# OPTIMIZATION OF THE TIEC/AMTEC CASCADE CELL

by

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ABSTRACT

A mathematical modeling of a system consisting of a cascade of a thermionic energy conversion (TIEC) device and an alkali metal thermal to electric converter (AMTEC) device has been performed to optimize the efficiency of the cell. The TIEC is heated by electron bombardment, which converts heat partially into electricity and rejects the remaining. The AMTEC utilizes this reject heat of the TIEC. Cascading these two cells provides lots of advantages. A mathematical model of the cascade converter has been developed to analyze the effects of key parameters such as power level, heat fluxes, and temperatures. In this effort, a 12-node system of nonlinear simultaneous equations has been constructed which is solved by MATHCAD and a locally optimized efficiency has been derived. Thus, efficiency of the cascaded cell is improved, so that it is greater than the highest efficiency among the TIEC and AMTEC and lower than the sum of their individual efficiencies. The results were compared with the previous program written for the same problem.

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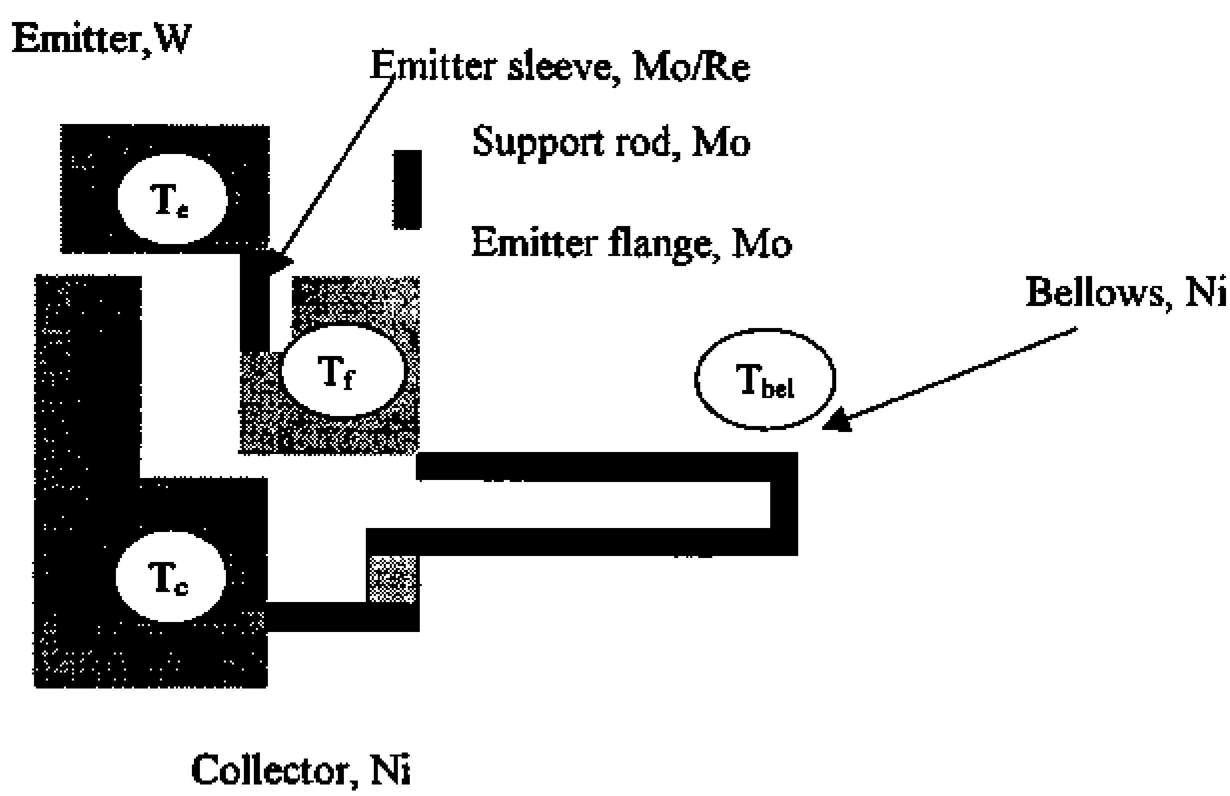
# CHAPTER I

# INTRODUCTION

The conversion of thermal and/or mechanical energy to electrical energy has been historically the mainstay for power systems. With the advent of Faraday's law of electromagnetic induction and the steam engine, dynamic conversion systems evolved rapidly, and were perfected with the engineering details mostly for terrestrial and aircraft devices. As a potential alternative to the dynamic conversion systems, static thermal to electrical conversion systems have been investigated, particularly for space power systems. With the discovery of the seeback effect, the thermo-electromotive force, which occurs in a material or materials under the effect of a temperature gradient, has been the driving force for static thermo-electric generators. A large number of static devices have been evolved and studied [1]. The goal for static conversion systems has been to identify static converters with efficiencies that are competitive with dynamic systems. Alternatively, two compatible static devices may be linked together in a tandem form to increase the overall efficiency of a space power system. The purpose of this work is to optimize the efficiency of a device that cascades two static devices. Also the temperature profile of the cascaded cell has to be maintained, thus resulting in a more efficient cascade. The cascade consists of a thermo-ionic energy converter (TIEC) and an alkali metal thermal to electric converter (AMTEC).

Thermo-ionic Energy Converter (TIEC) is a static device (see Figure 1.1), which converts heat energy through the surface emission of electrons. It consists of an emitter




The input for TIEC is 318 watt and the efficiency of the cell is determined by the equation:

$$\eta = \frac{(o^*T_e - c).e}{(n^*T_{e\text{-}b})^*A_{ee} + \varepsilon\sigma(T_e^4 - T_c^4)} = 9.61\%$$

where $Q_{electric} = (o^*T_e - c).e = 5.933$ watt

$Q_{electron} = (n^*T_e.b)^*A_{ce} = 23.47$ watt

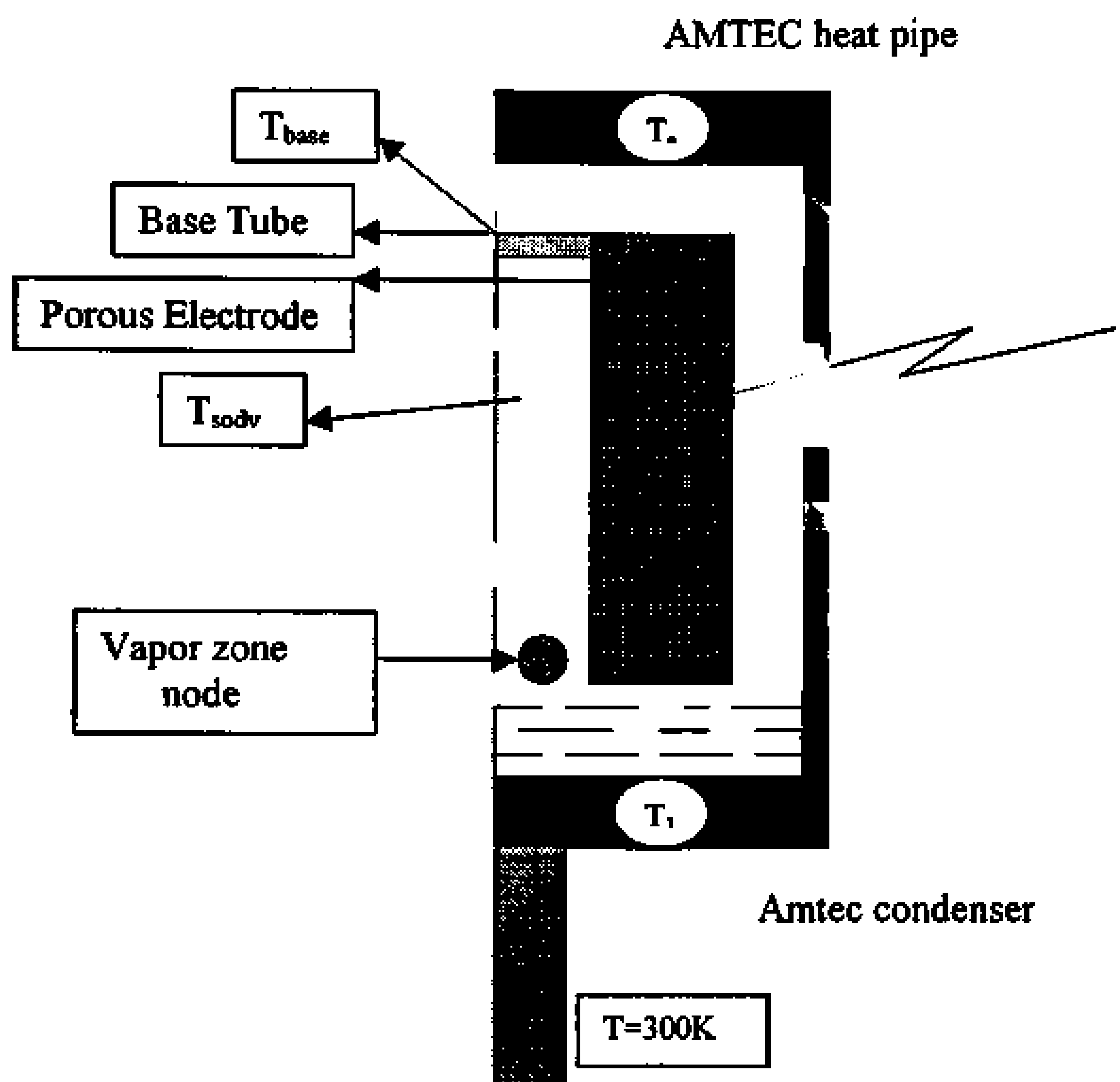
and $Q_{rade} = \varepsilon\sigma(T_e^4 - T_c^4) = 38.206$ watt

Figure 1.1

Schematic of Thermal Model of Thermo-ionic Converter (TIEC) (cathode) which receives heat from a suitable source and emits electrons from the other side. A collector (anode) collects these electrons and is cooled to a lower temperature than the emitter to limit the back emission of the electrons [2]. The anode and the cathode are connected by external electrical leads to supply the generated power to the load.

Alkali Metal Thermal to Electrical Converter (AMTEC) is a device for the direct conversion of heat to electric power with no moving parts (see Figure 1.2). It has the potential to produce electric power with many times higher efficiency and power density than current devices like Radioisotope Thermoelectric Generator (RTG), General Purpose Heat Source modules (GPHS) [3]. AMTEC uses Beta-Alumina Solid Electrolyte (BASE) separator in a high pressure and high temperature region of liquid sodium (Na) at 900K-1300K and a low-pressure region in which the Na activity is controlled by a condenser at 350K-700K. The AMTEC converts the work of the isothermal expansion of sodium vapor directly to electric power. AMTEC has many advantages for terrestrial and space power applications including no moving parts with the resulting potential for low maintenance and high durability and efficiency.

The typical operating temperatures of TIEC are 1600K-2000K at the emitter node and 800K-1100K at the collector. The operating temperatures of AMTEC at hot source are 900K-1100K and about 400K at the cooler region. The strengths and weaknesses of both TIEC and AMTEC are, for AMTEC the strengths include good efficiency (>25%) and weaknesses include an operating temperature of less than 1300K at the hot region and about 400K at the cooler region. The strengths of TIEC include ability to use a very hot sources (>2000K), while the weaknesses include moderate efficiencies (10-15%).




The input for AMTEC is 38.206 watt and the efficiency of the cell is determined by the equation:

$$\eta = \frac{Q_{amtecelec}}{Q_{rade}} = 11.81\%$$

where

$Q_{amtecelec} = q * T_{base} - e = 4.514$ watt

and $Q_{rade} = \varepsilon\sigma(T_e^4 - T_c^4) = 38.206$ watt

Figure 1.2

Schematic of Thermal Model of Alkali Metal Thermal to Electrical Converter (AMTEC)

Thus the TIEC-AMTEC cascade is optimized in order to minimize the weaknesses and to increase the efficiency of the system.

TIEC/AMTEC assembly: An assembly of TIEC and AMTEC has been cascaded with a transition piece between the collector of TIEC and the heat input region of AMTEC (see Figure 1.3). The transition piece is surrounded by a trim collar (heater/cooler) which can be heated by a resistance heating element, or cooled by water or by air, in order to maintain the desired temperature in the transition piece. The heat rejected by the collector of the TIEC is picked up by the heat input region of the AMTEC through the transition piece and thus both the TIEC and AMTEC are operated.

Lodhi, Schuller and Hausgen [1] had developed a model by writing nine nodal equations (using conservation of energy) at strategic locations within the cascaded cell. The nodal points selected on the cascade system were at the emitter, flange, bellows, collector, transition piece, trim collar, AMTEC evaporator (in the high temperature zone), AMTEC condenser (in the low temperature zone) and the calorimeter.

The number of nodal points has been increased from nine to twelve. The new nodal points included were at insulator between transition piece and AMTEC evaporator, base tube, vapor zone (in the low-pressure region between the base tube and the condenser). The variable identified in this cell is temperature at various points of the cell. In this, twelve nodal points have been selected. Mathematical equations for the energy balance and heat fluxes have been developed using this variable and are described in Chapter V. Through this mathematical model the optimized efficiency of the cell is achieved.

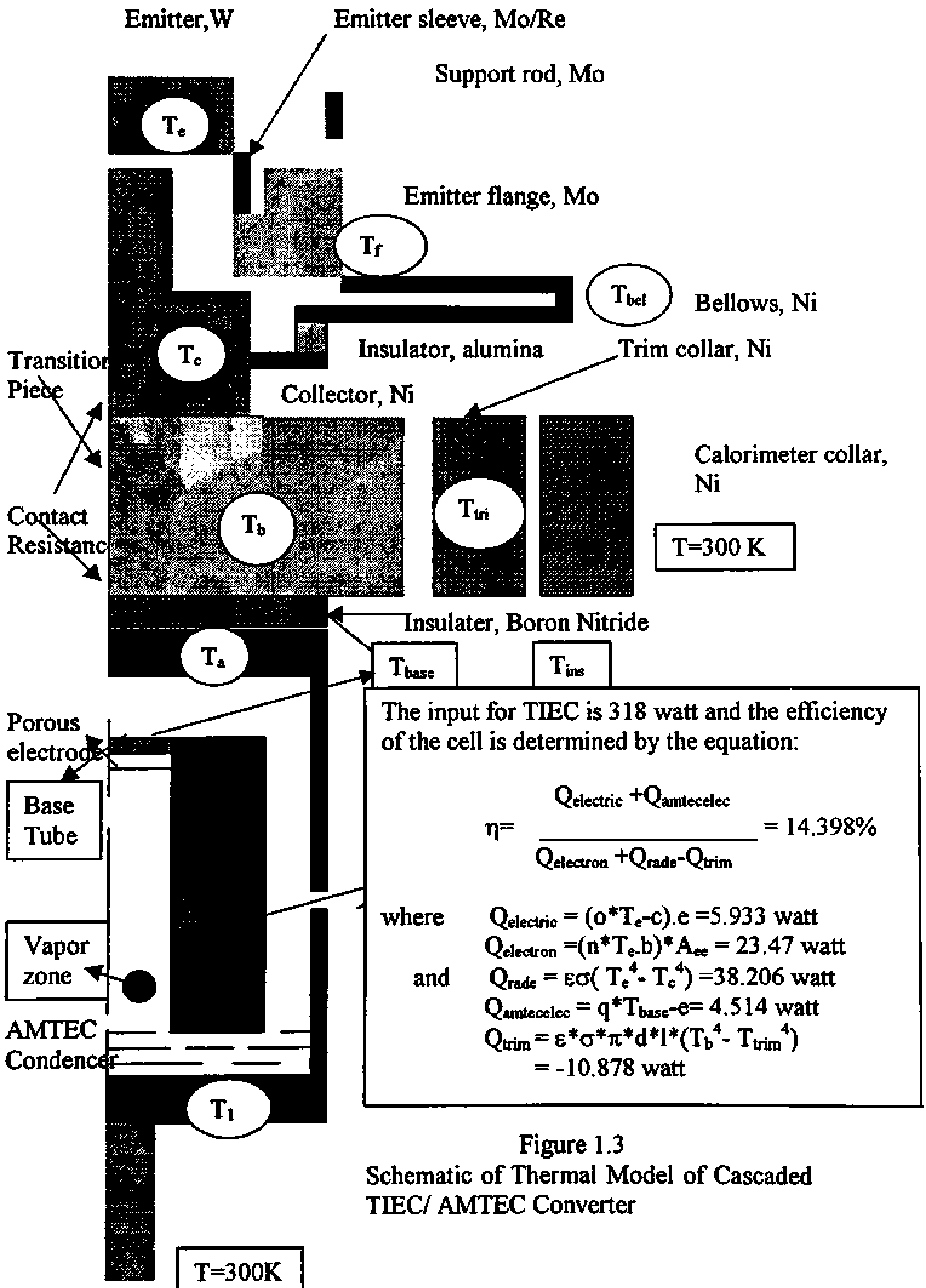



Figure 1.3
Schematic of Thermal Model of Cascaded TIEC/ AMTEC Converter

When we are cascading two cells, the efficiency of the cascaded cell should be more than the highest efficiency of the individual cells and less than the sum of the efficiencies of the two cells. For this process a mathematical model has been written in Mathcad, version 7.0. The results obtained were compared with the previous program that had only nine nodal points.

# CHAPTER II

# THERMO-IONIC ENERGY CONVERTER

Thermo-ionic Energy Converter (TIEC) is a device, which converts the thermal energy to electrical energy. There are two types of TIEC, one which works with vacuum and the other works with plasma.

The working principle of vacuum TIEC involves in conversion of a gas of free electrons in the inter-electrode space, in a vacuum. These electrons create a negative space charge between the electrodes, and as a result, a retarding field develops which deflects some of the emitted electrons back to the cathode. The gas chosen in TIEC is cesium(Cs). Cesium has the lower ionization potential among candidate substances and its adsorption reduces the surface work function of the electrodes below that of liquid cesium or any metal.

The cell operates at a temperature of 2000K-1800K at the emitter end, $T_e$ (see Figure 1.1). The cell has been designed to work independent of the heating source. The heat from emitter zone is either radiated or conducted to the other parts of the cell. A portion of the heat is radiated to the surroundings, $T_{amb}$. The heat gained by the flange from the emitter is in two folds, one through radiation and the other through conduction. The temperature at flange is $T_f$. The gas, to get ionized and to dissipate the electrons, uses some amount of heat. The electrons get accelerated and travel through the electrodes towards the collector. The electricity is drawn from the electrodes at this point. The heat energy is also radiated to the collector.

The heat energy from the flange node is again conducted to the bellows, radiated to surroundings and to collector. The bellows being located away from the emitter and flange, much of heat is not radiated. Thus, the temperature at the bellows, $T_{bel}$, is considerably lesser than that of the temperatures at other nodes. From bellows most of the heat is given out to surroundings. The temperature at the collector, $T_c$, is expected to be between 1000K-1250K. Thus, most of the heat energy is given out to the surroundings. Release of heat energy at such high temperatures makes the cell inefficient. That is, operating a cell at such high temperatures to generate about 10% of electricity makes it inefficient. Hence, utilization of this heat energy that is rejected out from the collector will make it more useful.

The converter efficiency of TIEC is calculated as the ratio of the contact potential difference which is equal to the voltage on the load to the sum of the energy carried away by the electrons from the cathode and the radiation heat transfer from emitter to collector. Thus, the efficiency, $\eta_{TIEC}$ is:

$$\eta_{TIEC} = \frac{Q_{electric}}{Q_{electron}+Q_{radc}}$$

Where $Q_{electric} = (O^* \, T_e\text{-}c)^* A_{ee}$

$O$ = constant = 0.0135 watt/K*cm$^2$,

$T_e$ = Emitter temperature in K,

$A_{ee}$= Area of the emitter in cm$^2$,

c = power per unit area in watt/cm$^2$,

$Q_{electron} = (n^*T_e\text{-}b)^* A_{ee}$

n = constant = 0.0423 watt/K*cm$^2$,

$b = 58.6$ watt/cm$^2$,

$Q_{radc} = \sigma*\varepsilon*(T_e^4 - T_c^4)$

$\sigma = 5.67*10^{-12}$.watt/cm$^2$*K$^4$,

$\varepsilon$ = emissivity = 0.18,

$T_c$ = collector temperature in K.

# CHAPTER III

# ALKALI-METAL THERMAL-TO-ELECTRICAL CONVERTER

Alkali- Metal Thermal-to-Electrical converter (AMTEC) (see Figure 1.2) is a high temperature regenerative concentration cell for elemental sodium, which converts thermal energy directly into electrical energy. The efficient operation of AMTEC cell involves several heat and mass transfer processes. AMTEC is a relatively new type device, based upon the principle of alkali-metal concentration cell, conceived in late sixties. These are static conversion devices, which can provide efficiencies close to the theoretical Carnot efficiency. AMTEC operates at a temperature of 1200K-1000K at hot side and 500K-400K at the cold side.

To understand the working of AMTEC, its operating cycle is illustrated in Figure 1.2. A closed vessel is divided into a high temperature ($T_a$) region in contact with a heat source and a low temperature ($T_l$) at the cooler region. The critical material in the operation of the AMTEC is the beta'-alumina solid electrolyte (BASE), a sodium ion conductor whose ionic conductivity is much larger than its electronic conductivity. In an AMTEC cell, the BASE separates the hot (high-pressure) region filled with a small quantity of liquid sodium in contact with the heat source, from the cold (low-pressure) region occupied with sodium vapor. A porous electrode covers the low-pressure (outer) side of the BASE. Electrical leads in contact with the porous electrode and the high-temperature liquid sodium exit through the wall of the device and is connected to an external load. The pressure differential across the BASE forces ionization of sodium atoms on the hot side. The ions diffuse through the BASE toward the low pressure side in response to the pressure differential gradient of free Gibbs energy while the electrons circulate through the external load, producing electrical work. Electrons and sodium ions recombine at the interface between the BASE and the porous electrode. The resulting sodium ions absorb their heat of vaporization, move through the electrode and the vapor space, then release their heat of vaporization at the low-temperature side of the condenser surface. A wick structure or an electromagnetic pump will return the liquid condensate to the high temperature side of the BASE tube to complete its circulation.

The heat distribution in the AMTEC is as follows, The temperature at the hot plate, $T_a$, should be between 1200K-1000K. The heat source for AMTEC may be solar heat or any other heating sources. The heat energy from this point is given out to the sodium liquid. Because of convective heat transfer, the sodium starts boiling and slowly gets converted to vapor. The heat energy allows the sodium to move towards the low-pressured BASE. As the sodium approaches the BASE it gets ionized and the sodium particles will separate out. The temperature at the BASE, $T_{base}$, is approximately equal to that of the boiling point temperature of sodium. Hence some part of heat energy is used in transporting sodium, some part in generating electricity and some of the heat energy is wasted out as parasitic losses. Also some part of heat energy is transferred to the sodium vapor in convection mode. The heat passed onto the vacuumed vapor zone from the BASE is linearly distributed to the condenser and to the surroundings through the wall of the cell. The temperature at the condenser, $T_l$, is expected to be above 400K, which is above room temperature.

The efficiency of AMTEC, $\eta_{AMTEC}$, is defined as the ratio of the amount of electric energy generated to the amount of heat supplied to the AMTEC.

Thus, the efficiency of AMTEC is:

$$\eta_{AMTEC} = \frac{Q_{amtecelec}}{Q_{rade}}$$

Where $Q_{amtecelec} = (q^* T_{base} - e)$,

$q$ = constant = $1.2*10^{-12}$.watt/K,

$T_{base}$ = BASE tube temperature in K,

$e$ = 8.9 watt,

$Q_{rade} = \sigma^*\varepsilon^*(T_e^4 - T_c^4)$

$\sigma$ = $5.67*10^{-12}$.watt/cm$^2$*K$^4$,

$\varepsilon$ = emissivity = 0.18,

$T_c$ = collector temperature in K.

CHAPTER IV

# CASCADE OF TIEC/AMTEC CELL

The purpose of cascading TIEC and AMTEC is to derive an optimum power output with less heat energy wasted. A schematic diagram of such a cascade is shown in Figure 1.3. Cascading of direct energy conversion devices when the waste heat of a high temperature device is used to operate a bottoming low temperature device would allow the development of a highly efficient, compact and light weight power source as well as bimodal system to address future civilian and defense missions [5]. One such system uses a Cs- TIEC to operate a bottoming Sodium- AMTEC. The optimum collector temperature, $T_c$, of TIEC is typically in the 1200K to 1000K ranges and AMTEC hot pipe temperature, $T_a$, will also fall in the same range.

The construction of such system involves using a transition piece between the collector of TIEC and the heat pipe of AMTEC. The transition piece is surrounded by a trim collar (heater) which can be heated by a resistance heating element, or cooled by water or air, to maintain the desired temperature and heat flux in the transition region. The TIEC is heated up by the electron bombardment on the upper end of the emitter, which converts a portion of heat energy into electricity and rejects the rest from the lower end of the collector, into the transition piece. The remaining heat rejected at the bottom of the collector is picked up by the AMTEC via the transition piece.

The heat rejected from the collector of TIEC is absorbed by the transition piece. Some portion of the heat energy may be used to heat the trim collar. The heat energy supplied to trim collar will be through radiation. If an excess amount of heat is supplied to the transition piece, then some heat is passed onto the trim collar. If the collector of TIEC does not supply sufficient amount of heat energy to the transition piece, then the trim collar will supply the required heat energy. Hence, the heat contribution by the trim collar to the total cell may be included or excluded. A calorimeter is used by the transition piece to measure the amount of heat energy passing through it. Thus, certain amount of heat energy is radiated to the calorimeter from the trim collar, and the heat energy from the calorimeter is rejected to the surroundings. A certain amount of heat energy is supplied to the AMTEC heat pipe. The heat flow of the cascaded cell is illustrated in Figure 4.1.

The efficiency of the cascaded cell should be theoretically the sum of the efficiencies of both TIEC and AMTEC, but this does not happen practically. Hence the efficiency of the cascaded cell should be larger than the highest efficiency of the individual cells and lesser than the sum of the efficiencies of both the cells. The efficiency of the cascaded cell, $\eta_{Cascade}$, is:

$$\eta_{TIEC} = \frac{Q_{electric} + Q_{amtecelec}}{Q_{electron} + Q_{rade} - Q_{trim}}$$

Where, $Q_{electric} = (O * T_e - c) * A_{ee}$,

$O$ = constant = 0.0135 watt/K*cm$^2$,

$T_e$ = Emitter temperature in K,

$A_{ee}$ = Area of the emitter in cm$^2$,

$c$ = power per unit area in watt/cm$^2$,

$Q_{amtecelec} = (q * T_{base} - e)$,

$q$ = constant = $1.2 * 10^{-12}$.watt/K,

$T_{base}$ = BASE tube temperature in K,

e = 8.9 watt,

$Q_{electron} = (n*T_e-b)*A_{ee}$

n = constant = 0.0423 watt/K*cm$^2$

b = 58.6 watt/cm$^2$,

$Q_{radc} = \sigma*\varepsilon*(T_e^4- T_c^4)$,

$\sigma = 5.67*10^{-12}$.watt/cm$^2$*K$^4$,

$\varepsilon$ = emissivity = 0.18,

$T_c$ = collector temperature in K,

$Q_{trim} = \sigma*\varepsilon*\pi*d_{trans}*l_{trans}*(T_b^4- T_{trim}^4)$,

$\sigma = 5.67*10^{-12}$.watt/cm$^2$*K$^4$,

$\varepsilon$ = emissivity = 0.2,

$d_{trans}$= diameter of the transition piece in cm,

$l_{trans}$= length of the transition piece in cm,

$T_{trim}$= trim collar temperature in K,

$T_b$ = transition piece temperature in K.

The mathematical model is written for the cascaded cell and is aimed at optimizing the efficiency of the cell.

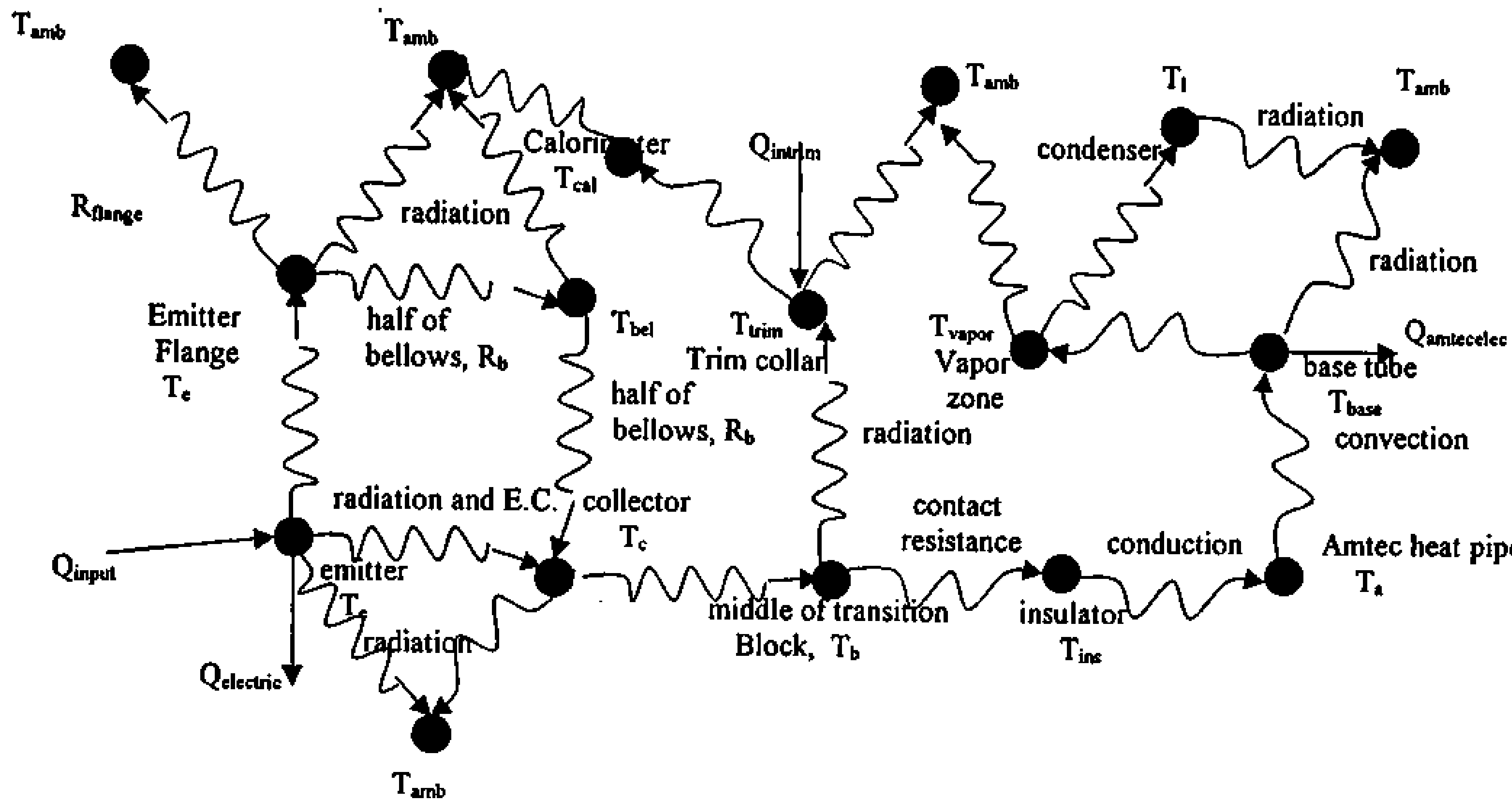



Figure 4.1
Thermal Circuit for Calculating Temperatures and
Power Flows in Thermo-ionic-AMTEC Cascade Converter

# CHAPTER V

# MATHEMATICAL MODEL AND FEASIBLE SOLUTION

A mathematical model has been developed in order to maximize the efficiency of the cascaded cell satisfying all the constraints associated with. The total cell has been divided into twelve nodal points and at each nodal point the heat balance equations have been written. These heat balance equations are the constraints to the problem. As a general rule heat always travel from hot region to cold, hence the other constraints of the problem are $T_e > T_f > T_c > T_b > T_a > T_{base} > T_{sodv} > T_l$.

The objective function is:

$$\text{Maximize} \quad \frac{Q_{electric} + Q_{amtecelec}}{Q_{electron} + Q_{rade} - Q_{trim}}.$$

The heat balance constraints are:

1. $16.13*10^{-12}*T_e^4-9.348*10^{-12}*T_{amb}^4-3.206*10^{-12}*T_c^4-3.576*10^{-12}*T_f^4+0.1792*T_e-0.1234*T_f <=239.14$

2. $3.576*10^{-12}*T_e^4-32.022321*10^{-12}*T_f^4+6.5225208*10^{-12}*T_{bel}^4+15.154*10^{-12}*T_{amb}^4+6.7688*10^{-12}*T_c^4+0.123*T_e-0.2298T_f+0.01*T_{amb}+0.08*T_{bel}+5.73 <= 0$

3. $6.5225208*10^{-12}*T_f^4-69.210247*10^{-12}*T_{bel}^4+37.106*10^{-12}*T_{amb}^4+0.086*T_f+0.086*T_c-0.172*T_{bel}+4.153126*10^{-12}*T_b^4+21.4486*10^{-12}*T_{cal}^4 <= 0$

4. $4.1626*10^{-12}*T_e^4-16.7148*10^{-12}*T_c^4+6.7688*10^{-12}*T_f^4+5.7834*10^{-12}*T_{amb}^4+0.0423*T_e+0.086*T_{bel}-0.9887*T_c+0.9072*T_b <= 58.6$

5. $0.9027*T_c-3.1807*T_b+2.278*T_{ins}-43.673955*10^{-12}*T_b^4+34.476377*10^{-12}*T_{trim}^4+9.197578*10^{-12}*T_{bel}^4 <= 0$

6. $34.476377*10^{-12}*T_b^4-81.304527*10^{-12}*T_{trim}^4+46.82815*10^{-12}*T_{cal}^4+90 <= 0$

7. $21.4486*10^{-12}*T_{bel}^4-222.3387*10^{-12}*T_{cal}^4+46.8281*10^{-12}*T_{trim}^4+154.06196*10^{-12}*T_{amb}^4 <= 0$

8. $1.8224*T_b-7.2992*T_{ins}+34.476377*10^{-12}*T_{trim}^4-37.556844*10^{-12}*T_{ins}^4+5.4772*T_a+2.7930738*10^{-12}*T_{amb}^4 <= 0$

9. $5.4772*T_{ins}*T_a^2+5.4772*T_{ins}*T_{sod}^2+10.9544*T_{ins}*T_{sod}*T_a-291829.48*T_a^3+875499.39*T_{sod}^2*T_a-875493.9*T_{sod}*T_a^2-291829.48*T_{sod}^3 <= 0$

10. $291829.48*T_a^3-875488.44*T_a^2*T_{sod}+875488.44*T_a*T_{sod}^2-291829.48*T_{sod}^3-2.068977*T_{base}+0.0068977*T_{sodv}+53.2 <= 0$

11. $0.006977*T_{base}-0.0197813*T_{sodv}+0.0058273*T_{amb}+0.006977*T_l <= 0$

12. $2.05*T_{base}+0.006977*T_{sodv}-0.406977*T_l+0.4*T_{amb} <= 44.30$.

The constraints listed above are the twelve nodal points which are located at the emitter, flange, bellows, collector, transition piece, trim collar, calorimeter, insulator, AMTEC evaporator (at the high temperature region), base tube, vapor zone (low pressure region) and condenser (at the low temperature region) respectively. At each of these points a detailed study of heat distribution is done and the resulting equations are constrained to the problem.

Apart from these above constraints there are other parameters which will also significantly affect the efficiency of the cell. These are the terms involved in parasitic losses (p and d), electric power output (q and e) of AMTEC and sodium heat transportation (r, f and G), see Figure 5.1. These parameters affect the AMTEC cells efficiency, and thus the efficiency of the cascaded cell, see Figures 5.2, 5.3 and 5.4. Once the program has been run and checked for feasible solution, then these variables are included and again the program is run. Thus, by applying the search method for the problem with these constraints a local optimal solution can be achieved.

The objective function and the constraints are programmed in Mathcad and are allowed to run for several iterations with a minimum error factor of $1.507*10^{-12}$, see Appendix. The Mathcad program will simultaneously solve for the temperature profile and the search method. Each time when the program runs in iterations and solves the problem, the parameters p, d, q, e, r, f and G are changed in an order. Thus, a solution is achieved which satisfies all the constraints, this solution is known as the feasible solution. The model had given a feasible solution of 14.398% satisfying all the constraints. The parameters listed above are changed slightly and the program is run each time. Only one parameter is changed at a time and the results are checked. If the results are not within the constrained values of temperature and efficiency then these values are again changed. Finally the optimized efficiency is achieved as shown above where all the constraints are satisfied (see Table 5.1). The constraints being:

1. largest value of $\eta_{TIEC}$ or $\eta_{AMTEC} < \eta_{hybrid} < \eta_{TIEC} + \eta_{AMTEC}$
2. $400 < T_{cond} < 500$
3. $1100 < T_{base} < 1150$

The temperature profile is shown in the Figure 5.5. This solution of 14.398% is greater than the highest of the individual cells where the efficiencies of individual cells are, for TIEC 9.66% and for AMTEC 11.81% and lesser than the sum of the efficiencies of the both the cells, 21.47%. Hence, the solution is acceptable.

## Table 5.1 COMPARISON TABLE

| S.No | P (Watt/K) | D (Watt) | E (Watt) | Q (Watt/K) | R (amp/K) | G (watt/K) | F (amp) | $R_l$ (K/Watt) | $T_{cond}$ | $\eta_{TIEC}$ | $\eta_{AMTEC}$ | $\eta_{hybrid}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | $5*10^{-2}$ | 34.3 | 8.9 | $1.2*10^{-2}$ | $2.0*10^{-2}$ | 1.0 | 10.0 | 1.0 | 360 | 9.6 | 11.76 | 14.373 |
| 2 | $6*10^{-2}$ | - | - | - | - | - | - | - | 362 | - | 8.1 | 12.461 |
| 3 | $4*10^{-2}$ | - | - | - | - | - | - | - | 358 | - | 16.341 | 16.78 |
| 4 | $5*10^{-2}$ | 33.3 | - | - | - | - | - | - | 358 | - | 15.93 | 16.569 |
| 5 | - | 35.3 | - | - | - | - | - | - | 358 | - | 16.75 | 17.001 |
| 6 | - | 34.3 | 7.9 | - | - | - | - | - | 356 | - | 20.78 | 19.109 |
| 7 | - | - | 9.9 | - | - | - | - | - | 359 | - | 14.14 | 15.624 |
| 8 | - | - | 8.9 | $1.1*10^{-2}$ | - | - | - | - | 359 | - | 13.5 | 15.3 |
| 9 | - | - | - | $1.3*10^{-2}$ | - | - | - | - | 357 | - | 19.096 | 18.23 |
| 10 | - | - | - | $1.2*10^{-2}$ | $1.0*10^{-2}$ | - | - | - | 355 | - | 22.305 | 19.922 |
| 11 | - | - | - | - | $3.0*10^{-2}$ | - | - | - | 360 | - | 11.76 | 14.373 |
| 12 | - | - | - | - | $4.0*10^{-2}$ | - | - | - | 362 | - | 8.1 | 12.461 |
| 13 | - | - | - | - | $5.0*10^{-2}$ | - | - | - | 364 | - | 2.74 | 9.621 |
| 14 | - | - | - | - | $2.0*10^{-2}$ | 2.0 | - | - | 362 | - | 8.1 | 12.46 |
| 15 | - | - | - | - | - | 0.5 | - | - | 359 | - | 14.29 | 15.705 |
| 16 | - | - | - | - | - | 1.0 | 5.0 | - | 359 | - | 13.27 | 15.165 |
| 17 | - | - | - | - | - | - | 15.0 | - | 359 | - | 15.33 | 16.245 |
| 18 | - | - | - | - | - | - | 10.0 | 0.5 | 343 | - | 11.75 | 14.365 |
| 19 | - | - | - | - | - | - | - | 2.0 | 396 | - | 11.8 | 14.39 |
| 20 | $5*10^{-2}$ | 34.3 | 8.9 | $1.2*10^{-2}$ | $2.0*10^{-2}$ | 1.0 | 10.0 | 2.5 | 413 | 9.6 | 11.87 | 14.398 |

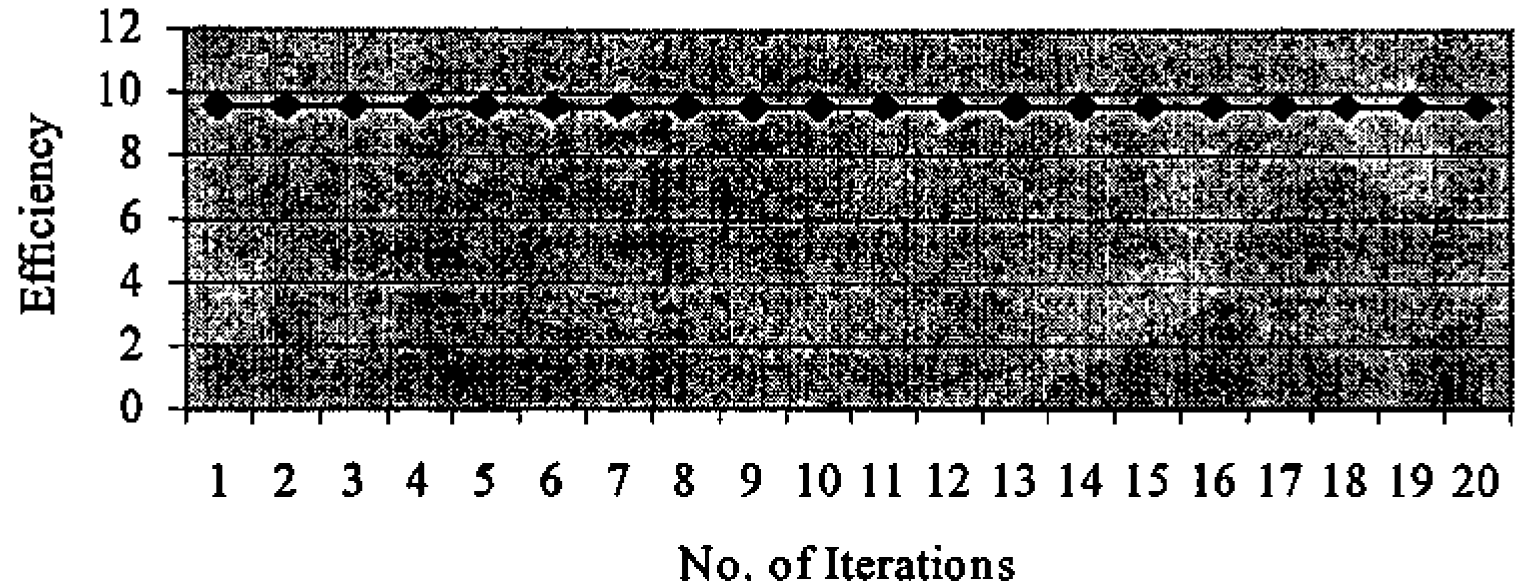



Figure 5.1
Affect of parameters on efficiency of TIEC

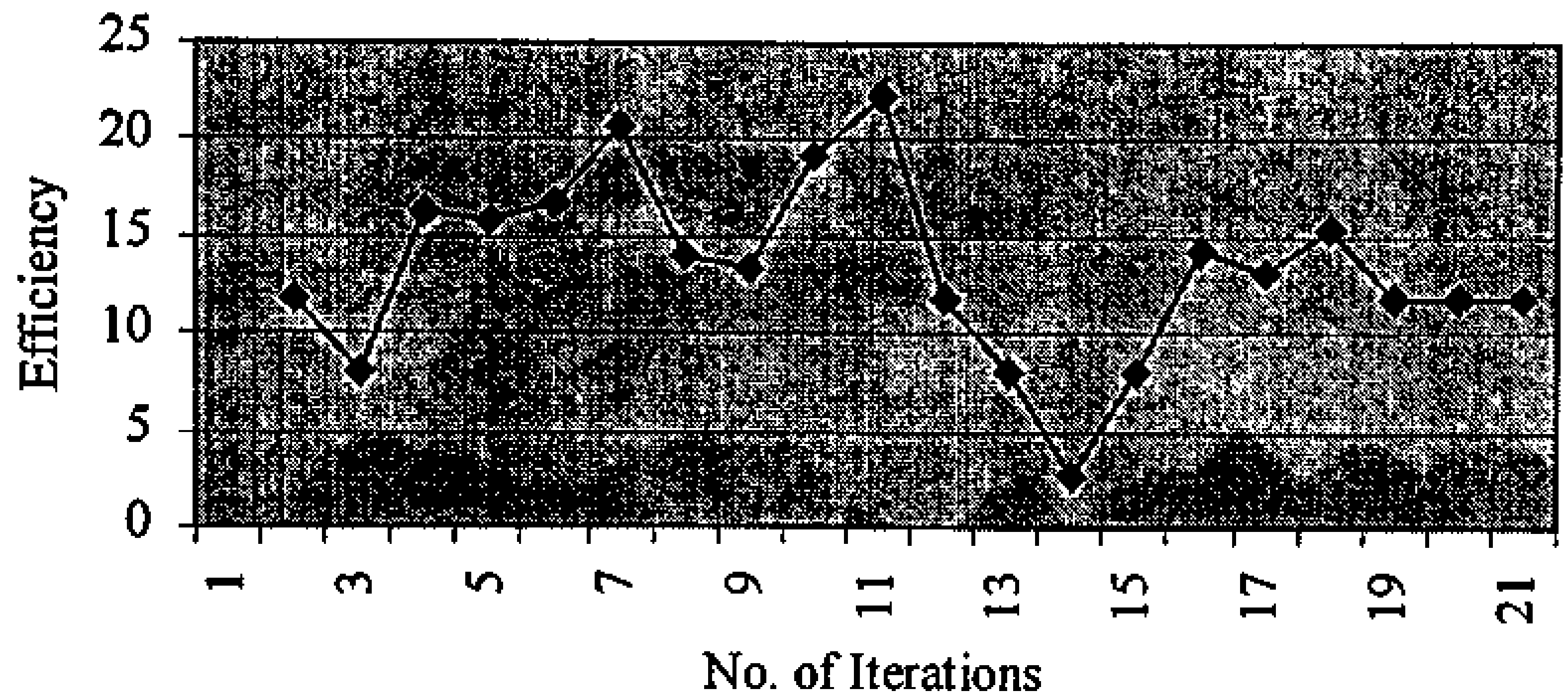



Figure 5.2
Affect of parameters on efficiency of AMTEC

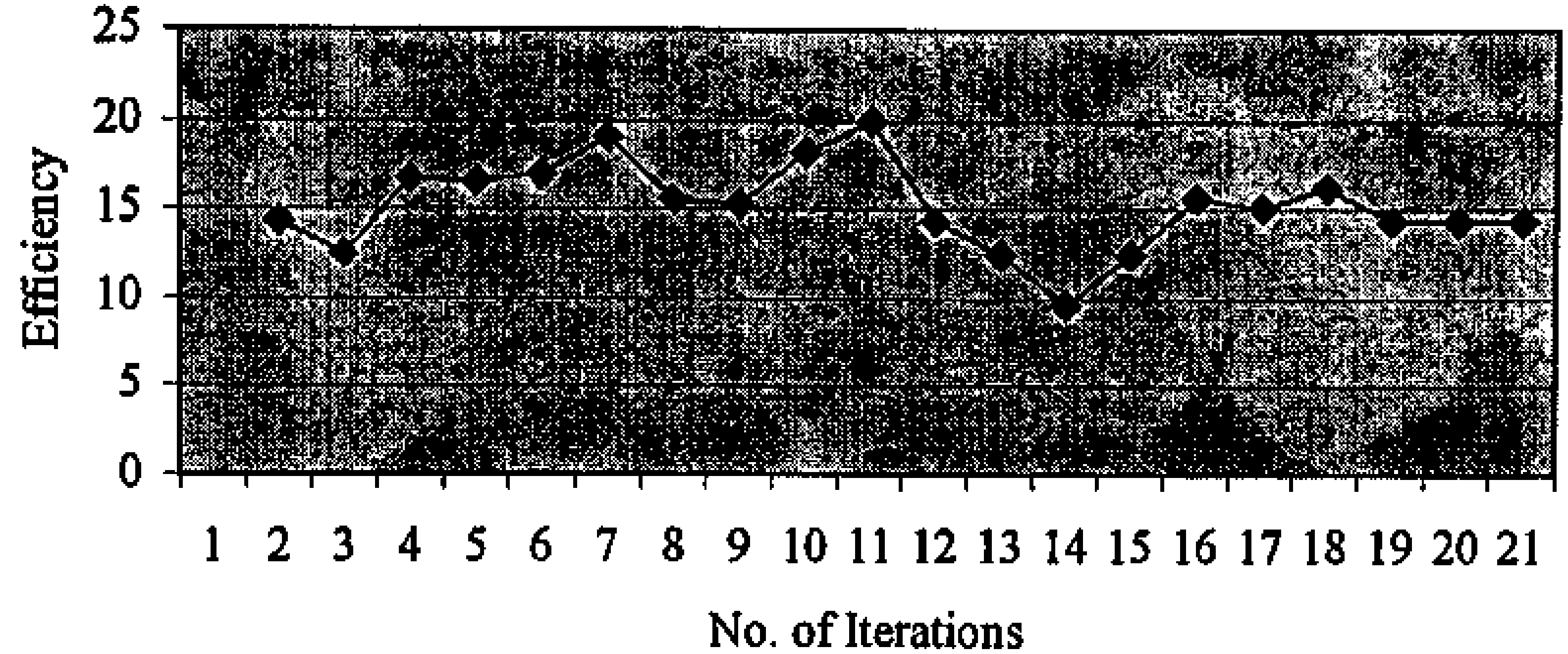



Figure 5.3
Affect of parameters on efficiency of Cascaded Cell

$$
\begin{bmatrix} T_e \\ T_f \\ T_{bel} \\ T_c \\ T_b \\ T_{trim} \\ T_{ins} \\ T_a \\ T_{base} \\ T_{sodv} \\ T_l \\ T_{cal} \end{bmatrix} = \begin{array}{c|l} & 0 \\ \hline 0 & 1940 \\ 1 & 1310 \\ 2 & 1036 \\ 3 & 1225 \\ 4 & 1184 \\ 5 & 1229 \\ 6 & 1167 \\ 7 & 1159 \\ 8 & 1118 \\ 9 & 635.941 \\ 10 & 413.746 \\ 11 & 878.045 \end{array} \cdot K
$$

The constraints for the temperature profile are:

1) $T_e > T_f > T_c > T_b > T_{ins} > T_a > T_{base} > T_{sodv} > T_l$
2) $1100 > T_{base} > 1150$
3) $400 > T_l > 500$
4) $T_f > T_{bel}$
5) $T_e <= 2000$

All these constraints are satisfied.

Figure 5.4
Temperature Profile

# CHAPTER VI

# COMPARISON OF THE RESULTS OF MATHCAD PROGRAM AND EXCEL SPREADSHEET MODEL

After solving the problem in Mathcad, the same problem was tested using Excel spreadsheet model. The problem is defined in Excel 97, the objective function being,

Maximize: (0.0135*$T_e$+0.012*$T_{base}$-29.16)/(0.0423*$T_e$+3.2067252*POWER(10,-12)* POWER($T_e$,4)-3.2067252*POWER($T_c$,4)+34.476377*POWER(10,-12)* POWER($T_b$,4)-34.476377*POWER(10,-12)*POWER($T_{trim}$,4)).

The changing cells are the temperatures. The constraints are defined and are allowed to pass through iterations. After the iteration process, the result obtained by this procedure is found to be not feasible. It is observed that the trim collar and calorimeter constraints are not satisfied, but the Mathcad program satisfied the same constraints. The reason for this could be in two fold, first, the Excel program could solve the TIEC model but it could not solve the AMTEC and the Cascaded models (see the results chapter). This may be because of the mathematical complexities involved, and two, some assumptions are made while framing the objective function (efficiency). These assumptions are taken from the literature survey, and there may be some assumptions missing while taken from the literature survey. As the Excel model has failed, the results of the Mathcad program have been again checked manually by solving the constrained equations. This showed that the Mathcad model's results are reliable with every constraint holding true. When a similar procedure was followed with the Excel model results are not satisfactory.

Further work is encouraged to find solutions that are feasible globally.

# CHAPTER VII

# RESULTS

The results of the mathematical model are presented in two parts. The first is the Mathcad program. The Mathcad program gives a solution, which is feasible locally. For any unconstrained nonlinear program there can be several solutions, and for the maximization problem, among these solutions there can be a maximum value which is known as global optimal solution. The search method here is constrained to a local region, and an optimal solution in this region is achieved, which is the local optimal solution. It is found that the efficiency of the TIEC is 9.66% and that of the AMTEC is 11.81%. The efficiency of the cascaded cell was found to be 14.398%, as given by the Mathcad program. These results have satisfied all the constraints defined in the mathematical model. The heat balance equations have been framed and are allowed to run in iterations, and the result showed that the temperature profile is satisfactory. The temperature profile is shown in Figure 5.5. Then after the temperature profile is proper, the other parameters were identified by search method (see Figure 5.1), and finally a feasible solution is achieved. It is required that the efficiency of the cascade cell has to be greater than the highest efficiency among TIEC or AMTEC, and lower than the sum of the efficiency of TIEC and AMTEC. This constraint of the model is also fulfilled.

The second fold of the solution is presented using Excel spreadsheet modelling. It was found that the solutions obtained by this method were not feasible completely. However the TIEC model gave a feasible solution of 10.38% (efficiency). But the AMTEC and cascaded models could not give feasible solutions by this method. The reasons for this are discussed in the appendix section. The Excel spreadsheet model has failed to satisfy two constraints. In particular, the Excel model has failed to satisfy the trim collar and the transition piece constraints. In trim collar node's heat balance equation, the heat rejection was found to exceed the limits specified by the constraint. The actual value of heat rejection is 90 Watts but the value given by this model is 90.5, which is unacceptable. In the transition piece's nodal equation the heat input and output should be equal, but this model gives an excess of 14.8 Watts for the output, which again is unacceptable. Because of these discrepancies the temperature profile is also found to be unacceptable.

In order to verify the results of the two models, manually the constraint equations were solved. The resulting temperature profiles from both models are taken and these temperatures are fitted in the constraints. It was observed that the Mathcad program's temperatures exactly fulfill the constraints where as the Excel spreadsheet model's temperatures do not. The same mathematical equations could be solved by Mathcad program and could not be solved by the Excel model. This implies that either Excel 97 package cannot support such problems because of high mathematical complexities or there must be some assumptions, which are missed while framing the objective function. From this we could infer that the Mathcad program is reliable in satisfying the constraints part. It is thus an acceptable approach for this optimization model.

The results achieved by this project were comparatively better than the results of the previous work, which included only nine nodal points [1].

# CHAPTER VIII

## CONCLUSION

The basic purpose of optimizing the cascaded cell is to limit weight, area and material property constraints and finally improve the efficiency and the cost effectiveness. Of these the temperature profile was obtained (see Figure 5.5). The efficiency of the cascaded cell is 14.398%. When compared to the previous work involving nine nodal points, a detailed temperature profile is obtained. The efficiency of the cascaded cell has increased from 13.4% to 14.398%, see Chapter VII. Though the solution is locally optimized still further work needs to be done to globally optimize the efficiency. This project is a good beginning towards the global optimization of the cascaded cell. In this direction the Mathcad program is an acceptable approach for optimization of the cell to the extent that it could not handle any more number of constraint equations than used in this program. For a program containing more equations than the present work does need some other approach with larger space and capability.

## REFERENCES


1. M.A.K. Lodhi, Michael Schuller, and Paul Hausgen, "Mathematical Modeling For a Thermionic-AMTEC Cascade system," Space Technology and Applications International Forum, American Institute of Physics, pp. 1285-1290, 1996.

2. F.G. Baksht, G.A. Dyuzhev, A.M. Martsinovskiy, B. Ya. Moyzhes, G. Ye. Pikus, and E.B. Sonin, "Thermionic Converters and Low Temperature Plasma," Technical Information Center/ U.S. Department of Energy, pp No. 1-18, 1978.

3. A. Schock, "Design Evolution and Verification of the General Purpose Heat Source," Proceedings, $15^{th}$ Intersociety Energy Conversion Engineering Conference, AIAA, NY, 2:1032.

4. A. Schock, H. Noravian, C. Or, and V. Kumar, "Design and Analysis of Radioisotope Poweer System Based on Revised Multitube AMTEC Cell Design," Space Technology and Applications International Forum, American Institute of Physics, pp. 1411-1423, 1997.

5. A. Ya. Ender, I.N Kolyshkin, V.I. Kuznestsov, E.V. Yakovlev, and D.V. Paramonov, "Cascaded Space Solar Power System with High Temperature Cs-Ba Thermionic Converter and AMTEC," Space Technology and Application International Forum., American Institute of Physics, pp. 1565-1570, 1998.

6. Gabor Miskokzy, Bob Sievers, Bob svedberg, Micheal Schuller, Tom Vanhagen, and Joe Smith, "Design and Preliminary Testing of a Thermionic-AMTEC Cascade," Space Technology and Applications International Forum, American Institute of Physics, pp.1291-1297, 1996.

7. Joe. N Smith. Jr, "Thermal Analyses of Cascaded Thermionic-AMTEC Converter," Private communication through 1994 report, General Atomics Court, San Diego, CA.

8. Mark L. Underwood, Roger M. Williams, Barbara Jeffries-Nakumura, M. Amy Ryan, and Dennis O'Connor, "Performance Projections of Alternative AMTEC Systems and Devices," Space Technology and Applications International Forum, American Institute of Physics, pp. 472-481, 1991.

9. Douglas J. Wilde, *Optimum Seeking Method.* Englewood Cliffs, N.J: Prentice Hall, Inc., 1964.

10. Fredrick S. Hiller and Gerald J. Lieberman, *Introduction to Operations Research.* New York: McGraw-Hill, Inc., 1995.

# APPENDIX

## MATHCAD PROGRAM

$r_1 := 1 \cdot cm$ $r_2 := 1.5 \cdot cm$ $r_3 := 2.15 \cdot cm$ $r_4 := 1.9 \cdot cm$ $r_5 := 3.75 \cdot cm$

$r_7 := 2.65 \cdot cm$ $r_6 := r_4$ $r_8 := 4.925 \cdot cm$ $r_{10} := 2.85 \cdot cm$ $l_1 := .8 \cdot cm$

$l_2 := .4 \cdot cm$ $l_3 := .2 \cdot cm$ $l_4 := .7 \cdot cm$ $l_5 := 1.2 \cdot cm$ $l_6 := 1 \cdot cm$ $l_9 := .25 \cdot cm$

$i := 1 .. 11$

$r_{11} := 1.59 \cdot cm$

$k := 1 .. 9$

$r_i :=$

| |
|---|
| 1 |
| 1.5 |
| 2.15 |
| 1.9 |
| 3.75 |
| 1.9 |
| 2.65 |
| 1.525 |
| 4.925 |
| 2.85 |
| 1.59 |

$l_k :=$

| |
|---|
| .8 |
| .4 |
| .2 |
| .7 |
| 1.2 |
| 1 |
| .9 |
| 3.3 |
| .25 |

$l := l \cdot cm$

$r := r \cdot cm$

cylinder to cylinder view factor

$$R(ri, ro) := \frac{ro}{ri} \qquad L(l, ri) := \frac{l}{ri}$$

$$A(L, R) := L^2 + R^2 - 1 \qquad B(L, R) := L^2 - R^2 + 1$$

$$Fcoi(l, ro, ri) := \frac{1}{R(ri,ro)} - \frac{1}{\pi \cdot R(ri,ro)} \cdot \left[ acos\left(\frac{B(L(l,ri),R(ri,ro))}{A(L(l,ri),R(ri,ro))}\right) \ldots \right.$$
$$+ \frac{-1}{2 \cdot L(l,ri)} \cdot \left[ \sqrt{(2 + A(L(l,ri),R(ri,ro)))^2 - (2 \cdot R(ri,ro))^2} \cdot acos\left(\left(\frac{\quad}{R\ldots}\right.\right.\right.$$
$$\left.\left. + B(L(l,ri),R(ri,ro)) \cdot asin\left(\frac{1}{R(ri,ro)}\right) + \frac{-\pi \cdot A(L(l,ri), \ldots}{2} \right]\right]$$

Disk to Disk View Factor($F_{1\text{-}2}$)

$$M1(rad1, h) := \frac{rad1}{h}$$

$$M2(rad2, h) := \frac{rad2}{h}$$

$$X(M1, M2) := \frac{1 + M2^2}{M1^2} + 1$$

$$Fd12(rad1, rad2, h) := \left[ \frac{X(M1(rad1,h), M2(rad2,h)) - \sqrt{-4 \cdot \left(\frac{M2(rad2,h)}{M1(rad1,h)}\right)^2 + X(M1(rad1,h), M2(rad2,h))^2}}{2} \right.$$

**View Factor Calculations**

$$F_{elfg} := \frac{1 \cdot \left[ l_1 \cdot \left( 1 - \frac{r_2}{r_1} \cdot Fcoi(l_1, r_2, r_1) \right) \right] + l_2 \cdot \left( 1 + \frac{r_2}{r_1} \cdot Fcoi(l_2, r_2, r_1) \right)}{2 \cdot (l_1 + l_2)}$$

$$F_{elff} := \frac{l_1}{l_1 + l_2} \cdot \frac{1}{2} \cdot \left[ \frac{r_2}{r_1} \cdot (Fcoi(l_1, r_2, r_1)) - \frac{r_3}{r_1} \cdot Fcoi(l_1, r_3, r_1) \right]$$

$$F_{fllb} := \frac{1}{2 \cdot l_3} \cdot \left[ (l_3 + l_4) \cdot \left[ 1 - \frac{r_5}{r_3} \cdot Fcoi[(l_3 + l_4), r_5, r_3] \right] - l_4 \cdot \left( 1 - \frac{r_5}{r_3} \cdot Fcoi(l_4, r_5, r_3) \right) \right]$$

$$F_{flfb} := \frac{(r_5)^2}{(r_3)^2 - (r_4)^2} \cdot (Fd12(r_5, r_3, l_4) - Fd12(r_5, r_4, l_4)) - \frac{(r_4)^2}{(r_3)^2 - (r_4)^2} \cdot (Fd12(r_4, r_3, l_4) - Fd12(r_4, r_4, l_4))$$

$$F_{fblb} := .5 \cdot \left( 1 - \frac{r_5}{r_4} \cdot Fcoi(l_4, r_5, r_4) \right)$$

$$F_{btp} := \frac{(r_7)^2}{(r_5)^2 - (r_6)^2} \cdot (Fd12(r_7, r_5, l_5) - Fd12(r_7, r_6, l_5)) - \frac{(r_8)^2}{(r_5)^2 - (r_6)^2} \cdot (Fd12(r_8, r_5, l_5) - Fd12(r_8, r_6, l_5))$$

$$F_{bc} := \frac{(r_9)^2}{(r_5)^2 - (r_6)^2} \cdot (Fd12(r_9, r_5, l_6) - Fd12(r_9, r_6, l_6)) - \frac{(r_{10})^2}{(r_5)^2 - (r_6)^2} \cdot (Fd12(r_{10}, r_5, l_6) - Fd12(r_{10}, r_6, l_6))$$

$F_{fllb} = 0.251$ $F_{bc} = 0.516$ $F_{fblb} = 0.396$ $F_{elff} = 0.08$

$F_{flfb} = 0.61$ $F_{btp} = 0.247$ $F_{elfg} = 0.365$

this version has been modified for a convertor with a 10 mil gap, and with a contact resistances at both ends of the transition piece and a thermal path through the emitter sleeve and the bellows. The geometrical parameters are listed first:**********************modified with two nodal points included***************

$\sigma := 5.67\cdot 10^{-12}\cdot\frac{watt}{cm^2\cdot K^4}$ $\quad \varepsilon_{ti} := 0.18$ $\quad \varepsilon_{trim} := 0.2$ $\quad \varepsilon_{cal} := 0.2$ $\quad \varepsilon_{empara} := 0.2$

$d_{trans} := 5.08\cdot cm$ $\quad d_{trim} := 6.9\cdot cm$ $\quad l_{trans} := 1.905\cdot cm$ $\quad \varepsilon_b := 0.2$

$A_{ee} := 1\cdot cm^2$ $\quad A_{trans1} := 5.31\cdot cm^2$

$A_{emedge} := 2\cdot\pi\cdot r_1\cdot(l_1 + l_2)$ $\quad A_{emedge1} := 2\cdot\pi\cdot r_1\cdot l_1$ $\quad A_{backer} := \pi\cdot(r_1)^2$ $\quad A_{er} := \pi\cdot(r_1)^2$

$A_{emedge} = 7.54\cdot cm^2$ $\quad A_{emedge1} = 5.027\cdot cm^2$ $\quad A_{backer} = 3.142\cdot cm^2$ $\quad A_{er} = 3.142\cdot cm^2$

$A_{flange} := 20.1\cdot cm^2$ $\quad A_{iflange} := 2\cdot\pi\cdot r_4\cdot(l_3 + l_4 - l_2)$ $\quad A_{flipl} := 2\cdot\pi\cdot r_3\cdot l_3$ $\quad A_{fbl} := 2\cdot\pi\cdot r_4\cdot l_4$

$A_{iflange} = 5.969\cdot cm^2$ $\quad A_{flipl} = 2.702\cdot cm^2$ $\quad A_{fbl} = 8.357\cdot cm^2$

$A_{bel} := \pi\cdot\left[(r_5)^2 - (r_4)^2\right]$ $\quad A_{cone} := 1.414\cdot\pi\cdot\left[(r_2)^2 - (r_1)^2\right]$ $\quad A_{flipf} := \pi\cdot\left[(r_3)^2 - (r_4)^2\right]$

$A_{bel} = 32.837\cdot cm^2$ $\quad A_{cone} = 5.553\cdot cm^2$ $\quad A_{flipf} = 3.181\cdot cm^2$

$A_{tp} := \pi\cdot\left[(r_7)^2 - (r_8)^2\right]$ $\quad A_{call} := 2\cdot\pi\cdot r_9\cdot l_8$ $\quad A_{calf} := \pi\cdot\left[(r_9)^2 - (r_{10})^2\right]$

$A_{tp} = 14.756\cdot cm^2$ $\quad A_{call} = 102.117\cdot cm^2$ $\quad A_{calf} = 50.684\cdot cm^2$

$A_{cal} := A_{call} + A_{calf}$ $\quad A_{colrad} := 5.1\cdot cm^2$ $\quad A_{ins} := 2\cdot\pi\cdot r_{11}\cdot l_9$

$A_{ins} = 2.498\cdot cm^2$

$A_{cal} = 152.801\cdot cm^2$ $\quad A_{trans2} := 8.04\cdot cm^2$

$d_v := 1.59\cdot cm$ $\quad l_v := 6.14\cdot cm$

$A_{base} := 36.722\cdot cm^2$ $\quad \varepsilon_{elec} := 0.7$ $\quad A_{vapor} := \pi\cdot d_v\cdot l_v$ $\quad A_{vapor} = 30.67\cdot cm^2$

$N := 0$ the folowing are the calculated thermal resistances:

$R_s := 8.1 \cdot \frac{K}{watt}$ $R_b := 23 \cdot \frac{K}{watt}$ $k_{interface} := .17 \cdot \frac{watt}{cm^2 \cdot K}$

n and b for electron cooling

$n := .0423 \cdot \frac{watt}{K \cdot cm^2}$ $b := 58.6 \cdot \frac{watt}{cm^2}$

o and c for TI electric power density

$o := .0135 \cdot \frac{watt}{K \cdot cm^2}$ $c := 20.26 \cdot \frac{watt}{cm^2}$

$h_1 := 0.1031 \cdot \frac{watt}{K \cdot cm^2}$ $h_2 := 0.00019 \cdot \frac{watt}{cm^2 \cdot K}$

p and d for AMTEC parasitic loss

$p := 5.0 \cdot 10^{-2} \cdot \frac{watt}{K}$ $d := 34.3 \cdot watt$

q and e for AMTEC electric power

$e := 8.9 \cdot watt$ $q := 1.2 \cdot 10^{-2} \cdot \frac{watt}{K}$

r and f for AMTEC current density

$r := 2.0 \cdot 10^{-2} \cdot \frac{amp}{K}$ $G := 1 \cdot \frac{watt}{amp}$

$T_{res} := 573 \cdot K$ $f := 10 \cdot amp$

these are starting points for the calaculations:

$T_e = 1800 \cdot K$ $T_c := 1250 \cdot K$ $T_a := 1170 \cdot K$ $T_l := 350 \cdot K$ $T_{cal} := 900 \cdot K$

$T_{amb} := 325 \cdot K$ $T_f := 1280 \cdot K$ $T_{bel} := 1100 \cdot K$ $T_{trim} := 500 \cdot K$ $T_{ins} := 1180 \cdot K$

$T_{sod} := 1150 \cdot K$ $T_{sodv} := 1000 \cdot K$ $T_b := 1200 \cdot K$ $T_{base} := 1100 \cdot K$

these are the input parameters:

$Q_{input} := 318 \cdot watt$ $Q_{intrim} := 90 \cdot watt$ $R_{flange} := 100 \cdot \frac{K}{watt}$

$R_{res} := 100 \cdot \frac{K}{watt}$ $R_{ins} := 100 \cdot \frac{K}{watt}$ $R_l := 2.5 \cdot \frac{K}{watt}$

$C_s := 0.1$ $s := 0.15$ $h_{lv} := 3870000 \cdot \frac{joule}{kg}$ $K := 0.50 \cdot \frac{watt}{cm \cdot K}$ $p_c := 25.55 \cdot 10^6 \cdot Pa$

$p_l := 75000 \cdot Pa$ $\rho := 776 \cdot 10^{-6} \cdot \frac{kg}{cm^3}$ $\sigma_a := 0.12 \cdot 10^{-4} \cdot \frac{joule}{cm^2}$ $T_x := \frac{T_a + T_{sod}}{2}$

**this is the solve block containing the ten nodal equations in ten unknown temperatures:**

given

**at emitter node:**

$$Q_{input} = \sigma \cdot \varepsilon_{empara} \cdot \left[A_{backer} \cdot \frac{1}{1+N} + A_{emedge} \cdot (1 - F_{elff}) - A_{emedge1} \cdot F_{elfg}\right] \cdot \left(T_e^4 - T_{amb}^4\right) \ldots$$
$$+ (o \cdot T_e - c) \cdot A_{ee} \ldots$$
$$+ \sigma \cdot \varepsilon_{ti} \cdot A_{er} \cdot \left(T_e^4 - T_c^4\right) + (n \cdot T_e - b) \cdot A_{ee} + \frac{T_e - T_f}{R_s} \ldots$$
$$+ \sigma \cdot \varepsilon_{empara} \cdot \left(A_{emedge} \cdot F_{elfg} + A_{emedge1} \cdot F_{elff}\right) \cdot \left(T_e^4 - T_f^4\right)$$

**at flange node:**

$$\frac{T_e - T_f}{R_s} \ldots$$
$$+ \sigma \cdot \varepsilon_{empara} \cdot \begin{pmatrix} A_{emedge} \cdot F_{elfg} \ldots \\ + A_{emedge1} \cdot F_{elff} \end{pmatrix} \cdot \left(T_e^4 - T_f^4\right)$$
$$= \frac{T_f - T_{bel}}{R_b \cdot 0.5} \ldots$$
$$+ \sigma \cdot \varepsilon_{empara} \cdot \begin{bmatrix} A_{cone} \cdot \left(1 - \frac{A_{emedge}}{A_{cone}} \cdot F_{elfg}\right) \ldots \\ + \pi \cdot \left(r_3^2 - r_2^2\right) \cdot \left(1 - \frac{2 \cdot r_1 \cdot l_1}{r_3^2 - r_2^2} \cdot F_{elff}\right) \ldots \\ + 2 \cdot \pi \cdot r_3 \cdot l_3 \cdot (1 - F_{fllb}) \ldots \\ + \pi \cdot \left(r_3^2 - r_4^2\right) \cdot (1 - F_{flfb}) \ldots \\ + 2 \cdot \pi \cdot r_4 \cdot l_4 \cdot (1 - F_{fblb}) \end{bmatrix}$$
$$+ \frac{T_f - T_{amb}}{R_{flange}} + \sigma \cdot \varepsilon_{empara} \cdot A_{iflange} \cdot \left(T_f^4 - T_c^4\right) \ldots$$
$$+ \sigma \cdot \varepsilon_{empara} \cdot \begin{pmatrix} A_{flipl} \cdot F_{fllb} \ldots \\ + A_{flipf} \cdot F_{flfb} \ldots \\ + A_{fbl} \cdot F_{fblb} \end{pmatrix} \cdot \left(T_f^4 - T_{bel}^4\right) \ldots$$
$$+ \frac{T_f - T_{res}}{R_{res}}$$

at bellows node:

$$\frac{T_f - T_{bel}}{R_b \cdot 0.5} \ldots + \sigma \cdot \varepsilon_{empara} \cdot \begin{pmatrix} A_{flipl} \cdot F_{fllb} \ldots \\ + A_{flipf} \cdot F_{flfb} \ldots \\ + A_{fbl} \cdot F_{fblb} \end{pmatrix} \cdot \left(T_f^4 - T_{bel}^4\right) \ldots + \frac{T_c - T_{bel}}{R_b \cdot 0.5} + \sigma \cdot \varepsilon_{trim} \cdot A_{tp} \cdot F_{btp} \cdot \left(T_b^4 - T_{bel}^4\right) = \sigma \cdot \varepsilon_{empara} \cdot \begin{bmatrix} A_{bel} \cdot \left(2 - F_{btp} - F_{bc}\right) \ldots \\ + \left[ - \begin{pmatrix} A_{flipl} \cdot F_{fllb} \ldots \\ + A_{flipf} \cdot F_{flfb} \ldots \\ + A_{fbl} \cdot F_{fblb} \end{pmatrix} \right] \end{bmatrix} \cdot \left(T_{bel}^4 - T_{an}\right) + \sigma \cdot \varepsilon_{cal} \cdot A_{bel} \cdot F_{bc} \cdot \left(T_{bel}^4 - T_{cal}^4\right)$$

at collector node:

$$\begin{bmatrix} \sigma \cdot \varepsilon_{ti} \cdot A_{er} \cdot \left(T_e^4 - T_c^4\right) + \left(n \cdot T_e - b\right) \cdot A_{ee} \ldots \\ + \sigma \cdot \varepsilon_{empara} \cdot A_{iflange} \cdot \left(T_f^4 - T_c^4\right) \end{bmatrix} = \frac{T_c - T_{bel}}{R_b \cdot 0.5} + A_{trans1} \cdot k_{interface} \cdot \left(T_c - T_b\right) \ldots + \sigma \cdot \varepsilon_{empara} \cdot A_{colrad} \cdot \left(T_c^4 - T_{amb}^4\right)$$

at middle of tranistion piece:

$$A_{trans1} \cdot k_{interface} \cdot \left(T_c - T_b\right) = \frac{A_{trans2} \cdot k_{interface} \cdot \left(T_b - T_{ins}\right)}{0.6} \ldots + \sigma \cdot \varepsilon_{trim} \cdot \pi \cdot d_{trans} \cdot l_{trans} \cdot \left(T_b^4 - T_{trim}^4\right) \ldots + \sigma \cdot \varepsilon_b \cdot A_{bel} \cdot F_{btp} \cdot \left(T_b^4 - T_{bel}^4\right)$$

at trim collar node:

$$\sigma \cdot \varepsilon_{trim} \cdot \pi \cdot d_{trans} \cdot l_{trans} \cdot \left(T_b^4 - T_{trim}^4\right) + Q_{intrim} = \sigma \cdot \varepsilon_{cal} \cdot \pi \cdot d_{trim} \cdot l_{trans} \cdot \left(T_{trim}^4 - T_{cal}^4\right)$$

at insulator node:

$$\frac{A_{trans2} \cdot k_{interface} \cdot \left(T_b - T_{ins}\right)}{0.75} \ldots + \sigma \cdot \varepsilon_{trim} \cdot \pi \cdot d_{trans} \cdot l_{trans} \cdot \left(T_{trim}^4 - T_{ins}^4\right) = \frac{A_{trans2} \cdot k_{interface} \cdot \left(T_{ins} - T_a\right)}{0.25} + \frac{T_{ins} - T_a}{R_{ins}} \ldots + \sigma \cdot \varepsilon_{trim} \cdot A_{ins} \cdot 0.986 \cdot \left(T_{ins}^4 - T_{amb}^4\right)$$

at AMTEC node:

$$\frac{A_{trans2} \cdot k_{interface}}{0.25} \cdot \left(T_{ins} - T_a\right) = \left[ C_s^{\ 3} \cdot \frac{\left(K \cdot h_{lv} \cdot \rho\right)}{\sigma_a \cdot T_x^{\ 2}} \cdot \left(\frac{p_l}{p_c}\right)^{3 \cdot s} \right] \cdot \left(T_a - T_{sod}\right)^3 \cdot A_{trans2}$$

at Base Tube node:

$$\left[ C_s^{\ 3} \cdot \frac{\left(K \cdot h_{lv} \cdot \rho\right)}{\sigma_a \cdot T_x^{\ 2}} \cdot \left(\frac{p_l}{p_c}\right)^{3 \cdot s} \right] \cdot \left(T_a - T_{sod}\right)^3 \cdot A_{trans2} = h_2 \cdot A_{base} \cdot \left(T_{base} - T_{sodv}\right) + \left(T_{base} \cdot r - f\right) \cdot G \ldots \\ + \left(q \cdot T_{base} - e\right) + \left(p \cdot T_{base} - d\right)$$

at the Vapor zone node:

$$h_2 \cdot A_{base} \cdot \left(T_{base} - T_{sodv}\right) = h_2 \cdot A_{vapor} \cdot \left(T_{sodv} - T_{amb}\right) + h_2 \cdot A_{base} \cdot \left(T_{sodv} - T_l\right)$$

at condensor node:

$$\left(r \cdot T_{base} - f\right) \cdot G + \left(p \cdot T_{base} - d\right) + h_2 \cdot A_{base} \cdot \left(T_{sodv} - T_l\right) = \frac{T_l - T_{amb}}{R_l}$$

at calorimeter node:

$$\sigma \cdot \varepsilon_{cal} \cdot A_{bel} \cdot F_{bc} \cdot \left(T_{bel}^{\ 4} - T_{cal}^{\ 4}\right) \ldots \\ + \sigma \cdot \varepsilon_{cal} \cdot \pi \cdot d_{trim} \cdot l_{trans} \cdot \left(T_{trim}^{\ 4} - T_{cal}^{\ 4}\right) = \sigma \cdot \varepsilon_{cal} \cdot \left(A_{cal} - A_{bel} \cdot F_{bc}\right) \cdot \left(T_{cal}^{\ 4} - T_{amb}^{\ 4}\right)$$

$$\begin{bmatrix} T_e \\ T_f \\ T_{bel} \\ T_c \\ T_b \\ T_{trim} \\ T_{ins} \\ T_a \\ T_{base} \\ T_{sodv} \\ T_1 \\ T_{cal} \end{bmatrix} := \mathrm{Minerr}\left(T_e, T_f, T_{bel}, T_c, T_b, T_{trim}, T_{ins}, T_a, T_{base}, T_{sodv}, T_1, T_{cal}\right)$$

$$ERR = 1.503 \cdot 10^{-12}$$

$$\begin{bmatrix} T_e \\ T_f \\ T_{bel} \\ T_c \\ T_b \\ T_{trim} \\ T_{ins} \\ T_a \\ T_{base} \\ T_{sodv} \\ T_1 \\ T_{cal} \end{bmatrix} = \cdot K$$

| | 0 |
|---|---|
| 0 | $1.94 \cdot 10^3$ |
| 1 | $1.31 \cdot 10^3$ |
| 2 | $1.036 \cdot 10^3$ |
| 3 | $1.225 \cdot 10^3$ |
| 4 | $1.184 \cdot 10^3$ |
| 5 | $1.229 \cdot 10^3$ |
| 6 | $1.167 \cdot 10^3$ |
| 7 | $1.159 \cdot 10^3$ |
| 8 | $1.118 \cdot 10^3$ |
| 9 | 635.941 |
| 10 | 413.746 |
| 11 | 878.045 |

$$Q_{bellpara} := \sigma \cdot \varepsilon_{empara} \cdot \left[ A_{bel} \cdot \left(2 - F_{btp} - F_{bc}\right) \cdots + \left[ - \begin{pmatrix} A_{flipl} \cdot F_{fllb} \cdots \\ + A_{flipf} \cdot F_{flfb} \cdots \\ + A_{fbl} \cdot F_{fblb} \end{pmatrix} \right] \right] \cdot \left(T_{bel}^4 - T_{amb}^4\right)$$

$$Q_{bellpara} = 44.892 \cdot watt \qquad Q_{colrad} := \sigma \cdot \varepsilon_{empara} \cdot A_{colrad} \cdot \left(T_c^4 - T_{amb}^4\right)$$

$$Q_{colrad} = 12.975 \cdot watt \qquad Q_{electric} := \left(o \cdot T_e - c\right) \cdot A_{ee} \qquad Q_{electron} := \left(n \cdot T_e - b\right) \cdot A_{ee}$$

$$Q_{amtecthru} := \left(r \cdot T_{base} - f\right) \cdot G \qquad Q_{electric} = 5.933 \cdot watt \qquad Q_{electron} = 23.47 \cdot watt$$

$$Q_{amtecthru} = 12.357 \cdot watt$$

$$Q_{flngpara} := \sigma \cdot \varepsilon_{empara} \cdot \left[ A_{cone} \cdot \left(1 - \frac{A_{emedge}}{A_{cone}} \cdot F_{elfg}\right) \dots + \pi \cdot \left(r_3^2 - r_2^2\right) \cdot \left(1 - \frac{2 \cdot r_1 \cdot l_1}{r_3^2 - r_2^2} \cdot F_{elff}\right) \dots + 2 \cdot \pi \cdot r_3 \cdot l_3 \cdot \left(1 - F_{fllb}\right) \dots + \pi \cdot \left(r_3^2 - r_4^2\right) \cdot \left(1 - F_{flfb}\right) \dots + 2 \cdot \pi \cdot r_4 \cdot l_4 \cdot \left(1 - F_{fblb}\right) \right] \cdot \left(T_f^4 - T_{amb}^4\right)$$

$$Q_{flngpara} = 60.357 \cdot watt$$

$$Q_{flangcond} := \frac{T_f - T_{amb}}{R_{flange}}$$

$$Q_{flangcond} = 9.846 \cdot watt$$

$$Q_{calnew} := \sigma \cdot \varepsilon_{cal} \cdot \left(A_{cal} - A_{bel} \cdot F_{bc}\right) \cdot \left(T_{cal}^4 - T_{amb}^4\right)$$

$$Q_{calnew} = 89.852 \cdot watt$$

$$Q_{amtecpara} := \left(p \cdot T_{base} - d\right)$$

$$Q_{amtecpara} = 21.592 \cdot watt$$

$$Q_{emitterbacklosses} := \sigma \cdot \varepsilon_{empara} \cdot \left[ A_{backer} \cdot \frac{1}{1 + N} \dots + A_{emedge} \cdot \left(1 - F_{elff}\right) - A_{emedge1} \cdot F_{elfg} \right] \cdot \left(T_e^4 - T_{amb}^4\right)$$

$$Q_{emitterbacklosses} = 132.383 \cdot watt$$

$$Q_{condenser} := h_2 \cdot A_{base} \cdot \left(T_{sodv} - T_1\right)$$

$$Q_{condenser} = 1.55 \cdot watt$$

$$Q_{ins} := \frac{A_{ins} \cdot k_{interface}}{0.6} \cdot \left(T_{ins} - T_a\right)$$

$$Q_{ins} = 5.414 \cdot watt$$

$$Q_{trim} := \sigma \cdot \varepsilon_{trim} \cdot \pi \cdot d_{trans} \cdot l_{trans} \cdot \left(T_b^4 - T_{trim}^4\right)$$

$$Q_{trim} = -10.878 \cdot watt$$

$$Q_{cal} := \sigma \cdot \varepsilon_{cal} \cdot \pi \cdot d_{trim} \cdot l_{trans} \cdot \left(T_{trim}^4 - T_{cal}^4\right)$$

$$Q_{base} := h_2 \cdot A_{base} \cdot \left(T_{base} - T_{sodv}\right)$$

$$Q_{cal} = 79.122 \cdot watt$$

$$Q_{base} = 3.362 \cdot watt$$

$$Q_{rade} := \sigma \cdot \varepsilon_{ti} \cdot A_{er} \cdot \left(T_e^4 - T_c^4\right)$$

$$Q_{amtecelec} := \left(q \cdot T_{base} - e\right)$$

$$Q_{rade} = 38.206 \cdot watt$$

$$Q_{amtecelec} = 4.514 \cdot watt$$

**heat balance:**

$$\left(Q_{input} + Q_{intrim}\right) - Q_{flngpara} - Q_{bellpara} - Q_{cal} - Q_{amtecthru} - Q_{amtecelec} \cdots + \left[\left(-Q_{amtecpara} - Q_{electric} - Q_{emitterbacklosses} - Q_{colrad} - Q_{flangcond}\right) - Q_{ins} - Q_{condenser}\right] = 1$$

$$\eta_{hybrid} := \frac{Q_{electric} + Q_{amtecelec}}{Q_{electron} + Q_{rade} - Q_{trim}}$$

$$\eta_{hybrid} = 14.398 \cdot \%$$

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