter was assigned a value of one. Both phases were then given the same effective density and viscosity.

A three-phase simulation model such as that described above was built to perform the work described in this thesis. The objectives of the study were three-fold. First, Land's trapping function concepts were incorporated into the study and their effects on relative permeabilities were included. Second, the effects of numerical dispersion were minimized by the use of upstream saturations to calculate relative permeabilities; this concept was expanded to include upstream values for the solvent relative permeabilities within the block containing the solvent front. Third, the simulation results were compared to Buckley-Leverett results for the purpose of validating the model.

CHAPTER II

THE PARTIAL DIFFERENTIAL EQUATION

The computer simulation model developed for this thesis was based on a partial differential equation (PDE) for fluid flow through porous media. Figure 2-1 illustrates the process which is occurring. Mass flow rates at reservoir conditions were used, and the model was developed in Darcy units. These units are centimeter-gram-second (CGS) for all parameters except permeability (darcies) and pressure atmospheres).

Darcy's equation for horizontal flow and the material balance equation were used in deriving the PDE. Fluid flow in one dimension was used. Three equations were derived separately for water, oil and solvent. By combining these three equations, the final PDE was obtained, which accounts for three-phase flow.

Several assumptions were used in developing the PDE. These are listed as follows:

1. Black Oil Model - The fluid composition at the surface remains constant with respect to time.
2. Reservoir width is constant.
3. No formation subsidence occurs.
4. No deformation of the reservoir occurs.
5. The reservoir is horizontal.
6. Reservoir temperature is constant.

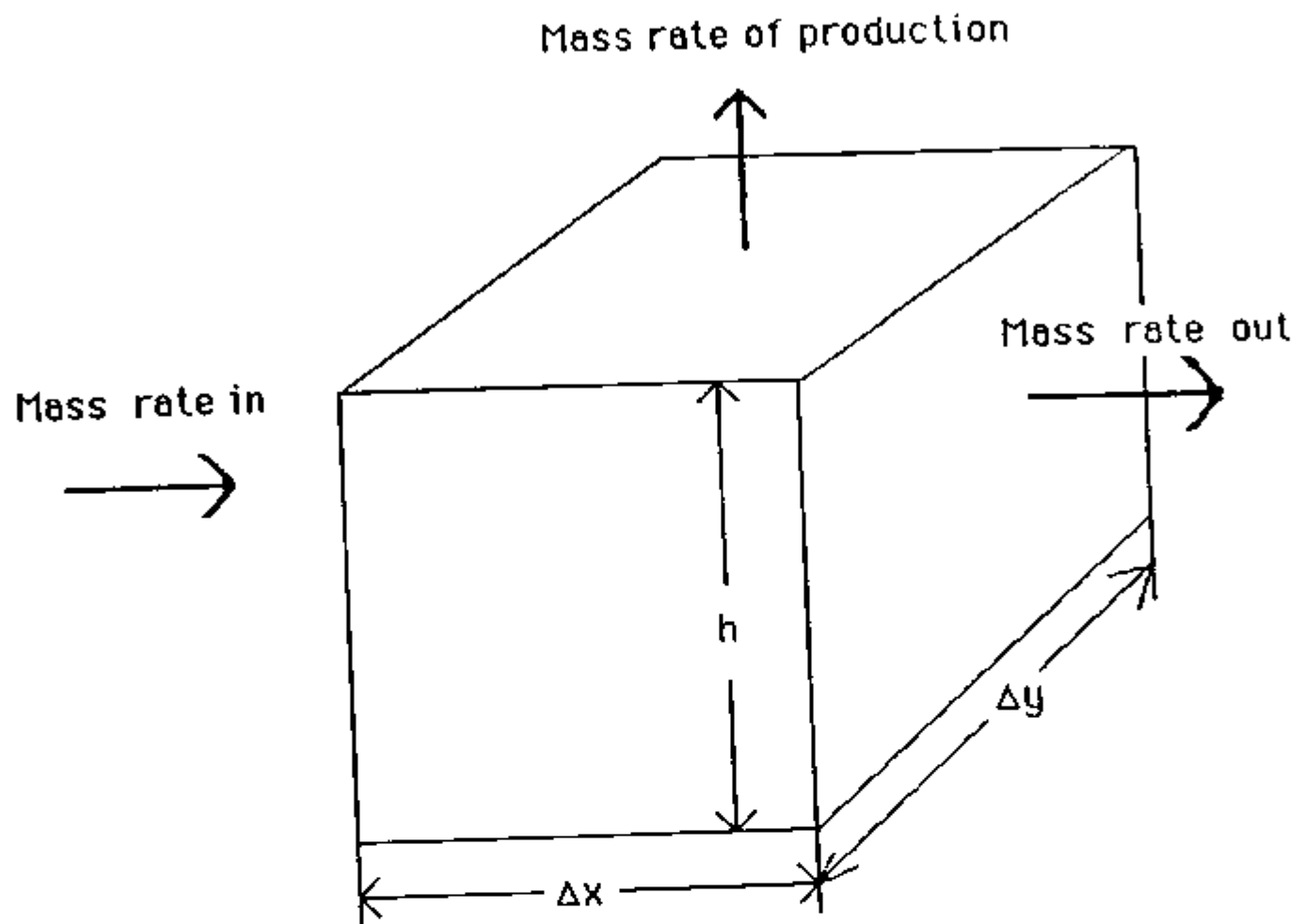


Figure 2-1
Mass Rate Fluid Flow Diagram

7. Oil and water are assumed slightly compressible.
8. The reservoir contains no hydrocarbon gas.
9. Effects of capillary pressure are negligible.
10. Fluid composition is constant with respect to position.

For each phase flowing, the mass material balance equation at reservoir conditions is:

$$\begin{aligned} &(\text{Mass Rate In}) - (\text{Mass Rate Out}) - (\text{Mass Rate of Production}) \\ &= \text{Mass Rate of Accumulation} \end{aligned}$$

or

$$\text{In} - \text{Out} - q_{fs}\rho_{fs} = \frac{\partial M_f}{\partial t} \quad (2-1)$$

where M_f is the total mass of fluid in the block, the terms In and Out refer to mass rates, and the other symbols used are described in the Nomenclature section.

Working first with the left-hand side (LHS) of the equation, it can be shown that:¹⁶

$$\text{In} - \text{Out} = -\Delta x \frac{\partial Q_{mf}}{\partial x} \quad (2-2)$$

From Darcy's Law for horizontal flow:

$$Q_{mf} = \frac{-K K_{rf}\rho_{fs}A}{B_f\mu_f} \frac{\partial p}{\partial x} \quad (2-3)$$

where

$$A = \Delta y h. \quad (2-4)$$

Substituting the result shown in Equation (2-3):

$$In - Out = \Delta x \frac{\partial}{\partial x} \left[\frac{K K_{rf} \rho_{fs} \Delta y h}{B_f \mu_f} \right] \frac{\partial P}{\partial x}. \quad (2-5)$$

Based on the second and tenth assumptions (constant reservoir width and constant fluid composition), it is seen that

$\partial \rho_{fs} / \partial x$ and $\partial \Delta y / \partial x$ are equal to zero.

$$In - Out = \Delta x \Delta y \rho_{fs} \frac{\partial}{\partial x} \left[\frac{K h K_{rf}}{B_f \mu_f} \right] \frac{\partial P}{\partial x}. \quad (2-6)$$

Expansion of the right-hand side (RHS) of the equation is as follows:

$$\text{Mass of fluid in the block} = (\text{Pore Volume}) (\rho_{fs}) S_f \quad (2-7)$$

where S_f is the saturation of fluid in the block.

$$M_f = \frac{\Delta x \Delta y h \phi \rho_{fs} S_f}{B_f} \quad (2-8)$$

The mass rate of accumulation can be written as:

$$\frac{\partial M_f}{\partial t} = \frac{\partial}{\partial t} \left[\frac{\Delta x \Delta y h \phi \rho_{fs} S_f}{B_f} \right]. \quad (2-9)$$

From assumptions 3 and 4 (no subsidence or deformation of the formation), it is seen that thickness (h) and areal extent ($\Delta x \Delta y$) are not functions of time. Because of the black-oil assumption, surface fluid composition (ρ_{fs}) is not a function of time, either. Equation (2-9) can be rewritten as:

$$\frac{\partial M_f}{\partial t} = \Delta x \Delta y h \rho_{fs} \frac{\partial}{\partial t} \left[\frac{\phi}{B_f} S_f \right] \quad (2-10)$$

Since reservoir temperature and fluid composition are constant with respect to time, porosity (ϕ) and formation volume factor (B_f) are functions of pressure only. Applying multiplicative rules of derivatives and the chain rule:

$$\frac{\partial}{\partial t} \left[\frac{\phi}{B_f} S_f \right] = S_f \frac{\partial}{\partial t} \left[\frac{\phi}{B_f} \right] + \left[\frac{\phi}{B_f} \right] \frac{\partial}{\partial t} S_f \quad (2-11)$$

$$\frac{\partial}{\partial t} \left[\frac{\phi}{B_f} S_f \right] = \frac{\phi}{B_f} \frac{\partial S_f}{\partial t} + S_f \left[\frac{d}{dP} \left[\frac{\phi}{B_f} \right] \right] \frac{\partial P}{\partial t} \quad (2-12)$$

$$\frac{d \left[\phi / B_f \right]}{dP} \frac{\partial P}{\partial t} = \left[\frac{d \left[\phi / B_f \right]}{dP} + \frac{1}{B_f} \frac{d\phi}{dP} \right] \frac{\partial P}{\partial t} \quad (2-13)$$

$$\frac{d \left[\phi / B_f \right]}{dP} \frac{\partial P}{\partial t} = \left[\frac{1}{B_f} \frac{d\phi}{dP} - \frac{\phi}{B_f^2} \frac{dB_f}{dP} \right] \frac{\partial P}{\partial t} \quad (2-13a)$$

Factoring out ϕ/B_f , Equation (2-13) becomes:

$$\frac{d(\phi/B_f)}{dP} \frac{\partial P}{\partial t} = \frac{\phi}{B_f} \left[\frac{1}{\phi} \frac{d\phi}{dP} - \frac{1}{B_f} \frac{dB_f}{dP} \right] \frac{\partial P}{\partial t} \quad (2-14)$$

Since pore volume compressibility is considered constant and the liquids are slightly compressible,

$$\frac{d(\phi/B_f)}{dP} \frac{\partial P}{\partial t} = \frac{\phi}{B_f} [C_{pv} + C_f] \frac{\partial P}{\partial t} \quad (2-15)$$

Substituting Equations (2-15) and (2-12) back into Equation (2-10) yields:

$$\frac{\partial M_f}{\partial t} = \Delta x \Delta y h \rho_{fs} \left[\frac{\phi}{B_f} \frac{\partial S_f}{\partial t} + S_f \left[\frac{\phi}{B_f} \right] [C_{pv} + C_f] \frac{\partial P}{\partial t} \right] \quad (2-16)$$

Substituting Equation (2-16) and (2-6) into the original material balance equation and multiplying by $[B_f/(\Delta x \Delta y \rho_{fs})]$ results in:

$$B_f \frac{\partial}{\partial x} \left[\frac{KhK_{rf}}{B_f \rho_f} \frac{\partial P}{\partial x} \right] - \frac{q_{fs} B_f}{\Delta x \Delta y} = h \phi \frac{\partial S_f}{\partial t} + [h \phi S_f (C_{pv} + C_f)] \frac{\partial P}{\partial t} \quad (2-17)$$

Since the assumptions used in developing Equation (2-17) are relevant for any slightly compressible fluid, this partial differential equation applies for both water and undersaturated oil. In the analytical model, the solvent, carbon dioxide, is injected into a reservoir above the

minimum miscibility pressure (MMP) of the hydrocarbon-solvent system. Pressure gradients are particularly low in the solvent zone because of the high mobility of the solvent. The relatively high pressure coupled with the low pressure gradients in the reservoir allows the solvent to behave essentially as a liquid at reservoir conditions. Thus, Equation (2-17) is also used for the solvent.

By adding the three equations for water, oil, and solvent, a one-dimensional equation can be written as follows:

$$\begin{aligned}
 & B_o \frac{\partial}{\partial x} \left[\frac{KhK_{ro}}{B_o \mu_o} \frac{\partial P}{\partial x} \right] + B_w \frac{\partial}{\partial x} \left[\frac{KhK_{rw}}{B_w \mu_w} \frac{\partial P}{\partial x} \right] & (2-18) \\
 & + B_s \frac{\partial}{\partial x} \left[\frac{KhK_{rs}}{B_s \mu_s} \frac{\partial P}{\partial x} \right] - \frac{q_{os}B_o + q_{ws}B_w + q_{ss}B_s}{\Delta x \Delta y} \\
 & = h\phi \left[\frac{\partial S_o}{\partial t} + \frac{\partial S_w}{\partial t} + \frac{\partial S_s}{\partial t} \right] \\
 & + h\phi \left[S_o C_{pv} + S_w C_{pw} + S_s C_{ps} + S_o C_o + S_w C_w + S_s C_s \right] \frac{\partial P}{\partial t} .
 \end{aligned}$$

Since only water, oil, and solvent are present, their saturations add to one. It follows, then, that:

$$S_o C_{pv} + S_w C_{pw} + S_s C_{ps} = C_{pv} \quad (2-19)$$

and

$$\frac{\partial S_o}{\partial t} + \frac{\partial S_w}{\partial t} + \frac{\partial S_s}{\partial t} = 0. \quad (2-20)$$

The final PDE is now:

$$\begin{aligned}
 & B_o \frac{\partial}{\partial x} \left(\frac{KhK_{ro}}{B_o \mu_o} \frac{\partial P}{\partial x} \right) + B_w \frac{\partial}{\partial x} \left(\frac{KhK_{rw}}{B_w \mu_w} \frac{\partial P}{\partial x} \right) & (2-21) \\
 & + B_s \frac{\partial}{\partial x} \left(\frac{KhK_{rs}}{B_s \mu_s} \frac{\partial P}{\partial x} \right) - \frac{q_{os} B_o + q_{ws} B_w + q_{ss} B_s}{\Delta x \Delta y} \\
 & = h \phi \left(C_{pv} + S_o C_o + S_w C_w + S_s C_s \right) \frac{\partial P}{\partial t} .
 \end{aligned}$$

CHAPTER III

THE FINITE DIFFERENCE EQUATION

In order to use Equation (2-21), the partial differential equation (PDE), as a numerical model, a finite difference equation (FDE) must be developed which approximates the PDE. For the one-dimensional model developed for this study, the reservoir is divided into variable increments of length (Δx), and the end increments are set to form closed boundaries (see Figure 3-1). The mass center of each block is chosen as a distinct location or node. The subscripts (i) are used to denote location, while the superscripts (n) refer to time level. The subscripts and symbols used in this chapter are defined in the Nomenclature section. A known value is indicated with a superscript of (n), while an unknown value is indicated with a superscript of (n+1). This subscripting and superscripting convention is illustrated in Figure 3-2.

As mentioned above, a FDE is an approximation of a PDE. Specifically, the PDE developed in Chapter II (Equation (2-21)) will be approximated by a FDE.

As an example of the approach used to discretize the PDE, the oil phase approximation is shown as follows:

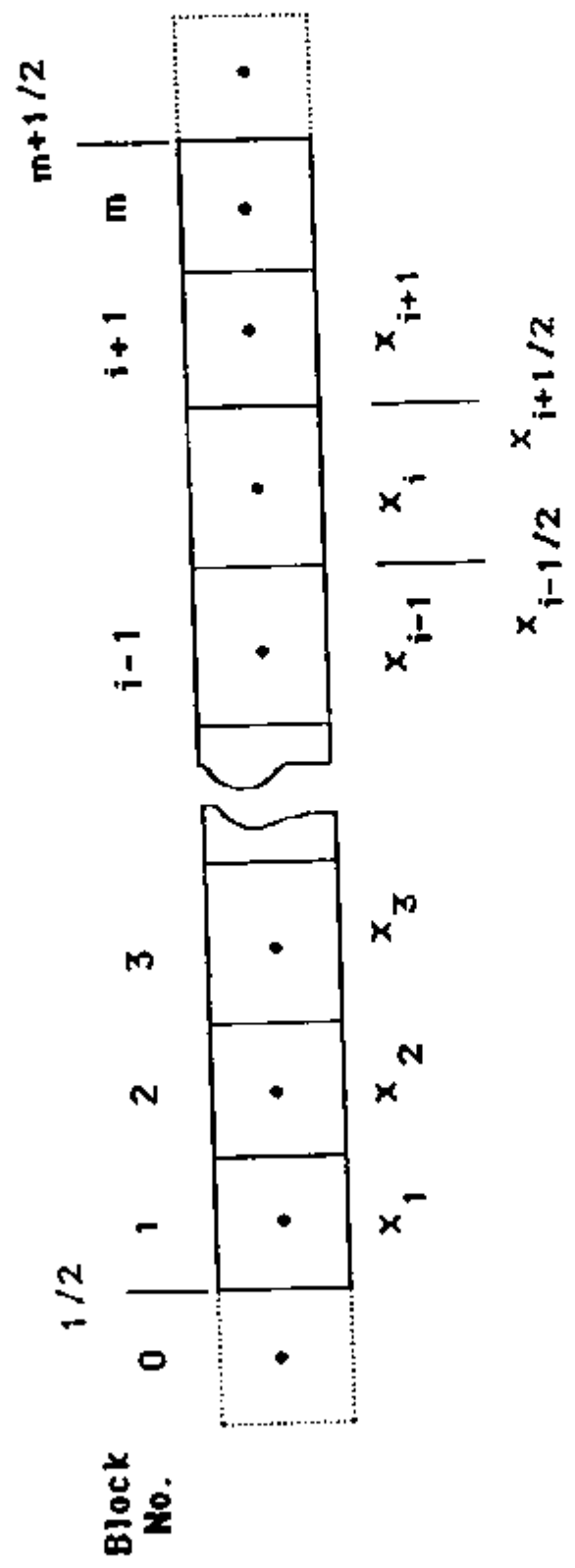


Figure 3-1
Grid Block Diagram

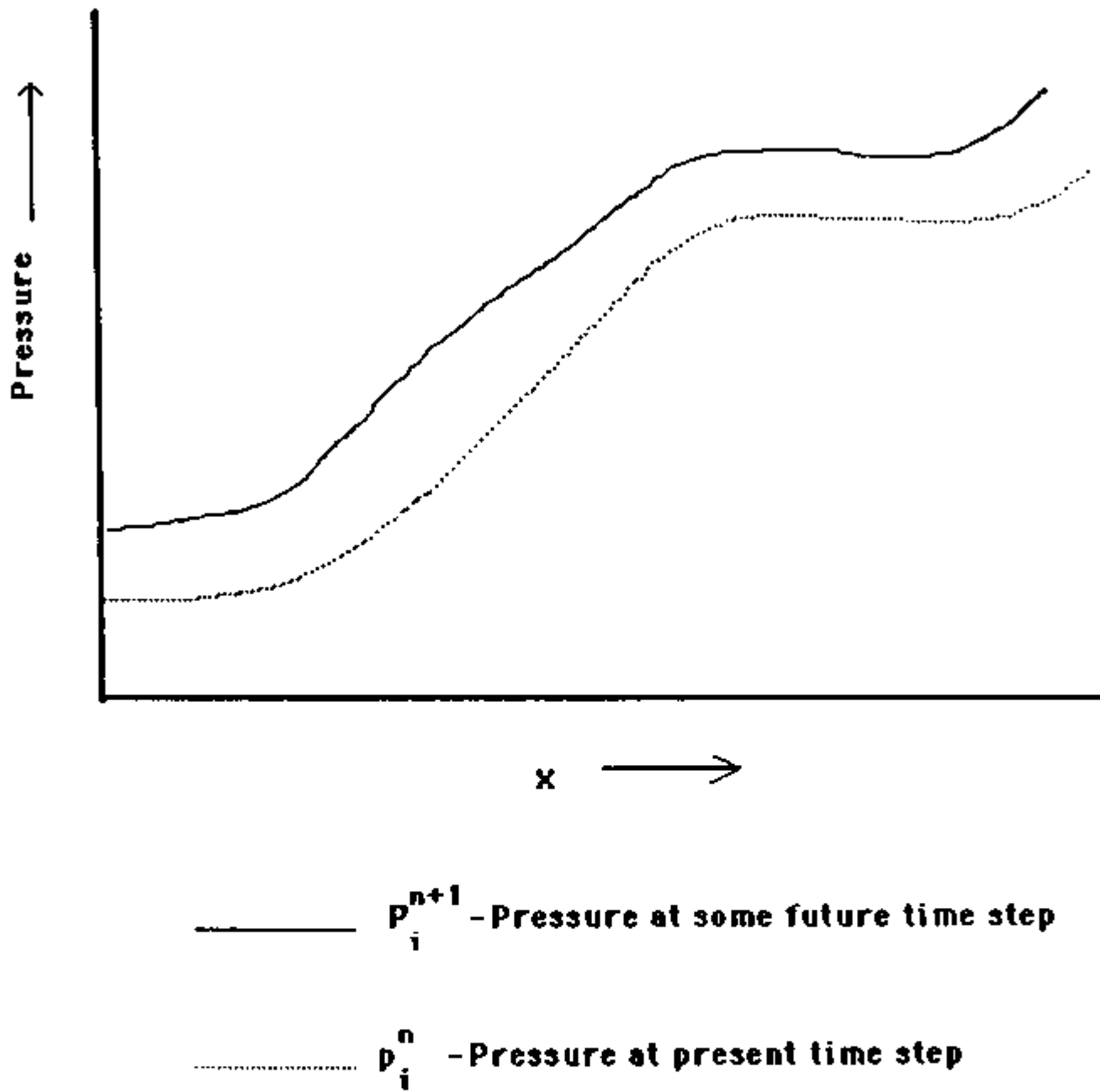


Figure 3-2

Pressure vs. Distance and Time

$$B_o \frac{\partial}{\partial x} \left[\frac{KhK_{ro}}{B_o \mu_o} \frac{\partial P}{\partial x} \right] \quad (3-1)$$

$$\approx \frac{B_{oi}}{\Delta x_i} \left\{ \left[\frac{KhK_{ro}}{B_o \mu_o} \right]_{i+1/2} \left[\frac{P_{i+1} - P_i}{\left[\frac{\Delta x_i + \Delta x_{i+1}}{2} \right]} \right] - \left[\frac{KhK_{ro}}{B_o \mu_o} \right]_{i-1/2} \left[\frac{P_i - P_{i-1}}{\left[\frac{\Delta x_i + \Delta x_{i-1}}{2} \right]} \right] \right\}$$

and,

$$B_o \frac{\partial}{\partial x} \left[\frac{KhK_{ro}}{B_o \mu_o} \frac{\partial P}{\partial x} \right] \approx B_{oi} \left\{ \frac{2(Kh)_{i+1/2}}{\Delta x_i (\Delta x_{i+1} + \Delta x_i)} \left[\frac{K_{ro}}{B_o \mu_o} \right]_{i+1/2} [P_{i+1} - P_i] \right. \quad (3-2)$$

$$\left. - \frac{2(Kh)_{i-1/2}}{\Delta x_i (\Delta x_i + \Delta x_{i-1})} \left[\frac{K_{ro}}{B_o \mu_o} \right]_{i-1/2} [P_i - P_{i-1}] \right\}$$

The form of the HKX terms that results from series averaging the interblock permeability-thickness products is shown as follows.

$$HKX_i = \frac{2(Kh)_{i+1/2}}{\Delta x_{i+1} + \Delta x_i} \quad (3-3A)$$

and

$$HKX_i = 2 \left[\frac{(Kh)_i (Kh)_{i+1}}{\Delta x_i (Kh)_{i+1} + \Delta x_{i+1} (Kh)_i} \right] \quad (3-3B)$$

$$HKX_{i-1} = \frac{2(Kh)_i (Kh)_{i-1}}{\Delta x_i (Kh)_{i-1} + \Delta x_{i-1} (Kh)_i} \quad (3-4)$$

Also, for convenience, the following definitions are made:

$$\alpha_{o_{i-1/2}} = \left[\frac{K_{ro}}{B_o \mu_o} \right]_{i+1/2} \quad (3-5)$$

and

$$\alpha_{o_{i-1/2}} = \left[\frac{K_{ro}}{B_o \mu_o} \right]_{i-1/2} \quad (3-6)$$

Substituting results from Equations (3-3B) through (3-6) into Equation (3-2) yields:

$$\begin{aligned} B_o \frac{\partial}{\partial x} \left[\frac{KhK_{ro}}{B_o \mu_o} \frac{\partial P}{\partial x} \right] & \quad (3-7) \\ \approx \frac{B_{o_i} HKX_i \alpha_{o_{i-1/2}}}{\Delta x_i} (P_{i+1} - P_i) - \frac{B_{o_i} HKX_{i-1} \alpha_{o_{i-1/2}}}{\Delta x_i} (P_i - P_{i-1}) \end{aligned}$$

Using the same method for the water and solvent phases, it can be shown that:

$$\begin{aligned} B_w \frac{\partial}{\partial x} \left[\frac{KhK_{rw}}{B_w \mu_w} \frac{\partial P}{\partial x} \right] & \quad (3-8) \\ \approx \frac{B_{w_i} HKX_i \alpha_{w_{i-1/2}}}{\Delta x_i} (P_{i+1} - P_i) - \frac{B_{w_i} HKX_{i-1} \alpha_{w_{i-1/2}}}{\Delta x_i} (P_i - P_{i-1}) \end{aligned}$$

and

$$\begin{aligned} B_s \frac{\partial}{\partial x} \left[\frac{KhK_{rs}}{B_s \mu_s} \frac{\partial P}{\partial x} \right] & \quad (3-9) \\ \approx \frac{B_{s_i} HKX_i \alpha_{s_{i-1/2}}}{\Delta x_i} (P_{i+1} - P_i) - \frac{B_{s_i} HKX_{i-1} \alpha_{s_{i-1/2}}}{\Delta x_i} (P_i - P_{i-1}) \end{aligned}$$

where

$$\alpha_{w_{i+1/2}} = \left[\frac{K_{rw}}{B_w \mu_w} \right]_{i+1/2} \quad (3-10)$$

$$\alpha_{w_{i-1/2}} = \left[\frac{K_{rw}}{B_w \mu_w} \right]_{i-1/2} \quad (3-11)$$

$$\alpha_{s_{i+1/2}} = \left[\frac{K_{rs}}{B_s \mu_s} \right]_{i+1/2} \quad (3-12)$$

$$\alpha_{s_{i-1/2}} = \left[\frac{K_{rs}}{B_s \mu_s} \right]_{i-1/2} \quad (3-13)$$

To further simplify the FDE, the following coefficients are defined:

$$A_{o1} = \frac{B_{o1} H K X_{i-1} \alpha_{o_{i-1/2}}}{\Delta x_i} \quad (3-14)$$

$$C_{o1} = \frac{B_{o1} H K X_i \alpha_{o_{i+1/2}}}{\Delta x_i} \quad (3-15)$$

$$A_{w1} = \frac{B_{w1} H K X_{i-1} \alpha_{w_{i-1/2}}}{\Delta x_i} \quad (3-16)$$

$$C_{w1} = \frac{B_{w1} H K X_i \alpha_{w_{i+1/2}}}{\Delta x_i} \quad (3-17)$$

$$A_{s1} = \frac{B_{s1} H K X_{i-1} \alpha_{s_{i-1/2}}}{\Delta x_i} \quad (3-18)$$

$$C_{s1} = \frac{B_{s1} H K X_i \alpha_{s_{i+1/2}}}{\Delta x_i} \quad (3-19)$$

The above coefficients complete the simplification of the finite difference forms of the flux terms represented by the first three terms in the final PDE (Equation (2-21)). The next term in the PDE is the production term, and it can be converted to a finite difference form by just subscripting those terms that vary blockwise in the FDE. The "Q-Term" can then be defined as follows:

$$QT_i = \frac{q_{os_i} B_{oi} + q_{ws_i} B_{wi} + q_{ss_i} B_{si}}{\Delta x_i \Delta y} \quad (3-20)$$

The right-hand side of the PDE (Equation (2-21)) contains one first-order derivative. It can be approximated as follows:

$$\frac{\partial P}{\partial t} \approx \frac{P_i^{n+1} - P_i^n}{\Delta t} \quad (3-21)$$

The coefficient of the time derivative will be called σ and will be defined as follows:

$$\sigma_i = \frac{h_i \phi_i [C_{pv} + S_{oi} C_o + S_{wi} C_w + S_{si} C_{s_i}]}{\Delta t} \quad (3-22)$$

The Δt term is included in addition to the terms that constitute the coefficient of the time derivative, and the water and oil compressibility terms are considered constant.

Substituting the above definitions into the partial differential equation:

$$C_{0i}(P_{i+1}-P_i)^w - A_{0i}(P_i-P_{i-1})^w + C_{wi}(P_{i+1}-P_i)^w - A_{wi}(P_i-P_{i-1})^w \quad (3-23)$$

$$+ C_{si}(P_{i+1}-P_i)^w - A_{si}(P_i-P_{i-1})^w - QT_i = \sigma_i(P_i^{n+1}-P_i^n)$$

where the superscript w denotes the time level. If this equation were to be solved explicitly, the superscript would be (n) . However, for stability reasons, the model is solved in implicit form, and the superscript, w , is $(n+1)$.

For further simplification:

$$C_i = C_{0i} + C_{wi} + C_{si} \quad (3-24)$$

$$A_i = A_{0i} + A_{wi} + A_{si} \quad (3-25)$$

Equation (3-23) can now be written:

$$C_i(P_{i+1}-P_i)^{n+1} - A_i(P_i-P_{i-1})^{n+1} - QT_i = \sigma_i(P_i^{n+1}-P_i^n) \quad (3-26)$$

Rearranging terms and defining B_i and D_i as follows:

$$B_i = -A_i - C_i - \sigma_i \quad (3-27)$$

and

$$D_i = QT_i - \sigma_i P_i^n \quad (3-28)$$

the finite difference equation can be written:

$$A_i P_{i-1}^{n+1} + B_i P_i^{n+1} + C_i P_{i+1}^{n+1} = D_i \quad (3-29)$$

Solving the Finite Difference Equation

The first step in solving the set of finite difference equations is to state the boundary conditions. For this case, no flow (closed) boundaries are assumed. Since there is no flow across the boundaries:

$$\left[\frac{\partial P}{\partial x} \right]_{1/2} = 0 \quad (3-30)$$

and

$$\left[\frac{\partial P}{\partial x} \right]_{n+1/2} = 0 \quad (3-31)$$

where M is the number of grid divisions used. For the left boundary, the finite difference equivalent of the closed boundary (Equation (3-30)) is

$$\frac{P_1^{n+1} - P_0^{n+1}}{\Delta x} = 0 \quad (3-32)$$

and

$$P_0^{n+1} = P_1^{n+1} \quad (3-33)$$

Therefore, the first finite difference equation (for $i=1$), with the boundary condition included, can be written as follows:

$$(A_1 + B_1)P_1^{n+1} + C_1P_2^{n+1} = D_1 \quad (3-34)$$

Similarly, it can be shown that the right boundary can be written as follows:

$$A_M P_{M-1}^{n+1} + (B_M + C_M) P_M^{n+1} = D_M. \quad (3-35)$$

The set of simultaneous equations for M blocks would then be written as follows:

$$\begin{aligned} B_1' P_1^{n+1} + C_1 P_2^{n+1} &= D_1 \\ A_2 P_1^{n+1} + B_2 P_2^{n+1} + C_2 P_3^{n+1} &= D_2 \\ A_3 P_2^{n+1} + B_3 P_3^{n+1} + C_3 P_4^{n+1} &= D_3 \\ &\vdots \\ A_M P_{M-1}^{n+1} + B_M'' P_M^{n+1} &= D_M \end{aligned} \quad (3-36)$$

where $B_1' = A_1 + B_1$ and $B_M'' = B_M + C_M$.

Breitenbach, et al.,¹⁷ list some common methods by which these equations may be solved, and list their advantages and disadvantages. Gaussian elimination, which is used in the model, is best in test cases and small problems. Due to the amount of storage needed, the numerous arithmetic operations associated with the method, and possibilities of large round-off errors, this method is not used in large models.

Successive over relaxation (SOR) is a widely used iterative method which involves calculating an "over relaxation factor." Though accurate, the proper value of the over relaxation factor is critical in making the calculations and is also time consuming to obtain. The SOR method may use a

modification of the "Power Method" of Varga¹⁸ to calculate the over relaxation factor.

CHAPTER IV

CALCULATION OF FLUID AND ROCK PROPERTIES

Results from a simulator are only as accurate as the data input. Formation volume factors and viscosity, which are not dependent on saturations and are not highly pressure sensitive, may be measured in a laboratory. The information may then be entered as a table or as a correlation based on pressure and temperature. Determining relative permeability, however, is a more difficult task. While numerous correlations have been developed for two-phase fluid flow, the complexity of three-phase flow defies simple empirical relationships.

Determining Relative Permeability

As stated above, one of the most difficult properties to determine is relative permeability. Several papers have been published relating permeability to pore size, saturations and production history. Burdine¹⁹ developed a formula relating relative permeability to pore size distribution and the ratio of tortuosities by first determining the irreducible wetting phase saturation. Corey¹ presents a relationship for gas and oil relative permeabilities based on Burdine's equation for tortuosity. Corey's equations for a gas and oil system are expressed as follows:

$$K_{ro} = S_o^*{}^4 \quad (4-1)$$

$$K_{rg} = [1 - S_o^*]^2 [1 - S_o^{*2}] \quad (4-2)$$

where the saturations used are reduced saturations, defined as follows:

$$S_o^* = \frac{S_o - S_{or}}{1 - S_{or}} \quad (4-3)$$

Corey, et al.,²⁰ expanded on this previous work to provide isoperms for three-phase relative permeabilities.

Effect of Trapped Oil on Relative Permeability

As stated earlier, the relative permeability is dependent not only on saturations, but also on the process which enabled it to get there. As can be seen in Figure 4-1, a hysteresis effect is noted on the relative permeability curve of the non-wetting phase. Papers written indicate that this phenomenon occurs because oil is trapped during an imbibition process.^{2, 21, 22} The only way for this trapped oil to become free, or mobile, would be to decrease the water saturation.

For strongly water-wet rock, Land² demonstrated that

$$\frac{1}{S_{or}^*} - \frac{1}{S_{oi}^*} = C \quad (4-4)$$

where C is a "trapping constant", and is a property of the rock. Reduced saturations are used and are defined as

Relative Permeability Curves

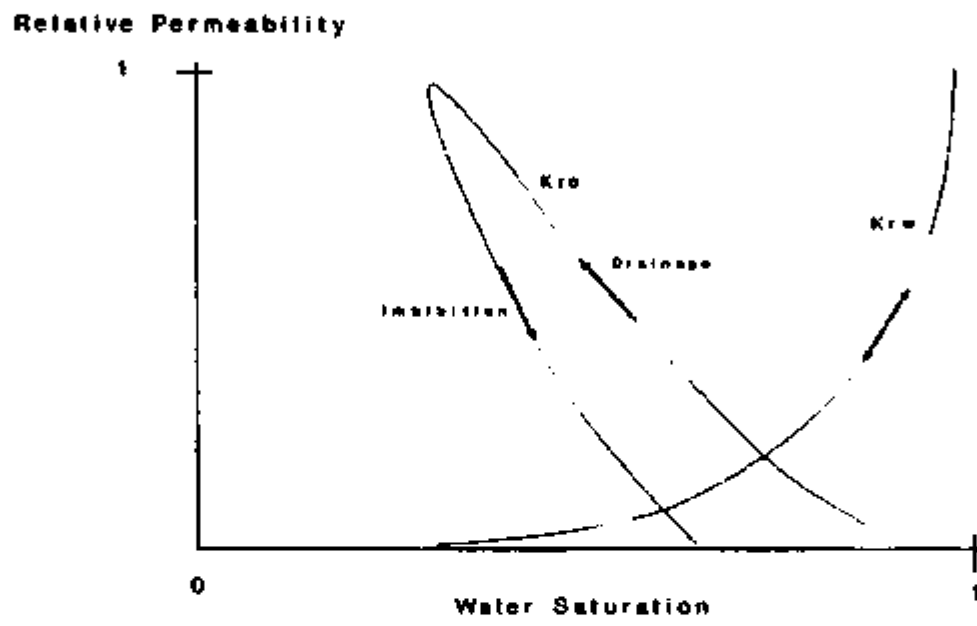


Figure 4-1

Relative Permeability Curves

follows:

$$S_{or}^* = \frac{S_{or}}{(1 - S_{w1})} \quad (4-5)$$

$$S_{oi}^* = \frac{S_{oi}}{(1 - S_{w1})} \quad (4-6)$$

where S_{or} is the residual oil saturation to waterflood, and S_{oi} is the initial oil saturation (before waterflood).

When a reservoir is at irreducible water saturation, there is no trapped oil. At residual oil saturation, after a waterflood (an imbibition process), there is no free oil. When S_o^* has a value between S_{oi}^* and S_{or}^* , the reduced mobile oil saturation is given by:²

$$S_{of}^* = \frac{1}{2} \left\{ [S_o^* - S_{or}^*] + \sqrt{[S_o^* - S_{or}^*]^2 + \frac{4}{c} [S_o^* - S_{or}^*]} \right\} \quad (4-7)$$

where S_{of}^* is the reduced free oil saturation, and is determined as:

$$S_{of}^* = \frac{S_{of}}{(1 - S_{w1})} \quad (4-8)$$

$$S_o^* = \frac{S_o}{(1 - S_{w1})} \quad (4-9)$$

The reduced trapped oil saturation is then equal to:

$$S_{oT}^* = \frac{1}{2} \left\{ (S_o^* + S_{or}^*) - \sqrt{(S_o^* - S_{or}^*)^2 + \frac{4}{c} (S_o^* - S_{or}^*)} \right\} \quad (4-10)$$

where

$$S_{oT}^* = \frac{S_{oT}}{(1 - S_{wi})} \quad (4-11)$$

See Figure 4-2 for a graphical representation of S_{of}^* versus S_{or}^* .

For a strongly water-wet Berea sandstone undergoing a drainage process, Land² states that Corey's equations apply.

$$K_{rw} = S_w^{*4} \quad (4-12)$$

$$K_{ro} = S_o^{*2} (1 - S_w^{*2}) \quad (4-13)$$

where S_w^* is the reduced free water saturation and is equal to:

$$S_w^* = \frac{(S_w - S_{wi})}{1 - S_{wi}} \quad (4-14)$$

For an imbibition process, the above equation still applies, while the relative permeability to oil is:

$$K_{ro} = S_{oF}^{*2} (1 - (1 - S_{oF}^*)^2) \quad (4-15)$$

In the case of a miscible flood, ignoring the mixing zone, two-phase flow can be assumed behind and in front of

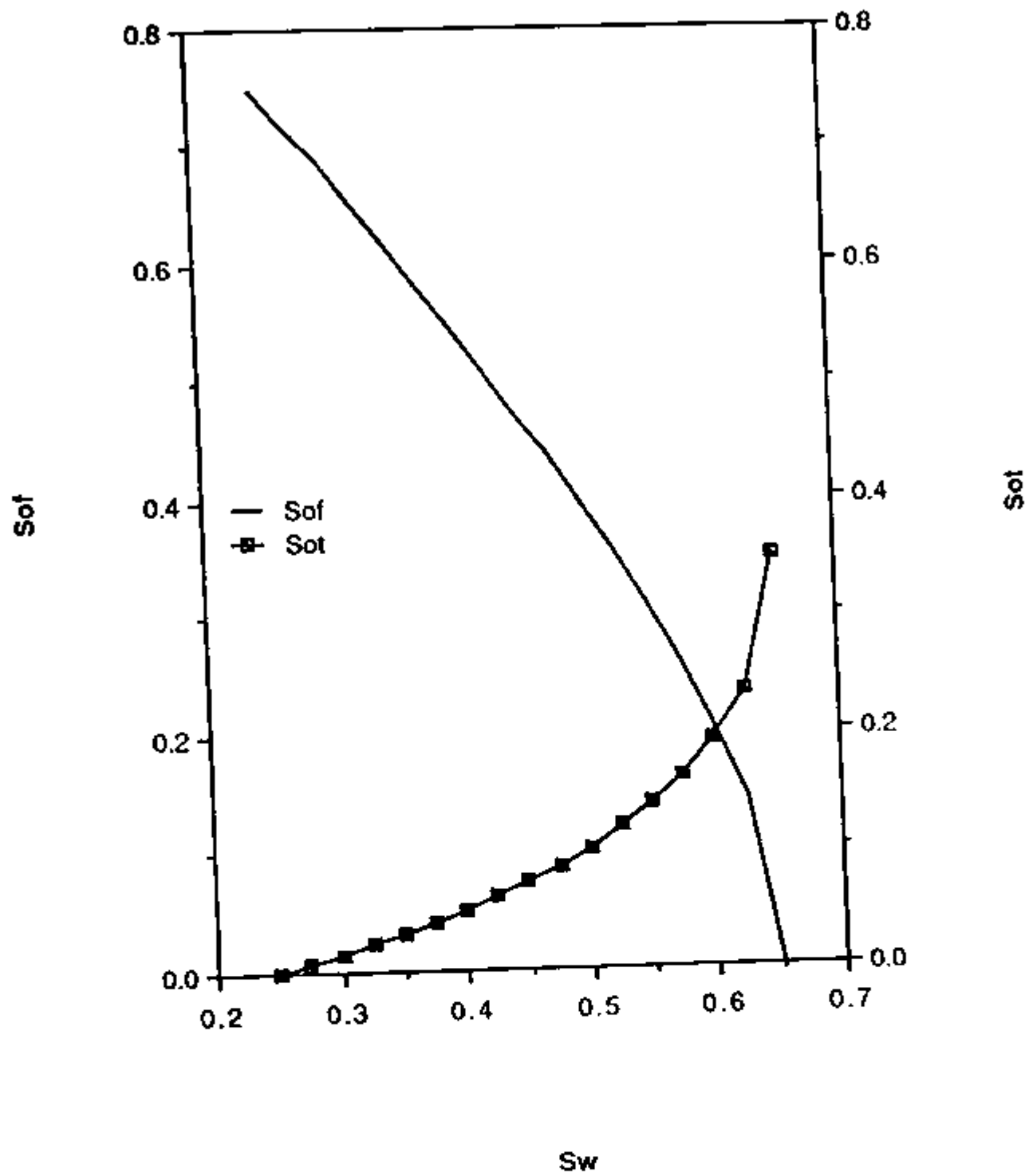


Figure 4-2

Trapped and Mobile Saturations vs. Water Saturation

the solvent front. Ahead of the front the oil has been miscibly displaced, and is immiscibly displacing water. Behind the solvent front, only carbon dioxide and water are flowing. At the solvent-oil interface, the solvent will have the same relative permeability as the free oil, and Equation (4-15) may be used for solvent if reduced solvent saturation is substituted for reduced oil saturation.

$$K_{rs} = S_{SOL}^* \left[1 - (1 - S_{SOL})^2 \right] \quad (4-16)$$

where

$$S_{SOL}^* = \frac{S_{SOL}}{(1 - S_{wi})} \quad (4-17)$$

CHAPTER V
COMPUTER MODEL

For a commercial model, the simulator must be run from the time of initial production. The output is then history matched with known results. For practical purposes, the simulator shown in the Appendix was run from the start of tertiary recovery. Values for saturation profiles and pressure distributions were input as constants in this model.

For convenience in comparing simulator results with the Buckley-Leverett results, the reservoir was assumed homogeneous. Constants input into the program include:

1. Porosity
2. Thickness
3. Length
4. Width
5. Permeability
6. Water and Oil Compressibilities
7. Initial Water and Oil Saturations
8. Initial Reservoir Pressure.

Reservoir temperature was assumed to be constant and linear correlations as a function of pressure were used to calculate formation volume factors. Slight corrections based on pressure were also used for the viscosity values.

Solvent compressibility was computed based on its formation volume factor.

As previously mentioned, saturations behind the solvent front were determined using a trapped oil function. Ahead of the solvent front, saturations are determined explicitly. From Equation (2-17), it can be shown that the changes in oil and water saturations are:

$$S_o^{n+1} - S_o^n \approx \frac{\Delta t}{h\phi} \left[\left[B_o \frac{\partial KhK_{ro}}{\partial x B_o \nu_o} \frac{dP}{dx} \right] + \frac{q_{os} B_o}{\Delta x \Delta y} \right] \quad (5-1)$$

$$- S_o^n [C_{pv} + C_o] (P_i^{n+1} - P_i^n)$$

$$S_w^{n+1} - S_w^n \approx \frac{\Delta t}{h\phi} \left[\left[B_w \frac{\partial}{\partial x} \frac{KhK_{rw}}{B_w \nu_w} \frac{dP}{dx} \right] + \frac{q_{ws} B_w}{\Delta x \Delta y} \right] \quad (5-2)$$

$$- S_w^n [C_{pv} + C_w] (P_i^{n+1} - P_i^n)$$

Corey's equations are used for calculating the relative permeabilities for water and oil in the block containing the solvent front. Equations (4-14) and (4-15), which involve mobile oil calculations, are used to determine the relative permeabilities of solvent and of oil ahead of the solvent front. Upstream saturation values are used to calculate the relative permeabilities at block interfaces. The preferred use of upstream saturations was established by Luo²³ as well as others.

For comparison with the Buckley-Leverett, the reservoir volume flow rate at the producing end was set equal to the reservoir volume injection rate. Fractions of oil and water flowing are based on the relative permeabilities and viscosities at the producing end. The oil-water ratio (OWR) is then equal to:

$$\text{OWR} = \frac{B_w(1-f_w)}{f_w B_o} \quad (5-3)$$

Two main calculations are involved in the program. These two calculations are the calculation of the new pressures and saturations, (See flow chart on Figure 5-1), and then the checking of the results with pre-set tolerances. For stability purposes, a check is programmed to see if the injection amount will cause the solvent front to cross a block boundary during a time step. If this occurs, the time step is divided into two sub-time steps. The first of these is computed to bring the solvent front exactly to the block interface and the remaining time allows the front to enter the next block. Also, it should be noted the time step cut is made before the pressure calculations are performed. In order for this process to be accurate, the time step is kept small. Pressure changes with respect to time are reduced which results in small changes in fluid formation volume factor. Figure 5-1 illustrates the program flow chart.

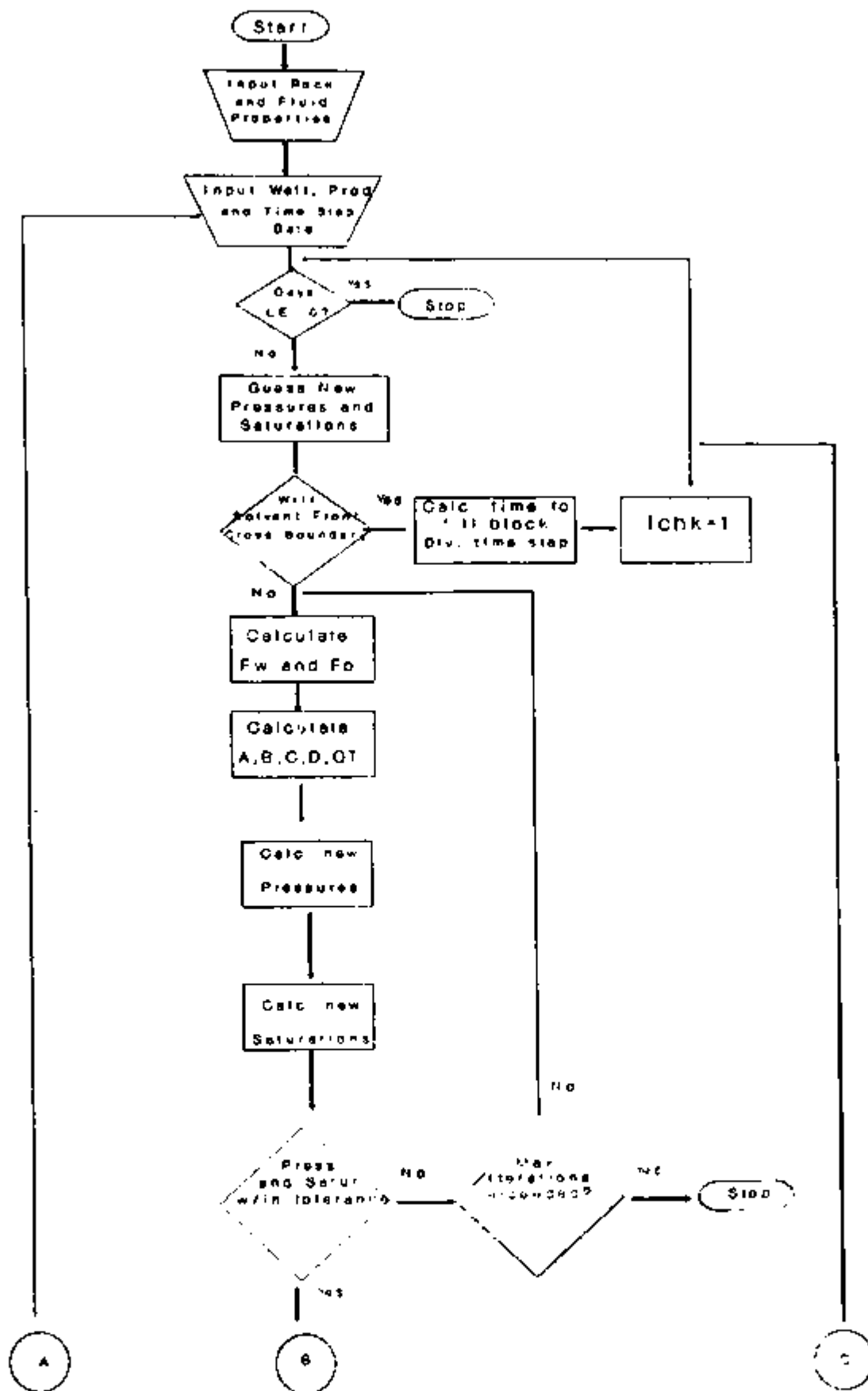


Figure 5-1
Simplified Flow Diagram

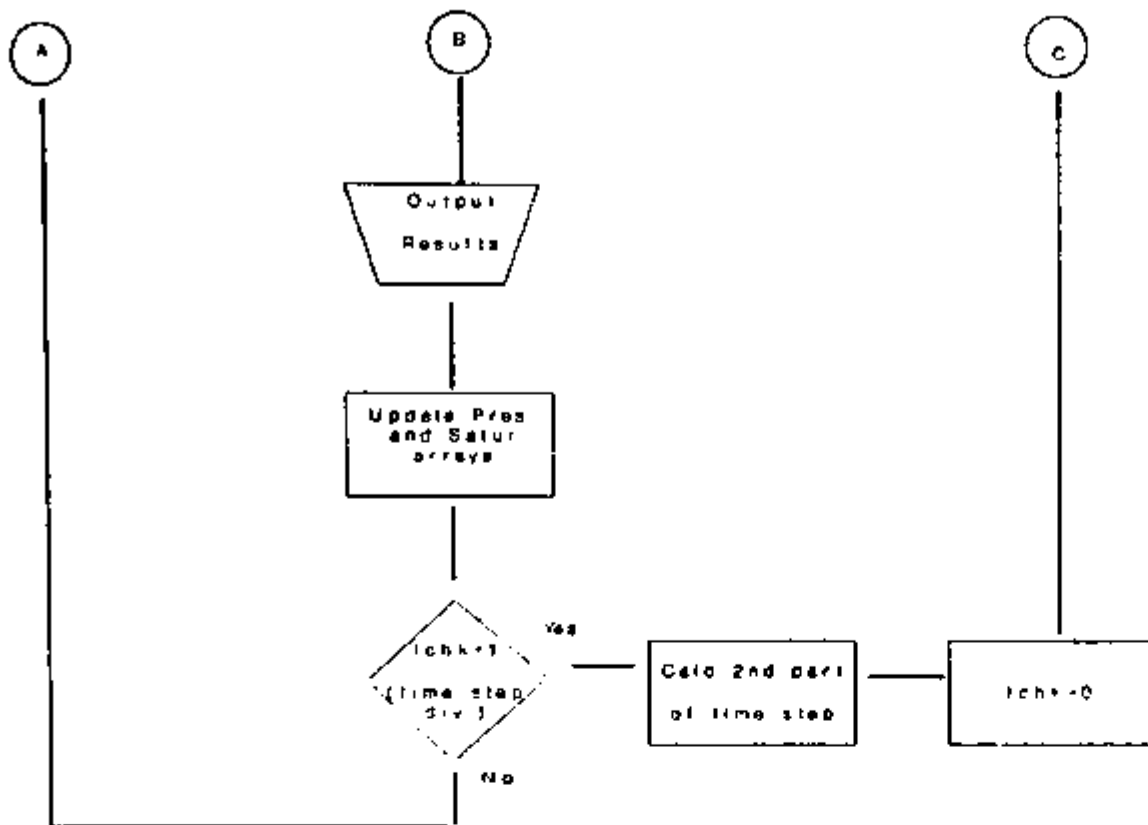


Figure 5-1 (continued)
Simplified Flow Diagram

CHAPTER VI

RESULTS AND CONCLUSIONS

Table 6-1 lists the values used for the simulator and the Buckley-Leverett model. Relative permeabilities, saturations, and pressures are tabulated in Tables 6-2 to 6-4 for various injection times. Table 6-2 shows the results shortly after the start of injection. Table 6-3 shows the results just before solvent breakthrough. Pressure profiles as a function of time are illustrated in Figure 6-1.

Figures 6-2 through 6-7 present saturation profiles from the simulator and from the Buckley-Leverett model, at various times. Using the Buckley-Leverett model, the solvent forms a sharp front, as opposed to the simulator, which shows a decrease in solvent saturation close to the front. This is a result of averaging the solvent volume over the total pore volume in the partially invaded block. The solvent front calculated by the Buckley-Leverett model is consistently ahead of the simulator's solvent front, partially caused by using a constant solvent formation volume factor.

The effects of numerical dispersion on the shape of the oil bank are apparent on Figures 6-3 and 6-5. Oil bank breakthrough occurs at approximately 17 days on the simulator, although minute quantities of oil are produced as early as five days after injection. Production from the Buckley-

Values Used in Calculations

Table 6-1

Number of blocks (M) = 50	(B)
Length of each block = 10 ft.	(B)
Width of each block = 10 ft.	(B)
Thickness of each block = 10 ft.	(B)
Porosity of each block = 18%	(B)
Residual oil saturation to waterflood = .375	(B)
Irreducible water saturation = .25	(B)
Absolute permeability = 450 md	(B)
Oil formation volume factor = 1.2 RVB/STB	(BL)
Water formation volume factor = 1.01	(BL)
Solvent formation volume factor = 3.19×10^{-3} RCF/SCF	(BL)
Compressibility of oil = 5.5×10^{-6} psi ⁻¹	(S)
Compressibility of water = 3.0×10^{-6} psi ⁻¹	(S)
Compressibility of pore volume = 5.5×10^{-6} psi ⁻¹	(S)
Initial reservoir pressure = 2500 psi	(B)
Oil viscosity = 1.0 cp	(BL)
Water viscosity = .5 cp	(BL)
Solvent viscosity = .04 cp	(BL)
Injection rate = 20,000 SCF/Day	(B)

BL - used for Buckley-Leverett Calculation

S - used in Simulator Calculations

B - used for Buckley-Leverett and Simulator

Table 6-2

Simulator Results at 2.05 days

Block	Pressure (psi)	Sw	So	Ss	Krw	Kro	Krs
1	2842.61	0.4375	0.0822	0.4803	0.0039	0	0.3571
2	2842.39	0.4841	0.4164	0.0995	0.0095	0.032	0.0044
3	2830.22	0.5224	0.4776	0	0.0174	0.1366	0
4	2818.75	0.5359	0.4641	0	0.0211	0.109	0
5	2805.77	0.549	0.451	0	0.0253	0.0845	0
6	2791.25	0.5629	0.4371	0	0.0303	0.0611	0
7	2775.15	0.5792	0.4208	0	0.0371	0.0376	0
8	2757.63	0.5988	0.4012	0	0.0468	0.0155	0
9	2739.68	0.617	0.383	0	0.0573	0.0025	0
10	2722.99	0.6243	0.3757	0	0.062	0.0001	0
11	2707.25	0.625	0.375	0	0.0625	0	0
12	2691.62	0.625	0.375	0	0.0625	0	0
13	2675.98	0.625	0.375	0	0.0625	0	0
14	2660.37	0.625	0.375	0	0.0625	0	0
15	2664.76	0.625	0.375	0	0.0625	0	0
16	2629.17	0.625	0.375	0	0.0625	0	0
17	2613.59	0.625	0.375	0	0.0625	0	0
18	2598.01	0.625	0.375	0	0.0625	0	0
19	2582.45	0.625	0.375	0	0.0625	0	0
20	2566.89	0.625	0.375	0	0.0625	0	0
21	2551.35	0.625	0.375	0	0.0625	0	0
22	2535.81	0.625	0.375	0	0.0625	0	0
23	2520.28	0.625	0.375	0	0.0625	0	0
24	2504.76	0.625	0.375	0	0.0625	0	0
25	2489.25	0.625	0.375	0	0.0625	0	0
26	2473.74	0.6249	0.3751	0	0.0624	0	0
27	2458.23	0.6248	0.3752	0	0.0624	0	0
28	2442.72	0.6248	0.3752	0	0.0624	0	0
29	2427.2	0.6247	0.3753	0	0.0623	0	0
30	2411.69	0.6247	0.3753	0	0.0623	0	0
31	2396.17	0.6246	0.3754	0	0.0622	0	0
32	2380.65	0.6245	0.3755	0	0.0622	0	0
33	2365.13	0.6245	0.3755	0	0.0622	0	0
34	2349.61	0.6244	0.3756	0	0.0621	0	0
35	2334.09	0.6244	0.3756	0	0.0621	0.0001	0
36	2381.56	0.6243	0.3757	0	0.062	0.0001	0
37	2303.03	0.6242	0.3758	0	0.062	0.0001	0
38	2287.5	0.6242	0.3758	0	0.062	0.0001	0
39	2271.96	0.6241	0.3759	0	0.0619	0.0001	0
40	2256.43	0.6241	0.3759	0	0.0619	0.0001	0
41	2240.89	0.624	0.3769	0	0.0618	0.0001	0
42	2225.35	0.6239	0.3761	0	0.0618	0.0001	0
43	2209.8	0.6239	0.3761	0	0.0618	0.0001	0
44	2194.26	0.6238	0.3762	0	0.0617	0.0001	0
45	2178.71	0.6238	0.3762	0	0.0617	0.0001	0
46	2163.16	0.6237	0.3763	0	0.0616	0.0002	0
47	2147.6	0.6236	0.3764	0	0.0616	0.0002	0
48	2132.05	0.6236	0.3764	0	0.0616	0.0002	0
49	2116.49	0.6235	0.3765	0	0.0615	0.0002	0
50	2100.92	0.6235	0.3765	0	0.0615	0.0002	0

Simulator Results at 18.22 days

Block	Pressure (psi)	Sw	So	Ss	Krw	Kro	Krs
1	2763.93	0.4375	0.0822	0.4803	0.0039	0	0.3571
2	2763.7	0.4375	0.0822	0.4803	0.0039	0	0.3571
3	2763.48	0.4375	0.0822	0.4803	0.0039	0	0.3571
4	2763.25	0.4375	0.0822	0.4803	0.0039	0	0.3571
5	2763.03	0.4375	0.0822	0.4803	0.0039	0	0.3571
6	2762.8	0.4375	0.0822	0.4803	0.0039	0	0.3571
7	2762.57	0.4375	0.0822	0.4803	0.0039	0	0.3571
8	2762.34	0.4375	0.0822	0.4803	0.0039	0	0.3571
9	2762.11	0.4375	0.0822	0.4803	0.0039	0	0.3571
10	2761.88	0.4375	0.0822	0.4803	0.0039	0	0.3571
11	2761.66	0.4499	0.403	0.1471	0.005	0.0172	0.0136
12	2756.08	0.4735	0.5265	0	0.0079	0.2548	0
13	2748.52	0.482	0.518	0	0.0091	0.2326	0
14	2740.37	0.4869	0.5131	0	0.01	0.2198	0
15	2731.84	0.491	0.509	0	0.0107	0.2097	0
16	2723	0.4945	0.5055	0	0.0113	0.2008	0
17	2713.86	0.4978	0.5022	0	0.0119	0.1929	0
18	2704.45	0.5007	0.4993	0	0.0125	0.1858	0
19	2694.77	0.5034	0.4966	0	0.013	0.1793	0
20	2684.84	0.506	0.494	0	0.0136	0.1734	0
21	2674.68	0.5084	0.4916	0	0.0141	0.1678	0
22	2664.29	0.5106	0.4894	0	0.0146	0.1627	0
23	2653.68	0.5128	0.4872	0	0.0151	0.1578	0
24	2642.85	0.5148	0.4852	0	0.0155	0.1532	0
25	2631.82	0.5168	0.4832	0	0.016	0.1488	0
26	2620.58	0.5187	0.4813	0	0.0165	0.1446	0
27	2609.15	0.5206	0.4794	0	0.0169	0.1405	0
28	2597.51	0.5224	0.4776	0	0.0174	0.1366	0
29	2585.69	0.5242	0.4758	0	0.0179	0.1329	0
30	2573.67	0.5259	0.4741	0	0.0183	0.1292	0
31	2561.46	0.5276	0.4724	0	0.0188	0.1257	0
32	2549.07	0.5293	0.4707	0	0.0192	0.1222	0
33	2536.49	0.5309	0.4691	0	0.0197	0.1189	0
34	2523.72	0.5326	0.4674	0	0.0202	0.1156	0
35	2510.76	0.5342	0.4658	0	0.0206	0.1123	0
36	2497.63	0.5358	0.4642	0	0.0211	0.1091	0
37	2484.3	0.5375	0.4625	0	0.0216	0.1059	0
38	2470.79	0.5391	0.4609	0	0.0221	0.1027	0
39	2457.09	0.5408	0.4592	0	0.0226	0.0996	0
40	2443.2	0.5425	0.4575	0	0.0231	0.0964	0
41	2429.11	0.5442	0.4558	0	0.0237	0.0931	0
42	2414.83	0.546	0.454	0	0.0243	0.0899	0
43	2400.33	0.5479	0.4521	0	0.0249	0.0865	0
44	2385.63	0.5499	0.4501	0	0.0256	0.0829	0
45	2370.69	0.552	0.448	0	0.0263	0.0791	0
46	2355.51	0.5544	0.4456	0	0.0271	0.075	0
47	2340.05	0.5571	0.4429	0	0.0281	0.0704	0
48	2324.3	0.5604	0.4396	0	0.0293	0.0651	0
49	2308.19	0.5644	0.4356	0	0.0309	0.0588	0
50	2291.66	0.5697	0.4303	0	0.033	0.0508	0

Table 6-4

Simulator Results at 71.75 days

Block	Pressure (psi)	Sw	So	Ss	Krw	Kro	Krs
1	2676.67	0.4375	0.0822	0.4803	0.0039	0	0.3571
2	2676.43	0.4375	0.0822	0.4803	0.0039	0	0.3571
3	2676.2	0.4375	0.0822	0.4803	0.0039	0	0.3571
4	2675.96	0.4375	0.0822	0.4803	0.0039	0	0.3571
5	2675.73	0.4375	0.0822	0.4803	0.0039	0	0.3571
6	2675.5	0.4375	0.0822	0.4803	0.0039	0	0.3571
7	2675.26	0.4375	0.0822	0.4803	0.0039	0	0.3571
8	2675.03	0.4375	0.0822	0.4803	0.0039	0	0.3571
9	2674.79	0.4375	0.0822	0.4803	0.0039	0	0.3571
10	2674.56	0.4375	0.0822	0.4803	0.0039	0	0.3571
11	2674.33	0.4375	0.0822	0.4803	0.0039	0	0.3571
12	2674.09	0.4375	0.0822	0.4803	0.0039	0	0.3571
13	2673.86	0.4375	0.0822	0.4803	0.0039	0	0.3571
14	2673.63	0.4375	0.0822	0.4803	0.0039	0	0.3571
15	2673.39	0.4375	0.0822	0.4803	0.0039	0	0.3571
16	2673.16	0.4375	0.0822	0.4803	0.0039	0	0.3571
17	2672.92	0.4375	0.0822	0.4803	0.0039	0	0.3571
18	2672.69	0.4375	0.0822	0.4803	0.0039	0	0.3571
19	2672.46	0.4375	0.0822	0.4803	0.0039	0	0.3571
20	2672.22	0.4375	0.0822	0.4803	0.0039	0	0.3571
21	2671.99	0.4375	0.0822	0.4803	0.0039	0	0.3571
22	2671.76	0.4375	0.0822	0.4803	0.0039	0	0.3571
23	2671.52	0.4375	0.0822	0.4803	0.0039	0	0.3571
24	2671.29	0.4375	0.0822	0.4803	0.0039	0	0.3571
25	2671.06	0.4375	0.0822	0.4803	0.0039	0	0.3571
26	2670.82	0.4375	0.0822	0.4803	0.0039	0	0.3571
27	2670.59	0.4375	0.0822	0.4803	0.0039	0	0.3571
28	2670.35	0.4375	0.0822	0.4803	0.0039	0	0.3571
29	2670.12	0.4375	0.0822	0.4803	0.0039	0	0.3571
30	2669.89	0.4375	0.0822	0.4803	0.0039	0	0.3571
31	2669.65	0.4375	0.0822	0.4803	0.0039	0	0.3571
32	2669.42	0.4375	0.0822	0.4803	0.0039	0	0.3571
33	2669.19	0.4375	0.0822	0.4803	0.0039	0	0.3571
34	2668.95	0.4375	0.0822	0.4803	0.0039	0	0.3571
35	2668.72	0.4375	0.0822	0.4803	0.0039	0	0.3571
36	2668.49	0.4375	0.0822	0.4803	0.0039	0	0.3571
37	2668.25	0.4375	0.0822	0.4803	0.0039	0	0.3571
38	2668.02	0.4375	0.0822	0.4803	0.0039	0	0.3571
39	2667.79	0.4375	0.0822	0.4803	0.0039	0	0.3571
40	2667.55	0.4375	0.0822	0.4803	0.0039	0	0.3571
41	2667.32	0.4375	0.0822	0.4803	0.0039	0	0.3571
42	2667.09	0.4375	0.0822	0.4803	0.0039	0	0.3571
43	2666.85	0.4375	0.0822	0.4803	0.0039	0	0.3571
44	2666.62	0.4375	0.0822	0.4803	0.0039	0	0.3571
45	2666.39	0.4375	0.0822	0.4803	0.0039	0	0.3571
46	2666.15	0.4375	0.0822	0.4803	0.0039	0	0.3571
47	2665.92	0.4375	0.0822	0.4803	0.0039	0	0.3571
48	2665.69	0.4375	0.0822	0.4803	0.0039	0	0.3571
49	2665.45	0.449	0.4095	0.1415	0.005	0.0239	0.0122
50	2659.42	0.4708	0.5292	0	0.0075	0.267	0

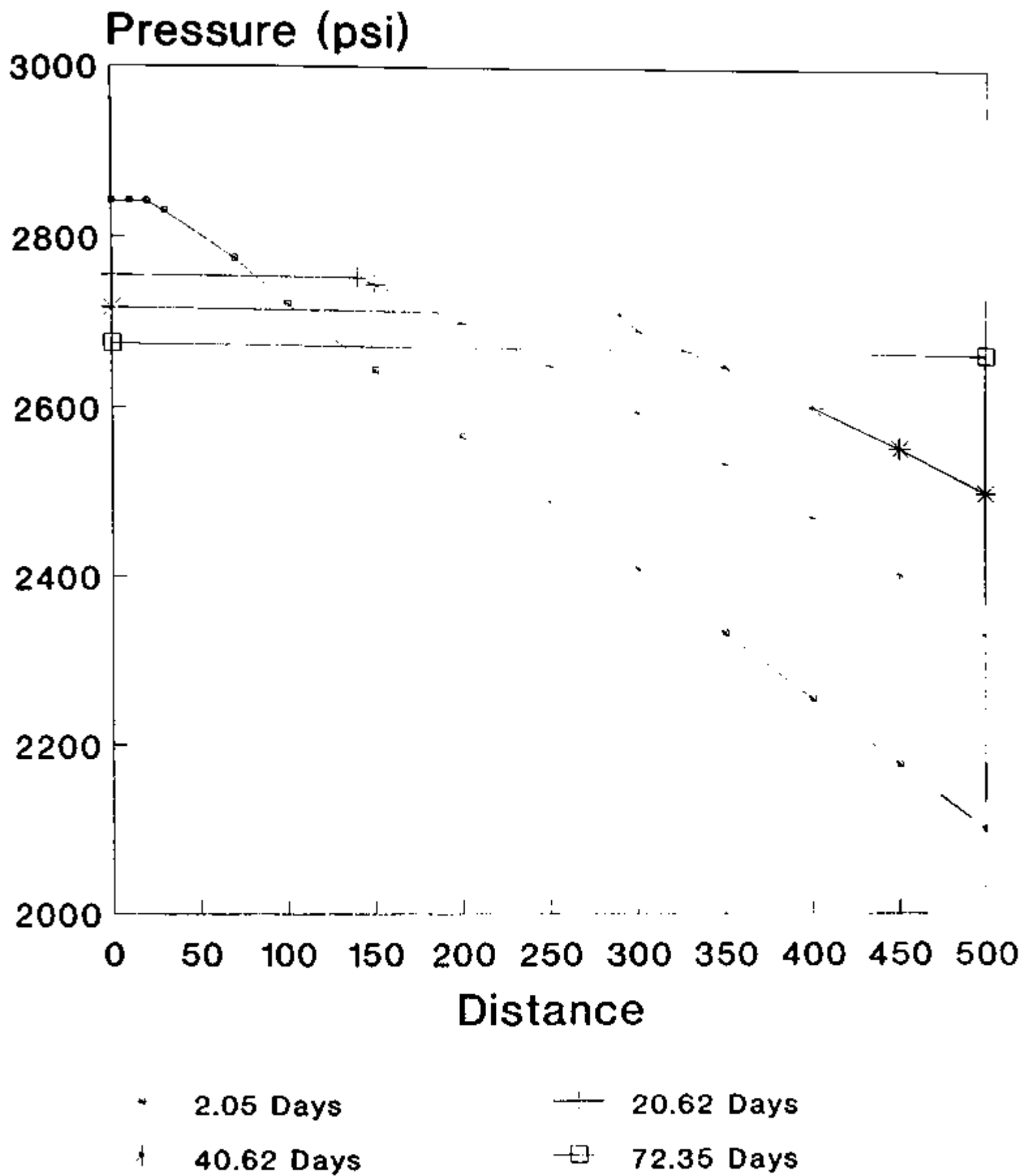


Figure 6-1
Pressure Profiles

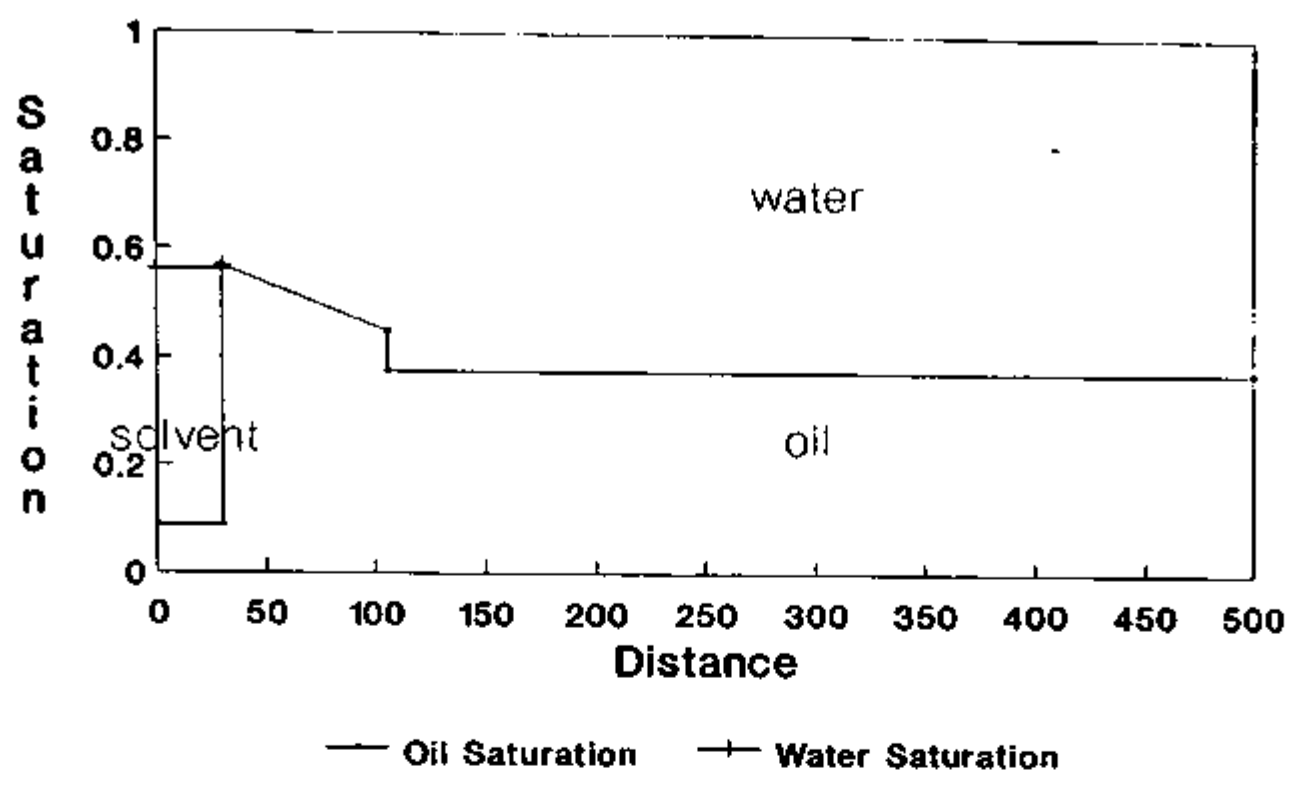


Figure 6-2
Saturation Profile from Buckley-Leverett (4.05 Days)

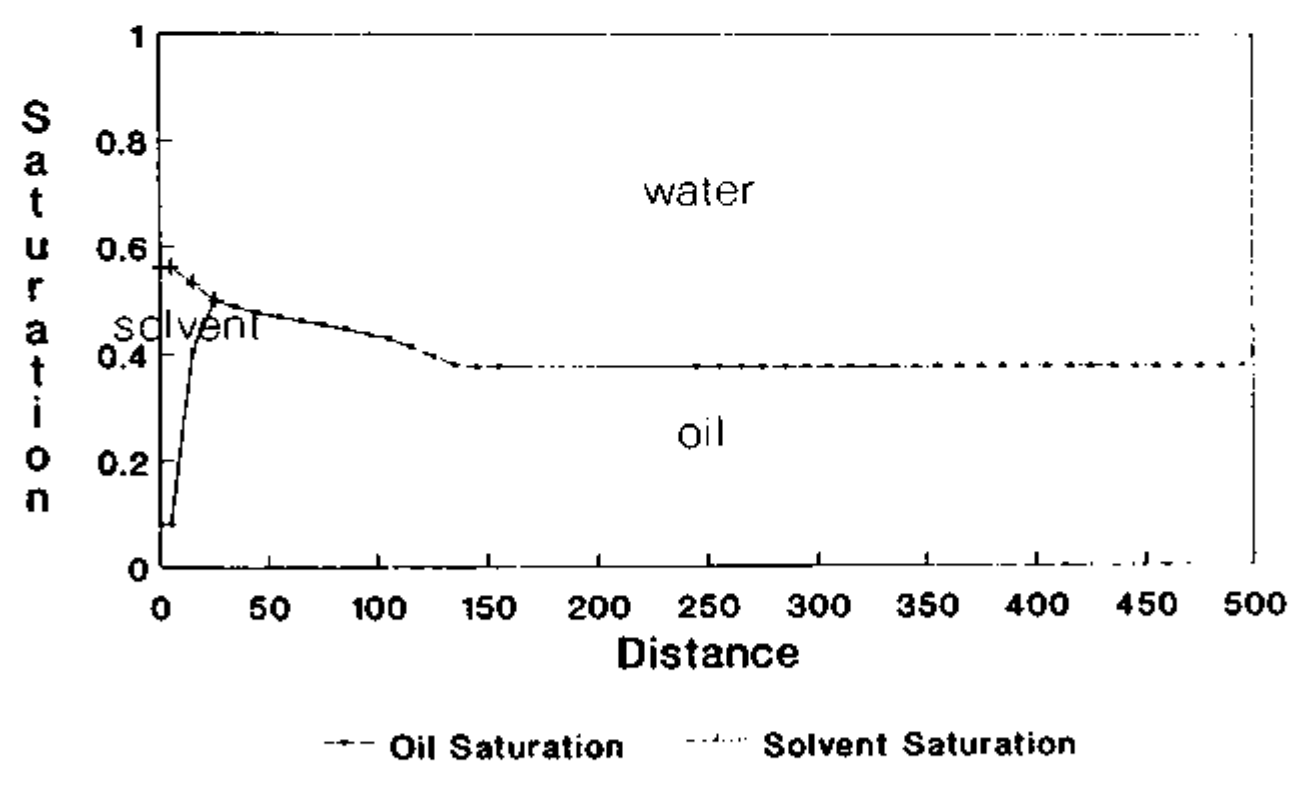


Figure 6-3
Saturation Profile from Simulator (4.05 Days)

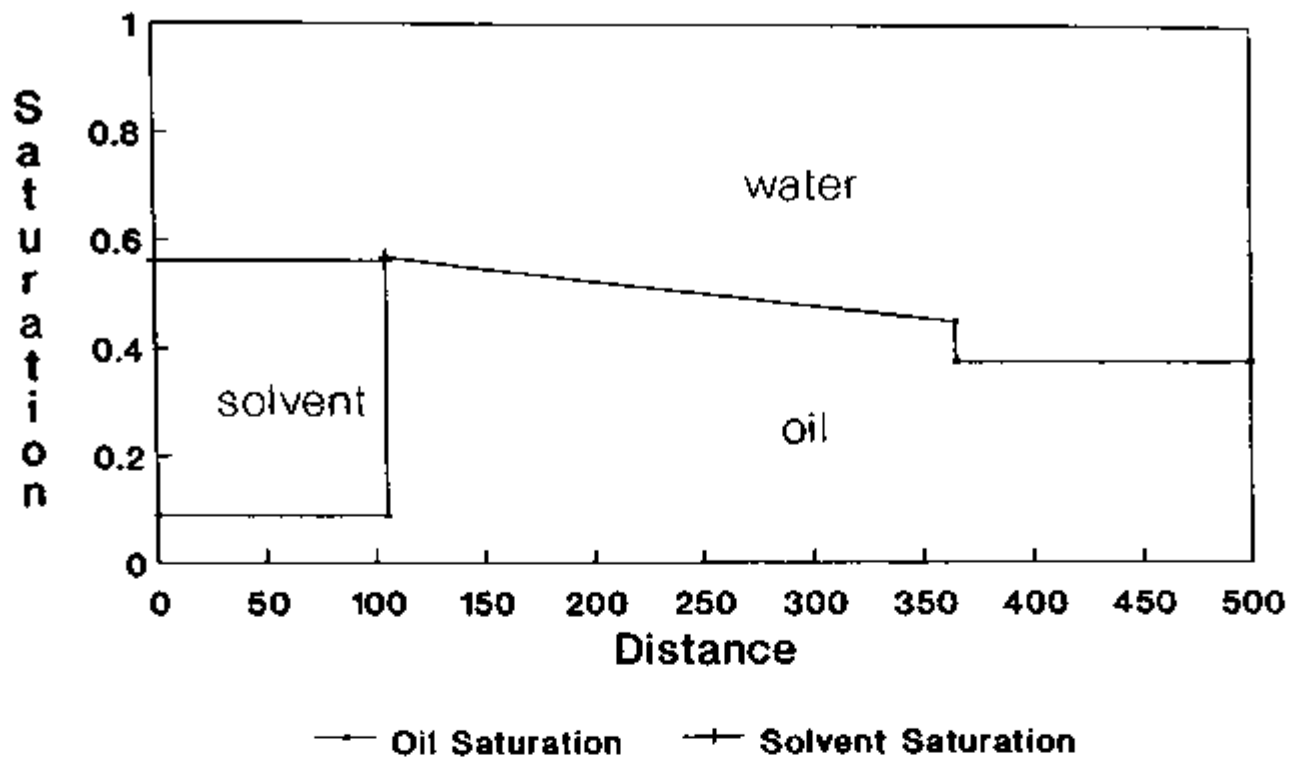


Figure 6-4

Saturation Profile from Buckley-Leverett (14.22 Days)

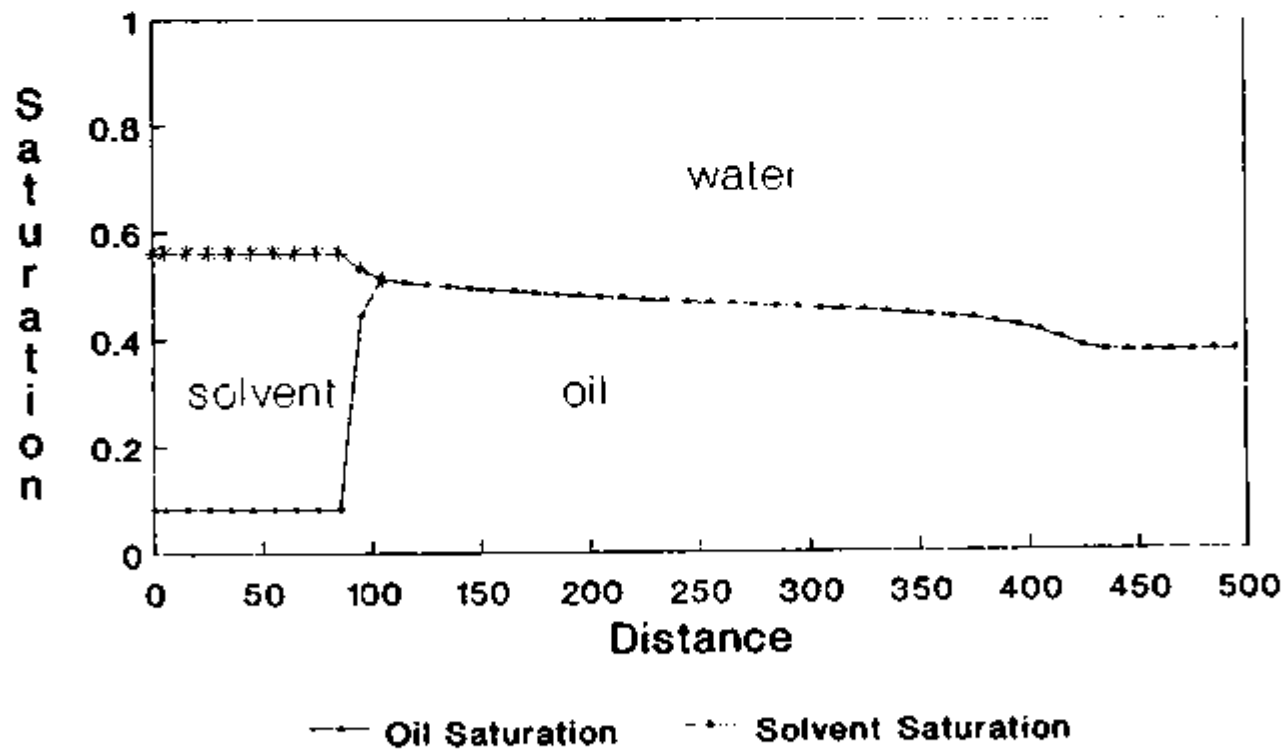


Figure 6-5

Saturation Profile from Simulator (14.22 Days)

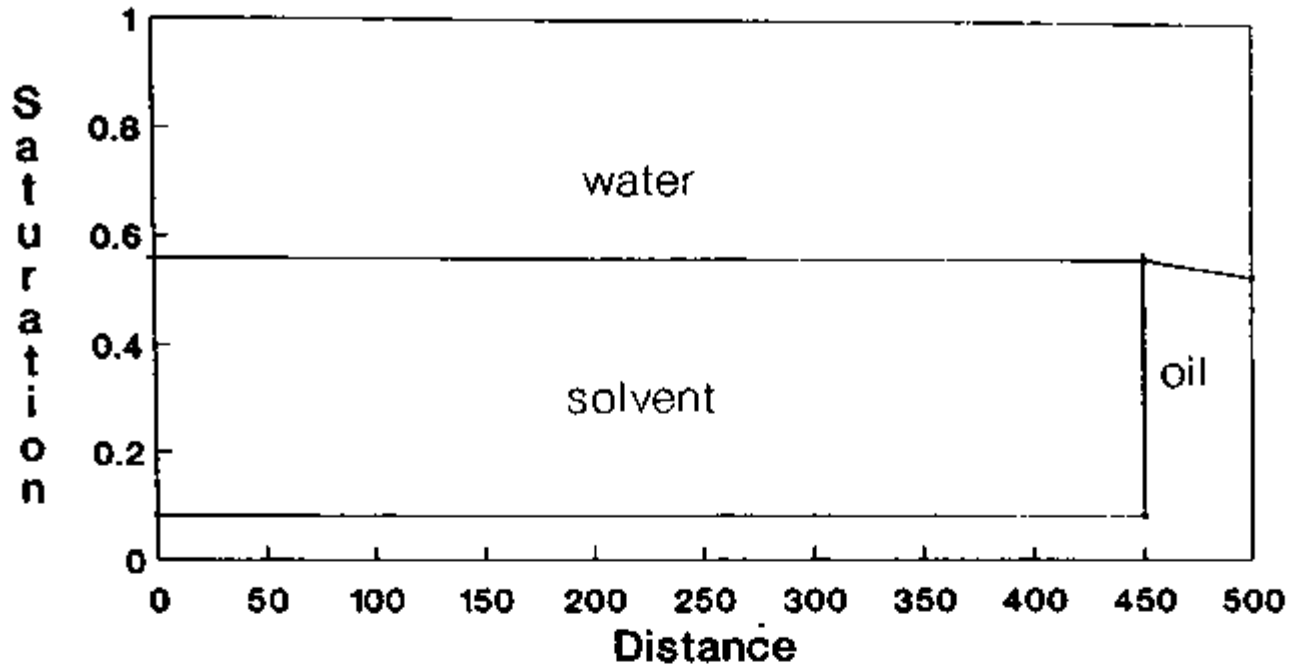


Figure 6-6

Saturation Profile from Buckley-Leverett (60.5 Days)

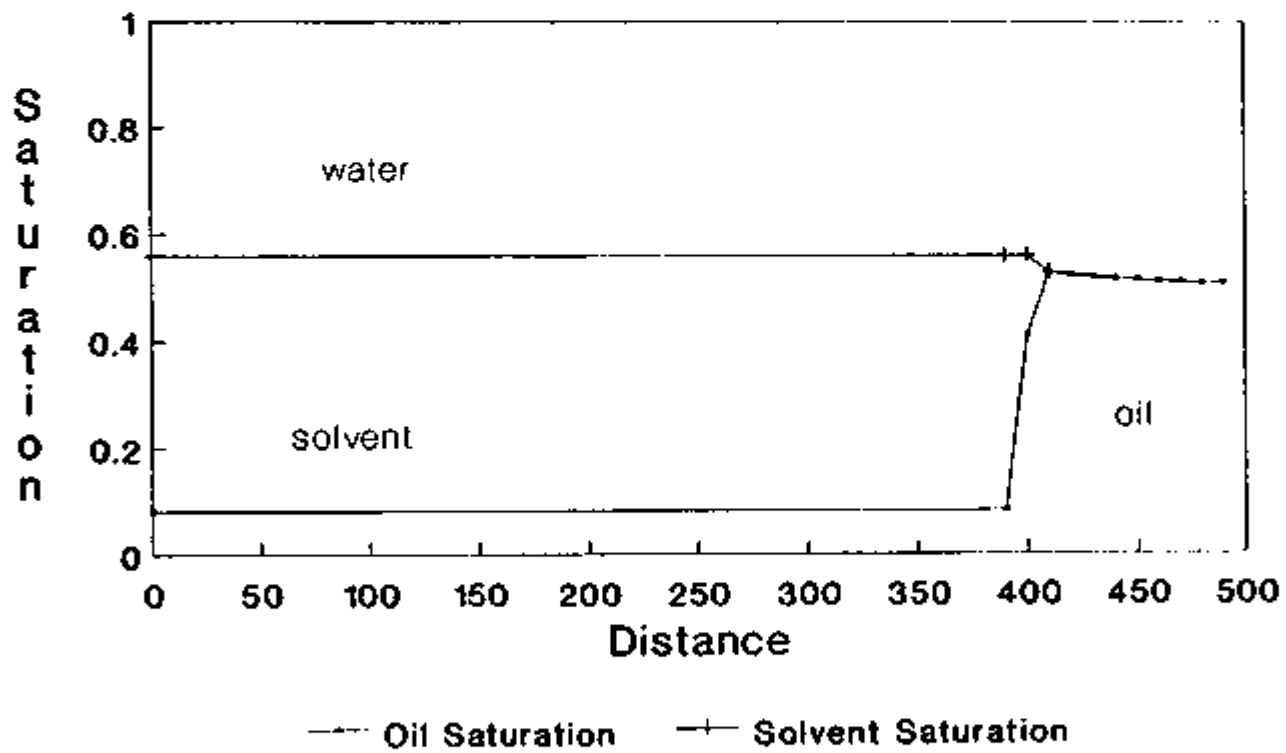


Figure 6-7

Saturation Profile from Simulator (60.5 Days)

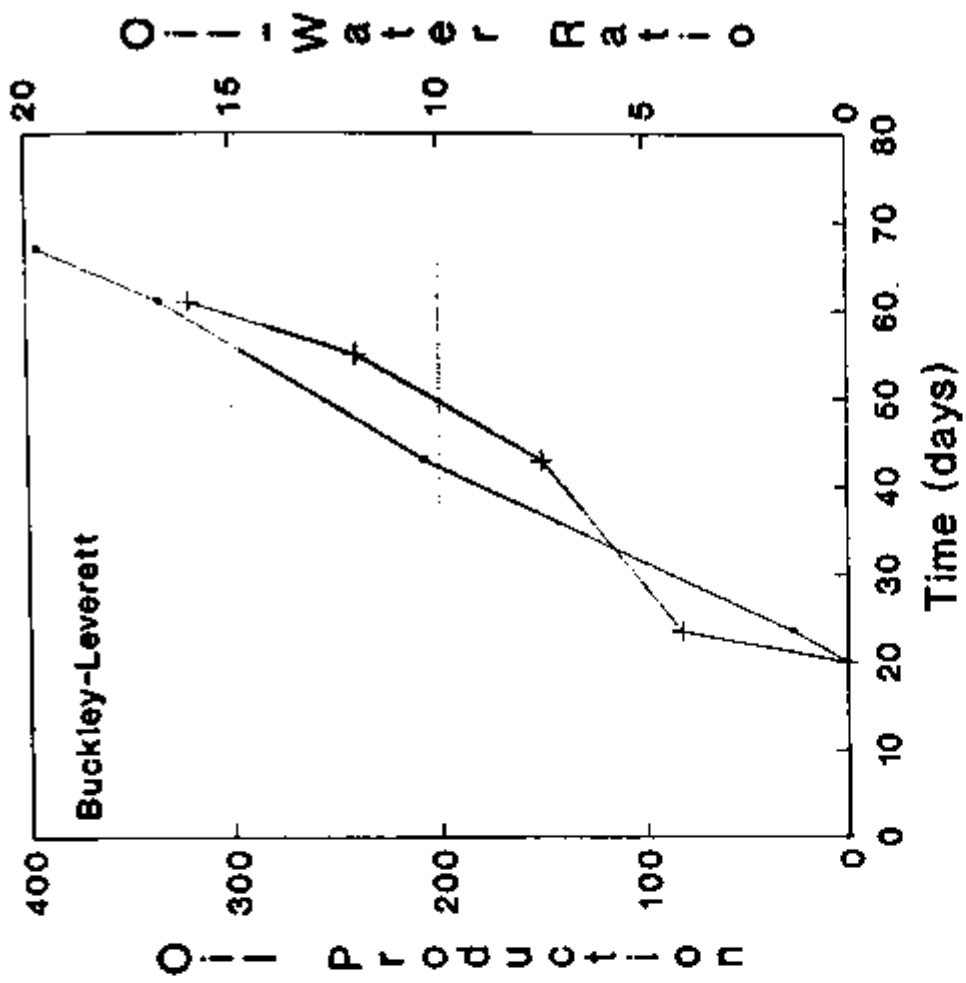
Leverett model does not begin until the solvent front reaches a length of 146.5 feet, or for an elapsed time of 19.8 days. Total recovery using the Buckley-Leverett method was 391 STB and was obtained after 67.7 days. Simulator calculations showed that 381 STB of oil were recovered in 72.5 days. Figures 6-8 and 6-9 graphically show production data from both methods, with the major difference occurring in the producing oil water ratio.

Sources of Error

Several errors may occur when running a simulation model. One occurs because the model is an approximation of the partial differential equation. Some of the assumptions used in developing the model are only partly valid. Also, all reservoir properties are seldom known, and many correlations must be developed. Errors also may be caused by the computer because of round off and truncation, but these should be very small because the program was run in double precision. If some of these errors occur early in the program, they may be magnified with each subsequent time step. For this reason, initial time steps are small and increase only to an amount that will not cause instability with the given injection rate.

Conclusions

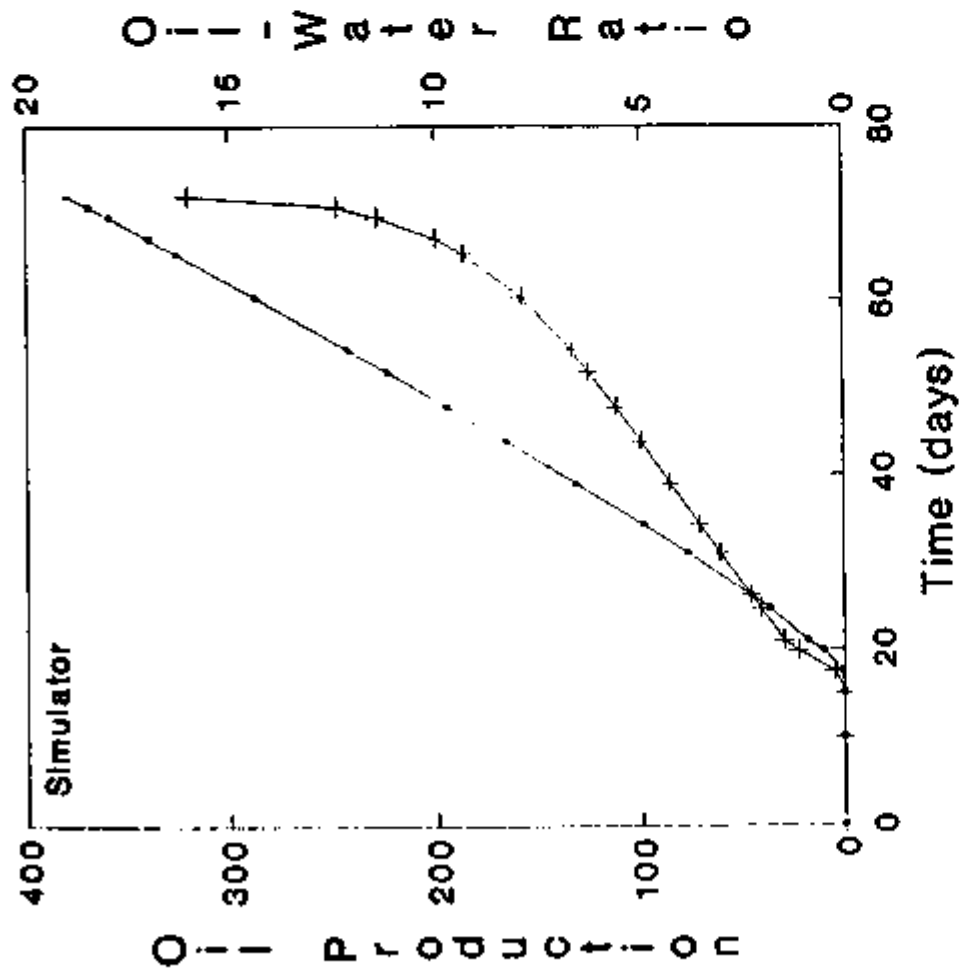
The model was designed for one-dimensional, three-phase flow. Though miscible conditions exist at the front, only



--- Oil Production — Oil-Water Ratio

Figure 6-9

Cumulative Oil Production (STB) and OWR vs. Time - Buckley-Leverett



--- Oil Production — Oil-Water Ratio

Figure 6-8

Cumulative Oil Production (STB) and OWR vs. Time - Simulator

carbon dioxide flows behind the solvent front, and only water and oil flow ahead of the solvent front. The effects of any mixing zone were ignored, as were the effects of dispersion, diffusion, and viscous fingering.

The model is sensitive to large changes in any phase volume during a time step. Therefore, time steps were kept small enough to insure that ten percent pore volume in any block was not exceeded during the time step. Large pressure changes during a time step also resulted in instability because of changes in the solvent compressibility and fluid formation volume factors. For this reason, accurate correlations were developed for the solvent and other fluid properties.

The results calculated by the simulation model compared favorably with Buckley-Leverett results. Major differences that occurred could be attributed to changing formation volume factors in the simulator, whereas the Buckley-Leverett model ignores the effects of fluid and pore volume compressibility, both of which were contained in the model.

Results from the simulator show a decrease in solvent saturation toward the oil-solvent front caused by averaging solvent saturation in the partially invaded block. Oil bank progress in the simulator shows the effects of numerical dispersion, causing the simulation front to lead the Buckley-Leverett front. Thus, oil production occurs earlier in the simulation model than in the Buckley-Leverett model.

However, ultimate recovery is achieved more rapidly in the Buckley-Leverett model because of the diffused nature of the simulator results.

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C
 C SIMULATION OF A MISCIBLE CARBON DIOXIDE FLOOD
 C INCORPORATING THE TRAPPED OIL FUNCTION
 C USING THE BLACK OIL MODEL
 C 3 PHASE (OIL, WATER AND SOLVENT)
 C 1-DIMENSIONAL

C PATRICIA CULPEPPER
 C

IMPLICIT DOUBLE PRECISION(A-H,O-Z)
 DIMENSION QOS(100),QWS(100),QTOT(100),H(100),PHI(100),DX(100),
 +PN1(100),PN2(100),SW(100),SO(100),QT(100),PO(100),OSO(100),
 +AO(100),AW(100),AS(100),CO(100),CS(100),CW(100),OSW(100),
 +D(100),A(100),C(100),B(100),GAMMA(100),OSS(100)
 +,BETA(100),SS(100),SW2(100),SO2(100),SS2(100)
 +,KAB(100),R(100),TUFF(100),HKX(100)
 INTEGER BLKSOL,BLKWAT,CMAX,CNT
 DOUBLE PRECISION KRO,KRW,KRS,NUM,KAB

C DATA INPUT

READ (5,*) M,CPV,DY,COP,CWP,SOR,PIN,SOI,SWI,TEM
 READ (5,*) (H(I),PHI(I),DX(I),KAB(I), I=1,M)
 CUMO=0
 CUMW=0
 OOIP=0
 OWIP=0
 FWO=1
 FOO=0
 CVG=30.48**3.*5.615
 IMARK=0

C INITIALIZE CONDITIONS

DO 10 I=1,M
 QOS(I)=0
 QWS(I)=0
 QTOT(I)=0
 KAB(I)=KAB(I)*.001
 PN1(I)=PIN/14.7
 PN2(I)=PIN/14.7
 PO(I)=PIN/14.7
 SW2(I)=1.-SOR
 OSW(I)=1.-SOR
 SW(I)=1.-SOR
 SO(I)=SOR
 SO2(I)=SOR
 OSO(I)=SOR
 SS(I)=0
 SS2(I)=0
 OSS(I)=0

10 CONTINUE

C PRINTING INPUT DATA

WRITE (6,7) M,DY,CPV,COP,CWP,CSP,SOR,SOI,PIN,SWI,TEM
 7 FORMAT(' NUMBER OF DIVISIONS = ',I3/' WIDTH OF RESERVOIR'
 1' (FEET) = ',F4.1/' COMPRESSIBILITY OF PORE VOLUME (PSI-1)'
 2' = ',E10.2/' OIL COMPRESSIBILITY (PSI-1) = ',E10.2/
 3' WATER COMPRESSIBILITY (PSI-1) = ',E10.2/' COMPRESSIBILITY OF'
 4' SOLVENT (PSI-1) = ',E10.2/' RESIDUAL OIL SATURATION '
 5' TO WATERFLOOD (FRACTION) = ',F3.2/' INITIAL OIL SATURATION ',
 6' (FRACTION) = ',F3.2/' RESERVOIR PRESSURE AT END OF WATERFLOOD',
 7' (PSI) = ',F5.0/' IRREDUCIBLE WATER SATURATION ',
 8' (FRACTION) = ',F3.2/' RESERVOIR TEMPERATURE (F) = ',F6.1//)

15 WRITE (6,15)
 FORMAT (5X,' BLOCK',3X,' LENGTH',3X,' THICKNESS',3X,' POROSITY'
 C ,3X,' PERMEABILITY (MD)'//)

DO 20 I=1,M
 PKAB=KAB(I)*1000
 WRITE (6,25) I,DX(I),H(I),PHI(I),PKAB

```

PN1(I)=PN2(I)+DPX
PN2(I)=PN1(I)
DSWX=DTR*(SW2(I)-OSW(I))
OSW(I)=SW2(I)
SW(I)=SW2(I)+DSWX
IF (SW(I) .GT. (1.-SOT)) SW(I)=1.-SOT
SW2(I)=SW(I)
DSSX=DTR*(SS2(I)-OSS(I))
OSS(I)=SS2(I)
SS(I)=SS2(I)+DSSX
IF (SS(I) .GT. SSOLV) SS(I)=SSOLV
OSO(I)=1.-OSW(I)-OSS(I)
SO(I)=1.-SW(I)-SS(I)
SO2(I)=1.-SW2(I)-SS2(I)
77 CONTINUE
QINJ=QINJ+(QSOLV*DT)
78 CALL SATS(SOR, SWI, SOI, SOT, SSOLV, PPAVG, FOFR, SOAVG, SOFR, CCC)
IF (SSOLV .LT. OSSOLV) WRITE (6, *) 'WARNING-SOLVENT DECREASE'
C HAVE RETURNED WITH TRAPPED OIL SATURATION
C CALCULATING DISTANCE TO SOLVENT FRONT
XSUM=0
XX=0
IF (MCHK .EQ. 1) GO TO 82
81 IF (ICLK .EQ. 1) GO TO 83
IF (DIFF .LT. QSOLV*DT) THEN
ICLK=1
DAYCUM=DAYCUM-DAYS
QINJ=QINJ-QSOLV*DT
DTSAV=DT
DAYS1=DIFF/QSOLV
DAYS2=DT-DAYS1
DAYS=DAYS1/86400
DT=DAYS1
BLKSOL=BLKSOL+1
GO TO 73
END IF
82 DO 80 I=1,M
PAV=PN1(I)
CALL FVF(BO, BW, BS, PAV, TEM)
PROD=PHI(I)*H(I)*DX(I)*DY*SSOLV/BS
XX=XX+PROD
IF (XX .GE. QINJ) THEN
DIFF=XX-QINJ
GO TO 83
END IF
80 CONTINUE
C SOLVENT FRONT HAS REACHED OUTLET END
IF (BLKSOL .EQ. M) THEN
WRITE (6,13)
13 FORMAT(/ 'SOLVENT FRONT HAS REACHED OUTLET END' /
1 'NORMAL TERMINATION OF PROGRAM')
GO TO 500
END IF
C CALCULATING LENGTH OF WATER FRONT
83 WRITE (6,*) FOFR, SOAVG, SOFR
WELGE=(1.)/(SOAVG-SOR)
XLEN=0
PHISUM=0
HSUM=0
K=0
DO 85 I=BLKSOL+1,M
K=K+1
BLKWAT=I
XLEN=XLEN+DX(I)
PHISUM=PHISUM+PHI(I)
HSUM=HSUM+H(I)

```

```

      HAVG=HSUM/K
      PROD=QINJ*WELGE/ (PHIAVG*HAVG*DY) -XLEN
85  CONTINUE
100  WP=PN1 (M)
      SSS=SS (M)
      SSW=SW (M)
      SSO=SO (M)
      CALL RPERM (KRO, KRW, KRS, SWI, SSW, SSO, SSS, CCC, SOR, I, BLKSOL)
      CALL VISC (UO, UW, US, WP)
      CALL FVF (BO, BW, BS, WP, TEM)
      FWN= (1+ (UW*KRO) / (UO*KRW) ) ** -1.
      FON=1. -FWN
      GO TO 92
92   SUM=0
      DO 95 I=BLKSOLV, BLKWAT
95   SUM=SUM+PN2 (I)
      AVG=SUM/ (BLKWAT-BLKSOL)
C    IF ( ABS (AVG-PPAVG) .GE. 1 ) GO TO 78
C  PRESSURE DIFFERENCE TOO HIGH-RECALCULATE VISCOSITY

```

56

C ENTRY FO COEFFICIENT ITERATIONS

```

      AO (1) =0
      AW (1) =0
      AS (1) =0
      CW (M) =0
      CO (M) =0
      CS (M) =0

      DO 200 I=1, (M-1)
      PAV=.5 * (PN2 (I) +PN2 (I+1) )
      CALL VISC (UO, UW, US, PAV)
      CALL FVF (BO, BW, BS, PAV, TEM)
      SSW=SW (I)
      SSO=SO (I)
      SSS=SS (I)
      CALL RPERM (KRO, KRW, KRS, SWI, SSW, SSO, SSS, CCC, SOR, I, BLKSOL)
      ALPHO=KRO/ (BO*UO)
      ALPHW=KRW/ (BW*UW)
      ALPHS=KRS/ (BS*US)
      PAV=PN2 (I)

      IF (I .GT. (BLKSOL+1) ) ALPHS=0.
      CO (I) =BO*ALPHO*HKX (I) /DX (I)
      CW (I) =BW*ALPHW*HKX (I) /DX (I)
      CS (I) =BS*ALPHS*HKX (I) /DX (I)
      PAV=PN2 (I+1)
      CALL FVF (BO, BW, BS, PAV, TEM)
      IF (I .LE. BLKSOL-1) THEN
        ALPHO=0
        ALPHW=0
      END IF
      AO (I+1) =BO*ALPHO*HKX (I) /DX (I+1)
      AW (I+1) =BW*ALPHW*HKX (I) /DX (I+1)
      AS (I+1) =BS*ALPHS*HKX (I) /DX (I+1)
      A (I) =AO (I) +AW (I) +AS (I)
      C (I) =CO (I) +CW (I) +CS (I)
202  FORMAT ( ' A(I) = ', F12.5, ' C(I) = ', F12.5)
200  CONTINUE
      A (M) =AO (M) +AW (M) +AS (M)
      C (M) =CO (M) +CW (M) +CS (M)
      WRITE (6, 99) BLKSOL, BLKWAT

```



```

99   FORMAT(' BLKSOL= ', I3, ' BLKWAT = ', I3)
      DO 250 I=1,M
          SIG=CPV+SO(I)*COP+SW(I)*CWP+SS(I)*CSP
          SIG=SIG*H(I)*PHI(I)/DT
          B(I)=-A(I)-C(I)-SIG
          D(I)=-SIG*PO(I)
          QOS(I)=0
          QWS(I)=0
          QT(I)=0
          IF (QTOT(I) .LE. 0) THEN
C INJECTION WELL CARBON DIOXIDE ONLY
              PAV=PN2(I)
              CALL FVF(BO,BW,BS,PAV,TEM)
              QT(I)=(QTOT(I)*BS)/(DX(I)*DY)
          ELSE
              PAV=PN2(I)
              CALL FVF(BO,BW,BS,PAV,TEM)
              FW=(FWN+FVN)/2.
              FO=(FON+FON)/2.
              QWS(M)=- (QSS*BS) *FW*.32774
              QOS(M)=- (QSS*BS) *FO*.32774
              PAV=PN2(M)
              CALL FVF(BO,BW,BS,PAV,TEM)
              QWS(M)=QWS(M)/BW
              QOS(M)=QOS(M)/BO
              QT(I)=(QOS(I)*BO+QWS(I)*BW)/(DX(I)*DY)
              WRITE (6,*) QOS(I), QWS(I), FW, FO, QT(I)
          END IF
          D(I)=D(I)+QT(I)
250  CONTINUE
          BETA(1)=B(1)
          GAMMA(1)=D(1)/BETA(1)
          DO 260 I=2,M
              BETA(I)=B(I)-A(I)*C(I-1)/BETA(I-1)
              GAMMA(I)=(D(I)-A(I)*GAMMA(I-1))/BETA(I)
260  CONTINUE
          PN2(M)=GAMMA(M)
          DO 270 I=2,M
              K=M-I+1
              PN2(K)=GAMMA(K)-C(K)*PN2(K+1)/BETA(K)
270  CONTINUE
          CNT=CNT+1
          CMAX=30
          IF (CMAX .GE. CNT) GO TO 292
          WRITE (6,800)
800  FORMAT(' MAXIMUM NUMBER OF ITERATIONS EXCEEDED')
          STOP
292  JFLAG=0
          DO 300 I=1,M
              PIX=PN2(I)
              PFX=PIX
              PBX=PIX
              IF (I .GT. 1) PBX=PN2(I-1)
              IF (I .LT. M) PFX=PN2(I+1)
              XMULT=DT/(H(I)*PHI(I))
              DPF=PFX-PIX
              DPB=PIX-PBX
              DSW=CW(I)*DPF-AW(I)*DPB
              DSO=CO(I)*DPF-AO(I)*DPB
              DSS=CS(I)*DPF-AS(I)*DPB
              PAV=PN2(I)
              CALL FVF(BO,BW,BS,PAV,TEM)
              QMULT=1./(DX(I)*DY)
              QTOX=QMULT*QOS(I)*BO
              QTWX=QMULT*QWS(I)*BW
              QTSX=QMULT*TUFF(I)*BS

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    DSW=XMULT*(DSW-QTWX)
    DSO=XMULT*(DSO-QTOX)
    DSS=XMULT*(DSS-QTSX)
    DUM1=PN2(I)-PO(I)
    PCL=PN2(I)+.1
    CALL FVF(BOF,BWF,BSF,PCL,TEM)
    PCH=PN2(I)-.1
    CALL FVF(BOL,BWL,BSL,PCH,TEM)
    CSP=(BSL-BSF)/.2
    CSP=CSP/BS
    DSW=DSW-DUM1*SW2(I)*(CPV+CWF)
    DSO=DSO-DUM1*SO2(I)*(CPV+COP)
    DSS=DSS-DUM1*SS2(I)*(CPV+CSP)
    SO2(I)=OSO(I)+DSO
    SW2(I)=OSW(I)+DSW
    SS2(I)=OSS(I)+DSS
    IF (I .GT. BLKSOL+1) SS2(I)=0
    IF (SS2(I) .LE. 0) SS2(I)=0
    IF (SW2(I) .LE. SWI) SW2(I)=SWI
    IF (SW2(I) .GE. OSW(I)) SW2(I)=OSW(I)
    SO2(I)=1.-SW2(I)-SS2(I)
    IF (I .LE. BLKSOL) THEN
        SO2(I)=SOT
        SS2(I)=SSOLV
        SW2(I)=1.-SOT-SSOLV
    END IF
    IF (SO2(I) .LE. (1.-SWI)) GO TO 310
    JFLAG=1
    SO2(I)=1.-SWI
310    IF (SO2(I) .GE. SOT) GO TO 320
    JFLAG=1
    SO2(I)=SOT
320    SW2(I)=1-SO2(I)-SS2(I)
300    CONTINUE

295    R(1)=(B(1)*PN2(1)+C(1)*PN2(2)-D(1))/B(1)
    DO 330 I=2,(M-1)
        R(I)=(A(I)*PN2(I-1)+B(I)*PN2(I)+C(I)*PN2(I+1)-D(I))/B(I)
330    CONTINUE
    R(M)=(A(M)*PN2(M-1)+B(M)*PN2(M)-D(M))/B(M)
C CHECKING CONVERGENCE
    IPFLAG=0
    ISFLAG=0
    PTOL=1
    STOL=.002
    DO 350 I=1,M
        PCHECK=ABS(PN2(I)-PN1(I))
        IF (PCHECK .GT. PTOL) IPFLAG=1
        PN1(I)=PN2(I)
        SCHCK=ABS(SO2(I)-SO(I))
        IF (SCHCK .GT. STOL) ISFLAG=1
        SO(I)=SO2(I)
        SW(I)=SW2(I)
        SS(I)=SS2(I)
350    CONTINUE
380    IF (IPFLAG .EQ. 1 .OR. ISFLAG .EQ. 1) GO TO 100

C PRINT RESULTS
C
    LL=(IMARK/40)*40
    IF ((DAYCUM .GE. 65. ) .OR. (LL .EQ. IMARK)) THEN
    WRITE (6,422) DAYCUM
422    FORMAT(' CUMULATIVE TIME = ',F6.3 , ' DAYS')
    WRITE (6,424)

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424  FORMAT(5X,'I',1X,'PRESSURE (PSIA)',1X,'R (ATM)',1X,
1    ' WATER SAT',1X,' OIL SAT',1X,' SOLVENT SAT',1X
2    ', 'KRO',4X,' KRW',4X,' KRS')
      DO 390 I=1,M
          PCON=PN2(I)*14.7
          SSO=SO2(I)
          SSW=SW2(I)
          SSS=SS2(I)
          CALL RPERM(KRO,KRW,KRS,SWI,SSW,SSO,SSS,CCC,SOR,I,BLKSOL)
          WRITE (6,426) I,PCON,R(I),SW2(I),SO2(I),SS2(I),KRO,KRW,KRS
426  FORMAT(4X,I2,3X,F10.2,2X,E13.5,2X,F6.4,2X,F6.4,2X,F6.4
1    ,2X,F6.4,2X,F6.4,2X,F6.4)
390  CONTINUE
      WRITE (6,720) BLKSOL,BLKWAT
720  FORMAT(' SOLVENT BLOCK = ',I2,' WATER BLOCK = ',I2)
      END IF
      IF (ICLK .EQ. 1) THEN
          DT=DAYS2
          DAYS=DAYS2/86400
          ICHK=0
          MCHK=1
          WRITE (6,*) DT,DAYS
          DTO=DAYS1
          GO TO 73
      END IF
      IF (MCHK .EQ. 1) THEN
          DT=DTSAV
          MCHK=0
          DAYS=DT/86400
      END IF
      PAV=PN2(M)
      CUMO=CUMO+QOS(M)*DT
      CUMW=CUMW+QWS(M)*DT
      WIP=0
      OIP=0
      DO 333 I=1,M
          PAV=PN2(I)
          CALL FVF(BO,BW,BS,PAV,TEM)
          OIL=PHI(I)*DX(I)*H(I)*DY*SO2(I)/BO
          WATER=PHI(I)*DX(I)*H(I)*DY*SW2(I)/BW
          OIP=OIP+OIL
          WIP=WIP+WATER
333  CONTINUE
      IF (CUMO .NE. 0) OMBE1=(OIP-OIP)/CUMO
      OMBE2=(OIP+CUMO)/OIP
      WMBE1=(OWIP-WIP)/CUMW
      WMBE2=(WIP+CUMW)/OWIP
      ZZ=WIP/CVG
      YY=OIP/CVG
      Y=CUMO/CVG
      Z=CUMW/CVG
      PAV=PN2(M)
      CALL FVF(BO,BW,BS,PAV,TEM)
      OWR=(BW*(1.-FW))/(FW*BO)

      IF ((DAYCUM .GE. 65.) .OR. (LL .EQ. IMARK)) THEN
          WRITE(6,*) BO,BW,BS,UO,UW,US
          WRITE(6,130) YY,ZZ,Y,Z,OMBE1,OMBE2,WMBE1,OWR
130  FORMAT(' OIP= ',F12.1,' STB'/' WIP = ',F12.1,' STB' /
1    ' CUM. OIL PRODUCED = ',F12.1,' STB'/' CUM WATER PRODUCED = '
2    ',F12.1,' STB'/' OMBE1= ',F8.4/' OMBE2= ',F8.4/
3    ' WMBE1= ',F8.4,' OIL-WATER RATIO= ',F7.4)
      END IF
      IMARK=IMARK+1
      DTO=DTN
      FOO=FON

```

```

20 CONTINUE
25 FORMAT (6X,I3,7X,F5.1,5X,F5.1,10X,F4.3,5X,F10.2)
C CONVERSION TO CGS UNITS
  DTO=1
  DY=DY*30.48
  DO 30 I=1,M
    H(I)=H(I)*30.48
    DX(I)=DX(I)*30.48
    PAV=PN1(I)
    CALL FVF(BO,BW,BS,PAV,TEM)
    OOIP=OOIP+PHI(I)*DX(I)*H(I)*DY*SO(I)/BO
30    OWIP=OWIP+PHI(I)*DX(I)*H(I)*DY*SW(I)/BW
    OIL=OOIP/CVG
    WATER=OWIP/CVG
    WRITE(6,35) OIL,WATER
    HKX(M)=0
    HKB=KAB(1)*H(1)
    DO 33 I=1,(M-1)
      HKF=KAB(I+1)*H(I+1)
      DEN=DX(I)*HKF+DX(I+1)*HKB
      NUM=2*HKF*HKB
      HKX(I)=NUM/DEN
      HKB=HKF
32    FORMAT(' HKX(I) = ',F8.3)
33    CONTINUE
35    FORMAT(' ORIGINAL OIL IN PLACE = ',F12.2,' STB' /
1' ORIGINAL WATER IN PLACE = ', F12.2,' STB' /)
    CPV=CPV*14.7
    COP=COP*14.7
    CWP=CWP*14.7
    CSP=CSP*14.7
    PPAVG=PIN/14.7
C INPUTTING DATA
  PAV=PN1(1)
  CALL FVF(BO,BW,BS,PAV,TEM)
  CALL SATS(SOR,SWI,SOI,SOT,SSOLV,PAV,FOFR,SOAVG,SOFR,CCC)
  DIFF=PHI(1)*H(1)*DX(1)*DY*SSOLV/BS
60  READ(5,*)DAYS,N,NWLS
    IPASS=0
    IF(DAYS.LE.0) GO TO 500
    DT=DAYS*86400.
    IF(NWLS.LE.0) GO TO 72
    DO 70 I=1,NWLS
C ASSUME (+) PRODUCTION IS OIL AND WATER
C ASSUME (-) INJECTION IS SOLVENT ONLY- INPUT IN SCF/DAY
    READ(5,*)QSS,IW
    QTOT(IW)=QSS*1.84
    IF(QSS.LE.0) THEN
      QSOLV=-QSS*.32774
      QTOT(IW)=QSS*.32774
      TUFF(IW)=QSS*.32774
    END IF
    QTOT(M)=-QSS*.32774
70  CONTINUE
72  CNT=0

    DO 75 NT=1,N
73  DAYCUM=DAYCUM+DAYS
    CNT=0
    DTN=DT
    OSSOLV=SSOLV
    DTR=DTN/DTO
C GUESS AT NEW PRESSURES AND SATURATIONS
    DO 77 I=1,M
      DPX=DTR*(PN2(I)-PO(I))
      PO(I)=PN2(I)

```

```

      FWO=FWN
75      CONTINUE
C END OF PRESSURE CALCULATIONS
      GO TO 60
500     STOP
      END

```

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```

C SUBROUTINE TO CALCULATE SATURATIONS IN OIL BANK
C FROM INITIAL INJECTION UNTIL BREAKTHROUGH
C
      SUBROUTINE SATS (SOR, SWI, SOI, SOT, SSOLV, PPAVG, FOFR, SOAVG, SOFR, C)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      DOUBLE PRECISION KRO, KRW, NUM
      DIMENSION SLOPE (10), CHEX (10)
      WRITE (6, *) SOR, SWI, SOI
      CALL VISC (UO, UW, US, PPAVG)
C RETURN WITH VISCOSITY DATA
      A=SOR
      B=1.
      SORS=SOR/ (1.-SWI)
      SOIS=SOI/ (1.-SWI)
      C=(1./SORS)-(1./SOIS)
      DO 10 I=1,4
C TOLERANCE OF OIL SATURATION IS .0001
      J=1
      D=(B-A)/10
      SO=A
      IF ( J .EQ. 1) GO TO 15
5      J=J+1
15     SO=SO+D
      SWS=(1.-SO-SWI)/(1.-SWI)
      SOS=SO/(1.-SWI)
      SPRIME=SOS-SORS
      SOFS=.5*(SPRIME+(SPRIME**2+4/C*SPRIME)**.5)
      KRW=SWS**4
      KRO=SOFS**2*(1.-(1.-SOFS)**2)
      NUM=KRW*UO
      DEN=KRO*UW
C      WRITE (6, *) SWS, SOS, SPRIME, KRW, KRO, NUM, DEN
      FO=(1+NUM/DEN)**-1.
      SLOPE (J)=FO/(SO-SOR)
      IF (J .EQ. 1) GO TO 5
      IF ( SLOPE (J) .GE. SLOPE (J-1)) GO TO 5
C IF HAVE PASSED INFLECTION POINT
      A=SO-D
      B=SO
10     CONTINUE
      SOFR=B
      FOFR=FO
      SOAVG=( (SOR-SOFR)*(1.-FOFR)/(-FOFR))+SOFR
      SWS=(1.-SOAVG-SWI)/(1.-SWI)
      SOS=SOAVG/(1.-SWI)
      KRW=SWS**4
      SPRIME=SOS-SORS
      SOFS=.5*(SPRIME+(SPRIME**2+4/C*SPRIME)**.5)
      KRO=SOFS**2*(1.-(1.-SOFS)**2)
      NUM=KRW*UO
      DEN=KRO*UW
      FOAVG=(1+NUM/DEN)**-1.
C      WRITE (6, *) KRW, KRO, SOFS
C HAVE NOW CALCULATED SATURATIONS AT FRONT AND AVERAGE
C WATER SATURATION

```

C

C SOLVING FOR SOLVENT AND TRAPPED OIL SATURATIONS

62

```

A=SOR
B=1.
DO 20 I=1,4
  J=1
  D=(B-A)/10.
  SO=A
18  SOS=SO/(1.-SWI)
  SPRIME=(SOS-SORS)
  BB=(SPRIME**2.+4/C*SPRIME)**.5
  SSOLV=.5*(SPRIME+BB)*(1.-SWI)
  SOT=.5*(SOS+SORS-BB)*(1.-SWI)
  CHEX(J)=(FOAVG*SSOLV)-(SOAVG-SOT)
  IF(ABS(CHEX(J)).LE..001)GO TO 21
  IF(CHEX(J).LE.0)THEN
    SO=SO+D
    J=J+1
    GO TO 18
  END IF
  B=SO+D
  A=SO
20  CONTINUE
C PRINT RESULTS
21  DUM=1
C    WRITE (6,30) SOFR,FOFR,SOAVG,FOAVG,SOT,SSOLV
30  FORMAT (' OIL SATURATION AT FRONT = ',F4.3/' FRACTION OIL',
1' AT THE FRONT = ',F7.3/' AVERAGE OIL SATURATION IN THE BANK = '
2, F4.3/' AVERAGE FRACTION OF OIL FLOWING IN THE BANK = ',F7.3/
3' TRAPPED OIL SATURATION = ',F7.3/' SOLVENT SATURATION BEHIND '
4'THE FRONT = ',F7.3//)
  RETURN
  END

```

C SUBROUTINE TO CALCULATE VISCOSITIES

```

SUBROUTINE VISC(UO,UW,US,PAVG)
  IMPLICIT DOUBLE PRECISION(A-H,O-Z)
  PRES=PAVG*14.7
  UO=1-(2000-PRES)*.000015
  UW=.5-(2000-PRES)*.0000098
  US=.04-(2000-PRES)*.00000082
  RETURN
  END

```

C SUBROUTINE TO CALCULATE FORMATION VALUE FACTORS

```

SUBROUTINE FVF(BO,BW,BS,PAV,TEM)
  IMPLICIT DOUBLE PRECISION(A-H,O-Z)
  PRES=PAV*14.7
  TAV=(160-TEM)/60
  IF(TEM.GE.160)TAV=0.
  IF(PRES.LE.1500)PRES=1500.
  IF(PRES.LE.1500)THEN
    RATIO=(1500-PRES)/500
    A--(.148*RATIO)+2.08
    B--(.36*RATIO)+.78
    BS=(A-B)*TAV+B
  END IF
  IF(PRES.GE.1500.AND.PRES.LE.2000)THEN
    RATIO=(2000-PRES)/500
    A--(.27*RATIO)+2.35
    B--(.54*RATIO)+1.32
    BS=(A-B)*TAV+B
  END IF

  IF(PRES.GE.2000)THEN

```

```

        RATIO=(2500-PRES)/500
        A=-(.15*RATIO)+2.5
        B=-(.44*RATIO)+1.76
        BS=(A-B)*TAV+B
    END IF
    BS=1./(BS*178.09)
    BO=(2450+.25*PRES)/2500
    BW=(2495+.01*PRES)/2500
C     WRITE (6,*) BO,BW,BS
    RETURN
    END

C SUBROUTINE TO CALCULATE RELATIVE PERMEABILITIES
C EQUATIONS USED FOR RELATIVE PERMEABILITIES ARE CORY'S EQUATIONS
    SUBROUTINE RPERM(KRO,KRW,KRS,SWI,SSW,SSO,SSS,C,SOR,I,BLKSOL)
    IMPLICIT DOUBLE PRECISION(A-H,O-Z)
    DOUBLE PRECISION KRO,KRW,KRS
    INTEGER BLKSOL
    SWS=(SSW-SWI)/(1.-SWI)
    SOS=SSO/(1.-SWI)
    KRW=SWS**4
    SORS=SOR/(1.-SWI)
    SPRIME=SOS-SORS
    IF (SPRIME .LE. 0) GO TO 10
    SOFS=.5*(SPRIME+(SPRIME**2+4./C*SPRIME)**.5)
    KRO=SOFS**2*(1.-(1.-SOFS)**2)
    GO TO 20
10    KRO=0
    IF (I .EQ. BLKSOL+1) KRO=SOS**2*(1.-SWS**2)
20    IF (KRW .GE. 1) KRW=1
    IF (KRO .GE. 1) KRO=1
    KRS=(SSS/(1.-SWI))**2*(1.-(1.-{SSS/(1.-SWI)}))**2)
    IF (I .GE. BLKSOL+2) KRS=0
C     WRITE (6,*) KRO,KRW,KRS,I,BLKSOL
    RETURN
    END

```

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